Part 2a: Simulating statistical models

Textbook: pp. 41.

## Introduction to the simulation of statistical models • Statistical models describe the theoretical distributions for data, data that emerges in real applications. • These models are typically more complex than the examples that were considered in Part 1. However, the methods from Part 1 can be used as building blocks in the simulation of the more complex models. • In the rest of Part b, that is based on Ch 2, we will discuss widely used statistical models.

## Multivariate normal distribution

A standard model for data that may be collected in an experiment or an observational study:

**Definition 2.1** Let  $\mu \in \mathbb{R}^d$  be a vector and  $\Sigma \in \mathbb{R}^{d \times d}$  be a symmetric, positive definite matrix. Then a random vector  $X \in \mathbb{R}^d$  is normally distributed with mean  $\mu$  and covariance matrix  $\Sigma$ , if the distribution of X has density  $f : \mathbb{R}^d \to \mathbb{R}$  given by

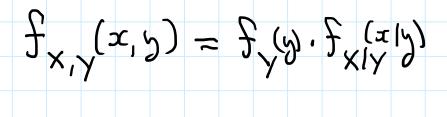
$$f(x) = \frac{1}{(2\pi)^{d/2} |\det \Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^{\top} \Sigma^{-1}(x-\mu)\right)$$
 (2.1)

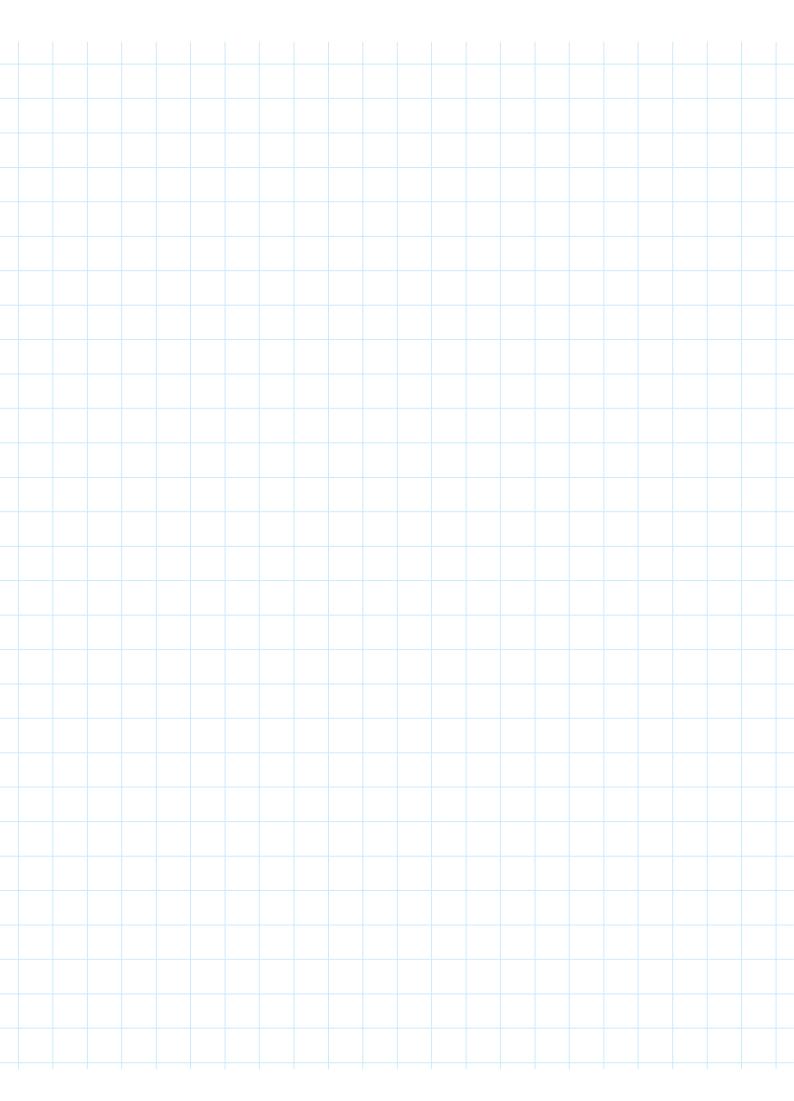
for all  $x \in \mathbb{R}^d$ .

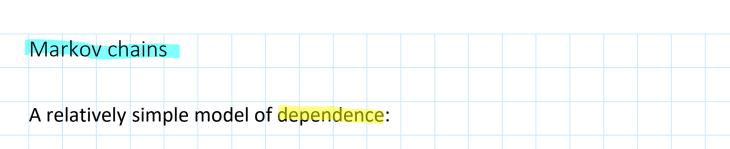
## Hierarchical models

The model is structured in layers. The distribution in a layer depends on the evaluation of the previous layer.

- In Bayesian models (discussed in Section 4.3) the distribution of the data depends on the value of one or more random parameters.
- In mixture models the distribution of samples depends on the random choice of mixture component.
- In Markov chains (discussed in Section 2.3) the distribution of the value at time t depends on the value of the Markov chain at time t-1.







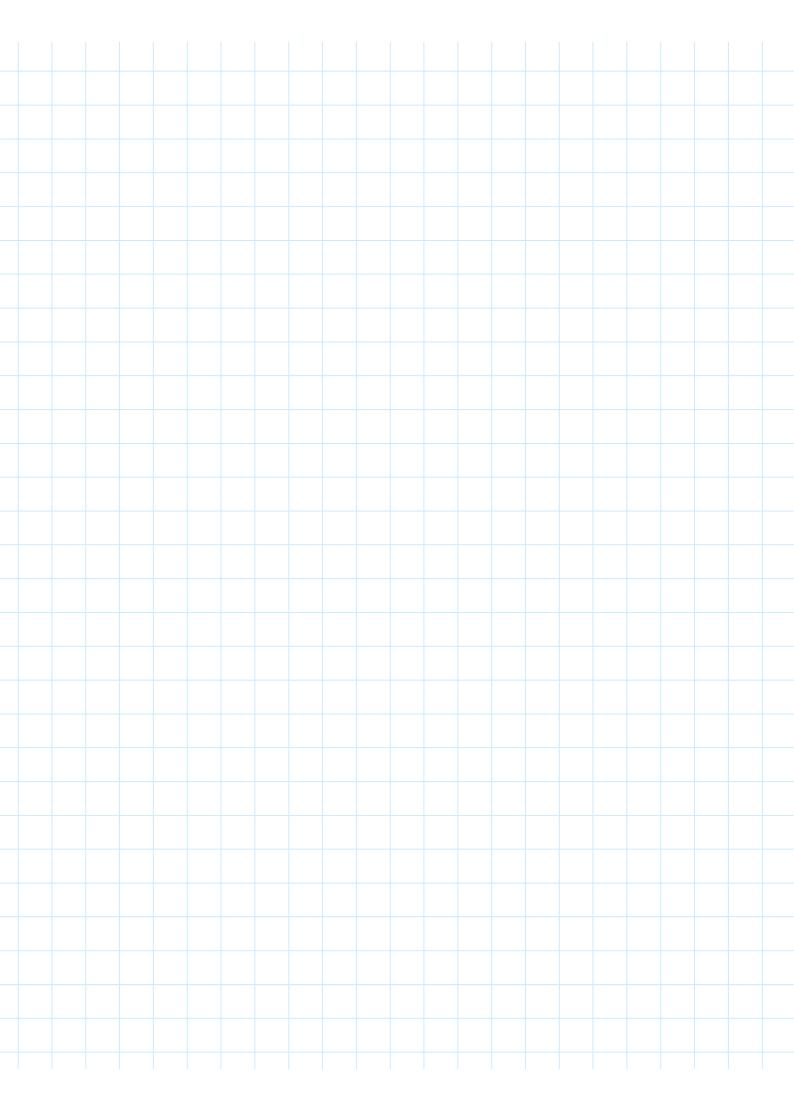
- Flexible enough to fit many applications.
- Simple enough to allow efficient computation and extensive mathematical theory.

**Definition 2.13** A stochastic process  $X = (X_j)_{j \in \mathbb{N}_0}$  with values in a set S is a *Markov chain*, if

$$P(X_{j} \in A_{j} | X_{j-1} \in A_{j-1}, X_{j-2} \in A_{j-2}, \dots, X_{0} \in A_{0})$$

$$= P(X_{j} \in A_{j} | X_{j-1} \in A_{j-1})$$
(2.2)

for all  $A_0, A_1, \ldots, A_j \subseteq S$  and all  $j \in \mathbb{N}$ , that is if the distribution of  $X_j$  depends on  $X_0, \ldots, X_{j-2}$  only through  $X_{j-1}$ . The set S is called the *state space* of X. The distribution of  $X_0$  is called the *initial distribution* of X.



		The Poisson process																		
				pro tion		in a	spa	ace.	The	pro	cess	is g	ove	rn b	y th	e Po	oisso	n		
		<b>Definition 2.34</b> A <i>Poisson process</i> on a set $D \subseteq \mathbb{R}^d$ with <i>intensity function</i> $\lambda$ : $\mathbb{R}^d$ $[0, \infty)$ is a random set $\Pi \subseteq D$ such that the following two conditions hold:  (a) If $A \subseteq D$ , then $ \Pi \cap A  \sim \text{Pois}(\Lambda(A))$ where $ \Pi \cap A $ is the number of poof $\Pi$ in $A$ and																		
		$\Lambda(A) = \int_A \lambda(x)  dx.$ (b) If $A, B \subseteq D$ are disjoint, then $ \Pi \cap A $ and $ \Pi \cap B $ are independent.															(2	2.5)		
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