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Effect of supply voltage asymmetry on IRP *p*–*q*-based switching compensator control

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Abstract: Results of a study on how the supply voltage asymmetry affects the reference signal for switching compensator control, in a situation when this signal is generated using the instantaneous reactive power (IRP) *p*–*q* theory are presented. According to the IRP *p*–*q* approach, the compensator should compensate IRP and the alternating component of the instantaneous active power of the load. However, that in the presence of the supply voltage asymmetry, even an ideal, unity power factor load has an instantaneous active power with a non-zero alternating component is demonstrated. According to IRP *p*–*q* theory-based approach, it should be compensated and this requires that a distorted current be injected into the distribution system. It means that in the presence of the supply voltage asymmetry, the algorithms based on the IRP *p*–*q* theory generate a non-sinusoidal reference signal for the compensator control, even when voltages and currents in the system are sinusoidal.

1 Introduction

The well known and the most common basic structure of a shunt switching compensator (SSC), controlled in an open feedback loop, is shown in Fig. 1. It is composed of the pulse-width modulation (PWM) voltage source inverter with a capacitor C as the dc voltage source, line inductors L and data acquisition and digital signal processing (DA&DSP) system, which provides a reference signal for the compensator control.

This device is referred to in literature under a few names, as an 'active power filter, 'power conditioner' or an 'active harmonic filter'. This difference in names can be irrelevant for a person that works in the area, but probably not for a novice which will need time to realise that these different names refer to the same device. Such a novice will need time to comprehend that in spite of the adjective 'active' in the name, this is not a source of energy, but dissipates it, meaning it is not an active, but a passive device. Such a novice will need time to comprehend that this device does not eliminate unwanted components of the supply current by filtering, but by injection of a compensating current, meaning it is not a filter, but a compensator. These common names do not characterise the device adequately to its properties. Observe also that the phrase 'power filter' suggests filtering of power which does not have much sense. Moreover, the phrase 'harmonic filter' does not characterise adequately the field of the device applications. It can compensate not only harmonics, but also the reactive current. It can balance the load and can compensate non-harmonic deviations of the supply current from a sinusoidal waveform, such as single spikes or notches. Even more, the concept of 'harmonics' is not necessary for such devices control.

The most dominating feature of such a device is fast switching which enables shaping the compensating current waveform. Therefore such compensators are referred to as switching compensators in this paper, though a better name might be perhaps coined. Anyway, a discussion on selection of a proper name for such compensators is desirable.

A SSC needs a reference signal for its control. This reference signal should be next reproduced by the compensator as its input current

$$\boldsymbol{j} \triangleq \begin{bmatrix} j_{\mathrm{R}} \\ j_{\mathrm{S}} \\ j_{\mathrm{R}} \end{bmatrix} \tag{1}$$

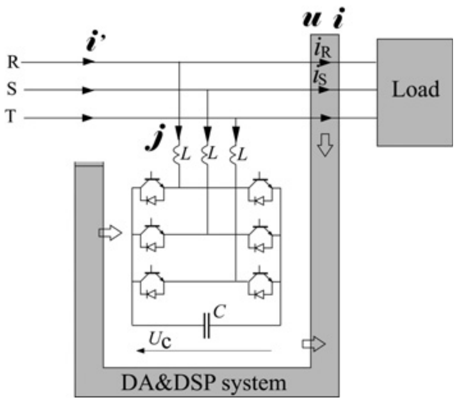

Figure 1 *SSC structure*

It requires that the component of load current that should be compensated is detected by analysis of the power properties of the compensated load.

One of the most common approaches to the reference signal generation for SSCs control is founded on the instantaneous reactive power (IRP) $p-q$ theory, developed by Nabae, Akagi and Kanazawa in 1983 [1].

Let us compile the main features of the IRP $p-q$ theory as needed in this paper when it is applied to a load supplied from a three-wire line, so that

$$i_{\mathrm{R}} + i_{\mathrm{S}} + i_{\mathrm{T}} \equiv 0 \tag{2}$$

at the assumption that

$$u_{\mathrm{R}} + u_{\mathrm{S}} + u_{\mathrm{T}} \equiv 0 \tag{3}$$

Since one current and one voltage are linear forms of the remaining two currents and voltages, such a load is described in terms of only two currents and two voltages, which can be arranged into vectors

$$\boldsymbol{i} \triangleq \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \end{bmatrix}, \quad \boldsymbol{u} \triangleq \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \end{bmatrix} \tag{4}$$

2 Compensation in terms of IRP $p-q$ theory

Power properties of electric loads are described with the IRP $p-q$ theory in terms of voltages and currents in Clarke coordinates, α, β and 0, meaning in terms of three orthogonal currents i_α, i_β and i_0.

Current i_0 in three-wire systems has zero value thus the system can be described in terms of the reduced vector of Clarke's currents as follows

$$\boldsymbol{i}_{\mathrm{C}} \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \end{bmatrix} \triangleq \boldsymbol{Ci} \tag{5}$$

Similarly defined is the reduced vector of Clarke's voltage:

$$\boldsymbol{u}_{\mathrm{C}} \triangleq \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \end{bmatrix} \triangleq \boldsymbol{Cu} \tag{6}$$

The reduced inverted Clarke transform matrix is equal to

$$\boldsymbol{C}^{-1} \triangleq \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix}^{-1} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \tag{7}$$

According to the IRP $p-q$ theory, the load properties are specified in terms of two instantaneous powers, active p and reactive q, defined as

$$p = u_\alpha i_\alpha + u_\beta i_\beta \tag{8}$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha \tag{9}$$

which can be written in the matrix form

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha, & u_\beta \\ -u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \triangleq \boldsymbol{U}_{\mathrm{C}} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{10}$$

This equation, when solved with respect to Clarke's currents i_α, i_β, has the form

$$\boldsymbol{i}_{\mathrm{C}} \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha, & -u_\beta \\ u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \triangleq \boldsymbol{U}_{\mathrm{C}}^{-1} \begin{bmatrix} p \\ q \end{bmatrix} \tag{11}$$

It means that the load currents in Clarke's coordinates α and β are determined by two instantaneous powers, that is the instantaneous active power, p and the IRP q.

Application of the IRP $p-q$ theory for compensation is based on the requirement [2–4] that the compensator should compensate the IRP, q, and the oscillating component of the instantaneous active power, $\tilde{p}$. This oscillating component can be filtered out from a signal proportional to the active power p with a high-pass filter.

The IRP $p-q$ theory-based approach to compensation is illustrated in Fig. 2.

The compensator current $\boldsymbol{j}$ should reproduce the reference signal $\boldsymbol{i}_{\mathrm{b}}$, generated in such a way that in the Clarke's coordinates

$$\boldsymbol{i}_{\mathrm{bC}} = \begin{bmatrix} (i_{\mathrm{b}})_\alpha \\ (i_{\mathrm{b}})_\beta \end{bmatrix} = \boldsymbol{U}_{\mathrm{C}}^{-1} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} \tag{12}$$

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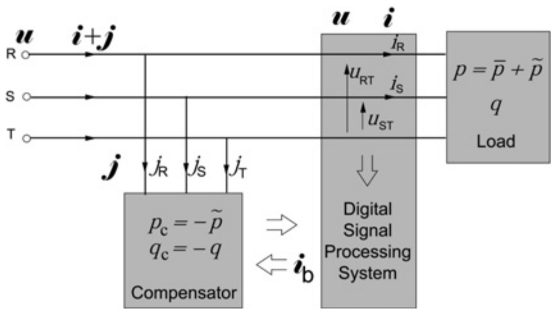

Figure 2 *Compensation according to IRP p–q theory*

Thus, after instantaneous powers $\tilde{p}$ and q of the load are calculated, the reference signal for the compensator is

$$\boldsymbol{i}_{\mathrm{b}} = \begin{bmatrix} z(i_{\mathrm{b}})_{\mathrm{R}} \\ (i_{\mathrm{b}})_{\mathrm{S}} \end{bmatrix} = \boldsymbol{C}^{-1}\boldsymbol{i}_{\mathrm{bC}} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \boldsymbol{U}_{\mathrm{C}}^{-1} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} \tag{13}$$

can be obtained.

Switches of the PWM inverter should be controlled such that the SSC input current $\boldsymbol{j}$ approximates the negative value of the unwanted component of the load current, meaning current $\boldsymbol{i}_{\mathrm{b}}$

$$\boldsymbol{j} = \boldsymbol{i}_{\mathrm{b}} \triangleq \begin{bmatrix} (i_{\mathrm{b}})_{\mathrm{R}} \\ (i_{\mathrm{b}})_{\mathrm{S}} \\ (i_{\mathrm{b}})_{\mathrm{T}} \end{bmatrix} \tag{14}$$

as accurately as possible.

There is awareness that the IRP $p-q$-based approach is not fully satisfactory. According to [5]: '... there are recognized limitations to this method including demonstrated poor performance in the presence of unbalance and voltage distortion ...' Consequently, other approaches have been developed. The synchronous reference frame (SRF) algorithm [6, 7] is one of them. There are also algorithms that stem from Fryze's power theory. The FDB method [8, 9] and the currents' physical components (CPC) [10–13] provide such algorithms. These algorithms were developed without clear explanation, however, why the IRP $p-q$-based approach may not fulfil expectations.

The concept of the IRP $p-q$ as a power theory was challenged in [14, 15]. It was shown that the IRP $p-q$ theory does not have properties as claimed in [16]. In particular, it cannot identify power properties of the load instantaneously. It does not identify power phenomena responsible for the power factor degradation, even in sinusoidal systems. It does not provide the physical nature of the IRP q.

Thus, a question occurs: can IRP $p-q$, being not founded on physical phenomena in electrical systems, provide reliable fundamentals for such systems compensation?

This question is answered in this paper by analysing how the reference signal for the SSC control is generated based on the IRP $p-q$ theory when the load current is asymmetrical, but due to distinctively different reasons:

(i) because the load is unbalanced,

(ii) because the supply voltage is asymmetrical,

while the load and the supply are idealised and identical with respect to all other features.

3 Compensation of unbalanced load at symmetrical voltage

Let us start with calculation of the reference signal for an SSC connected at terminals of an unbalanced, purely resistive load, with the equivalent circuit shown in Fig. 3, assuming that $Y_{\mathrm{RS}} = Y_{\mathrm{TR}} = 0$ and $Y_{\mathrm{ST}} = G$, while the distribution voltage is sinusoidal, symmetrical, of the positive sequence, and with the line voltage at terminal R equal to

$$u_{\mathrm{R}} = \sqrt{2}U \cos \omega_1 t \tag{15}$$

The reduced vector of Clarke's voltages is equal to

$$\boldsymbol{u}_{\mathrm{C}} \triangleq \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \boldsymbol{C}\begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \end{bmatrix} = \sqrt{3}\begin{bmatrix} U \cos \omega_1 t \\ U \sin \omega_1 t \end{bmatrix} \tag{16}$$

The vector of the line currents of an unbalanced resistive load is composed [12] of only the active and unbalanced currents

$$\boldsymbol{i} \triangleq \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \\ i_{\mathrm{T}} \end{bmatrix} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{u}} \tag{17}$$

The active current is proportional to the supply voltage and the load equivalent conductance

$$G_{\mathrm{e}} \triangleq \mathrm{Re}\{Y_{\mathrm{e}}\} \triangleq \mathrm{Re}\{Y_{\mathrm{RS}} + Y_{\mathrm{ST}} + Y_{\mathrm{TR}}\} = G \tag{18}$$

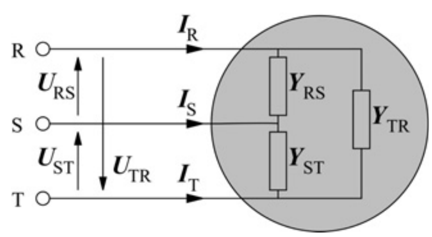

Figure 3 *Three-phase load*

hence

$$\boldsymbol{i}_{\mathrm{a}} \triangleq \begin{bmatrix} i_{\mathrm{Ra}} \\ i_{\mathrm{Sa}} \\ i_{\mathrm{Ta}} \end{bmatrix} = G_{\mathrm{e}}\boldsymbol{u} = G_{\mathrm{e}} \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \\ u_{\mathrm{T}} \end{bmatrix}$$

$$= G_{\mathrm{e}}\sqrt{2}\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} \mathrm{e}^{\mathrm{j}\omega_1 t} = G_{\mathrm{e}}\sqrt{2}\mathrm{Re}\{\boldsymbol{U}\mathrm{e}^{\mathrm{j}\omega_1 t}\} \quad (19)$$

where

$$\boldsymbol{U} \triangleq [\boldsymbol{U}_{\mathrm{R}}, \boldsymbol{U}_{\mathrm{S}}, \boldsymbol{U}_{\mathrm{T}}]^{\mathrm{T}} \quad (20)$$

In particular, the active current in phase R is

$$i_{\mathrm{Ra}} \triangleq i_{\mathrm{a}} = \sqrt{2}GU\cos\omega_1 t \quad (21)$$

The unbalanced current is proportional to the supply voltage and the load unbalanced admittance, equal to [12, 13]

$$\boldsymbol{A} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^*\boldsymbol{Y}_{\mathrm{RS}}) = -G \quad (22)$$

namely

$$\boldsymbol{i}_{\mathrm{u}} \triangleq \begin{bmatrix} i_{\mathrm{Ru}} \\ i_{\mathrm{Su}} \\ i_{\mathrm{Tu}} \end{bmatrix} = \sqrt{2}\mathrm{Re}\{\boldsymbol{A}\boldsymbol{U}^{\#}\mathrm{e}^{\mathrm{j}\omega_1 t}\} \quad (23)$$

where

$$\boldsymbol{U}^{\#} \triangleq [\boldsymbol{U}_{\mathrm{R}}, \boldsymbol{U}_{\mathrm{T}}, \boldsymbol{U}_{\mathrm{S}}]^{\mathrm{T}} \quad (24)$$

denotes the vector of line voltage complex rms (crms) values with switched $\boldsymbol{U}_{\mathrm{S}}$ and $\boldsymbol{U}_{\mathrm{T}}$ elements. Thus, the unbalanced current in line R is

$$i_{\mathrm{Ru}} \triangleq i_{\mathrm{u}} = -\sqrt{2}GU\cos\omega_1 t \quad (25)$$

The reduced vector of Clarke's currents is

$$\boldsymbol{i}_{\mathrm{C}} = \boldsymbol{C}\begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \end{bmatrix} = \boldsymbol{C}\sqrt{2}GU \times \begin{bmatrix} \cos\omega_1 t - \cos\omega_1 t \\ \cos(\omega_1 t - 120^\circ) - \cos(\omega_1 t + 120^\circ) \end{bmatrix} = 2\sqrt{3}GU\begin{bmatrix} 0 \\ \sin\omega_1 t \end{bmatrix} \quad (26)$$

The instantaneous active power of the load is

$$p = u_\alpha i_\alpha + u_\beta i_\beta = 3GU^2(1 - \cos 2\omega_1 t) = P - D\cos 2\omega_1 t \quad (27)$$

where D is the load unbalanced power, while the IRP q of the load at such supply is

$$q = u_\alpha i_\beta - u_\beta i_\alpha \quad (28)$$

According to the IRP p–q theory-based approach, the compensator should inject the current into the supply lines which in Clarke's coordinates is given by

$$\boldsymbol{j}_{\mathrm{C}} \triangleq \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \boldsymbol{U}_{\mathrm{C}}^{-1}\begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} = \boldsymbol{U}_{\mathrm{C}}^{-1}\begin{bmatrix} D\cos 2\omega_1 t \\ -D\sin 2\omega_1 t \end{bmatrix} \quad (29)$$

or in a more explicit form

$$\begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \frac{D}{u_\alpha^2 + u_\beta^2}\begin{bmatrix} u_\alpha, & -u_\beta \\ u_\beta, & u_\alpha \end{bmatrix}\begin{bmatrix} \cos 2\omega_1 t \\ -\sin 2\omega_1 t \end{bmatrix} = \sqrt{3}I_{\mathrm{u}}\begin{bmatrix} \cos\omega_1 t \\ -\sin\omega_1 t \end{bmatrix} \quad (30)$$

The compensator current

$$\boldsymbol{j} \triangleq \begin{bmatrix} j_{\mathrm{R}} \\ j_{\mathrm{S}} \end{bmatrix} = \boldsymbol{C}^{-1}\boldsymbol{j}_{\mathrm{C}} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \sqrt{3}I_{\mathrm{u}}\begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \cos\omega_1 t \\ -\sin\omega_1 t \end{bmatrix} = \sqrt{2}I_{\mathrm{u}}\begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t + 120^0) \end{bmatrix} \quad (31)$$

Thus, the compensator indeed injects a negative unbalanced current into the supply lines which compensates the unbalanced current of the load. It means that at load current asymmetry caused by the load imbalance, the IRP p–q-based approach to the reference signal generation meets expectations.

4 Compensation of balanced load at asymmetrical voltage

Now, let us investigate how the reference signals are affected by the supply voltage asymmetry. It is reasonable to 'clean up' the load for this purpose from all other causes of the power factor degradation. Therefore it is assumed that the load is purely resistive, linear and balanced, meaning such as shown in Fig. 4.

The supply voltage is assumed to be sinusoidal, but asymmetrical, meaning it is composed of positive and negative sequence components. To avoid confusion with lower indices used for denoting harmonic order, n, upper indices are used for denoting quantities of the positive and

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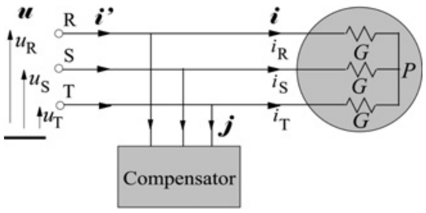

Figure 4 *Balanced resistive load with compensator*

negative sequences, 'p' and 'n', namely

$$\boldsymbol{u} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} = \begin{bmatrix} u_{\mathrm{R}}^{\mathrm{p}} \\ u_{\mathrm{S}}^{\mathrm{p}} \\ u_{\mathrm{T}}^{\mathrm{p}} \end{bmatrix} + \begin{bmatrix} u_{\mathrm{R}}^{\mathrm{n}} \\ u_{\mathrm{S}}^{\mathrm{n}} \\ u_{\mathrm{T}}^{\mathrm{p}} \end{bmatrix} \tag{32}$$

with, for the sake of simplicity

$$u_{\mathrm{R}}^{\mathrm{p}} \triangleq \sqrt{2}U^{\mathrm{p}} \cos \omega_1 t, \quad u_{\mathrm{R}}^{\mathrm{n}} \triangleq \sqrt{2}U^{\mathrm{n}} \cos \omega_1 t \tag{33}$$

Such a balanced resistive load is the best-possible load and, of course, it does not require any compensation, but this is only a 'cleaned up' load.

The following results are obtained when the IRP $p-q$ theory is applied for such load compensation. The reduced vector of Clarke's voltages is

$$\begin{aligned} \boldsymbol{u}_{\mathrm{C}} &\triangleq \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{S}} \end{bmatrix} \\ &= \sqrt{2}\boldsymbol{C} \begin{bmatrix} U^{\mathrm{p}} \cos \omega_1 t + U^{\mathrm{n}} \cos \omega_1 t \\ U^{\mathrm{p}} \cos(\omega_1 t - 120^\circ) + U^{\mathrm{n}} \cos(\omega_1 t + 120^\circ) \end{bmatrix} \\ &= \sqrt{3} \begin{bmatrix} (U^{\mathrm{p}} + U^{\mathrm{n}}) \cos \omega_1 t \\ (U^{\mathrm{p}} - U^{\mathrm{n}}) \sin \omega_1 t \end{bmatrix} \end{aligned} \tag{34}$$

The supply current for the load considered is

$$\boldsymbol{i} = G\boldsymbol{u} = G\boldsymbol{u}^{\mathrm{p}} + G\boldsymbol{u}^{\mathrm{n}} \tag{35}$$

thus, the reduced vector of Clarke's currents is

$$\boldsymbol{i}_{\mathrm{C}} \triangleq \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \boldsymbol{C} \begin{bmatrix} i_{\mathrm{R}} \\ i_{\mathrm{S}} \end{bmatrix} = \sqrt{3}G \begin{bmatrix} (U^{\mathrm{p}} + U^{\mathrm{n}}) \cos \omega_1 t \\ (U^{\mathrm{p}} - U^{\mathrm{n}}) \sin \omega_1 t \end{bmatrix} \tag{36}$$

The instantaneous active power of the load at such asymmetrical supply is

$$\begin{aligned} p &= u_\alpha i_\alpha + u_\beta i_\beta \\ &= 3G[U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2U^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t] \end{aligned} \tag{37}$$

Thus, the instantaneous active power p of balanced resistive loads supplied with asymmetrical voltage is not constant, but changes around its mean value with the alternating component equal to

$$\tilde{p} = 6GU^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t \tag{38}$$

The IRP q of such loads is equal to zero.

The alternating component of the instantaneous active power p is non-zero, thus, according to the IRP $p-q$ theory-based approach the compensator should compensate it with the current which in Clarke's coordinates is given by

$$\begin{aligned} \boldsymbol{j}_{\mathrm{C}} &\triangleq \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \boldsymbol{U}_{\mathrm{C}}^{-1} \begin{bmatrix} -\tilde{p} \\ -q \end{bmatrix} \\ &= \boldsymbol{U}_{\mathrm{C}}^{-1} \begin{bmatrix} -6GU^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t \\ 0 \end{bmatrix} \end{aligned} \tag{39}$$

or in a more explicit form

$$\begin{aligned} \begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} &= \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha, & -u_\beta \\ u_\beta, & u_\alpha \end{bmatrix} \begin{bmatrix} -6GU^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t \\ 0 \end{bmatrix} \\ &= \frac{-6GU^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \\ &= \frac{-6\sqrt{3}GU^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} (U^{\mathrm{p}} + U^{\mathrm{n}}) \cos \omega_1 t \\ (U^{\mathrm{p}} - U^{\mathrm{n}}) \sin \omega_1 t \end{bmatrix} \end{aligned} \tag{40}$$

Taking into account that the denominator in the last formula is equal to

$$\begin{aligned} u_\alpha^2 + u_\beta^2 &= 3(U^{\mathrm{p}} + U^{\mathrm{n}})^2 \cos^2 \omega_1 t + 3(U^{\mathrm{p}} - U^{\mathrm{n}})^2 \sin^2 \omega_1 t \\ &= 3[U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2U^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t] \end{aligned} \tag{41}$$

meaning it changes in time, the reference signal, and consequently, the compensator current is periodic, but rather complex. In particular, the compensator line R current is

$$\begin{aligned} j_{\mathrm{R}} &= \sqrt{\frac{2}{3}} j_\alpha \\ &= \frac{-2\sqrt{2}G(U^{\mathrm{p}} + U^{\mathrm{n}})U^{\mathrm{p}} U^{\mathrm{n}} \cos \omega_1 t \cos 2\omega_1 t}{U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2U^{\mathrm{p}} U^{\mathrm{n}} \cos 2\omega_1 t} \end{aligned} \tag{42}$$

Thus, in spite of sinusoidal supply voltages and load currents, the compensator currents are not sinusoidal. It means that the IRP $p-q$ theory-based algorithm generates erroneous reference signals for the compensator control. Compensator currents reproduce these erroneous reference signals and cause the supply current distortion.

This erroneous signal is generated in spite of the load ideal properties and consequently, unity power factor, because the instantaneous active power p, in a presence of the supply voltage asymmetry has a non-zero oscillating component. An SSC, controlled according to the IRP $p-q$ theory, attempts to compensate it and in effect, an undesirable reference signal is generated.

One could ask a question, however: 'does this component occurs because of the IRP $p-q$ theory properties, or does the instantaneous active power p at asymmetrical supply voltage indeed have such a component?'

To answer this question let us calculate the instantaneous active power, or simply, the instantaneous power $p(t)$, in such a situation without Clarke's transform. For such a balanced load with the phase conductance G, supplied with asymmetrical voltage, the instantaneous power, that is the rate of energy W flow between the load and the supply source, is equal to

$$\begin{aligned} p &\triangleq \frac{\mathrm{d}W}{\mathrm{d}t} = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}G\boldsymbol{u} = G[\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}]^{\mathrm{T}}[\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}] \\ &= G[\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{p}}] \end{aligned} \tag{43}$$

The first two terms are constant components of the instantaneous power

$$G\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{p}} = G\|\boldsymbol{u}^{\mathrm{p}}\|^2 \triangleq P^{\mathrm{p}} \tag{44}$$

$$G\boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{n}} = G\|\boldsymbol{u}^{\mathrm{n}}\|^2 \triangleq P^{\mathrm{n}} \tag{45}$$

where P^{p} and P^{n} are active powers of the positive and negative sequence voltages. The last term can be rearranged as follows

$$\begin{aligned} G(\boldsymbol{u}^{\mathrm{pT}}\boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{nT}}\boldsymbol{u}^{\mathrm{p}}) &= G(u^{\mathrm{p}}_{\mathrm{R}}u^{\mathrm{n}}_{\mathrm{R}} + u^{\mathrm{p}}_{\mathrm{S}}u^{\mathrm{n}}_{\mathrm{S}} + u^{\mathrm{p}}_{\mathrm{T}}u^{\mathrm{n}}_{\mathrm{T}}) \\ &\quad + G(u^{\mathrm{n}}_{\mathrm{R}}u^{\mathrm{p}}_{\mathrm{R}} + u^{\mathrm{n}}_{\mathrm{S}}u^{\mathrm{p}}_{\mathrm{S}} + u^{\mathrm{n}}_{\mathrm{T}}u^{\mathrm{p}}_{\mathrm{T}}) \\ &= 2G(u^{\mathrm{p}}_{\mathrm{R}}u^{\mathrm{n}}_{\mathrm{R}} + u^{\mathrm{p}}_{\mathrm{S}}u^{\mathrm{n}}_{\mathrm{S}} + u^{\mathrm{p}}_{\mathrm{T}}u^{\mathrm{n}}_{\mathrm{T}}) \\ &= 4GU^{\mathrm{p}}U^{\mathrm{n}}[\cos\omega_1 t \times \cos\omega_1 t \\ &\quad + \cos(\omega_1 t - 120^\circ) \times \cos(\omega_1 t + 120^\circ) \\ &\quad + \cos(\omega_1 t + 120^\circ) \times \cos(\omega_1 t - 120^\circ)] \\ &= 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t \end{aligned} \tag{46}$$

Consequently

$$p = \frac{\mathrm{d}W}{\mathrm{d}t} = P^{\mathrm{p}} + P^{\mathrm{n}} + 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t \tag{47}$$

Thus, indeed, the supply voltage asymmetry causes oscillations of the instantaneous active power p. When the constant value of the instantaneous active power p is the objective of compensation, then in the presence of the supply voltage asymmetry, an undesirable reference signal has to be generated.

Such oscillations can be caused not only by the supply voltage asymmetry, but also by the supply voltage harmonics.

Let us assume that the symmetrical supply voltage contains the fifth-order harmonic, thus

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5 = \begin{bmatrix} u_{1\mathrm{R}} \\ u_{1\mathrm{S}} \\ u_{1\mathrm{S}} \end{bmatrix} + \begin{bmatrix} u_{5\mathrm{R}} \\ u_{5\mathrm{S}} \\ u_{5\mathrm{S}} \end{bmatrix} \tag{48}$$

while the load is balanced and resistive with the phase conductance G, thus

$$\boldsymbol{i} = G\boldsymbol{u} = G(\boldsymbol{u}_1 + \boldsymbol{u}_5) \tag{49}$$

The instantaneous active power of the load is

$$\begin{aligned} p &\triangleq \frac{\mathrm{d}W}{\mathrm{d}t} = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{i} = \boldsymbol{u}^{\mathrm{T}}G\boldsymbol{u} = G(\boldsymbol{u}_1 + \boldsymbol{u}_5)^{\mathrm{T}}(\boldsymbol{u}_1 + \boldsymbol{u}_5) \\ &= G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_1) \end{aligned} \tag{50}$$

The first two terms are constant components of the instantaneous power

$$G\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_1 = G\|\boldsymbol{u}_1\|^2 \triangleq P_1 \tag{51}$$

$$G\boldsymbol{u}_5^{\mathrm{T}}\boldsymbol{u}_5 = G\|\boldsymbol{u}_5\|^2 \triangleq P_5 \tag{52}$$

where P_1 and P_5 are harmonic active powers of the fundamental and the fifth-order harmonics. The last term

$$\begin{aligned} G(\boldsymbol{u}_1^{\mathrm{T}}\boldsymbol{u}_5 + \boldsymbol{u}_5^{\mathrm{T}}u_1) &= G(u_{1\mathrm{R}}u_{5\mathrm{R}} + u_{1\mathrm{S}}u_{5\mathrm{S}} + u_{1\mathrm{T}}u_{5\mathrm{T}}) \\ &\quad + G(u_{5\mathrm{R}}u_{1\mathrm{R}} + u_{5\mathrm{S}}u_{1\mathrm{S}} + u_{5\mathrm{T}}u_{1\mathrm{T}}) \\ &= 2G(u_{1\mathrm{R}}u_{5\mathrm{R}} + u_{1\mathrm{S}}u_{5\mathrm{S}} + u_{1\mathrm{T}}u_{5\mathrm{T}}) \\ &= 4GU_1U_5[\cos\omega_1 t \times \cos 5\omega_1 t \\ &\quad + \cos(\omega_1 t - 120^\circ) \times \cos(5\omega_1 t + 120^\circ) \\ &\quad + \cos(\omega_1 t + 120^\circ) \times \cos(5\omega_1 t - 120^\circ)] \\ &= 6GU_1U_5\cos 6\omega_1 t \end{aligned} \tag{53}$$

Eventually, the instantaneous power of the load is

$$p(t) = \frac{\mathrm{d}W}{\mathrm{d}t} = P_1 + P_5 + 6GU_1U_5\cos 6\omega_1 t \tag{54}$$

meaning it has an oscillating component. The same was concluded in [17] using the FDB method. A compensator that will attempt to compensate this component will inject a distorted current into the distribution system. The IRP $p-q$ theory-based algorithm of the reference signal generation will provide an undesirable signal for the compensator control.

There are reported observations [5, 18] that the $p-q$ theory-based control algorithm works properly only at sinusoidal voltage. Unfortunately, a great majority of papers on the $p-q$ theory-based compensation assumes that zero reactive and constant instantaneous active power is the goal of compensation.

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5 Conclusions

The presented analysis demonstrates that if the constant instantaneous active power, p, is the control objective of a compensator, then in the presence of the supply voltage asymmetry, the compensator is a source of harmonic distortion of the supply current.

When the supply voltage is asymmetrical then the instantaneous power p of even an ideal, balanced resistive load has an oscillating component. This is the case even under sinusoidal conditions. Compensators controlled with IRP $p-q$ theory-based algorithm compensate this oscillating component by injection of current harmonics into the distribution system.

Since compensators should not cause supply current distortion, the constant instantaneous active power p is not an appropriate control objective when the supply voltage is asymmetrical.

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