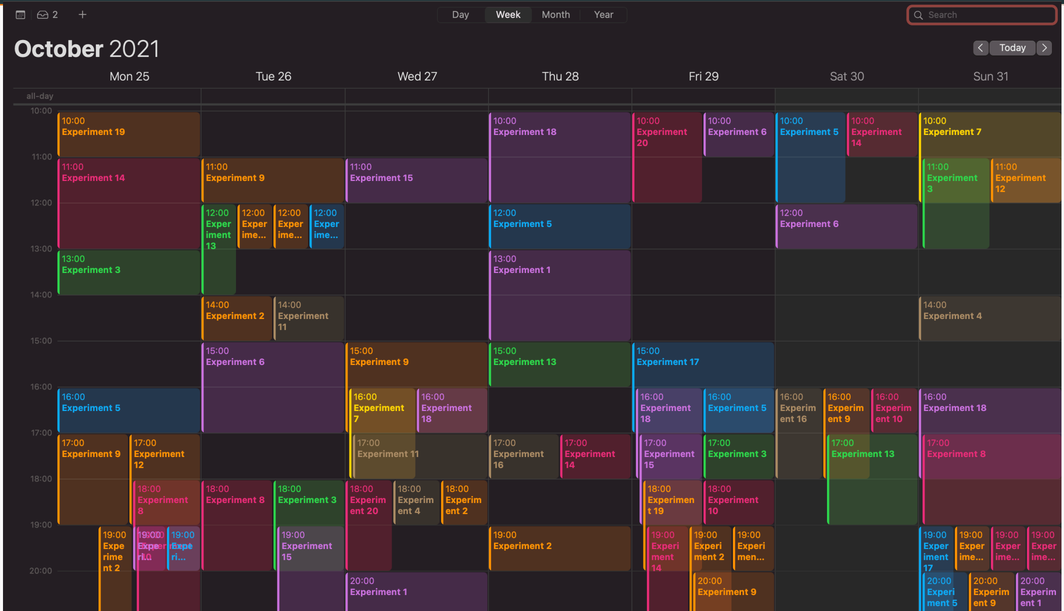
Temporal Unit Interval Independent Sets

Danny Hermelin, **Yuval Itzhaki**, Hendrik Molter, Rolf Niedermeier

Scheduling Lab Experiments



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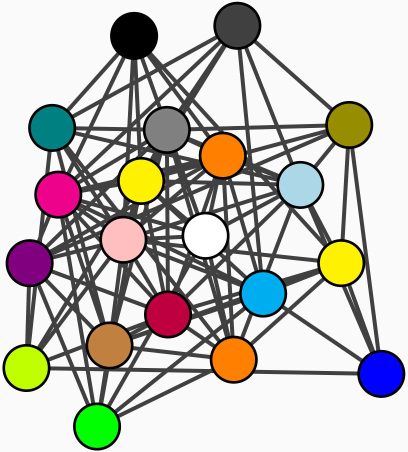

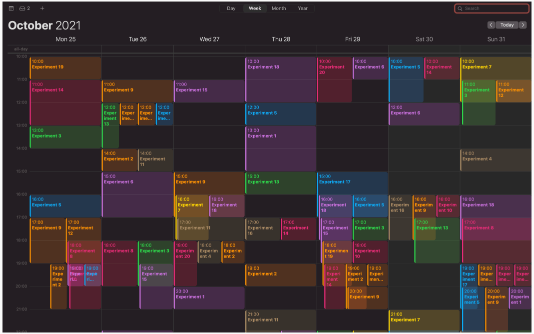

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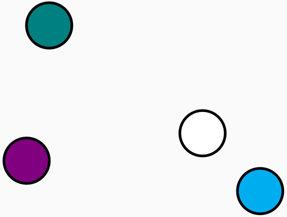
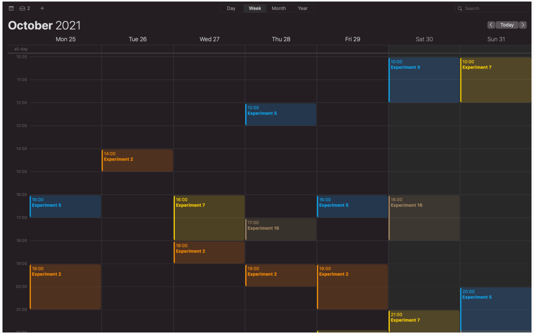




Temporal Δ Independent Set

Temporal Independent Set

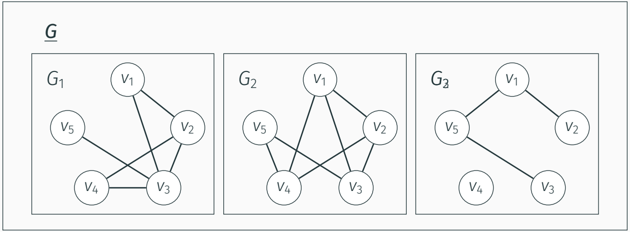
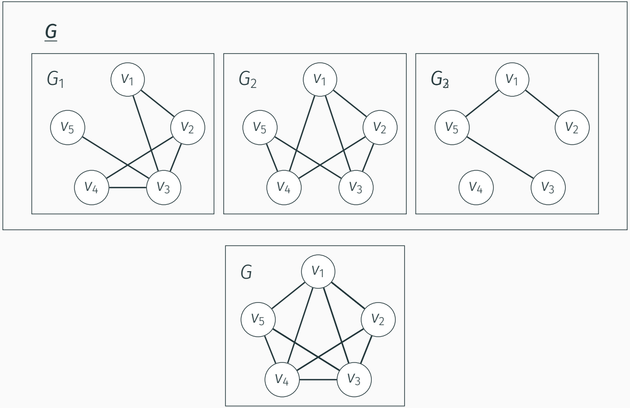


Figure 1: A temporal graph $\underline{G} = (V, \underline{E}, 3)$ with 3 time steps $[G_1, G_2, G_3]$.
The *life time* τ of $\underline{G}$ is 3.

Temporal Independent Set



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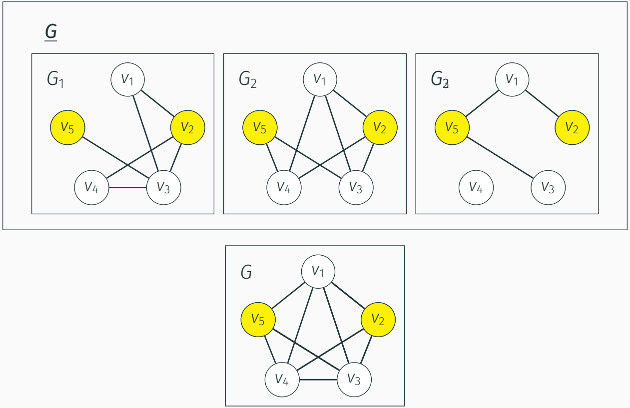

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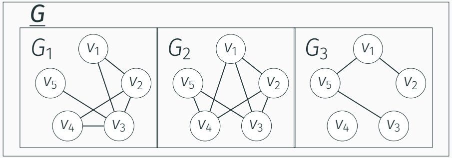
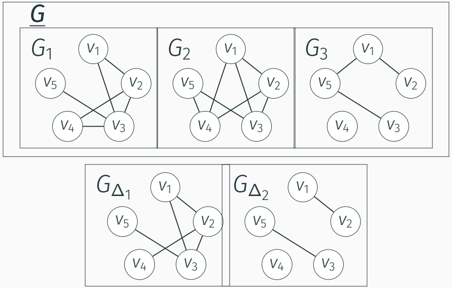


Figure 2: A Δ Independent Set in a temporal graph is an independent set in the intersection graph of every Δ consecutive layers.



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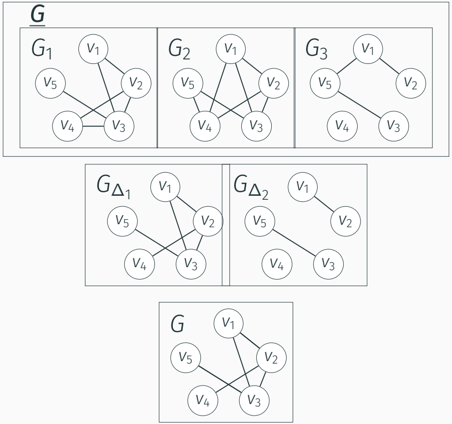
Temporal Δ Independent Set



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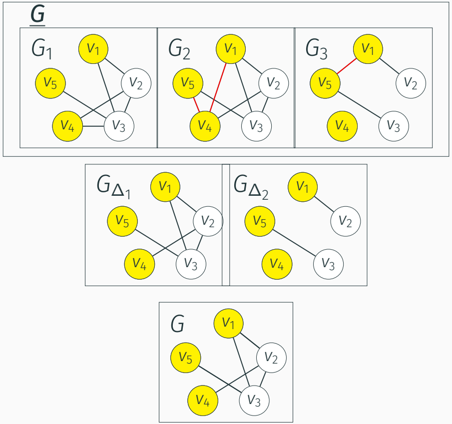

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Temporal Interval Graphs

Temporal Interval Graphs I

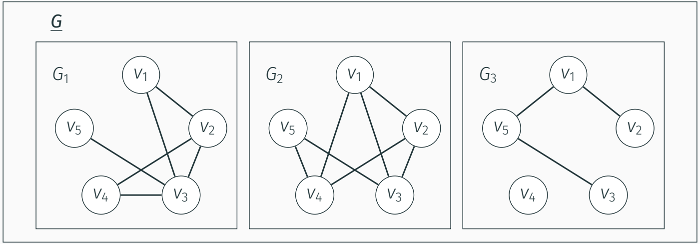

Figure 3: On temporal interval graphs, each layer has an interval representation ρ.

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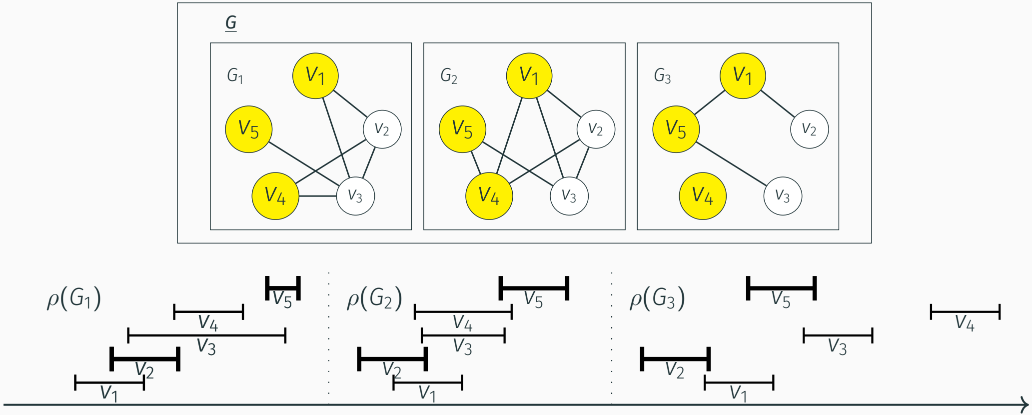

Figure 3: On temporal interval graphs, each layer has an interval representation ρ. The conflict graph has therefore a τ-track representation.

Temporal Interval Graphs II

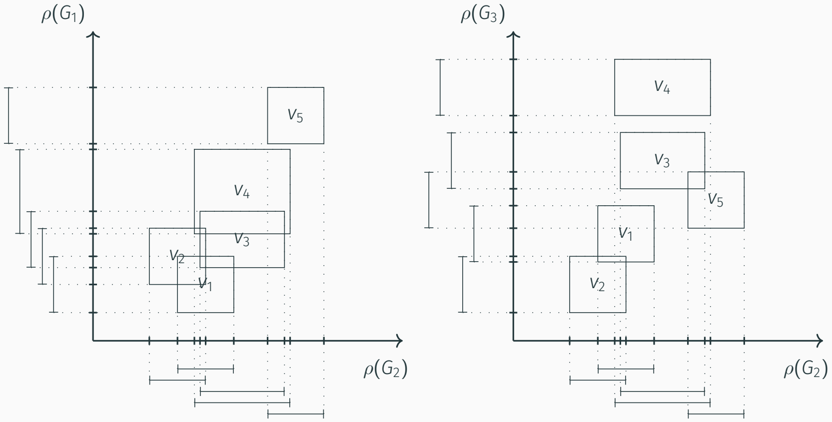

Figure 4: On temporal interval graphs,
the intersection graph of each Δ sliding window has a Δ-rectangle representation.

Problem Definition

TEMPORAL Δ INDEPENDENT SET (TΔIS) ON TEMPORAL UNIT-INTERVAL GRAPHS

Input: A temporal unit-interval graph $\underline{\mathcal{G}} = (V, \underline{E}, \tau)$ and two integer $\Delta, k \in \mathbb{N}$.

Question: Is there a k sized vertex subset which is an independent set in every edge-intersection graph of every Δ consecutive layers?

Our Results

- TΔIS on temporal unit-interval graph is NP-hard and W[1]-hard[1]
- TΔIS on temporal unit-interval graph can be approximated within a factor of $(\tau - \Delta + 1) \cdot 2^{\Delta}$
- Given an OPVD set, TΔIS can be solved in FPT time[2] of $2^{\ell}\, n^{\mathcal{O}(1)}$
- Computing the OPVD set is NP-hard and can be done in FPT time of $10^{\ell}\, n^{\mathcal{O}(1)}$

[1] With respect to the solution set size.
[2] With respect to the OPVD set size.

Order-Preserving Temporal Interval Graphs

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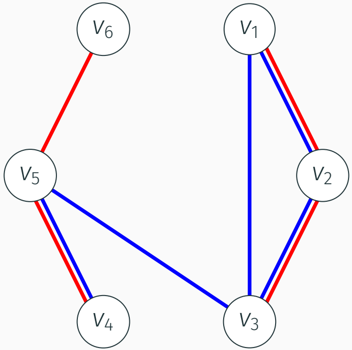
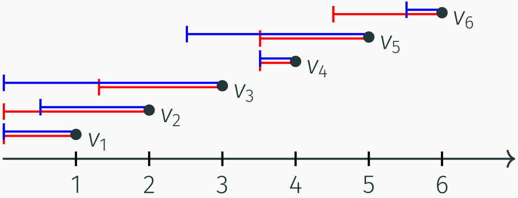


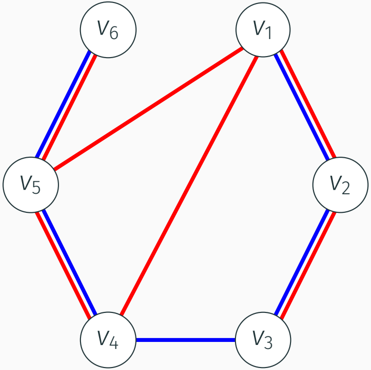
(a) Graph representations of the two interval graphs



(b) The normalized intersection models

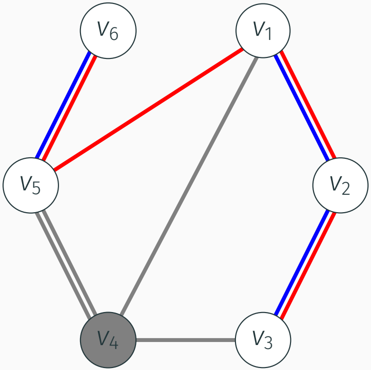
Figure 5: Two interval graphs, G_1 (blue) and G_2 (red), that have a common right-endpoints ordering $[v_1, v_2, v_3, v_4, v_5, v_6]$.

(Almost) Order-Preserving Temporal Interval Graphs



(a) Graph representations of the two interval graphs

Figure 6: Two interval graphs, G_1 (blue) and G_2 (red), that *do not* have a common right-endpoints ordering. The set $\{v_4\}$ is the *order-preserving vertex deletion set* (OPVD).



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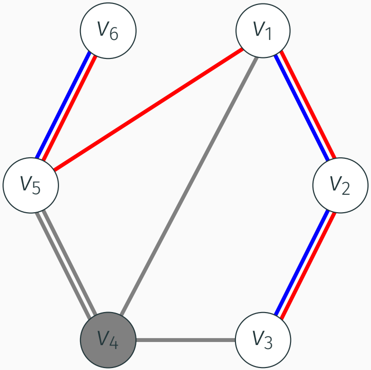

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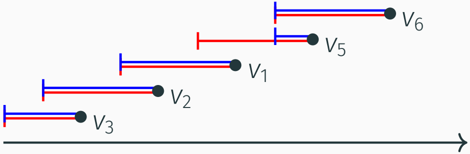

(b) Two intersection models of $G_1 - \{v_4\}$ and $G_2 - \{v_4\}$ which are compatible with $<_{V'}$

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Computing the OPVD of Temporal Unit-Interval Graphs

Reduction from Consecutive Ones Submatrix

Consecutive Ones Submatrix

Input: A Binary Matrix M

Question: Can we delete ℓ columns so that M has the C1P?

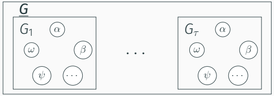
OPVD

Input: A temporal unit-interval graph.

Question: Is there an OPVD set of size ℓ?

$$\begin{bmatrix} m_{1,1} & \cdots & m_{1,n} \\ \vdots & \ddots & \vdots \\ m_{\tau n,1} & \cdots & m_{\tau n,n} \end{bmatrix}$$

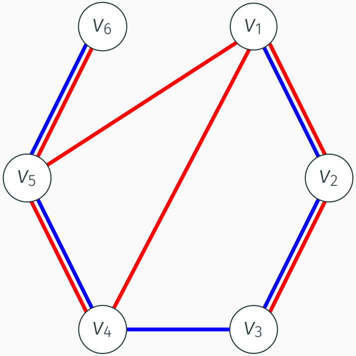
$$\Updownarrow$$



Computing an OPVD Set of Temporal Unit-Interval Graphs

The *Vertices vs Maximal Cliques* matrix of an order-preserving temporal unit-interval graph has the *consecutive ones property*.

$$\begin{array}{c c}
 & \begin{array}{cccccc} v_3 & v_2 & v_1 & v_4 & v_5 & v_6 \end{array} \\
\begin{array}{c} \{v_5, v_6\} \\ \{v_4, v_5\} \\ \{v_3, v_4\} \\ \{v_2, v_3\} \\ \{v_1, v_2\} \\ \{v_5, v_6\} \\ \{v_1, v_4, v_5\} \\ \{v_2, v_3\} \\ \{v_1, v_2\} \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$



A maximal submatrix with the consecutive ones properties can be computed in FPT time[3].

[3] With respect to the number of column deletions (Narayanaswamy et al., Algorithmica 2015).

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\hline
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\{v_1, v_2\} & 0 & 1 & 1 & & 0 & 0 \\
\{v_5, v_6\} & 0 & 0 & 0 & & 1 & 1 \\
\{v_1, v_4, v_5\} & 0 & 0 & 1 & & 1 & 0 \\
\{v_2, v_3\} & 1 & 1 & 0 & & 0 & 0 \\
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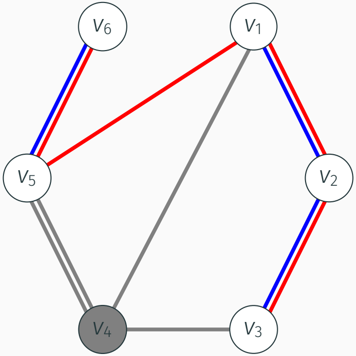
\end{array}
$$



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- What is the complexity of the computation of maximal order-preserving subgraph of a temporal interval graph?
- For which other graph classes can we extend order-preservation?
- For what other problems is order-preservation relevant?