3.4

$$
\begin{aligned}
m(t) &= \sum_{k=1}^{\infty} F_k(t) \\
&= F(t) + \sum_{k=1}^{\infty} F_{k+1}(t) \\
&= F(t) + \sum_{k=1}^{\infty} \int_0^t F_k(t-x) dF(x) \\
&= F(t) + \int_0^t \left(\sum_{k=1}^{\infty} F_k(t-x) \right) dF(x) \\
&= F(t) + \int_0^t m(t-x) dF(x)
\end{aligned}
$$

3.7

$$
\begin{gathered}
\because X \sim U(0,1) \\
m(t) = E[N(t)] \\
= \int_0^{\infty} E[N(t)|X_1] dF(X) \\
= \int_0^{\infty} E[1 + N(t-X)|X_1] dF(X) \\
= \int_0^t (1 + E[N(t-X)]) dF(X) \\
= t + \int_0^t m(t-s) ds = t + \int_0^t m(y) dy \\
\text{两边同时取微商} m^{'}(t) = 1 + m(t) \\
\therefore m(t) = e^t - 1 \\
\because \text{间隔时间加起来大于1的时刻为第} N(t) + 1 \text{到达} \\
\text{又} \because t \in [0,1] \\
\therefore \text{到达间隔时间超过1所需的的}(0,1)\text{均匀随机变量的期望数为} e
\end{gathered}
$$

3.10

(a)

$$\lim_{m\to\infty}\frac{S_1+\cdots+S_m}{N_1+\cdots+N_m}=\frac{\sum_{i=1}^{N_1+\cdots+N_m}X_i}{N_1+\cdots+N_m}$$
$$=\mathbb{E}[X]$$

(b)

$$\lim_{m\to\infty}\frac{S_1+\cdots+S_m}{N_1+\cdots+N_m}=\lim_{m\to\infty}\frac{S_1+\cdots+S_m}{m}\cdot\frac{m}{N_1+\cdots+N_m}$$
$$=\mathbb{E}[S_1]\cdot\frac{1}{\mathbb{E}[N]}$$
$$=\frac{\mathbb{E}\left[\sum_{i=1}^{N}X_i\right]}{\mathbb{E}[N]}$$

(c)

$$\mathbb{E}\left[\sum_{i=1}^{N}X_i\right]=\mathbb{E}[N]\mathbb{E}[X]$$

3.17

$$g=h+g*F$$
$$=h+(h+g*F)*F$$
$$=h+h*F+g*F_2$$
$$=\ldots$$
$$=h+h*F+\ldots+h*F_n+g*F_{n+1}$$
$$\because n\to\infty,F_n\to 0,$$
$$g=h+h*m$$

(a)

$$P(t)=\int_0^\infty P(\text{在}t\text{时刻处于开状态}|Z_1+Y_1=s)dF(s)$$
$$=\int_0^t P(\text{在}T\text{时刻处于开状态}|Z_1+Y_1=s)dF(s)+\int_t^\infty P(\text{在}T\text{时刻处于开状态}|Z_1+Y_1=s)dF(s)$$

$$= \int_0^t P(t-s)dF(s) + \int_t^\infty P(Z_1 > t | Z_1 + Y_1 = s)ds$$
$$= \int_0^t P(t-s)dF(s) + P(Z_1 > t)$$

(b)

$$g(t) = \int_0^\infty E[A(t)|X_t = s]dF(s)$$
$$= \int_0^t E[A(t)|X_t = s]dF(s) + \int_t^\infty E[A(t)|X_t = s]dF(s)$$
$$= \int_0^t g(t-s)dF(s) + \int_t^\infty tdF(s)$$
$$= \int_0^t g(t-s)dF(s) + t(1 - F(t))$$
$$P(t) \to \frac{\int_0^\infty P(Z_t > t)dt}{\mu_F} = \frac{E[Z]}{E[Z] + E[Y]}$$
$$g(t) \to \frac{E[X^2]}{2\mu}$$

3.27

$$E[R_{N(t)+1}] = E[R_{N(t)+1}|S_{N(t)} = 0]\bar{F}(t) + \int_0^t E[R_{N(t)+1}|S_{N(t)} = s]\bar{F}(t-s)dm(s)$$
$$= E[R_1|X_1 > t]\bar{F}(t) + \int_0^t E[R_1|X_1 > t-s]\bar{F}(t-s)dm(s)$$
$$\to \int_0^t E[R_1|X_1 > t]\bar{F}(t)\frac{dt}{\mu}$$
$$\because \bar{F}(t) = \int_t^\infty f(s)ds;$$
$$= \int_0^t \int_t^\infty E[R_1|X_1 = s]dF(s)\frac{dt}{\mu}$$
$$= \int_0^\infty \int_0^s E[R_1|X_1 = s]dF(s)\frac{dt}{\mu}$$
$$= \int_0^\infty E[R_1|X_1 = s]dF(s)\frac{s}{\mu}$$
$$= \frac{E[R_1X_1]}{\mu}$$
$\because \mu = E[X_1], E[X_1R_1] < \infty,$可以得出在$t \to \infty$有$E[R_1|X_1 > t]\bar{F}(t) \to 0,$
$Var(X) > 0. \therefore E[X^2] > E^2[X]$，除非$X$以概率为1地是常数

3.32

$$(a)$$
$$P_0 = 1 - \lambda\mu_G$$
$$(b)$$
$$E[G] = \frac{\mu_G}{P_0} = \frac{\mu_G}{1 - \lambda\mu_G}$$
$$(c)$$
$$E[G] = E[N\mu_G]$$
$$E[N] = \frac{1}{1 - \lambda\mu_G}$$