

Stiff Set (1)

Yunwen Chen, Yangjie Xu, Meng Su, Dandan Che, Zhengqiang Li, Xiaoyan Liu

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Shenzhen Institute of Advanced Technology, Chinese Academy of Sciences

Problem

Explicit Euler Method

Implicit Euler Method

Implicit Midpoint Method

Explicit Runge-Kutta Method

Problem

$$\begin{cases} u_t' = 98u + 198v \\ v_t' = -99u - 199v \end{cases}$$

$$u(0) = 1, v(0) = 1$$

$$0 \leq t \leq T = 100$$

$$\begin{cases} u(t) &= -3e^{-100t} + 4e^{-t} \\ v(t) &= 3e^{-100t} - 2e^{-t} \end{cases}$$

How to evaluate numerical solution ?

For a vector \mathbf{x} , we define the norm

$$||\mathbf{x}|| = \max |\mathbf{x}_i|$$

consequently, we can evaluate the numerical solution by using

$$||\mathbf{u} - \mathbf{u}_I||$$

where \mathbf{u}_I is the numerical solution and \mathbf{u} is exact solution at the numerical solution points.

Explicit Euler Method

Explicit Euler Method

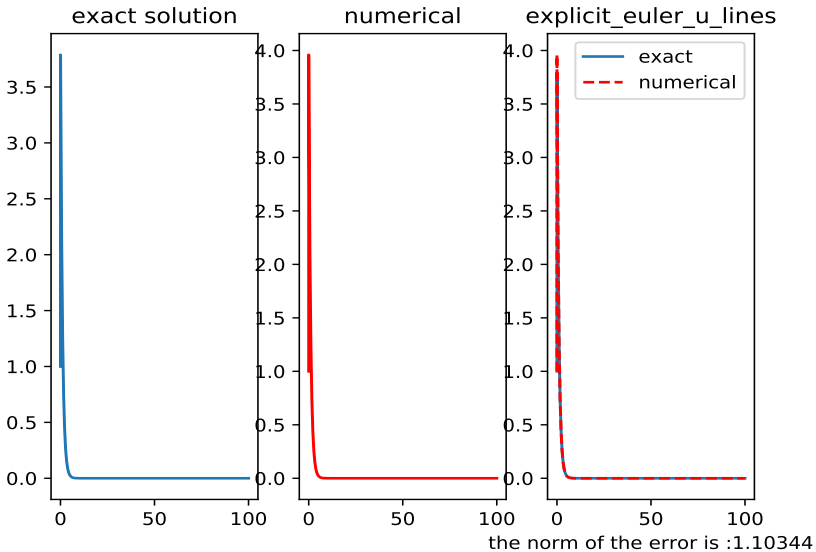
Explicit Euler method for this problem

$$\begin{cases} \frac{u_{n+1}-u_n}{h} = 98u_n + 198v_n \\ \frac{v_{n+1}-v_n}{h} = -99u_n - 199v_n \end{cases}$$

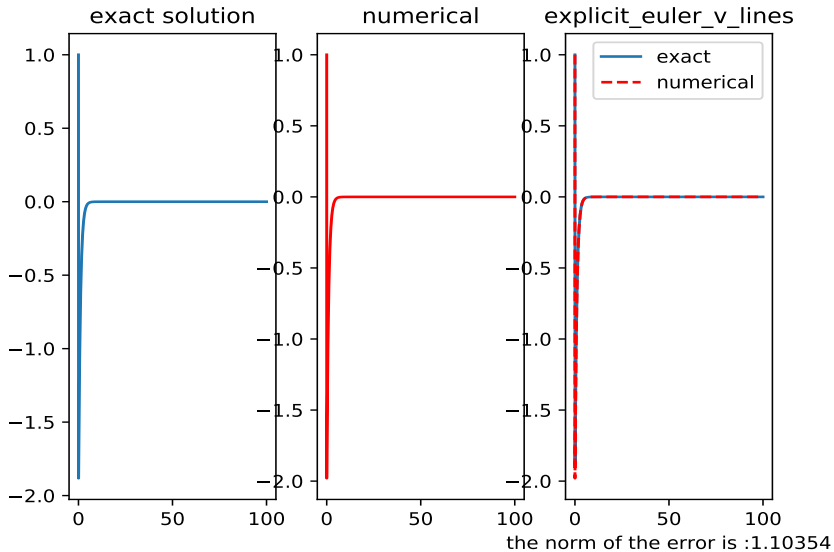
it can be presented as

$$\begin{cases} u_{n+1} = (98u_n + 198v_n)h + u_n \\ v_{n+1} = (-99u_n - 199v_n)h + v_n \end{cases}$$

Explicit Euler Result of $u(t)$



Explicit Euler Result of $v(t)$



Implicit Euler Method

Implicit Euler Method

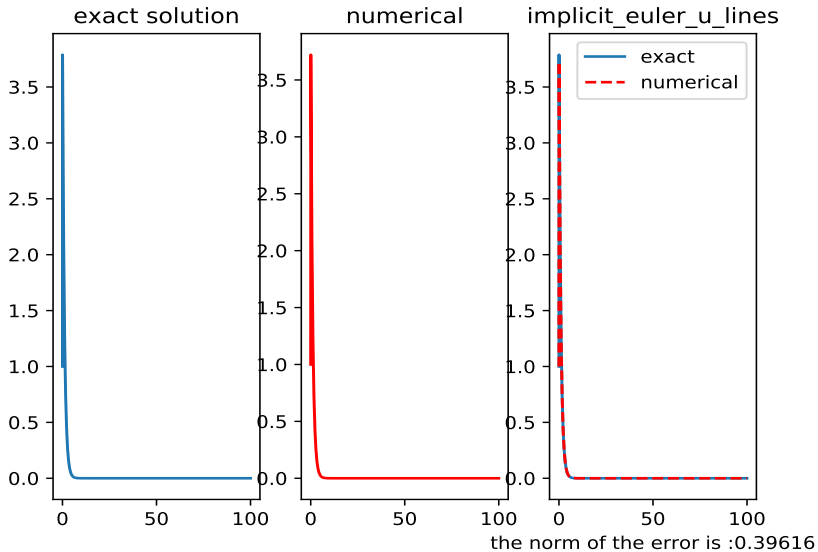
Implicit Euler method for this problem

$$\begin{cases} \frac{u_{n+1}-u_n}{h} = 98u_{n+1} + 198v_{n+1} \\ \frac{v_{n+1}-v_n}{h} = -99u_{n+1} - 199v_{n+1} \end{cases}$$

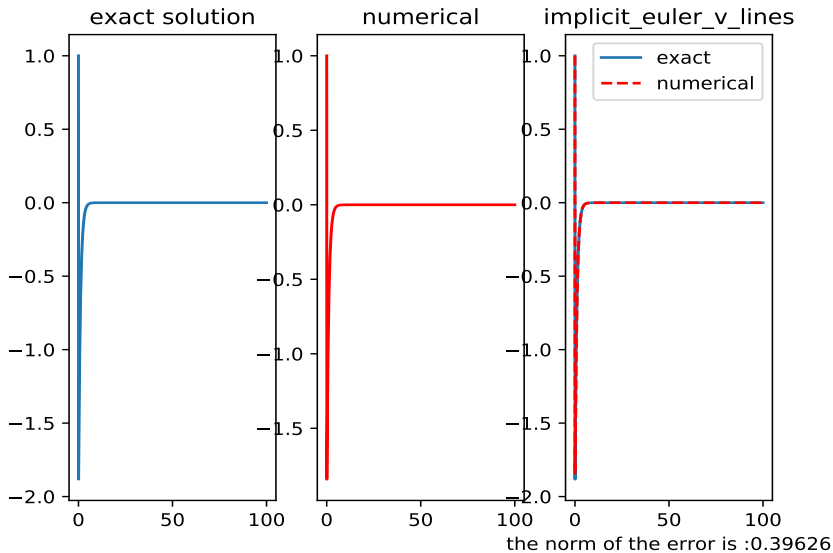
it can be presented as

$$\begin{cases} u_{n+1} = \frac{(1+199h)u_n + 198hv_n}{(1-98h)(1+199h) + (198h)(99h)} \\ v_{n+1} = \frac{-99hu_n + (1-98h)v_n}{(1-98h)(1+199h) + (198h)(99h)} \end{cases}$$

Implicit Euler Result of $u(t)$



Implicit Euler Result of $v(t)$



Implicit Midpoint Method

Implicit Midpoint Method

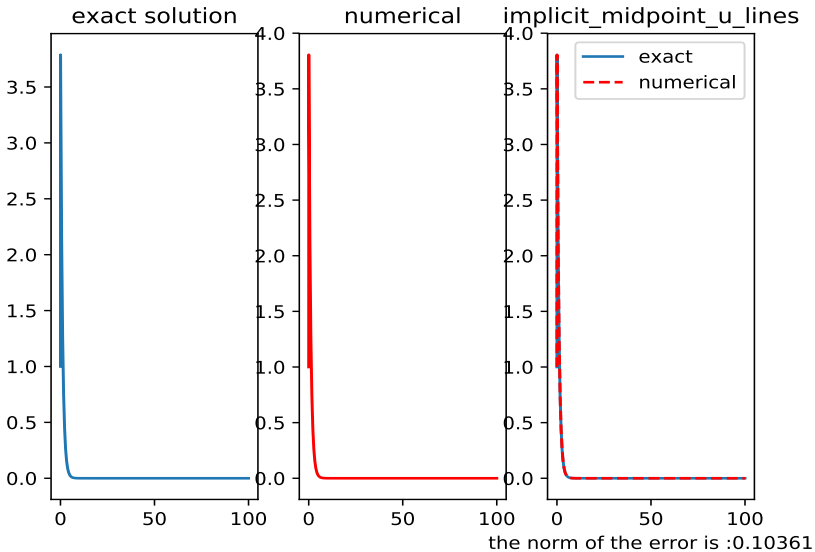
Implicit midpoint method for this problem

$$\begin{cases} \frac{u_{n+1}-u_n}{h} = 98 \frac{u_n+u_{n+1}}{2} + 198 \frac{v_n+v_{n+1}}{2} \\ \frac{v_{n+1}-v_n}{h} = -99 \frac{u_n+u_{n+1}}{2} - 199 \frac{v_n+v_{n+1}}{2} \end{cases}$$

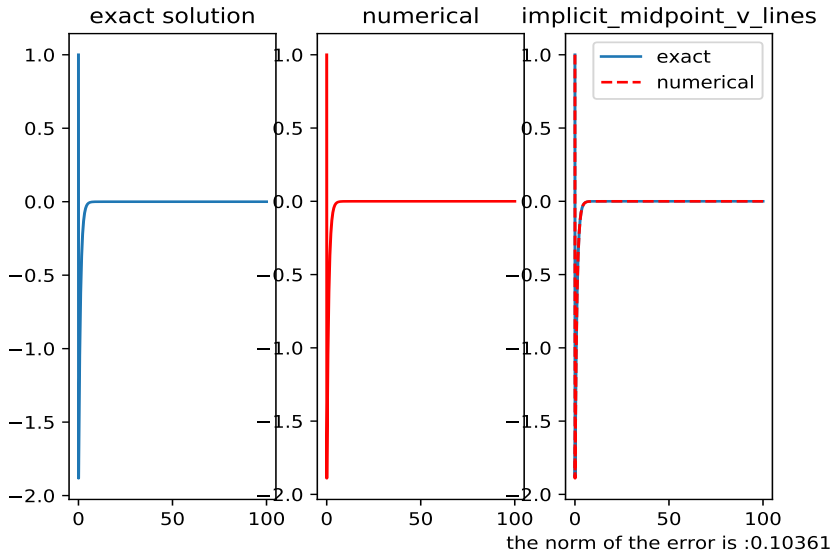
We use the iterator method to obtain the numerical solution

$$\begin{cases} u_{n+1}^{(s+1)} = 98h \frac{u_n+u_{n+1}^{(s)}}{2} + 198h \frac{v_n+v_{n+1}^{(s)}}{2} + u_n \\ v_{n+1}^{(s+1)} = -99h \frac{u_n+u_{n+1}^{(s)}}{2} - 199h \frac{v_n+v_{n+1}^{(s)}}{2} + v_n \end{cases}$$

Implicit Midpoint Result of $u(t)$



Implicit Midpoint Result of $v(t)$



Explicit Runge-Kutta Method

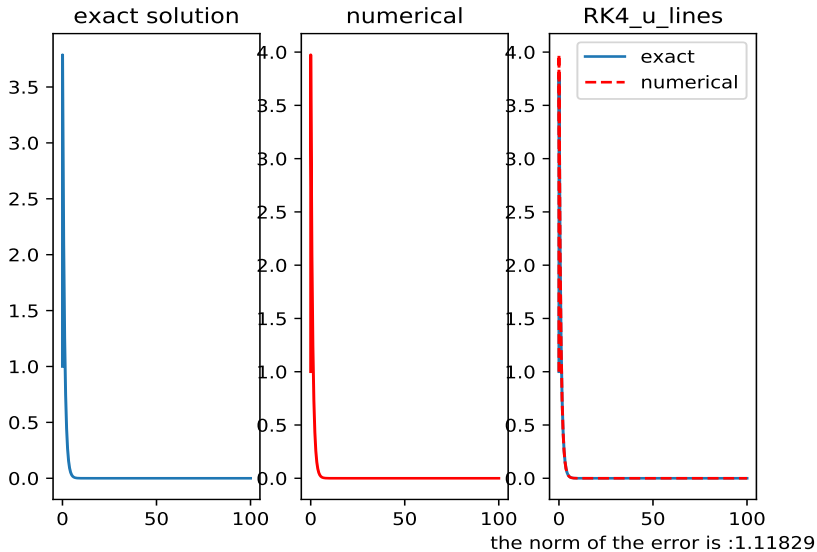
explicit Runge-Kutta Method

$$\begin{cases} u_{n+1} &= u_n + \frac{1}{6}(k_{u1} + 2k_{u2} + 2k_{u3} + k_{u4}) \\ v_{n+1} &= v_n + \frac{1}{6}(k_{v1} + 2k_{v2} + 2k_{v3} + k_{v4}) \end{cases}$$

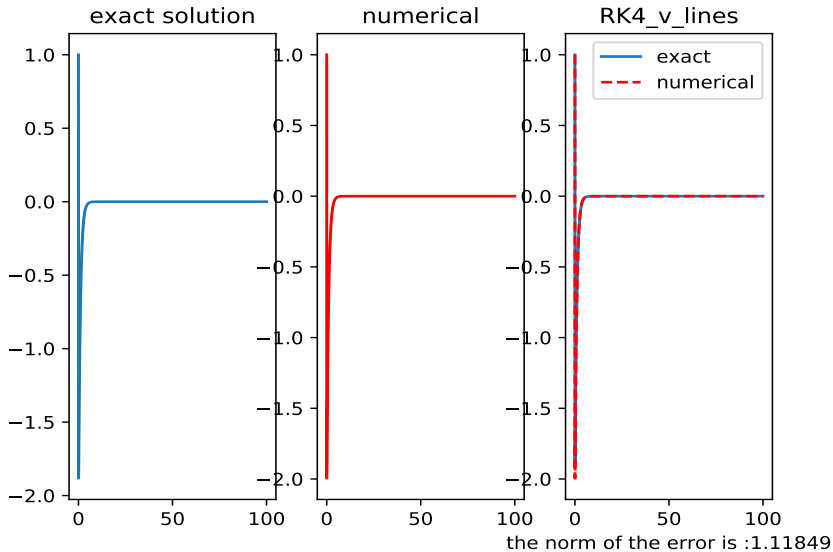
where

$$\begin{cases} f_u &= 98u + 198v \\ k_{u1} &= hf_u \\ k_{u2} &= h(f_u + \frac{1}{2}k_{u1}) \\ k_{u3} &= h(f_u + \frac{1}{2}k_{u2}) \\ k_{u4} &= h(f_u + \frac{1}{2}k_{u3}) \end{cases} \quad \begin{cases} f_v &= -99u - 199v \\ k_{v1} &= hf_v \\ k_{v2} &= h(f_v + \frac{1}{2}k_{v1}) \\ k_{v3} &= h(f_v + \frac{1}{2}k_{v2}) \\ k_{v4} &= h(f_v + \frac{1}{2}k_{v3}) \end{cases}$$

Runge Kutta Result of $u(t)$



Runge Kutta Result of $v(t)$



Thank you !