2019 年度日本政府(文部科学省) 奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE GOVERNMENT (MEXT) SCHOLARSHIP 2019

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数 学 (B)

MATHEMATICS(B)

注意 ☆試験時間は60分。

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS ((B)	١
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(2019)

Nationality		No.				
Name	(Please print full name, underlining far	mily na	me)	N	Marks	

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

- 1. Fill in the blanks with the correct numbers.
- (1) The total number of positive divisors of 2019 is \bigcirc and the whole sum of those divisors is \bigcirc .
- (2) For the three points O(0,0), A(3,0), and B(3,4) on the xy-plane, the equation of the angle bisector of $\angle AOB$ is $y = \begin{bmatrix} x \\ x \end{bmatrix}$.
- (3) For parabola $y = x^2$ and two points (-1,1) and (3,9) on it, its tangent line parallel to the line through the two points is the line $y = \boxed{ \textcircled{1} } x + \boxed{ \textcircled{2} }$, whose point of tangency is the point $\boxed{ \textcircled{3} }$, $\boxed{ \textcircled{4} }$.
- (4) When the line y = m(x 5) + 3 intersects the circle $x^2 + y^2 = r^2$ (r > 0) if and only if $0 \le m \le \boxed{\textcircled{1}}$, $r = \boxed{\textcircled{2}}$.
- (5) When $|x| \leq \frac{\pi}{2}$, the maximum of $\sin x + \cos x$ is and the minimum of that is
- (6) By $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.4771$, the number of digits of 6^{100} is $\boxed{\textcircled{1}}$, and its leading digit is $\boxed{\textcircled{2}}$.

- **2.** Let I(m,n) be a function of a pair (m,n) of natural numbers that is inductively defined by the following:
 - (i) I(m,1) = I(1,n) = 1 (for any (m,n));
 - (ii) I(m+1,n) + I(m,n+1) = I(m+1,n+1) (for any (m,n)).

Fill in the blanks with the answers to the following questions.

- (1) Express I(2, n) and I(3, n) in terms of n.
- (2) Find the value of I(5,3).
 - $(1) I(2,n) = \boxed{\bigcirc$
- $I(3,n) = \bigcirc$
- (2) I(5,3) =
- **3.** Let $f(x) = e^x$, g(x) = 1 + x, and $h(x) = 1 + x + \frac{1}{2}x^2$. Fill in the blanks with the answers to the following questions.
 - (1) When x < 0, arrange f'(x), g'(x), and h'(x) in ascending order.
 - (2) When x < 0, arrange f(x), g(x), and h(x) in ascending order.
 - (3) Compute $I_1 = \int_{-1}^{0} |f(x) g(x)| dx$ and $I_2 = \int_{-1}^{0} |f(x) h(x)| dx$.
 - $(1) \boxed{\textcircled{2}} < \boxed{\textcircled{3}}$
 - $(2) \boxed{\bigcirc{}} < \boxed{\bigcirc{}} < \boxed{\bigcirc{}}$
 - $(3) I_1 = \boxed{\bigcirc} \qquad \qquad I_2 = \boxed{\bigcirc}$