1 Lagrange

If we want to minimize the $f(x)$ subject to $g(x) \leq 0$ We can change this restrict optimization problem into a non-restrict optimization problem.

$$L(x, \lambda) = f(x) + \lambda g(x) \tag{1}$$

Once we find the x^* and λ^* minimize the L funciton, we have

$$0 = \frac{\partial L}{\partial x}|_{x^*} = \nabla f(x) + \lambda \nabla g(x) \tag{2}$$

$$0 = \frac{\partial L}{\partial \lambda}|_{\lambda^*} = g(x) \tag{3}$$

2 from Lagrange to the K-K-T condition

2.1 $g(x) > 0$

if x^* meet the condition $g(x^*) < 0$, we can regard the problem as the non restrict problem

$$L(x, \lambda) = f(x^*)$$

,i.e. we can set $\lambda = 0$ in this situation ,we have

$$\lambda g(x) = 0 \tag{4}$$

2.2 $g(x) = 0$

if $g(x^*) = 0$,we have

$$\nabla f(x^*) + \lambda \nabla g(x^*) = 0 \tag{5}$$

we must have $\lambda > 0$.
Proof:
if $\lambda < 0$,we know $\nabla f(x^*)$ and $\nabla g(x^*)$ are the same direct vectors. We can find a point $x^* + \eta \nabla f(x^*)$ who satisfied $g(x) < 0$

$$f(x^* + \eta \nabla f(x^*)) = f(x^*) + \eta \nabla f(x^*)^T \nabla f(x^*) < f(x^*) \tag{6}$$

where $\eta < 0$.Conlict with $f(x^*)$ is the minimum value.

It means vector $\nabla f(x^*)$ is opposite to vector $\nabla g(x^*)$ In this situation, we still have

$$\lambda g(x) = 0$$

3 K-K-T

$$\begin{cases} \lambda \geq 0 \\ g(x) \leq 0 \\ \lambda g(x) = 0 \end{cases} \tag{7}$$