* Num^l An^s

o Set-1 : Err^{rs}

• Rel^{ve} Err^r = $\left|\frac{\text{True} - \text{Approx}}{\text{True}}\right| = \left|\frac{\text{Err}^r}{\text{True}}\right|$. Oth^r trivia from Kani's.

o Set-2 (Fin^g Roots of Equs)

• Bisec^n Me^d : 1. Find $x_0 = a$, $x_1 = b$ s.t. $f(a)\cdot f(b) < 0$ 2. Next approx^n : $x_2 = \frac{a+b}{2}$

3. Check $f(x_2)$. Bas^d on sign, s.t. $f(x_2)\cdot f(a)$ or $f(x_2)\cdot f(b) < 0$, choose next int^l. Repeat it

→ Alw^s check cty of f in the ini^l int^l bef^{re} begi^g → Giv^s only 1 root in 1 go, in an int^l. Ch^{ge} int^l for more

• Regu^a Fal^i Me^d (Fal^e Pos^n) : 1. Choose x_0, x_1 s.t. $f(x_0)\cdot f(x_1) < 0$ 2. $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

3. If $f(x_0) f(x_2) < 0$, take x_2 as x_1, else take x_2 as x_0. (I think $x_0 < x_1$ alw^s)

→ Solve in Calc^r, then choose int^l → CALC- $C = f(A) : D = f(B) : X = \frac{AD - BC}{D - C} : f(X)$ } → We still need to find some x_0, x_1. i.e. A & B

• Sec^t Me^d (Mod^d RF) : 1. Ch^{se} x_0, x_1 as any 2 pts. May not con^n root.

2. Just do $x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$. Note, as you move forw^d, $x_0, x_1, \ldots$ are leav^g beh^d. Just keep mov^g ahead

$f(x)$ if req^d → See this has to come 1st & then B = X ! Rem^r

→ CALC :- $C = f(A) : D = f(B) : X = \frac{AD - BC}{D - C}$: A = B : B = X. Keep not^g X only.

↔ Can use tab^r repr^n for all the above. Even if not, chill! → Then NR form^a ⇒ $x_{n+1} = x_n - \frac{m f(x_n)}{f'(x_n)}$

If giv^n $x = \sqrt{2}$ is a root of mul^y m in $x^4 - 4x^2 + 4 = 0$ ... Also men^n it) — Calc :- $C = f(A) : D = f'(A) : X = A - \frac{C}{D} : A = X$. Put A_0. Put $f(x)$ also

• New^n Raph^n Me^d (Use only if $f(x)$ & $f'(x)$ are cts. Also men^n it) (Get this one by Calc, then choose nearby)

1. Find only 1 x_0 close eno^{gh} to the root (Not x_0, x_1 like ab^{ve}. Get this one by Calc, then choose nearby)

→ Check that $f'(x_0) \neq 0$. 2. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ [Pf : Let actual root $\alpha = x_0 + h$, then $f(\alpha) = f(x_0 + h) = 0$. By Tay^r Ser, $f(x_0 + h) = f(x_0) + h f'(x_0) + \ldots = 0$, ∴ $h \approx -\frac{f(x_0)}{f'(x_0)}$]

→ For find^g $\sqrt{N}$ (N>0). Consi^r $x^2 - N = 0$. ∴ Iter^n : $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$

→ Oth^r It^n Form^a : 1. $\frac{1}{N}$: $x_{n+1} = x_n(2 - N x_n)$ [Take $f(x) = \frac{1}{x} - N$] 2. $\frac{1}{\sqrt{N}}$: $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{N x_n}\right)$ [$f(x) = \frac{1}{x^2} - \frac{1}{N}$]

3. $\sqrt[k]{N}$: $x_{n+1} = \frac{1}{k}\left[(k-1)x_n + \frac{N}{x_n^{k-1}}\right]$ {$f(x) = x^k - N$}   4. $\frac{1}{\sqrt[p]{N}}$: $x_{n+1} = \frac{x_n(p + 1 - N x_n^p)}{p}$ {$f(x) = x^{-p} - N$}

→ Very fast me^d → Fails if ① $f'(x_i) = 0$ in a nbd of the root. ② $f'(\alpha) = 0$ at the root. This means 2 th^{gs} → 2 th^{gs}

2.1 $f(x)$ has mul^{le} roots at α 2.2 $f(x)$ has stat^y pt. at α ③ f, f' must be def^d & cts.

• Cgnce Crit^n : $\{x_n\}_0^\infty$ cgs to root α w. ord^r p, if $|x_{n+1} - \alpha| \leq \lambda |x_n - \alpha|^p$ f.s. $\lambda > 0$ — p : Or^r of Cgnce, λ : Asymp^c Err^r Const

→ If $\varepsilon_n = x_n - \alpha$, we need $|\varepsilon_{n+1}| \leq \lambda |\varepsilon_n|^p$. For p = 1, we req^{re} $\lambda < 1$ → Lin^r Cgnce.

→ If $|x_{n+1} - \alpha| \leq \lambda |x_n - \alpha|$ ∀ n ≥ 0 or if $|x_n - \alpha| \leq \lambda^n |x_0 - \alpha|$, we get Lin^r Cgnce.

• Cgnce of Bis^n Me^d : ∀ n, let $c_n = \frac{a_n + b_n}{2}$, ∴ $|c_{n+1} - \alpha| \leq \frac{|c_n - \alpha|}{2}$, ∴ Bis^m Me^d is alw^s Lin^r Cgt.

• Cgnce of Sec^t Me^d : Or^r of cgnce : $p = \frac{1 + \sqrt{5}}{2} \approx 1.618$; $\lambda = \left[\frac{f''(\alpha)}{2 f'(\alpha)}\right]^{p/(1+p)}$ → 0.618 {Pf by Tay^r's Ser^s expan^n $f(\varepsilon_i + \alpha) = \ldots$}

→ $\varepsilon_{i+1} \approx \varepsilon_i \varepsilon_{i-1}\left[\frac{f''(\alpha)}{2 f'(\alpha)}\right]$ → Rel^t b/w err^{rs} is "Err^r Eq^n". Holds only if α is a Simple Root. But see 2019 - T^t - 12 - Q 5 (c)

• Cgnce of New^n Raph^n : $p = 2$, $\lambda = \frac{f''(\alpha)}{2 f'(\alpha)}$, $\varepsilon_{i+1} = \left[\frac{f''(\alpha)}{2 f'(\alpha)}\right] \varepsilon_i^2$ → If α is a mul^{le} root i.e. $f'(\alpha) = 0$, cgnce is no lon^r quad^c, but lin^r

• Cgnce of Regu^a Fal^i : $\varepsilon_{i+1} = \left[\frac{f''(\alpha)}{2 f'(\alpha)}\right](x_0 - \alpha)\varepsilon_i$ {* Check p & λ !!!} ≡ $\varepsilon_i \varepsilon_0 \left[\frac{f''(\alpha)}{2 f'(\alpha)}\right]$ → ErrEq for p & λ, we need ε_0 in t. of ε_i

→ Us^d in Iter^n Me^d //

Thm : If $\phi(x)$ is cts in some [a, b] that contains the root & $|\phi'(x)| \leq c < 1$ in [a, b]. Then, ∀ $x_0 \in [a, b]$ the $\langle x_k \rangle$ given by $x_{k+1} = \phi(x_k)$, k = 0, 1, 2, ... cgs to the root α of "$x = \phi(x)$"

→ $|\phi'(x)| \leq c < 1$; Same as "Lipschitz Cond^n". C = Lip'z Const.

Chebyshev's Me^d ; 3rd or^r // $\phi(x) = x - \frac{f(x)}{f'(x)} - \frac{1}{2}\left[\frac{f(x)}{f'(x)}\right]^2 \frac{f''(x)}{f'(x)}$

It's Cheby, Not Weddel

We don't know as yet

o Set-3

• Dir^t Me^{ds} : — Diag^l Ma^x, — A = Low^r Δ^r : Forw^d Subs^n, — A = Upp^r Δ^r : Back^d Subs^n } Sys^m of Lin^r Equs

That's in ∫ pa^t

∴ Do this much only

→ Gauss^n Elim^n Me^d: 1. Rem^{ve} the lead var^s in succ^{ve} eqns, ∴ chg to upp^r Δ^r form 2. Back^d subs^n

└ $r = n$: Uni^c soln, $|A| \neq 0$; $r < n$ & $\underbrace{b_{r+1}, b_{r+2}, \ldots, b_n}_{\text{in } [A|B] \text{ only}} \neq 0$: No soln; $r < n$ & $b_{r+1}, \ldots, b_n = 0$: ∞ solns

Ech^n stage

- Part^l Pvtg: Check 1st col in 1st st^{ge}, get the big^t no. on top, Check 2nd col in 2nd st^{ge}, get big^t no. on top
- Full Pvtg: Check whole ma^x & get big^t no. on top. Nev^r us^d. Done when pvt ele^{ts} vanish

→ For sys of eqs, Full row red^d ech^n form is oft^d, NOT jus Upp^r Δ^r

→ Gauss Jor^n Elim^n Me^d: $[A|b] \xrightarrow{\text{Gauss Jor}^n} [I|d]$. Pvtg is us^d. Us^d mos^{ly} for A^{-1}. $[A|I] \xrightarrow{GJ} [I|A^{-1}]$

- Some^s, Gauss Elim for $A^{-1} \equiv [A|I] \to$ Ch^{ge} A to Upp^r $\Delta^r \to$ Ch^{ge} to I $\to [I|A^{-1}]$
- Indir^t Me^{ds} (Ite^{ve}): Us^d when coeff ma^x is "Sparse" or "Large".

→ Gauss-Sei^l Ite^n Me^d: $x^{(k+1)} = -\frac{1}{a_{11}}(a_{12}y^{(k)} + a_{13}z^{(k)} - b_1)$, $y^{(k+1)} = -\frac{1}{a_{22}}(a_{21}x^{(k+1)} + a_{23}z^{(k)} - b_2)$, |||ly z

└ Calc - $A = \frac{(b_1 - B - C)}{a_{11}}$; $B = \frac{(b_2 - A - C)}{a_{22}}$; $C = \frac{(b_3 - B - A)}{a_{33}}$ └ Me^d of "Succ^{ve} Displ^{ts}", use a bett^r val^e asap

- It^{ns} won't cgs if diag^s are not "heavy", ∴ Rearr^{ge} to get heavy diag^s & then beg^n: Take $x^{(0)} = y^{(0)} = z^{(0)} = 0$

○ Set-4 : Interpol^n

- Forw^d Diff^s: $\Delta y_0 = y_1 - y_0$ (1st FD) ; $\Delta^2 y_0 = \Delta(\Delta y_0) = \Delta y_1 - \Delta y_0 = y_2 - 2y_1 + y_0$ (2nd FD). |||ly, Δy_1, $\Delta^2 y_2$ etc. (Lead^g Diff^s ⟶)
- Back^d —"— : $\nabla y_1 = y_1 - y_0$ (1st BD) ; $\nabla^2 y_2 = \nabla y_2 - \nabla y_1$ (2nd BD) } Only not^n ch^{ge}, not the diff^s ∴ $\delta f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2})$
- Cen^l —"— : $\delta y_{1/2} = y_1 - y_0$, ∴ $\delta y_n = y_{n+\frac{1}{2}} - y_{n-\frac{1}{2}}$, |||ly $\delta^{(k)} y_n = \delta^{(k-1)} y_{n+\frac{1}{2}} - \delta^{(k-1)} y_{n-\frac{1}{2}}$

For "Effect of Err^s", see Kani's ↑

- Shift Op^r: $E f(x) = f(x+h)$; $E^2 f(x) = f(x+2h)$; $E^n f(x) = f(x+nh)$; $E^{-n} f(x) = f(x-nh)$
- Avg Op^r (μ): $\mu f(x) = \frac{1}{2}[f(x+\frac{h}{2}) + f(x-\frac{h}{2})]$ → Not^n • Diff^l Op^r: $Df(x) = f'(x)$ } Just read the prop^s af^r this 1ce fr^m not^s
- $E = 1 + \Delta$; $E = (1-\nabla)^{-1}$; $\delta = E^{1/2} - E^{-1/2}$; $\mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$; $hD = \log E$ (by Tay's Thm) ↳ ∴ $e^{hD} = E$

Thm: n^{th} diff^s of a poly^l of n^{th} deg^{ee} are const & all hi^r deg diff^s are 0, when x is equally spaced → IMP
[Conv also True!], i.e. if x is $=^{ly}$ spa^d & n^{th} diff^s of a fn are const ⇒ That fn is n^{th} deg poly.

- New^n's Bino^l Expan^n Form^a (For get^g miss^g val^s in mid^{le} for a fn): $y_n - ny_{n-1} + \frac{n(n-1)}{2}y_{n-2} + \cdots + (-1)^n y_0 = 0$ [By Bino^l Exp^n]

$\Delta^n y_0 = 0$, i.e. $(E-1)^n y_0 = 0$, ∴ ... ∴ $(n-1)^{th}$ deg poly ← If n val^s are known; take $\Delta^n y_0 = 0$ ~~Not n+1, coz (n+1)th val we re find~~

For sol^g prob^{ms}, expand this only & put $E^2 y_0 = y_2$ etc. (n^{th} diff $= 0 \equiv$ van^{sh})

- If $(n+1)$ val^s for a fn are giv^n, $(n+1)^{th}$ diff^{ce} will van^{sh} (not const) & for a poly^l of n^{th} deg, $(n+1)^{th}$ diff^{ce} will van^{sh} (∵ n^{th} will be const). ↳ ∴ $n+1$ val^s give a poly of n^{th} deg, but can give $< n$ also ↳ 2018 Q56-P2

Thm: New^n's Forw^d Interpo^n Form^a: Let $x_0, x_1, \ldots, x_n$ be eq^{ly} spa^d by dis^{ce} h. Then,

n^{th} deg poly^l for approx^n ← $\phi(x) = y_0 + \frac{u\Delta y_0}{1!} + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)\cdots(u-(n-1))}{n!}\Delta^n y_0$ $\left\{ u = \frac{x - x_0}{h} \right.$

└ Us^d for fin^g val^s close to beg^g of data set └ If $f(3.1)$ is req^d, Take $y_0 = f(3)$. Leave $f(1), f(2)$

└ Calc - $y_0 + A\Delta y_0 + \frac{A(A-1)}{2!}\Delta^2 y_0 + \cdots$ → Press CALC, put u as A └ Rem^r Cum^{ve} Freq^{cy} Type Prob^{ms}

Thm: New^n-Greg^y Back^d Int^n Form^a: For val^s of y near the end of data set.

$f(x) \approx \phi(x) = y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \cdots + \frac{u(u+1)\cdots(u+n-1)}{n!}\nabla^n y_n$ $\left\{ u = \frac{x - x_n}{h} \right.$

--- For^d Int^n / Back^d Int^n --->

Err^r in Int^n Form^a: 1. For^d Int^n: $R(x) = \frac{\Delta^{n+1} y_0}{(n+1)!} u(u-1)\cdots(u-n)$ $\left\{ u = \frac{x-x_0}{h} \right.$

2. Back^d Int^n: $R(x) = \frac{\nabla^{n+1} y_n}{(n+1)!} u(u+1)\cdots(u+n)$ $\left\{ u = \frac{x-x_n}{h} \right.$ ✓ Just the next term in corres^g form^a ∴ Easy

- Int^n w. ≠ Inter^{ls}: 1. Lag^e Int^n Form^a (Uni^c Int^g Poly^l): $f(x) = \sum_{i=0}^{n} l_i(x) f_i$, wh^{re},

Lag^e Int^n Coeff^s ← $l_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$; $\sum l_i(x) = 1$ & $l_i(x_j) = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$

└ Inv^{se} Int^n Form^a (giv^n y, find x. Take it as $g(y) \equiv f^{-1}(y)$. Can be done if f is cts & mono ↑ or ↓):

$g(y) = f^{-1}(y) = \sum_{i=1}^{n} \frac{\Pi_{n+1}(y)}{(y-y_i)\Pi'_{n+1}(y_i)} \cdot x_i$ { Just x & y take oppo^{te} roles, rest same as Lag^e Form^a }

But act^{ly} it's the last t^m, coz if all $\Delta^n y_0$ are us^d in the form^a, $\Delta^{n+1} y_0$ will be 0, but Err^r can't be 0 ∴ We take n+1 as the last n only & ∴ Err^r = Last Tm

• Part^l frac^{ns} us^g Lag's Form^a : Take num^r of the rat^l fn. Calc $f(x)$ at ~~$x = 1, 2, 3$~~ Val^s in deno^r. Put in Lag Form.

E.g. - $\frac{3x^2+x+1}{(x-1)(x-2)(x-3)}$; Take $f(x) = 3x^2+x+1$, $\therefore$

x	1	2	3
$f(x)$	5	15	31

$\therefore f(x) = 3x^2+x+1 = \frac{(x-2)(x-3)}{(1-2)(1-3)} \cdot 5 + \dots + \dots$

↳ $f(x)$ at 1, 2, 3 coz that's in deno^r

$\therefore \frac{3x^2+x+1}{(x-1)(x-2)(x-3)} = \frac{5}{2(x-1)} - \frac{15}{(x-2)} + \frac{31}{2(x-3)}$

For lin poly, it's err $= \frac{1}{2}(x-x_0)(x-x_1)f''(c)$. Fi^d the bd of $f''(x)$ & $\therefore$ get err bd & if h is invol^d, use $\frac{h^2}{8}M$

(3)

• $\boxed{\text{Trunc}^n \text{ Err}^r \text{ Bds}}$: $E_1(f, x) = \frac{1}{2}(x-x_0)(x-x_1)f''(\xi)$; Trun^n Err^r in Lin^r Int^n (i.e. 1st deg only)

If $|f''(x)| < M_2$ $\forall x \in [x_0, x_1]$, then $|E_1(f, x)| = |f(x) - P(x)| \leq \frac{1}{2} \text{Max}|(x-x_0)(x-x_1)| M_2 \rightarrow$ Err^r Bd

$\therefore |f(x) - P(x)| \leq \frac{1}{8}(x_1-x_0)^2 M_2 = \frac{1}{8}h^2 M_2 < \varepsilon$ (given), $\therefore$ get apt 'h'

• For n^{th} deg poly^l: $E_n(f, x) = \frac{\Pi_{n+1}(x)}{(n+1)!} f^{n+1}(\xi)$
↳ Err^r in Lag Int^n

$= (x-x_0)(x-x_1) \dots (x-x_n) \frac{f^{n+1}(\xi)}{(n+1)!}$

• Lag^e Int^n Form^a can be wri^n as : $f(x) = \sum_{r=0}^{n} \frac{\phi(x)}{(x-x_r)\phi'(x_r)} \times f(x_r)$, wh^e $\phi(x) = (x-x_0)(x-x_1) \dots (x-x_n)$

• Dr^w ba^k of Lag^e int^n : If an^r int^n val is inser^d, the coeff^s are req^d to be recalc^d

• Div^d Diff^s : $\underbrace{[x_0, x_1]}_{1^{st}\ DD} = \frac{y_1 - y_0}{x_1 - x_0}$, |||ly $[x_1, x_2]$ etc. & $\underbrace{[x_0, x_1, x_2]}_{2^{nd}\ DD} = \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0}$,

$\underbrace{[x_0, x_1, x_2, x_3]}_{3^{rd}\ DD} = \frac{[x_1, x_2, x_3] - [x_0, x_1, x_2]}{x_3 - x_0}$

• DDs are sym^l, i.e. ord^r doesn't matt^r ; $[x_2, x_0, x_1] = [x_0, x_1, x_2] = [x_1, x_2, x_0]$

• n^{th} DDs of a n^{th} deg poly^l are const ; Same as in New^n Me^d ; $[x_0, x_1] = \frac{\Delta y_0}{h}, \dots, [x_0, x_1, \dots, x_n] = \frac{1}{n! h^n}\Delta^n y_0$

2. New^n's DD Form^a : $y = f(x) = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)[x_0, x_1, x_2] + \dots +$
$(x-x_0)(x-x_1) \dots (x-x_n)[x_0, x_1, \dots, x_n]$

↳ Unlike Lag^e, we can dir^y ins^t new val^s

No n etc. $(x-x_0)(x-x_1)$ only

If you fi^d bd for $f^{n+1}(c)$, f is the giv^n fn! ← e.g. - $\sin x$, e^x etc. NOT int^n poly! That'll be err bd

○ Set - 5 : Num^l Integ^n

• New^n Cotes Form^a (Gen^l Quadr^{re} Form^a for equidis^t ord^s) : $I = \int_a^b f(x)dx = \int_{x_0}^{x_0+nh} f(x)dx = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - n^3 + n^2\right)\Delta^3 y_0 - \dots\right]$

Wri $f(x)$ as in New^n's Int^l Ppl^n form

• Trape^l Rule ($n=1$) : $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\dots+y_{n-1})\right]$ → Repl^g curve b/w x_0, x_1 by a Str^t Line
↳ Not h

↳ Err^r : $E = -\frac{1}{12}h^3\left[y_0'' + y_1'' + \dots + y_{n-1}''\right] = -\frac{(b-a)}{12}h^2 y''(\bar{x})$, wh^{re} $y''(\bar{x}) = \max y_i$ (Not $i\nu$)

For bd of f, ch^k if f is ↑ or ↓

↳ $\therefore$ Err^r in Trap^l $= \boxed{O(h^2)}$

• Simp^n's $\frac{1}{3}^{rd}$ Rule ($n = 2k$) (Even # of subin^{ts}) : We take a 2nd deg poly^l to approx cur^e b/w x_r & x_{r+2h}

$I = \int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\left[(y_0+y_n) + 2\underbrace{(y_2+y_4+\dots+y_{n-2})}_{\text{Even Ord}^s} + 4\underbrace{(y_1+y_3+\dots+y_{n-1})}_{\text{Odd Ord}^s}\right]$

↳ Err^r : $E = ~~-\frac{nh^5}{90}y^{iv}(\bar{x})~~ = -\frac{(b-a)}{180}h^4 y^{iv}(\bar{x})$ $\left\{\because nh = \frac{b-a}{2}\right\}$ $= \boxed{O(h^4)}$
here 180

• Simp^n's $\frac{3}{8}^{th}$ rule ($n = 3k$) (# of subin^s = mult^{le} of 3) : $I = \int_{x_0}^{x_0+nh} f(x) = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\dots+y_{n-2}+y_{n-1}) + 2(y_3+y_6+\dots+y_{n-3})\right]$

$-\frac{(b-a)}{80}h^4 y^{iv}(\bar{x})$ ✓

↳ $E = ~~-\frac{3h^5}{80}y^{iv}(\bar{x})~~ = \boxed{O(h^4)}$ - Or^r same as Simp^n's $\frac{1}{3}^{rd}$, but mag^{de} less in $\frac{1}{3}^{rd}$
Here 80, Not 180

→ App^{ns} : Giv^n velo^s, fi^d dis^{ce} ; giv^n coord^s, fi^d vol. of revo^n → Calc : Use TABLE for calc^d val^s

↳ Weddle's Rule ; Kani's

• Gauss^n Int^n : Allows uneq^l spac^g of ordi^s. Lim^{ts} have to be -1 to 1 : $\int_a^b \rightarrow \int_{-1}^{1}$ us^g $x = \frac{(b-a)}{2}u + \frac{(b+a)}{2}$

$\int_{-1}^{1} f(x)dx = w_1 f(x_1) + w_2 f(x_2) + \dots + w_n f(x_n) = \sum_{i=1}^{n} w_i f(x_i)$
w_i ↳ Wei^{ts} ; x_i ↳ Abscissae → Symm^l abt mid pt of int^l

→ 1 pt form^a ($n=1$) : $\int_{-1}^{1} f(x)dx = 2f(0)$

→ 2 pt form^a ($n=2$) : $\int_{-1}^{1} f(x)dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$ → Gauss Legendre Form^a
Exact till 3rd deg fn

→ 3 pt. form^a : $\int_{-1}^{1} f(x)dx = \frac{5}{9}f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}f(0) + \frac{5}{9}f\left(\sqrt{\frac{3}{5}}\right)$
Exact till 5th deg poly^l

Here, x_i's are the roots of $(n+1)^{th}$ Leg^{re} Poly^l $P_{n+1}(x)$ wh^e : $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$; $P_0(x) = 1$, $P_1(x) = x$

→ The 1st 5 Legre Polyls: $P_0(x)=1$; $P_1(x)=x$; $P_2(x)=\frac{1}{2}(3x^2-1)$; $P_3(x)=\frac{1}{2}(5x^3-3x)$; $P_4(x)=\frac{1}{8}(35x^4-30x^2+3)$

└ & corresd weits w_i are givn by: $w_i=\int_{-1}^{1}\prod_{\substack{j=0\\ j\neq i}}^{n}\left(\frac{x-x_j}{x_i-x_j}\right)dx$ └ Roots = $x_i=0,\pm\sqrt{\frac{3}{5}}$ → See!

o Set - 6 : Soln of ODEs

• Tays sers Med; $y(x)=y(x_0+h)=y_0+hy_0'+\frac{h^2}{2!}y_0''+\cdots$ whe $h=x-x_0$ ($\therefore x=x_0+h$)

└ y'',y''' etc. are derid frm givn f only [$y''=f_x+f_yf$; $y'''=\frac{d}{dx}(f_x+f_yf)$ etc.] → i.e. f_x+f_yy' ($\because y'=f$)

└ If askd to trunce afr x^4 term, just wrte x^5 term $<\varepsilon$ → Err & get suitle x. → Con $f_y y'$

→ Tay's Med for simuls 1st orr ODEs: Givn $y'=f_1(x,y,z)$; $z'=f_2(x,y,z)$ $y(x_0)=y_0$, $z(x_0)=z_0$

Take 2 Tayr sers: $y_1=y_0+hy_0'+\frac{h^2}{2!}y_0''+\cdots$ & $z_1=z_0+hz_0'+\frac{h^2}{2!}z_0''+\cdots$

→ Tay's Med for 2nd orr ODE: Exacly same as 1st orr. Get vals for $y',y'',y''',\ldots$, put in Tay's

• Eur's Med: Soln as a set of vals: $y_n=y(x_n)=y_{n-1}+hf(x_{n-1},y_{n-1})$ → Slow procss, Need smll h to get accuy, o/w errs get accu

└ Itern forma

• Modid Eur's Med: $y_r^{(n)}=y_{r-1}+\frac{h}{2}\left[f(x_{r-1},y_{r-1})+f(x_r,y_r^{(n-1)})\right]$ → Also, in 1st step, it's h, NOT $h/2$ ∴ $(n-1)$

x_r or $x_{r-1}+h$; x_r only! └ This term coms only in 2nd step

└ For it, we appy Trapel Rule in $[x_{r-1},x_r]$

• Runge Kutta Med: Don't need to calcte hir orr deris, give grear accuy, reqre fn vals only at some specc pts. It agrees w. Tay's series upto the term in h^r, whe r is the orr of that RK Med.

(1.) 1st orr RK: Eur's Med - $y_1=y_0+hy_0'$, $y_1=y_0+hy_0'+\frac{h^2}{2}y_0''+\cdots$ ∴ Match till h

(2.) 2nd orr RK: Modid Eur's Med - In Eulr's Forma, put $f(x_0+h,y_1^{(0)})=f(x_0+h,y_0+hf_0)$. Then expad the LHS & use $f(x_0+h,y_0+hf_0)=f(x_0,y_0)+h\left(\frac{\partial f}{\partial x}\right)_0+hf_0\left(\frac{\partial f}{\partial y}\right)_0+O(h^2)$ for RHS → Tayr Sers of 2 Vars

∴ We've: $y_1=y_0+\frac{1}{2}(k_1+k_2)$, whe $k_1=hf(x_0,y_0)$, $k_2=hf(x_0+h,y_0+k_1)$

↳ Same as Modd Eur only.

(3.) 3rd orr RK: $y_1=y_0+\frac{1}{6}(k_1+4k_2+k_3)$, whe $k_1=hf(x_0,y_0)$, $k_2=hf(x_0+\frac{h}{2},y_0+\frac{k_1}{2})$,

$k_3=hf(x_0+h,y_0+k')$ → $k'=hf(x_0+h,y_0+k_1)$

(4.) 4th orr RK: $k_1=hf(x_0,y_0)$; $k_2=hf(x_0+\frac{h}{2},y_0+\frac{k_1}{2})$; $k_3=hf(x_0+\frac{h}{2},y_0+\frac{k_2}{2})$; $k_4=hf(x_0+h,y_0+k_3)$

& $k=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$, ∴ $y_1=y_0+k$.

• Advge: The opn is idenl, whe the ODE is linr or non linr

• RK for a sysm of equs: $y''=f(x,y,y')$; $y(x_0)=y_0$; $y'(x_0)=y_0'$ → Put $\frac{dy}{dx}=p$, ∴ $\frac{dp}{dx}=f_2(x,y,p)$ & $\frac{dy}{dx}=\boxed{p=f_1(x,y,p)}$

→ f_2 for $\frac{dp}{dx}$ & usd in l_i's

Inil Vals: (x_0,y_0,p_0), Step sis: h,k,l, We have:

$k_1=hf_1(x_0,y_0,p_0)$

$k_2=hf_1(x_0+\frac{h}{2},y_0+\frac{k_1}{2},p_0+\frac{l_1}{2})$

$k_3=hf_1(x_0+\frac{h}{2},y_0+\frac{k_2}{2},p_0+\frac{l_2}{2})$

$k_4=hf_1(x_0+h,y_0+k_3,p_0+l_3)$

$y_1=y_0+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$

Not k_1,k_2

$l_1=hf_2(x_0,y_0,p_0)$

$l_2=hf_2(x_0+\frac{h}{2},y_0+\frac{k_1}{2},p_0+\frac{l_1}{2})$

$l_3=hf_2(x_0+\frac{h}{2},y_0+\frac{k_2}{2},p_0+\frac{l_2}{2})$

$l_4=hf_2(x_0+h,y_0+k_3,p_0+l_3)$

$p_1=p_0+\frac{1}{6}(l_1+2l_2+2l_3+l_4)$

|||ly, y_2, p_2

This is it $f_1(x,y,p)$ will alws be p, ∴ Jus put val of p here, there's no x,y in f_1

Here also l_1,l_2 only

& f_1 usd in k_i's not l_i's! Rem f_1 is jus p, so we'll jus have p_0, $p_0+\frac{l_1}{2}$, $p_0+\frac{l_2}{2}$ etc.

* Compu^n Prog^g

$0.625 \times 2 = 1.25$ 1
$0.25 \times 2 = 0.5$ 0 (Top to bott^m)
$0.5 \times 2 = 1$ 1

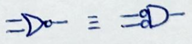
- Dec to Bin: Keep div^g by 2, Wr^te rem^ns bott^m-up. For frac^ns:
- Bin to Dec: Incr^g pow^rs of 2 get mult^d, start from left. For frac^ns, continue mult^g as $\_ \times 2^{-1} + \_ \times 2^{-2} + \dots$
- Dec ↔ Oct, Hexa: Just rem^r; it's done exac^ly as in Bin, with 8 & 16 resp^ly
- Bin ↔ Oct: Group in 3s & ch^ge • Bin ↔ Hexa: Gr^p in 4s & ch^ge • Hex ↔ Oct: Go via Bin
- Bin +^n: $\begin{array}{r} 1111 \\ +1010 \\ \hline 11001 \end{array}$ • Bin −^n: $\begin{array}{r} 1001 \\ -0111 \\ \hline 0010 \end{array}$ • ×^n, ÷^n as in Dec. • 1's, 2's C: −^n beco^s +^n
- 2C is most used
- Both used for signed repr^n (i.e. msb is for sign) 0 +ve, 1 −ve!
- 1C: 1→0, 0→1 • 2C: 1C + 1

E.g.- For (−5) in 2C form, wr^te its bin: $101 \xrightarrow{1C} 010 \xrightarrow{2C} 011 \xrightarrow{Sign} 1011$ → 2'C is act^ly only us^d for −ve #

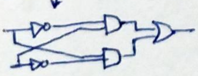
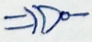
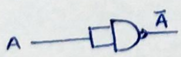
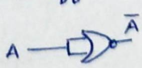
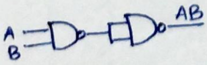
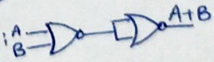
* Giv^n any no. (+ve/−ve), if you take its 2C, you get what^r, it's the −ve of orig^l no. in 2C form

- E.g.- Take 23, take its 2C. What^r bin no. you get, is (−23) in 2C form — Not orig^l bin form, but 2C form.
Take −35, take its 2C. What^r bin no. you get, is 35 in 2C form
- Taking 1C also does the same thing & you get the −ve in 1C form.
- For +^n, −^n us^g 1C & 2C, see Book 10 (Yellow) prog^g part. • $A-B \equiv A+(B)_{2C}$ • 9C, 10C same
- Props of Boo^n Al^a: 1. $A+B=B+A$, $AB=BA$ (Commu^ve) 2. $A+(B+C)=(A+B)+C$, $A(BC)=(AB)C$ [Asso^ve]
3. $A\cdot(B+C)=AB+AC$, $A+BC=(A+B)(A+C)$ [Pf: $A\cdot 1+BC = A(1+B)+BC = A\cdot1+AB+AC = A\cdot(1+C)+AB+AC = A^2+AC+AB+AC$]
4. $A+AB=A$, $A(A+B)=A$ [Absorp^n Laws] 5. $A+\bar{A}B=A+B$, $A(\bar{A}+B)=AB$ [Absorp^n Laws]
6. $AB+\bar{A}C+BC=AB+\bar{A}C$, $(A+B)(\bar{A}+C)(B+C)=(A+B)(\bar{A}+C)$ [Consensus Laws] ($BC=BC(A+\bar{A})$ & $(B+C)=B+C+A\bar{A}$)
7. De Morgan: $\overline{AB}=\bar{A}+\bar{B}$, $\overline{A+B}=\bar{A}\bar{B}$ 8. Duals: $A, A^* \equiv$ Repl^ce 1 by 0, 0 by 1 & (+) by (·), (·) by (+)
- SOP & POS: As name goes • Mint^m: A Prod^t hav^g all var^s, 0: Compl-i-ment, 1: Not
- Maxt^m: A sum hav^g all var^s, 1: Compl-i-ment, 0: Not — For CSOP, retain Min & mult^g $(x+\bar{x})$ ∀ miss^g x. For CPOS, ret^n Max & add $x\bar{x}$ ∀ miss^g x.
- Cano^l SOP/POS: A SOP/POS which only has Min/Max
- To f^d SOP fr^m Truth Table: Form Min ($A\bar{B}C$, $\bar{A}\bar{B}C$ etc.), wherever O/P = 1. Add all to get SOP
- —"— POS —"—: —"— Max ($A+B+\bar{C}$ etc.), wherever O/P = 0. Mul all & get POS.

Aid: SOP1 & POS0 (SOP: Check 1, POS: Check 0) • Each Max = Complement of corres^g Min

- Log^c Gates: NAND:  NOR: , Both are Univ^l Gates.

↳ For ch^g any fn to NAND only, write it in SOP form (not CSOP). Repl every gate by NAND
—"— NOR —"— POS —"— CPOS —"— NOR

- XOR: : $A\oplus B=\bar{A}B+A\bar{B}$: O/P is 1 only when odd no. of I/P are 1. 2 or 4 I/P = 1 ⇒ O/P = 0.
≡  • $A\oplus B\oplus C=A\bar{B}\bar{C}+\bar{A}B\bar{C}+\bar{A}\bar{B}C+ABC$ • Uses: Mod 2 add^n, Contr^d Inverter
- XNOR: ; $A\odot B=\overline{\bar{A}B+A\bar{B}}=AB+\bar{A}\bar{B}$: O/P 1 iff even no. of I/P are 1.
- NOT us^g NAND: , us^g NOR: 
- AND us^g NAND:  • OR us^g NOR: 

Rem^r: NAND takes AND & ~ it. |||ly NOR. Use it & oth^r alg^c mani^ns to make NAND/NOR only gates

- Conjunc^n: ∧ / * / · / AND • Disjunc^n: ∨ / + / OR • Cond^l op^r: $p\to q$ (if p then q) • Conjun^n of Taut is also a Taut
- Bicond^l Op^r: $p\leftrightarrow q$ (p iff q) • Taut^g: Iden^g True Stat^t • ※: Iden^g False Sta^t
- Equiv^ce Formu^e: $(p\vee q)\wedge\sim q \Rightarrow p$, [∵ $(p+q)p' \Rightarrow pp'+p'q \Rightarrow p'q \Rightarrow q$, |||ly Oth^r one] *Ch^ge ∧ & ∨ to · & + for ease
$(p\vee q)\wedge\sim p \Rightarrow q$
- $\boxed{p\to q \Rightarrow \sim p\vee q}$ → Most used • $p_1, p_2, \dots, p_n \vdash q \equiv [(p_1\wedge p_2\wedge p_3 \dots \wedge p_n)\Rightarrow q]$
True Arg^t ≡ Valid, False Arg^t ≡ Fallacy.
- Conju^ve Nor^l Form ≡ POS • Disjun^ve Nor^l Form ≡ SOP • Prin^l CNF ≡ Cano^l POS, |||ly PDNF ≡ CSOP
- Use $p\to q \Rightarrow \sim p\vee q$, De Morgan, oth^r laws & use the meth^d to get Min/Max
* $E\leftrightarrow F \Leftrightarrow (E\wedge F)\vee(\sim E\wedge\sim F)$
$E\leftrightarrow F \Leftrightarrow (\sim E\vee F)\wedge(E\vee\sim F)$