EE3025 Assignment-1

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Download all python codes from

```
https://github.com/yuvateja-ctrl/EE3025/tree/main/
    codes
```

and latex-tikz codes from

```
https://github.com/yuvateja-ctrl/EE3025
```

1 Digital Filter

1.1 Download the sound file from

```
wget https://raw.githubusercontent.com/gadepall/
EE1310/master/filter/codes/Sound_Noise.wav
```

1.2 Write the python code for removal of out of band noise and execute the code.
Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav')

#sampling frequency of Input signal
sampl_freq=fs

#order of the filter
order=4

#cutoff frquency 4kHz
cutoff_freq=4000.0

#digital frequency
Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and denominator
    polynomials respectively
b, a = signal.butter(order,Wn, 'low')

#filter the input signal with butterworth filter
output_signal = signal.filtfilt(b, a, input_signal)
#output_signal = signal.lfilter(b, a, input_signal)

#write the output signal into .wav file
sf.write('Sound_With_ReducedNoise.wav',
    output_signal, fs)

print(a)
print(b)
```

2 Difference equation

2.1 Write the difference equation of above Digital filter obtained in problem 1.2.
Solution:

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{2.0.1}$$

$$\begin{aligned} y(n) - 2.52y(n-1) + 2.56y(n-2) - 1.206y(n-3) \\ +0.22013y(n-4) = 0.00345x(n) + 0.0138x(n-1)+ \\ 0.020725x(n-2) + 0.0138x(n-3) + 0.00345x(n-4) \end{aligned} \tag{2.0.2}$$

2.2 Sketch x(n) and y(n).
Solution: The following code yields Fig 2.2

```
codes/inp_out.py
```

The filtered sound signal obtained through difference equation is found in

```
codes/Sound_de.wav
```

3 Z-transform

3.1

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{3.0.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{3.0.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{3.0.3}$$

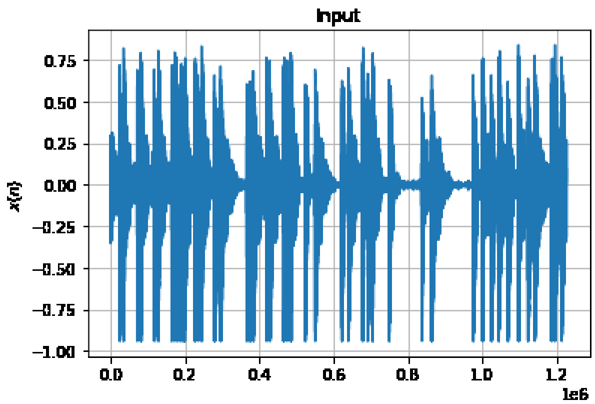

Fig. 0

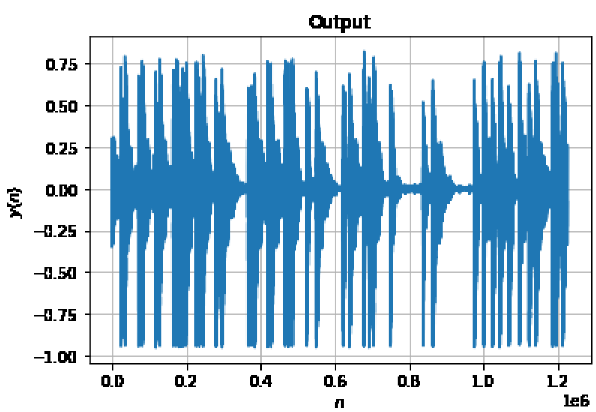

Fig. 0

Solution: From 3.0.1,

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{3.0.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{3.0.5}$$

resulting in 3.0.2. Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{3.0.6}$$

Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{3.0.7}$$

from 2.0.2 assuming that the Z-transform is a linear operation.

Solution: Applying 3.0.6 in 2.0.2 we get,

$$H(z) = \frac{Y(z)}{H(z)}$$
$$= \frac{b[0] + b[1]z^{-1} + b[2]z^{-2} + b[3]z^{-3} + b[4]z^{-4}}{a[0] + a[1]z^{-1} + a[2]z^{-2} + a[3]z^{-3} + a[4]z^{-4}} \tag{3.0.8}$$

Let

$$H\left(e^{\jmath w}\right) = H\left(z = e^{\jmath w}\right). \tag{3.0.9}$$

Plot $|H\left(e^{\jmath w}\right)|$.
Solution: The following code plots Fig 3.3.

```
codes/dtft.py
```

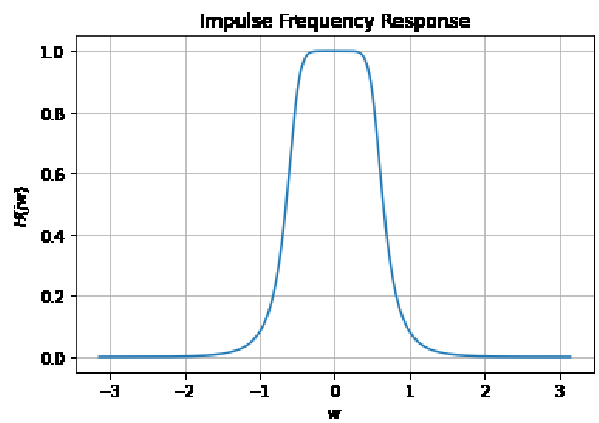

Fig. 0: $|H\left(e^{\jmath w}\right)|$

4 Impulse Response

4.1 From the difference equation eq. 2.0.2. Sketch h(n)
Solution: we know that output would be impulse response when the given input is impulse. From 2.0.1,
By substituting $x(n-k) = \delta(n-k)$, then $y(n-k)$ becomes $h(n-k)$ for all k=0,1,2,3,4. Now, the following code plots Fig 4.1

```
codes/impulseRes_def.py
```

4.2 Check whether h(n) obtained is stable.
Solution: The system is defined by the equation 2.0.2
By BIBO(Bounded Input Bounded Output) we know that if the input is bounded then the output should also be bounded.
Since x(n) is bounded, let B_x be some finite value

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |x(n-k)| \tag{4.0.1}$$

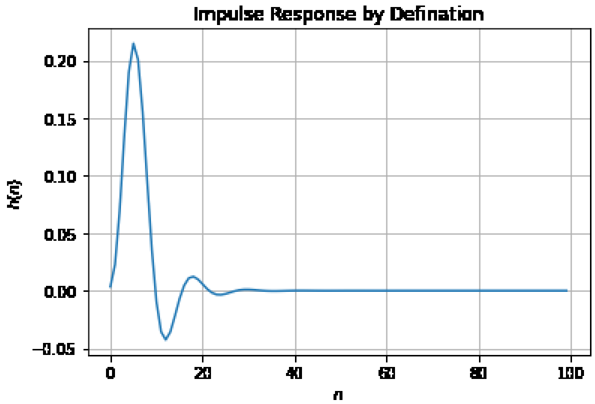

Fig. 0: $h(n)$

From convolution formula,

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \tag{4.0.2}$$

$$|y(n)| \leq \sum_{-\infty}^{\infty} |h(k)|\,|x(n-k)| \tag{4.0.3}$$

Let B_x be the maximum value x(n-k) can take, then

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |h(k)| \tag{4.0.4}$$

If

$$\sum_{-\infty}^{\infty} |h(k)| < \infty \tag{4.0.5}$$

Then

$$|y(n)| \leq B_y < \infty \tag{4.0.6}$$

Hence y(n) is bounded if both input x(n) and system tranfer function h(n) are bounded.
Here as audio input is also bounded we can say the system is said to be stable.

$$\sum_{n=-\infty}^{n=-\infty} |h(n)| < \infty \tag{4.0.7}$$

The above euation can be re written as,

$$\sum_{n=-\infty}^{n=-\infty} \left|h(n)z^{-n}\right|_{|z|=1} < \infty \tag{4.0.8}$$

$$\sum_{n=-\infty}^{n=-\infty} |h(n)| \left|z^{-n}\right|_{|z|=1} < \infty \tag{4.0.9}$$

From Triangle inequality,

$$\left|\sum_{n=-\infty}^{n=-\infty} h(n)z^{-n}\right|_{|z|=1} < \infty \tag{4.0.10}$$

$$\implies |H(n)|_{|z|=1} < \infty \tag{4.0.11}$$

Therefore, the Region of Convergence(ROC) should include the unit circle for the system to be stable. Since, h(n) is right sided the ROC is outside the outer most pole. From the equation 3.0.8 Poles of the given transfer equation is:

$$\begin{aligned} z(approx) = 0.69382 \pm 0.41i, \\ 0.56617835 \pm 0.134423 \end{aligned} \tag{4.0.12}$$

From the above poles, we can see that that the ROC of the system is $|z| > \sqrt{0.69382^2 + 0.41^2}$. From

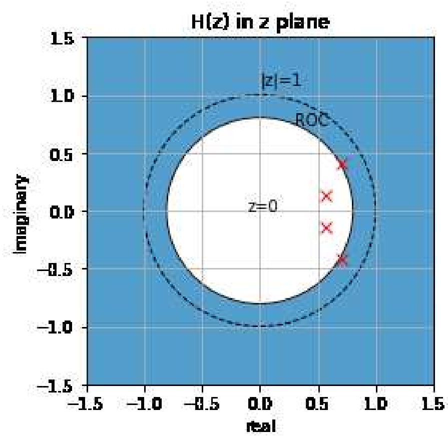

Fig. 0: ROC plot

the figure we can observe that ROC of the system includes unit circle $|z| = 1$. The code for plotting the figure is:

```
codes/roc.py
```

Which implies that the given IIR filter is stable, because h(n) is absolutely summable.And the given system is stable.
4.3 Compute Filtered output using convolution formula using h(n) obtained in 4.1

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \tag{4.0.13}$$

Solution: The following code plots Fig 4.3

```
/codes/out_conv.py
```

The filtered sound signal through convolution from this method is found in

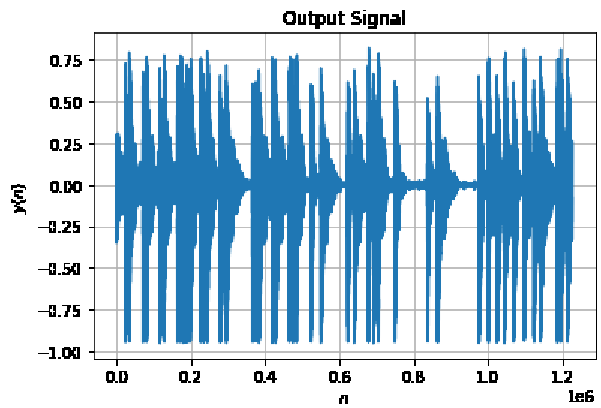

Fig. 0: $y(n)$ from the definition of convolution

```
codes/Sound_conv.wav
```

We can observe that the output obtained is same as y(n) obtained in Fig 2.2

5 FFT and IFFT

5.1 compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{5.0.1}$$

and $H(k)$ using h(n).

Solution: For this given IIR system with audio sample as x(n) and h(n) as impulse response h(n) obtained in 4.1 DFT of a Input Signal $x(n)$ is

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{5.0.2}$$

DFT of a Impulse Response $h(n)$ is

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{5.0.3}$$

The following code plots FFT of $x(n)$ and $h(n)$.

```
codes/xn_hn_fft.py
```

Magnitude and Phase plots obtained through above code is

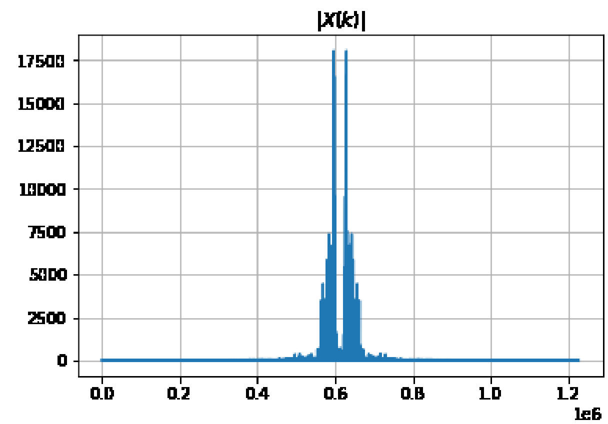

Fig. 0: Magnitude of X(k)

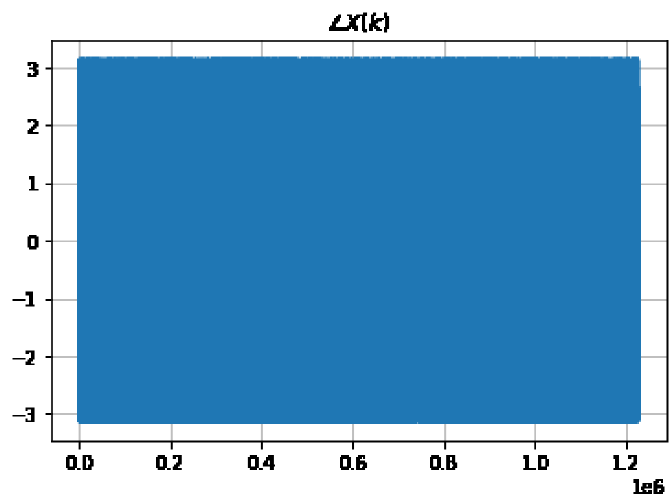

Fig. 0: Phase of X(k)

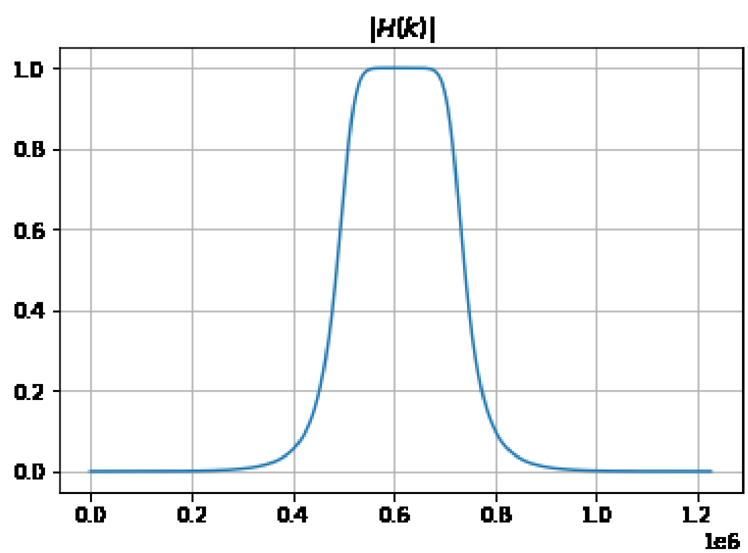

Fig. 0: Magnitude of H(k)

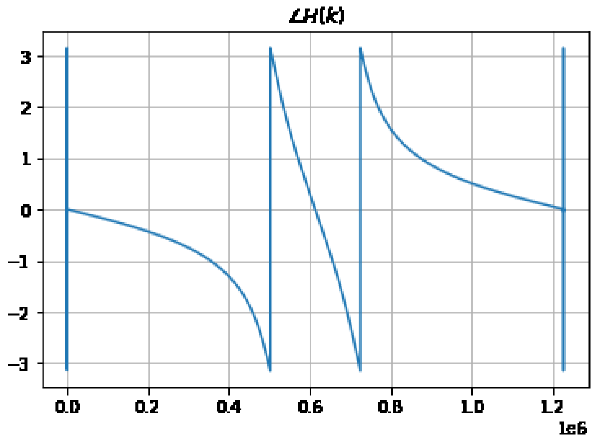

Fig. 0: Phase of H(k)

5.2 From

$$Y(k) = X(k)H(k) \tag{5.0.4}$$

Compute

$$y(n) \triangleq \sum_{k=0}^{N-1} Y(k)e^{j2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \tag{5.0.5}$$

Solution:
The following code plots Fig 5.2

codes/out_fft.py

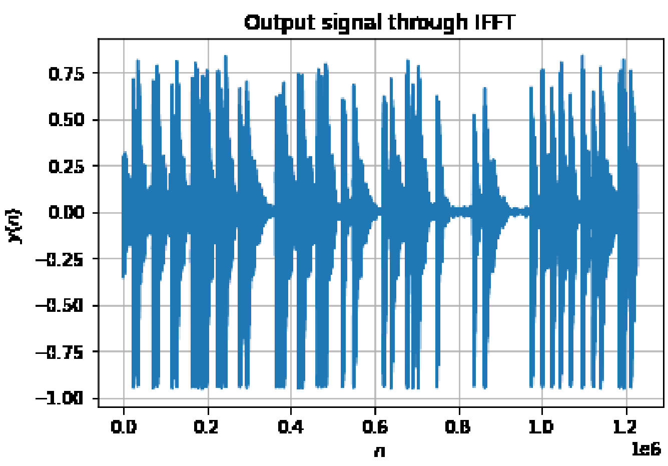

Fig. 0: $y(n)$

The filtered sound signal from this method is found in

codes/Sound_fft.wav

We can observe from the above plot that it is same as the y(n) observed in Fig 2.2