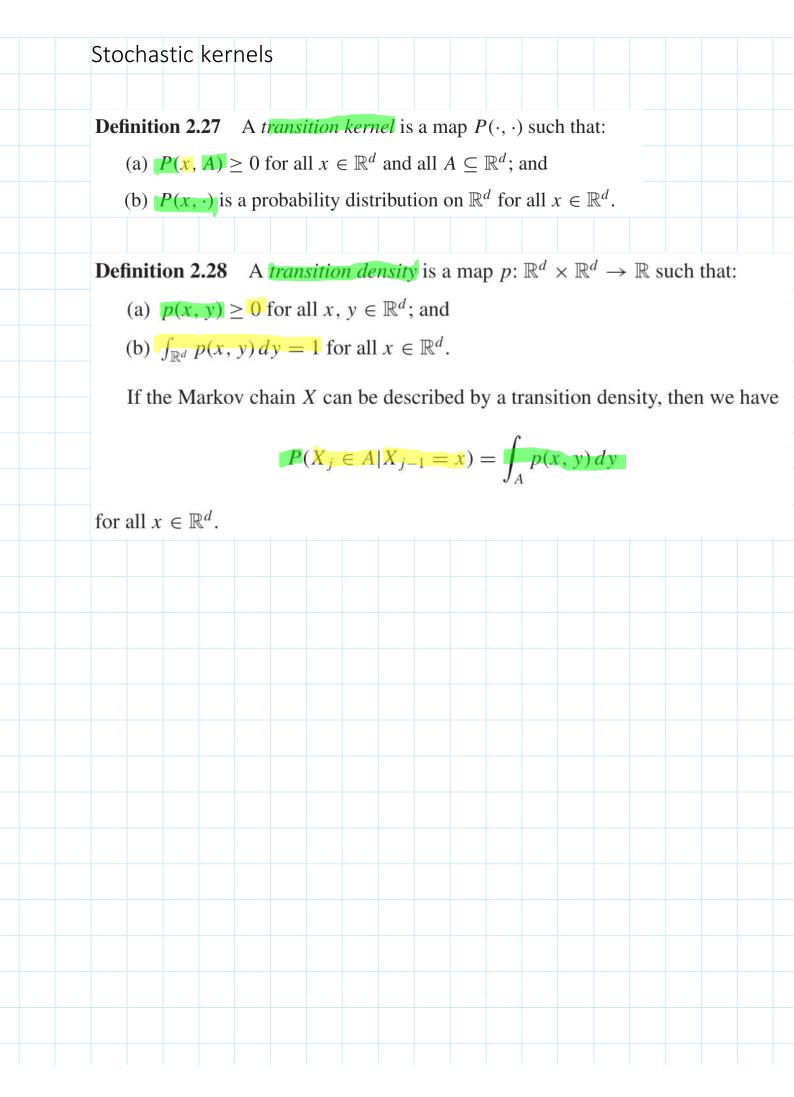
Part 2j: Continuous Markov chains

Textbook: pp. 56-58

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Simulation of a continuous Markov process **Algorithm 2.31** (Markov chains with continuous state space) input: a probability density $\pi \colon \mathbb{R}^d \to [0, \infty)$ a transition density $p: \mathbb{R}^d \times \mathbb{R}^d \to [0, \infty)$ randomness used: one sample $X_0 \sim \pi$ samples from the densities $p(x, \cdot)$ for $x \in \mathbb{R}^d$ output: a path of a Markov chain with initial distribution π and transition matrix P 1: generate $X_0 \sim \pi$ 2: output X_0 3: **for** $j = 1, 2, 3, \dots$ **do** generate $X_j \sim p(X_{j-1}, \cdot)$ output X_j 6: end for

The stationary distribution and convergence
Definition 2.30 A probability density π : $\mathbb{R}^d \to [0, \infty)$ is a stationary density for a
Markov chain on the state space \mathbb{R}^d with transition density p , if it satisfies
$\int_{S} \pi(x) p(x, y) dx = \pi(y)$
for all $y \in \mathbb{R}^d$.
Under appropriate regularity conditions of the transition
density $p(x, y)$ one can prove the convergence of the
marginal distributions to the stationary distribution.

An example **Example 2.29** On $S = \mathbb{R}$, let $X_0 = 0$ and $X_j = \frac{1}{2}X_{j-1} + \varepsilon_j$ for all $j \in \mathbb{N}$, where $\varepsilon_j \sim \mathcal{N}(0, 1)$ i.i.d. is a Markov chain with state space $S = \mathbb{R}$. $AR(1), \alpha = \frac{1}{2}, \sigma^2 = 1$ $T = N(0, T^2)$ Var(X) = Var(2 X; -1) + Var(2) => == == == +1 $\frac{1}{2} = \frac{1}{1 - 1/4} = \frac{4}{3}$ X: ~ N(0, 4) => X: ~ N(0, 4)