Part 2d: Bayesian models

Textbook: p. 46

Introduction to Bayesian models

- A Bayesian model is an hierarchical structure. The parameters are modeled as random. The distribution of the observations is determined by the realized values of the parameters.
- The marginal distribution of the parameters is called the *prior distribution*.
- Inference in Bayesian models is based on the *posterior distribution* = the conditional distribution of the parameters, given the observation.
- Here we consider the problem of simulating observations in a Bayesian model.
- In Part 4 we will deal with the problem of simulating from the posterior distribution of the parameters, given the observations.

An example of a Bayesian model

Observations: $X_1, X_2, \dots, X_n \sim$ i.i.d. Bernoulli.

Parameter: $p = P(X = 1)$. $P(X=0) = 1-p$

Prior distribution: $p \sim \text{Uniform}(0,1)$

Code for simulating a sample from this Bayesian model:

```
> rbinom(n,1,runif(1))
```

Another example of a Bayesian model

Example 2.5 Consider the Bayesian model where the data are described as i.i.d. samples $X_1, \ldots, X_n \sim \mathcal{N}(\mu, \sigma^2)$, and where the mean μ and the variance σ^2 are themselves assumed to be random with distributions $\sigma^2 \sim \mathrm{Exp}(\lambda)$ and $\mu \sim \mathcal{N}(\mu_0, \alpha\sigma^2)$. Since the variance σ^2 occurs in the distribution of μ, the model has the following dependence structure:

$$\sigma^2 \longrightarrow \mu \longrightarrow X_1, \ldots, X_n.$$

An algorithm for the simulation of a sample from this model:

1: generate $\sigma^2 \sim \mathrm{Exp}(\lambda)$
2: generate $\mu \sim \mathcal{N}(\mu_0, \alpha\sigma^2)$
3: **for** $i = 1, \ldots, n$ **do**
4: generate $X_i \sim \mathcal{N}(\mu, \sigma^2)$
5: **end for**