# 1 EM algorithm

Likelihood function:

$$\mathcal{L}(\Theta) = \Pi_{i=1}^{m}\ p(\boldsymbol{x}_i, \Theta) \tag{1}$$

log likelihood function:

$$\mathcal{LL}(\Theta) = \sum_{i=1}^{m} \log p(\boldsymbol{x}_i, \Theta) \tag{2}$$

$$\begin{aligned}
\mathcal{LL}(\Theta) &= \sum_{i=1}^{m} \log p(\boldsymbol{x}_i, \Theta) \\
&= \sum_{i=1}^{m} \log \sum_{\boldsymbol{z}_i} p(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta) \\
&= \sum_{i=1}^{m} \log \sum_{\boldsymbol{z}_i} Q_i(\boldsymbol{z}_i) \frac{P(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)}{Q_i(\boldsymbol{z}_i)} \\
&\geq \sum_{i=1}^{m} \sum_{\boldsymbol{z}_i} Q_i(\boldsymbol{z}_i) \log \frac{P(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)}{Q_i(\boldsymbol{z}_i)}
\end{aligned}$$

$$\begin{aligned}
c =& \frac{p(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)}{Q_i(\boldsymbol{z}_i)} \\
Q_i(\boldsymbol{z}) &= \frac{p(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)}{\sum_{\boldsymbol{z}_i} P(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)} \\
&= \frac{p(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)}{p(\boldsymbol{x}_i; \Theta)} \\
&= p(\boldsymbol{z}_i | \boldsymbol{x}_i; \Theta)
\end{aligned}$$

Repeat until convergence:
E-step:

$$Q_i(\boldsymbol{z}_i) = P(\boldsymbol{z}_i | \boldsymbol{x}_i; \Theta)$$

M-step:

$$\arg\max_{\Theta} \sum_{i=1}^{m} Q_i(\boldsymbol{z}) \log \sum_{z} \frac{p(\boldsymbol{x}_i, \boldsymbol{z}_i; \Theta)}{Q_i(\boldsymbol{z}_i)}$$