Erratum: Geometry of discrete quantum computing

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Sec. 5.4 of our paper [1] requires a clarification and two corrections.

Clarification: Unentangled vs. product states. In conventional quantum mechanics, using the real and complex numbers, a state is unentangled when it can be expressed as a product state or when equation (27) reports its purity to be 1. When using finite fields, it is possible for equation (27) to produce a purity of 1 for some entangled states. For example, consider the normalized entangled state $|\Psi_0\rangle$ over $\mathbb{F}_{7^2}$:

$$|\Psi_0\rangle = (1+2\mathrm{i})\,|011\rangle + (1+2\mathrm{i})\,|100\rangle - |101\rangle - |110\rangle + (-3+\mathrm{i})\,|111\rangle$$

Thus, in finite fields, the simplest way to calculate the number of unentangled states is to ignore equation (27) and to count the number of product states. This is exactly how the counting in section 5.4 was produced, but the paper did not clarify that counting using equation (27) would be incorrect.

Correction: The definition of maximally entangled states. The last paragraph in page 16 should read

In conventional quantum mechanics, a state is maximally entangled when equation (27) reports its purity to be 0. In the discrete case, states whose purity is a multiple of the field characteristic are incorrectly labeled as maximally entangled. For example, the normalized product state $|\Psi_1\rangle$ over $\mathbb{F}_{3^2}$:

$$|\Psi_1\rangle = |0\rangle \otimes \frac{|0\rangle - |1\rangle}{1-\mathrm{i}} \otimes \frac{|0\rangle - |1\rangle}{1-\mathrm{i}}$$

with the purity without divided by n:

$$\overline{P_{\mathfrak{h}}} = \sum_{j=1}^{n} \sum_{\mu=x,y,z} \left\langle \sigma_\mu^j \right\rangle^2 = 0 \ .$$

To correctly account for maximally entangled states, we modify the definition of maximally entangled states as follows:

$$\forall j, \forall \mu, \langle \sigma_\mu^j \rangle = 0$$

Correction: The counting of maximally entangled states. The number of maximally entangled states should also be updated according to the updated definition of maximally entangled states. Therefore, line 8 starting from “Repeating the computation” to the end of section 5.4 in page 17 should read

Repeating the computation, we find maximally entanglement states with frequencies for two qubits of

$$p\,(p-1)\,(p+1)^2 = \{96, 2688, 15\,840, 136\,800, \ldots\} \ .$$

The irreducible state counts for maximal entanglement are reduced by $(p+1)$, giving for $n=2$

$$p\left(p^2-1\right) = \{24, 336, 1320, 6840, \ldots\} \ .$$

For three-qubits, there are $p^3\left(p^4-1\right)(p+1)$ instances of pure maximally entangled states, while the general formula for four-qubit states remains unclear.

Acknowledgments

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References

[1] Andrew J Hanson, Gerardo Ortiz, Amr Sabry, and Yu-Tsung Tai. Geometry of discrete quantum computing. *Journal of Physics A: Mathematical and Theoretical*, 46(18):185301, 2013.