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A Correlation for the Pressure and Temperature Dependence of Viscosity[©]

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An expression is suggested for correlating viscosity and kinematic viscosity data over a wide range of pressure and temperature. This expression is obtained by expanding the natural logarithm of viscosity in a series about ambient pressure and temperature. This series expansion is truncated to obtain a relatively simple relation that contains several useful empiricisms as special cases. Coefficients in this expansion are conveniently determined using a regression analysis. The proposed relation describes ASME data for several oils to within a 3 percent average deviation for pressures to 350 MPa (50 ksi) and temperatures to 373 K (100°C). The average deviation is less than 15 percent for pressures to 1.0 GPa (150 ksi) and temperatures to 491.5 K (218°C). In both cases, the agreement is typically much better than those predicted by other correlations over the same range of pressure and temperature.

INTRODUCTION

The primary function of lubricating oils is to develop and maintain a film between surfaces in relative motion. The thickness and the load bearing capacity of this film depend

on the oil viscosity, which can change significantly over the range of operating conditions. In automotive engines, temperatures as high as 150°C cause a decrease in viscosity to one-hundredth of the value at ambient conditions (1). Additives containing viscosity index improvers are used in the formulation of multi-grade oils to reduce this sensitivity. These additives also cause a reversible decrease in viscosity at high shear rates ($10^5 - 10^6 \text{ s}^{-1}$) typical in some journal bearings (2). Viscosity can increase by a factor of seven at pressures near 69 MPa (10 ksi) which are typical in components like roller bearings and gears (3), (4). Here the focus is on the effect of pressure and temperature on viscosity.

Barus (5) suggested that the logarithm of viscosity is proportional to gauge pressure. This relation can be written in the form

$$\ln \left(\frac{\eta}{\eta_0} \right) = \alpha(p - p_0), \quad [1]$$

where η_0 is the value of viscosity at ambient pressure p_0 and the pressure coefficient α has the unit of inverse pressure. This relation can also be expressed in terms of log by replacing the pressure coefficient with 0.4343α . Correlations of this type have been used by Kouzel (6) for viscosity data

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NOMENCLATURE

A_n = viscosity coefficients for the reference state p_0, T_0
 \hat{A}_n = viscosity coefficients for the reference state p_1, T_1
 B_n = kinematic viscosity coefficients for the reference state p_0, T_0
 \bar{D} = average value of percentage deviations
 k = number of independent variables in the regression analysis
 n = number of data points
 p = absolute pressure, Pa (psi)
 p_0 = ambient pressure, Pa (psi)
 p_1 = reference pressure, Pa (psi)
 R^2 = correlation coefficient
 S = standard error in the dependent variable
 T = absolute temperature, K
 T_0 = ambient temperature, K

T_1 = reference temperature, K
 y_i = measured value of the dependent variable
 \bar{y} = mean value of y_i
 Y_i = predicted value of the dependent variable
 α = pressure coefficient, Pa^{-1} (psi^{-1})
 β = normalized temperature coefficient
 δ_i = difference between measured and predicted values ($y_i - Y_i$)
 η = viscosity, mPa·s (cP)
 η_0 = viscosity at p_0 and T_0 , mPa·s (cP)
 η_1 = viscosity at p_1 and T_1 , mPa·s (cP)
 ν = kinematic viscosity, St
 ν_0 = kinematic viscosity at p_0 and T_0 , St
 ν_1 = kinematic viscosity at p_1 and T_1 , St
 \ln = natural logarithm (\log_e)
 \log = common logarithm (\log_{10})

and by Fresco, et al. (7) for kinematic viscosity data. These expressions accurately describe data for a variety of liquids up to about 69 MPa (10 ksi). Roelands, et al. (8) and Sargent (9) have used more complicated functions to improve agreement with data at higher pressure.

Reaction rate concepts suggest that the logarithm of viscosity is inversely proportional to absolute temperature for Newtonian fluids (10). This Arrhenius relationship can be written in the form

$$\ln\left(\frac{\eta}{\eta_0}\right) = \beta\left(\frac{T_0}{T} - 1\right), \quad [2]$$

in terms of a dimensionless coefficient β , where η_0 is the value of viscosity at ambient temperature T_0 . Correlations of this type accurately describe viscosity and kinematic viscosity data for most liquids only over a limited range of temperatures. A more complicated dependence on T is needed to correlate data over a wider range.

Equations [1] and [2] suggest that viscosity and kinematic viscosity can be described as functions of the reduced variables p/p_0 and T_0/T . These variables are used in expressions for viscosity and kinematic viscosity proposed in the next section. These expressions are obtained using a series expansion about ambient conditions. Equations [1] and [2] appear as leading order terms in this expansion. This approach provides a systematic method for generating higher order corrections to these results. A truncated series expansion is used to correlate viscosity and kinematic viscosity data for a variety of fluids.

DATA CORRELATION

The preceding discussion suggests that viscosity is determined by an expression of the form

$$\ln\left(\frac{\eta}{\eta_0}\right) = f\left(\frac{p}{p_0}, \frac{T_0}{T}\right), \quad [3]$$

where η_0 is the viscosity at p_0 , T_0 . The function f can be expanded in a Taylor series (11) about ambient conditions to generate an explicit equation for viscosity. Truncating this expansion gives an expression of the form

$$\begin{aligned} \ln\left(\frac{\eta}{\eta_0}\right) = & A_1\left(\frac{p}{p_0} - 1\right) + A_2\left(\frac{T_0}{T} - 1\right) \\ & + A_3\left(\frac{p}{p_0} - 1\right)^2 + A_4\left(\frac{T_0}{T} - 1\right)^2 \\ & + A_5\left(\frac{p}{p_0} - 1\right)\left(\frac{T_0}{T} - 1\right), \end{aligned} \quad [4]$$

where the usual constant term vanishes since $\eta = \eta_0$ for $p = p_0$ and $T = T_0$. Note that Eqs. [1] and [2] appear as leading terms in this expansion with the correspondence $A_1 = \alpha p_0$ and $A_2 = \beta$. A similar expression

$$\begin{aligned} \ln\left(\frac{\nu}{\nu_0}\right) = & B_1\left(\frac{p}{p_0} - 1\right) + B_2\left(\frac{T_0}{T} - 1\right) \\ & + B_3\left(\frac{p}{p_0} - 1\right)^2 + B_4\left(\frac{T_0}{T} - 1\right)^2 \\ & + B_5\left(\frac{p}{p_0} - 1\right)\left(\frac{T_0}{T} - 1\right), \end{aligned} \quad [5]$$

is used here to correlate kinetic viscosity data. These expressions would involve higher powers of pressure and temperature if more terms are kept in each series expansion.

Coefficients in Eqs. [4] and [5] depend on the choice of reference state (p_0 , T_0). For example, the similar correlation

$$\begin{aligned} \ln\left(\frac{\eta}{\eta_1}\right) = & \hat{A}_2\left(\frac{p}{p_0} - 1\right) + \hat{A}_2\left(\frac{T_1}{T} - 1\right) \\ & + \hat{A}_3\left(\frac{p}{p_1} - 1\right)^2 + \hat{A}_4\left(\frac{T_1}{T} - 1\right)^2 \\ & + \hat{A}_5\left(\frac{p}{p_1} - 1\right)\left(\frac{T_1}{T} - 1\right), \end{aligned} \quad [6]$$

is obtained by expanding about (p_1 , T_1) where η_1 is the viscosity at $p = p_1$ and $T = T_1$. The two sets of parameters are related by the expressions

$$\begin{aligned} \ln\left(\frac{\eta_1}{\eta_0}\right) = & A_1\left(\frac{p_1}{p_0} - 1\right) + A_2\left(\frac{T_0}{T_1} - 1\right) + A_3\left(\frac{p_1}{p_0} - 1\right)^2 \\ & + A_4\left(\frac{T_0}{T_1} - 1\right)^2 + A_5\left(\frac{p_1}{p_0} - 1\right)\left(\frac{T_0}{T_1} - 1\right), \\ \hat{A}_1 = & A_1\frac{p_1}{p_0} + 2A_3\frac{p_1}{p_0}\left(\frac{p_1}{p_0} - 1\right) + A_5\frac{p_1}{p_0}\left(\frac{T_0}{T_1} - 1\right), \\ \hat{A}_2 = & A_2\frac{T_0}{T_1} + 2A_4\frac{T_0}{T_1}\left(\frac{T_0}{T_1} - 1\right) + A_5\frac{T_0}{T_1}\left(\frac{p_1}{p_0} - 1\right), \\ \hat{A}_3 = & A_3\left(\frac{p_1}{p_0}\right)^2; \hat{A}_4 = A_4\left(\frac{T_0}{T_1}\right)^2; \hat{A}_5 = A_5\frac{p_1 T_0}{p_0 T_1}, \end{aligned} \quad [7]$$

which depend on the pressure and temperature in the two reference states. Since these terms scale with the choice of reference state, parameters in Eqs. [4] and [5] are possible measures of chemical structure. After specifying the reference state, coefficients in these equations are determined using a multiple linear regression technique described in the Appendix.

Comparisons with viscosity and kinematic viscosity data obtained from the ASME study (12) are shown in Figs. 1–5. In these plots, circles denote experimental observations from ambient conditions to 1.0 GPa (150 ksi) pressure and 491.5 K (218°C) temperature. The lines represent correlations based on a regression analysis of all data for a given field. These correlations use ambient values $p_0 = 0.101$ MPa (14.7 psi) and $T_0 = 298$ K (24.9°C) as a reference state. Plots shown in Figs. 1–5 and statistics given in each caption suggest that

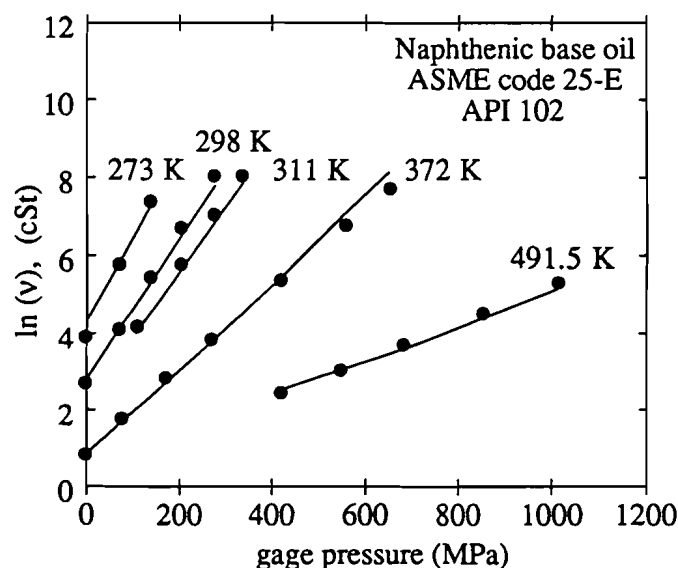
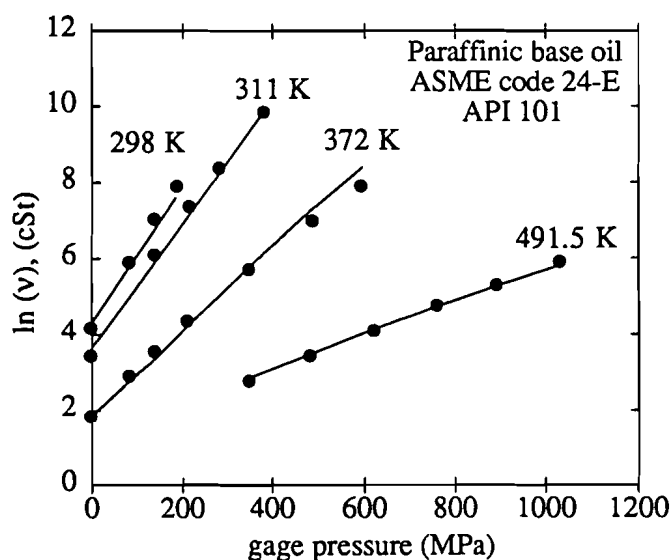
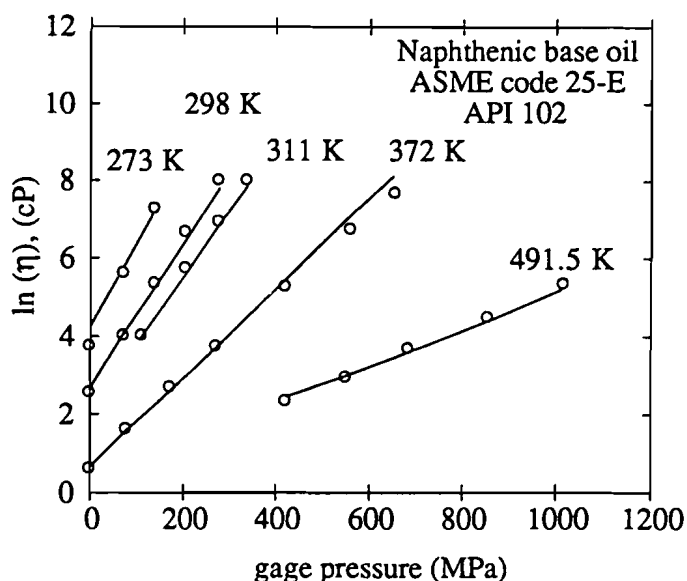
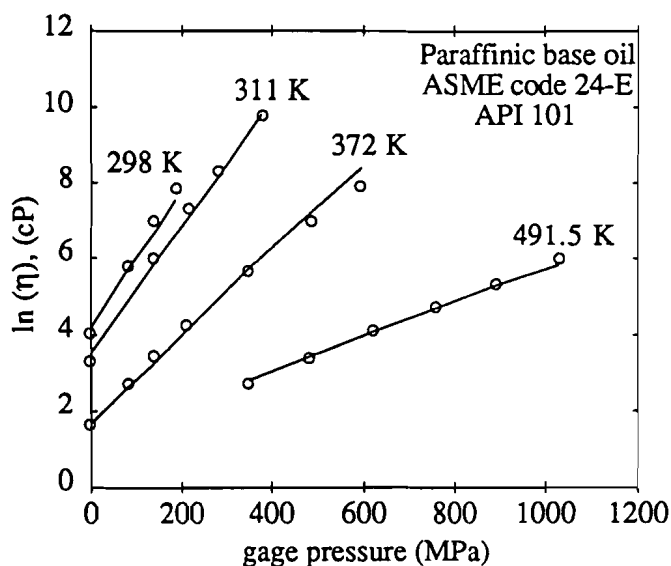


Fig. 1—Viscosity and kinematic viscosity data from (12) for a paraffinic oil. The accuracy of these correlations is suggested by the statistics $S = 0.215$ cP, $\bar{D} = 14.6$ percent, $R^2 = 0.991$, $n = 25$ for viscosity and $S = 0.221$ cSt, $\bar{D} = 14.2$ percent, $R^2 = 0.991$, $n = 25$ for kinematic viscosity.

Fig. 2—Viscosity and kinematic viscosity data from (12) for a naphthenic oil. The accuracy of these correlations is suggested by the statistics $S = 0.216$ cP, $\bar{D} = 14.0$ percent, $R^2 = 0.992$, $n = 24$ for viscosity and $S = 0.212$ cSt, $\bar{D} = 13.6$ percent, $R^2 = 0.992$, $n = 24$ for kinematic viscosity.

Eqs. [4] and [5] accurately describe viscosity and kinematic viscosity over a wide range of pressure and temperature.

COMPARISON WITH OTHER CORRELATIONS

Several researchers have proposed expressions for viscosity as a function of pressure and temperature. Some of the earlier work in this area is reviewed by Hersey and Hopkins (13). An example from this report is the expression

$$\ln\left(\frac{\eta}{\eta_0}\right) = A\left(\frac{p}{T + \theta_p} - \frac{p_0}{T_0 + \theta_p}\right) + B\left(\frac{1}{T + \theta_T} - \frac{1}{T_0 + \theta_T}\right), \quad [8]$$

suggested by Cameron (14) where A , B , θ_p , θ_T are constants. Subsequently, Appeldoorn (15) correlated data using the equation

$$\log\left(\frac{\eta}{\eta_0}\right) = A \log\left(\frac{T}{T_0}\right) + (p - p_0) \left[B + C \log\left(\frac{T}{T_0}\right) \right], \quad [9]$$

where A , B , C are constants. At about the same time, Roelands (16) used the expression

$$\frac{\log(\eta + 1.2)}{\log(\eta_0 + 1.2)} = \left(\frac{T - 138}{T_0 - 138}\right)^a \left[\frac{p - p_0}{(196.1)10^6} + 1\right]^b, \quad [10]$$

to describe data in terms of constants a and b where η , η_0 (cP), p , p_0 (Pa) and T , T_0 (°C) have the indicated units. These equations are all limited to pressures under 350 MPa and temperatures between 293–413 K (20–120°C). Pywell (17)

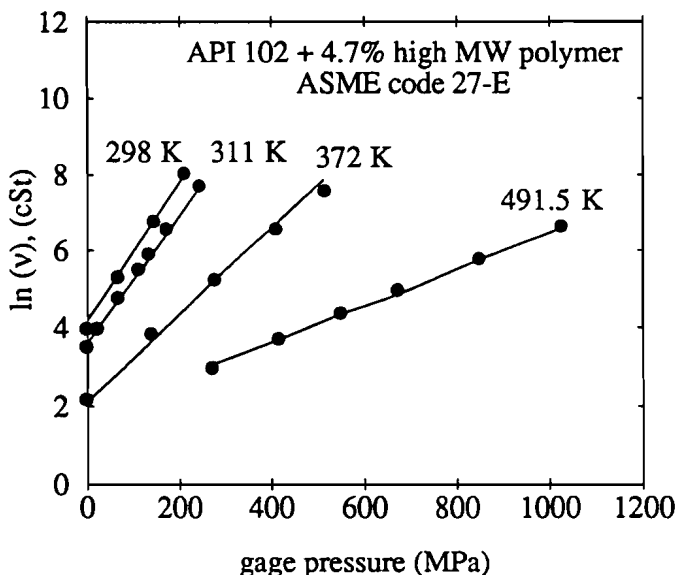
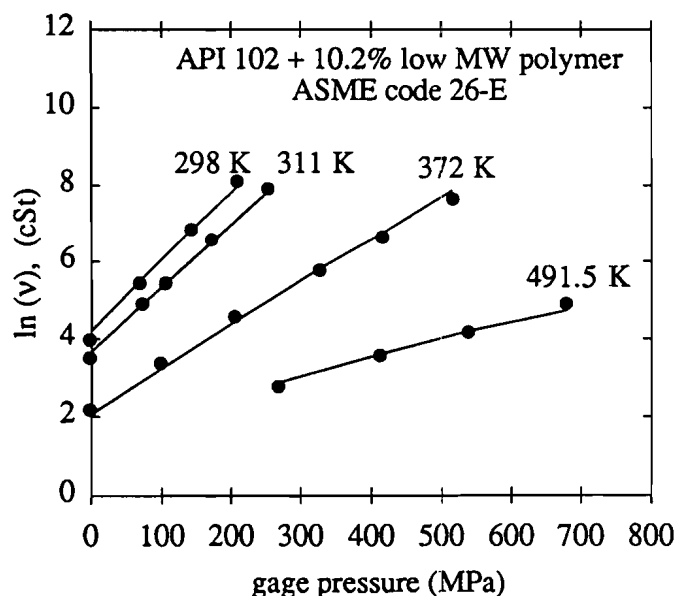
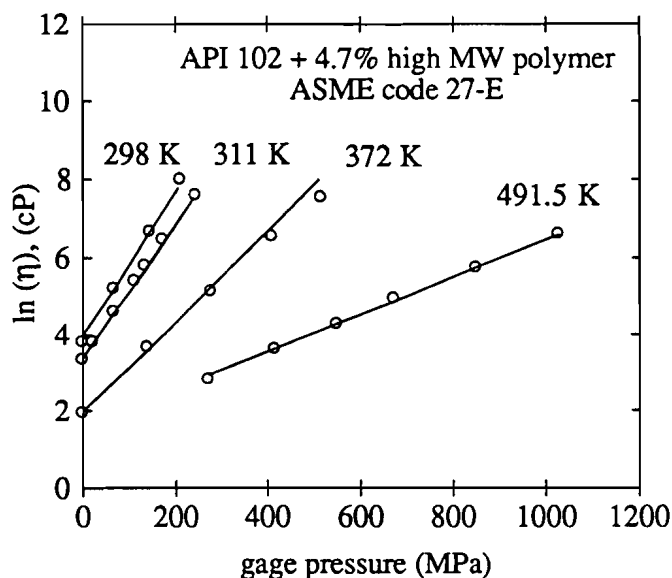
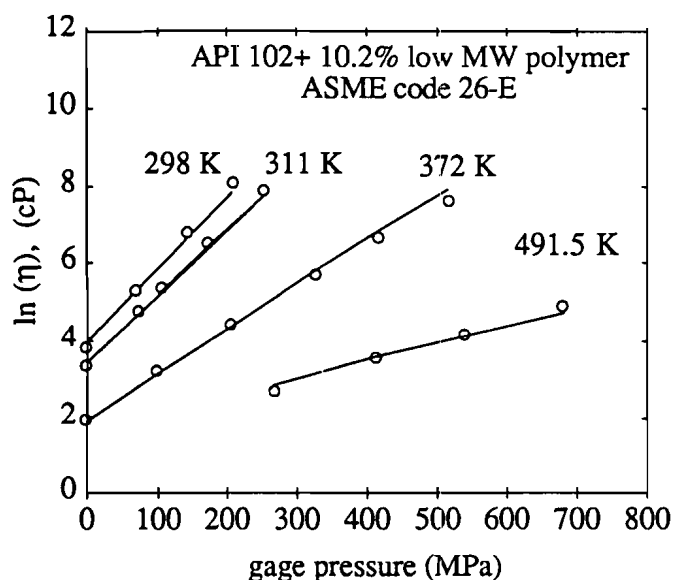


Fig. 3—Viscosity and kinematic viscosity data from (12) for a naphthenic oil with a low molecular weight polymer additive. The accuracy of these correlations is suggested by the statistics $S = 0.167$ cP, $\bar{D} = 11.1$ percent, $R^2 = 0.994$, $n = 24$ for viscosity and $S = 0.135$ cSt, $\bar{D} = 10.2$ percent, $R^2 = 0.995$, $n = 24$ for kinematic viscosity.

Fig. 4—Viscosity and kinematic viscosity data from (12) for a naphthenic oil with high molecular weight polymer additive. The accuracy of these correlations is suggested by the statistics $S = 0.169$ cP, $\bar{D} = 10.6$ percent, $R^2 = 0.992$, $n = 25$ for viscosity and $S = 0.130$ cSt, $\bar{D} = 9.3$ percent, $R^2 = 0.995$, $n = 24$ for kinematic viscosity.

later developed a correlation valid at pressures to 1.0 GPa, but coefficients in this expression are difficult to determine using a regression analysis.

ASME viscosity data (12) provide an established basis for comparing Eq. [4] with these expressions. Only data obtained from ambient conditions to pressures of 350 MPa (50.7 ksi) and temperatures of 373 K (100°C) are used in this comparison to meet stated limits on Eqs. [8]–[10]. Coefficients and statistics for correlations based on Eq. [4] are given in Table 1. Coefficients in Eqs. [8] and [9] were obtained using a similar procedure after adopting values for θ_p, θ_T recommended by Cameron (14). Constants in Eq. [10] are given by Roelands (16) for most of the oils in Table 1. These correlations are compared in Table 2 where average and extreme values of percentage deviation are given for each method. Results for average deviation are summarized by classification in Table 3. In terms of percent deviation,

Eq. [4] is more accurate than the other correlations of ASME viscosity data.

A similar comparison is not given here for correlations of kinematic viscosity data. These correlations are usually specialized to predict isothermal pressure coefficients. For example, Fresco *et al.* (7) correlate viscosity data at 311 K (38°C) and 372 K (99°C) for several ASME fluids listed in Table 1. These correlations are used to predict isothermal kinematic viscosity data at a specific pressure. These predictions are compared with results obtained using viscosity correlations proposed by Kouzel (6), Roelands *et al.* (8), Cameron and Chu (18) and Wooster (19). In this case, Eq. [5] gives results that are comparable to the best prediction for each oil.

CONCLUSIONS

Expressions are developed to correlate viscosity and ki-

TABLE 1—COEFFICIENTS AND STATISTICS FOR VISCOSITY CORRELATIONS OF ASME DATA (12) BASED ON EQ. [4].

ASME CODE	<i>n</i>	η_0 (cP)	A_1	A_2	A_3	A_4	A_5	<i>S</i> (cP)	R^2
1-A	15	17.88	$(1.48)10^{-3}$	11.78	$-(7.7)10^{-8}$	14.31	$(2.17)10^{-3}$	0.028	0.999
24-E	17	52.24	$(2.25)10^{-3}$	16.99	$-(1.0)10^{-7}$	25.37	$(3.71)10^{-3}$	0.068	0.999
25-E	15	11.76	$(2.20)10^{-3}$	13.31	$-(7.1)10^{-8}$	21.03	$(4.33)10^{-3}$	0.047	0.999
26-E	15	48.30	$(2.18)10^{-3}$	13.26	$-(9.1)10^{-8}$	17.96	$(3.93)10^{-3}$	0.020	0.999
27-E	15	47.17	$(2.13)10^{-3}$	12.98	$-(7.1)10^{-8}$	17.81	$(3.94)10^{-3}$	0.026	0.999
32-G	15	341.9	$(2.43)10^{-3}$	21.45	$-(1.1)10^{-7}$	24.88	$(3.83)10^{-3}$	0.048	0.999
33-G	15	1318	$(2.91)10^{-3}$	24.26	$-(1.6)10^{-7}$	25.26	$(4.94)10^{-3}$	0.123	0.999
34-G	15	360.1	$(2.47)10^{-3}$	20.86	$-(1.2)10^{-7}$	21.93	$(3.91)10^{-3}$	0.039	0.999
35-G	15	308.0	$(2.55)10^{-3}$	20.19	$-(5.7)10^{-8}$	19.87	$(5.06)10^{-3}$	0.026	0.999
36-G	17	111.7	$(3.16)10^{-3}$	22.07	$-(5.4)10^{-8}$	32.81	$(7.07)10^{-3}$	0.054	0.999
37-G	15	366.5	$(3.65)10^{-3}$	26.12	$-(3.5)10^{-8}$	35.89	$(8.43)10^{-3}$	0.023	0.999
38-G	15	1762	$(5.00)10^{-3}$	36.76	$-(3.8)10^{-9}$	66.75	$(1.88)10^{-2}$	0.068	0.999
39-G	15	381.0	$(3.78)10^{-3}$	26.58	$-(2.3)10^{-8}$	37.23	$(8.91)10^{-3}$	0.023	0.999
40-G	15	363.4	$(3.61)10^{-3}$	26.14	$-(2.4)10^{-8}$	37.14	$(8.36)10^{-3}$	0.027	0.999
41-G	15	309.5	$(2.41)10^{-3}$	20.31	$-(1.3)10^{-7}$	21.07	$(3.65)10^{-3}$	0.037	0.999

Each set of coefficients is obtained using a regression analysis of data at pressures to 350 MPa (50.7 ksi) and temperatures to 373 K (100°C). Ambient conditions $p_0 = 0.101$ MPa (14.7 psi) and $T_0 = 298$ K (24.9°C) are used as the reference state for these correlations.

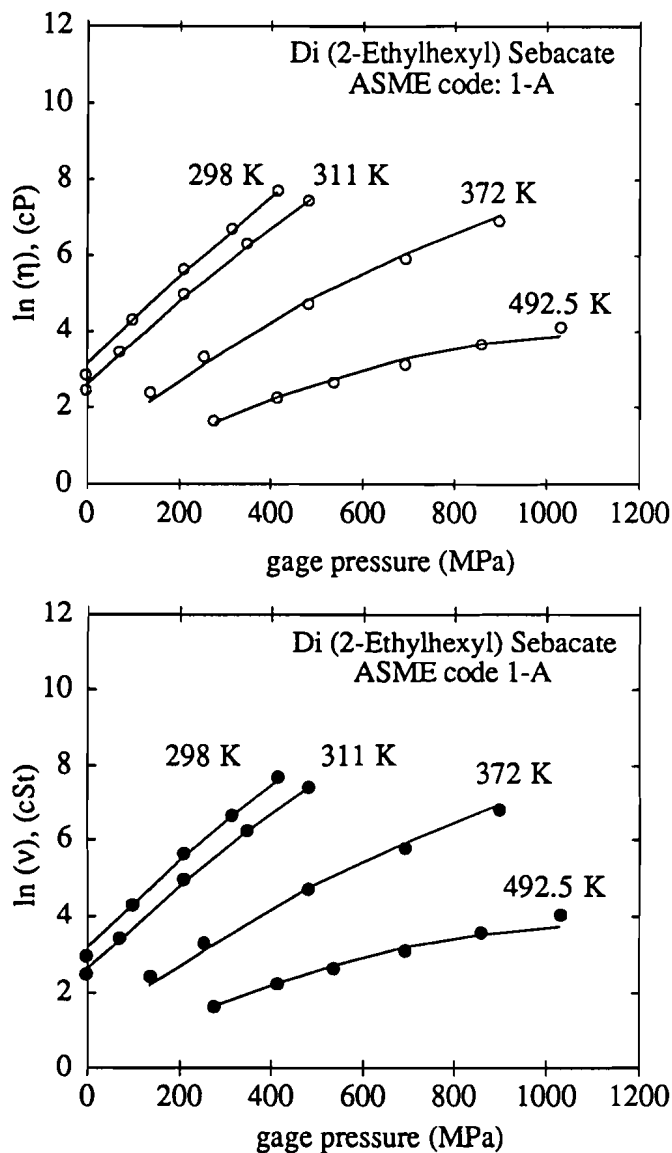


Fig. 5—Viscosity and kinematic viscosity data from (12) for a synthetic lubricant. The accuracy of these correlations is suggested by the statistics $S = 0.158$ cP, $\bar{D} = 10.6$ percent, $R^2 = 0.995$, $n = 21$ viscosity and $S = 0.153$ cSt, $\bar{D} = 10.3$ percent, $R^2 = 0.995$, $n = 21$ for kinematic viscosity.

nematic viscosity data over a wide range of pressure and temperature. These expressions contain the special cases suggested by Eqs. [1] and [2] along with higher order terms that significantly extend the useful range of these equations. The procedure used in this development can be extended to include other products of pressure and temperature in expressions for viscosity and kinematic viscosity. For example, a term involving p^2/T is obtained by truncating at the next higher order in the series expansion. This term is needed to correlate viscosity data for several silicone oils described by Bridgman (20) and in the ASME report (12). These observations show an inflection point in plots of $\ln(\eta)$ as a function of pressure. While this behavior is not predicted by Eqs. [4] and [5], these equations could be systematically refined to correlate data for these fluids.

Even without such modifications, Eqs. [4] and [5] accurately describe viscosity and kinematic viscosity data from the ASME report (12). These equations were used to correlate data over two different ranges of pressure and temperature for all fluids listed in Table 1. These correlations predict measured values to within an average deviation of 15 percent over the range from ambient conditions to 1.0 GPa (150 ksi) and 491.5 K (218°C). This agreement is even better for pressures to 350 MPa (50.7 ksi) and temperatures to 373 K (100°C). In this range, deviations from ASME viscosity data are less than 5 percent on average, and within 10 percent for all observations. These statistics are consistent with the small standard error and the large correlation coefficient that characterize both ranges. This agreement is significant given the variety of fluids described by these correlations.

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TABLE 2—A COMPARISON OF DIFFERENT CORRELATIONS FOR ASME VISCOSITY DATA (12) AT PRESSURES TO 350 MPa (50.7 ksi) AND TEMPERATURES TO 373 K(100°C).

ASME CODE	EQ. [4]	EQ. [8] CAMERON (14)	EQ. [9] APPELDOORN (15)	EQ. [10] ROELANDS (16)
1-A	1.7 (3.6)	18.8 (−60.7)	18.9 (−62.7)	—
24-E	4.6 (9.3)	31.7 (−131)	34.5 (−140)	14.8 (−41.9)
25-E	2.4 (−6.3)	25.2 (−68.0)	17.3 (−74.8)	8.4 (−23.6)
26-E	1.2 (3.2)	16.9 (−44.0)	8.4 (−22.4)	—
27-E	1.5 (4.3)	18.6 (−67.5)	7.5 (−18.3)	—
32-G	3.0 (−6.2)	24.0 (−52.7)	12.4 (−22.0)	4.3 (13.6)
33-G	0.9 (−1.9)	13.9 (−30.4)	10.3 (16.2)	3.2 (8.0)
34-G	2.4 (4.3)	20.8 (48.6)	10.8(20.4)	2.9 (8.7)
35-G	1.5 (2.9)	20.0 (−44.4)	8.4 (−14.1)	2.9 (−5.8)
36-G	2.3 (−7.9)	38.4 (−110)	30.6 (−74.8)	8.4 (−23.6)
37-G	1.4 (−3.0)	23.8 (−79.1)	10.8 (−23.9)	6.4 (−14.9)
38-G	3.8 (8.6)	78.7 (−169)	78.2 (−156)	24.3 (64.5)
39-G	1.6 (−2.9)	22.8 (−84.4)	9.4 (−25.3)	9.2 (−25.0)
40-G	1.9 (3.9)	30.4 (−72.0)	11.2 (23.5)	8.9 (−29.5)
41-G	2.6 (5.0)	18.8 (−53.0)	10.8 (−23.7)	3.6 (−9.9)

The value of \bar{D} is given for each correlation along with the worst percentage deviation in parentheses.

TABLE 3—PERCENTAGE DEVIATION BY CLASSIFICATION FOR DIFFERENT CORRELATIONS OF ASME VISCOSITY DATA (12)

CLASSIFICATION	NO.	EQ. [4]	EQ. [8] CAMERON (14)	EQ. [9] APPELDOORN (15)	EQ. [10] ROELANDS (16)
Naphthenic	7	2.3	34.0	24.0	9.7
Paraffinic	5	2.5	22.1	15.3	5.6
Polymer blend	2	1.3	17.9	7.9	—
Synthetic	1	1.7	18.8	18.9	—

These results are obtained by averaging values of \bar{D} over all lubricants of the same type in Table 2.

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REFERENCES

- (1) Fuller, D. D., *Theory and Practice of Lubrication for Engineers*, John Wiley & Sons, New York (1984).
- (2) Watson, R. W. and McDonnell, T. F., "Additives—The Right Stuff for Automotive Engine Oils," SAE No. 841208 (1986).
- (3) Barnes, H. A., Hutton, J. F. and Walters, K., *An Introduction to Rheology*, Elsevier, New York (1989).
- (4) Dowson, D. and Higginson, G. R., *Elastohydrodynamic Lubrication*, Pergamon Press, Oxford (1977).
- (5) Barus, C., "Isothermals, Isopiestic and Isometrics Relative to Viscosity," *Amer. Jour. Sci.*, **45**, pp 87–96 (1983).
- (6) Kouzel, B., "How Pressure Effects Liquid Viscosity," *Hydrocarbon Proc. and Petroleum Refiner*, **44**, 3, pp 120 (1965).
- (7) Fresco, G. P., Klaus, E. E. and Tewksbury, E. J., "Measurement and Prediction of Viscosity-Pressure Characteristics of Liquids," *Trans. ASME Jour. Lubr. Tech.*, pp 451–458 (1969).
- (8) Roelands, C. J. A., Vlugter, J. C. and Waterman, H. I., "The Viscosity-Temperature-Pressure Relationship of Lubrication Oils and Its Correlation With Chemical Constitution," *Trans. ASME Jour. Basic Eng.*, **85**, pp 601–607 (1963).
- (9) Sargent, L. B., "Pressure-Viscosity Coefficients of Liquid Lubricants," *ASLE Trans.*, **26**, 1, pp 1–10 (1983).
- (10) Tanner, R. I., *Engineering Rheology*, Clarendon Press, Oxford (1985), p 348.
- (11) Spiegel, M. R., *Mathematical Handbook of Formulas and Tables*, McGraw-Hill Book Company, New York (1968).
- (12) ASME, *Pressure-Viscosity Report*, ASME, New York (1953).
- (13) Hersey, M. D. and Hopkins, R. F., *Viscosity of Lubricants Under Pressure: Coordinated Data from Twelve Investigations*, ASME, New York (1954).
- (14) Cameron, A., "Determination of the Pressure-Viscosity Coefficient and Molecular Weight of Lubricating Oils by Means of the Temperature-Viscosity Equations of Vogel and Eyring," *Jour. Inst. Petr.*, **31**, pp 401–414 (1945).
- (15) Appeldoorn, J. K., "A Simplified Viscosity-Pressure-Temperature Equation," SAE No. 709A (1963).
- (16) Roelands, C. J. A., *Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils*, Ph.D. Dissertation, University of Delft (1966).
- (17) Pywell, R. F., "The Improved Prediction of Lubricating Oil Viscosity at High Pressures," *The Rheology of Lubricants*, Davenport, T. C. ed., John Wiley & Sons, New York, pp 118–129 (1972).
- (18) Chu, P. S. Y. and Cameron, A., "Pressure Viscosity Characteristics of Lubricating Oils," *Jour. Inst. Petr.*, **48**, pp 147–155 (1962).
- (19) Bingham, A. E., "Some Problems of Fluids for Hydraulic Power Transmission," *Proc. Inst. Mech. Engr.*, **165**, p 269, discussion by Wooster, R. C., (1951).
- (20) Bridgman, P. W., "Viscosities to 30,000 kg/cm²," *Proc. Amer. Acad. Arts & Sci.*, **77**, 4, pp 117–128 (1949).
- (21) Kleinbaum, D. G., Kupper, L. L. and Muller, K. E., *Applied Regression Analysis and Other Multivariable Methods*, PWS-Kent Publishing Company, Boston (1988), p 111.

APPENDIX

A step-up multiple linear regression technique (21) determines coefficients in Eqs. [4] and [5] using a least squares

algorithm. This method also evaluates the statistical significance of each variable used in the correlation. Implementations of this technique are readily available in spreadsheet programs and statistical analysis packages for a variety of computers. This software determines dependent variables $\ln(\eta)$ and $\ln(\nu)$ as linear combinations of five independent variables which include temperature, pressure and their products. An error estimate for predicted values of the dependent variable is given by the expression

$$S \equiv \sqrt{\frac{1}{n-k} \sum_{i=1}^n \delta_i^2} , \quad [\text{A1}]$$

in terms of the difference $\delta_i \equiv y_i - Y_i$ between measured values y_i and predicted values Y_i where n data points and k independent variables are used in the analysis. The average value of percentage deviations

$$\bar{D} \equiv \frac{100}{n} \sum_{i=1}^n \left| \frac{\delta_i}{y_i} \right| , \quad [\text{A2}]$$

gives a dimensionless error estimate for predicted values of the dependent variable. The ability of the correlation to predict the dependent variable is estimated using the parameter

$$R^2 \equiv 1 - \frac{\sum_{i=1}^n \delta_i^2}{\sum_{i=1}^n (Y_i - \bar{y})^2} , \quad [\text{A3}]$$

where \bar{y} is the average of all experimental observations (21). These statistics provide a basis for comparing different correlations of the same data.