Erratum: Geometry of discrete quantum computing

Andrew J. Hanson

School of Informatics and Computing, Indiana University, Bloomington, IN 47405, U.S.A

Gerardo Ortiz

Department of Physics, Indiana University, Bloomington, IN 47405, U.S.A

Amr Sabry

School of Informatics and Computing, Indiana University, Bloomington, IN 47405, U.S.A

Yu-Tsung Tai

Department of Mathematics, Indiana University, Bloomington, IN 47405, U.S.A
School of Informatics and Computing, Indiana University, Bloomington, IN 47405, U.S.A

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Sec. 5.4 of our paper [1] requires a clarification and a correction.

Clarification: Unentangled vs. product states. In conventional quantum mechanics, using the real and complex numbers, a state is unentangled when it can be expressed as a product state or when equation (27) reports its purity to be 1. When using finite fields, it is possible for equation (27) to produce a purity of 1 for some entangled states. For example, consider the normalized 3-qubit state $|\Psi_0\rangle$ over $\mathbb{F}_{7^2}$:

$$|\Psi_0\rangle = |011\rangle + (-2-\mathrm{i})\,|100\rangle + |101\rangle + |110\rangle \ .$$

Thus, in finite fields, the simplest way to calculate the number of unentangled states is to ignore equation (27) and to count the number of product states. This is exactly how the counting in section 5.4 was produced, but the paper did not clarify that counting using equation (27) would be incorrect.

Correction: Maximally entangled states. We correct the definition and the counting of maximally entangled states in the following corrected section 5.4.

5.4. Maximal entanglement

In conventional quantum mechanics, a state is maximally entangled when equation (27) reports its purity to be 0. In the discrete case, states whose purity is a multiple of the field characteristic are incorrectly labeled as maximally entangled. For example, consider the normalized 3-qubit product state $|\Psi_1\rangle$ over $\mathbb{F}_{3^2}$:

$$|\Psi_1\rangle = |0\rangle \otimes (|0\rangle - |1\rangle) \otimes (|0\rangle - |1\rangle) \ .$$

$|\Psi_1\rangle$ has the purity $\overline{P_{\mathfrak{h}}} = 0$, where

$$\overline{P_{\mathfrak{h}}} = \sum_{j=1}^{n} \sum_{\mu=x,y,z} \left\langle \sigma_\mu^j \right\rangle^2$$

is equation (27) eliminating the $1/n$ factor.

In continuous case, the zero purity actually means every summand is 0, i.e.,

$$\forall j, \forall \mu, \langle \sigma_\mu^j \rangle = 0 \ . \tag{40}$$

Because equation (40) avoids any wrapping around when adding expectation values, it defines maximally entangled states over finite fields.

Computing some examples for various n and small values of p, one can verify explicitly that unit-norm product states for $n = 2$, $p = \{3, 7, 11, 19, \ldots\}$ occur with frequency

$$(p+1)p^2(p-1)^2 = \{144, 14\,112, 145\,200, 2339\,280, \ldots\} \ ,$$

and for general n, $(p+1)p^n(p-1)^n$.

The irreducible state counts are reduced by $(p+1)$, giving

$$p^2(p-1)^2 = \{36, 1764, 12\,100, 116\,964, \ldots\} \ ,$$

and in general for n-qubits, there are $p^n\,(p-1)^n$ instances of product pure states.

Repeating the computation, we find maximally entanglement states with frequencies for two qubits of

$$p\,(p-1)\,(p+1)^2 = \{96, 2688, 15\,840, 136\,800, \ldots\} \ .$$

The irreducible state counts for maximal entanglement are reduced by $(p+1)$, giving for $n=2$

$$p\left(p^2-1\right)=\{24,336,1320,6840,\ldots\}\ .$$

For three-qubits, there are $p^3\left(p^4-1\right)(p+1)$ instances of pure maximally entangled states, while the general formula for four-qubit states remains unclear.

Acknowledgments

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References

[1] Andrew J Hanson, Gerardo Ortiz, Amr Sabry, and Yu-Tsung Tai. Geometry of discrete quantum computing. *Journal of Physics A: Mathematical and Theoretical*, 46(18):185301, 2013.