Part 2e: Mixture models

Textbook: p. 46-48

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Definition of a mixture model

Definition 2.6 Let P_1, \ldots, P_k be probability distributions on \mathbb{R}^d and let $\theta_1, \ldots, \theta_k > 0$ such that $\sum_{a=1}^k \theta_a = 1$. Then the *mixture* P_{θ} of the distributions P_1, \ldots, P_k with weights $\theta_1, \ldots, \theta_k$ is given by

$$P_{\theta}(\mathbf{A}) = \sum_{a=1}^{k} \theta_a P_a(\mathbf{A})$$

for all $A \subseteq \mathbb{R}^d$.

Lemma 2.8 Assume that P_1, \ldots, P_k have densities f_1, \ldots, f_k . Then the mixture distribution P_{θ} also has a density which is given by

$$f_{\theta} = \sum_{a=1}^{k} \theta_a f_a.$$

$$F_{\theta}(x) = P_{\theta}((-\infty, x))$$

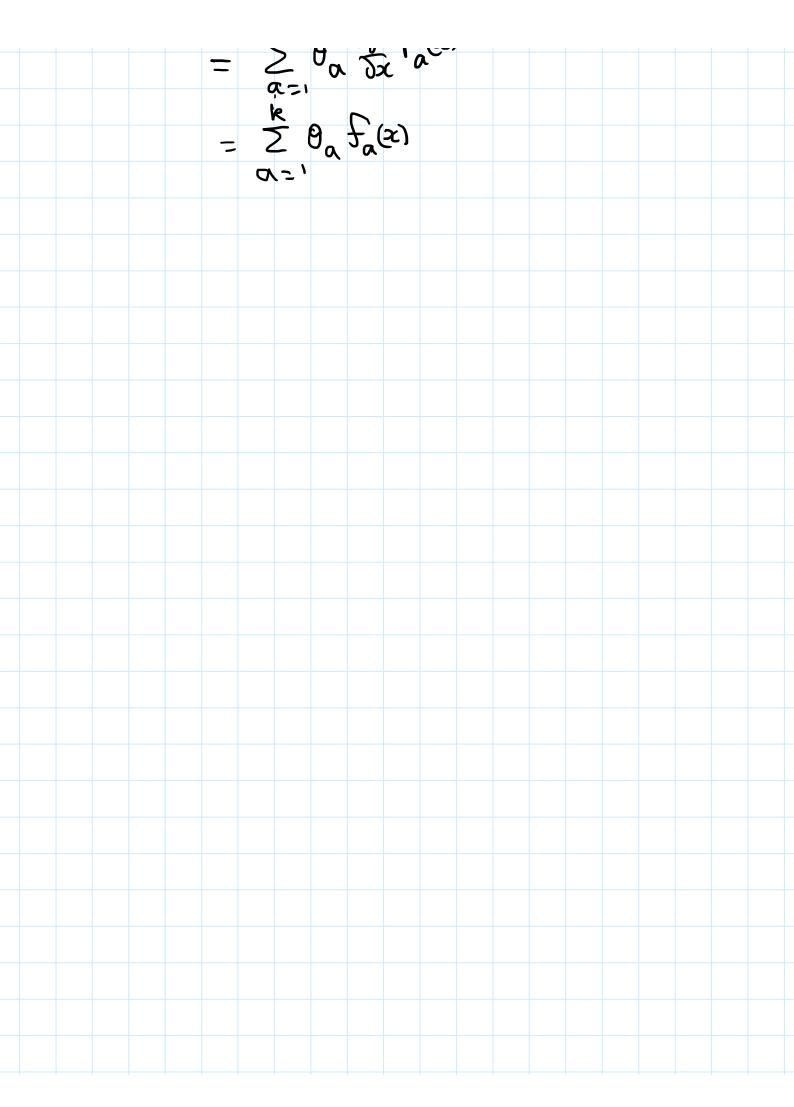
$$= \sum_{\alpha=1}^{\infty} P_{\alpha} P_{\alpha}((-\infty, x))$$

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Simulating a mixture model Algorithm 2.9 (mixture distributions) input: probability distributions P_1, \ldots, P_k weights $\theta_1, \ldots, \theta_k > 0$ with $\sum_{a=1}^k \theta_a = 1$ randomness used: $Y \in \{1, 2, \ldots, k\}$ with $P(Y = a) = \theta_a$ for all a samples $X \sim P_a$ for different $a \in \{1, \ldots, k\}$ output: $X \sim P_\theta$ 1: generate $Y \in \{1, 2, \ldots, k\}$ with $P(Y = a) = \theta_a$ for all a2: generate $X \sim P_Y$ 3: return X

Lemma 2.10 The sample *X* constructed by algorithm 2.9 is distributed according to the mixture distribution from definition 2.6, that is $X \sim P_{\theta}$.

$$(x,y) \sim f(x,y) = P_{y}(y) \cdot f(x,y) = \theta_{y} \cdot f(x,y)$$

$$x \sim \sum_{\alpha=1}^{k} f(x,\alpha) = \sum_{\alpha=1}^{k} p_{\alpha} f(\alpha)$$

$$x \sim \sum_{\alpha=1}^{k} f_{x,y}(x,\alpha) = \sum_{\alpha=1}^{k} p_{\alpha} f(\alpha)$$