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- Q.1) During the 1980's, the general consensus is the about 5% of the nation's children had autism. Some believed that increase certain chemical in the environment had led to an increase in autism.
- a) Write an appropriate test for this hypothesis, stating what are the necessary conditions for formulating the test.
- b) Give an appropriate test for this hypothesis, stating what is this situation.
- c) A recent study examined 38+ children and found that 46 showed signs of autism. Perform a test of the hypothesis and state the p-value.
- D) what are the conclusions? State how we the p-value.

Step-1 : Establish Null & Alternate hypothesis.

Null hypothesis : 5% of the nation children had autism.

Alternate hypothesis : More than 5% of the nation children had autism.

$$H_0 : P = 0.05$$

$$H_A : P > 0.05$$

This is a one tailed test since we are going to check only one end of the experiment.

Step-2 : Determine the test we are going to perform - the Z-test

(2)

Step-3:- Set value of alpha (α). Since in this problem the significance value is not given.
 Let by default assumption,
 $\alpha = 5\%$
 $\alpha = 0.05$

Step-4 :- Establish the decision rule.

For Z-Critical,

$Z_{\text{Critical}} < Z_{\text{Score}} \text{ (Test Score)}$

we will reject null hypothesis.

For P-value.

if,

$P\text{-Value} < \text{Significance Value}$.

we reject our null hypotheses.

Step-5 :- Gathering Data.

A recent study examined 344 children and found that 46 showed signs of autism. 5% of the nation's children had autism.

Step-6 :- Analyze the data.

We know,

$$P = 0.05, n = 384$$

$$Z\text{-Score} = \frac{\hat{P} - P}{\sqrt{\frac{P \cdot Q}{n}}}$$

Sample Proportion,

$$\hat{P} = \frac{46}{384}$$

$$\hat{P} = 0.11$$

$$n = 384$$

$$q = 1 - p$$

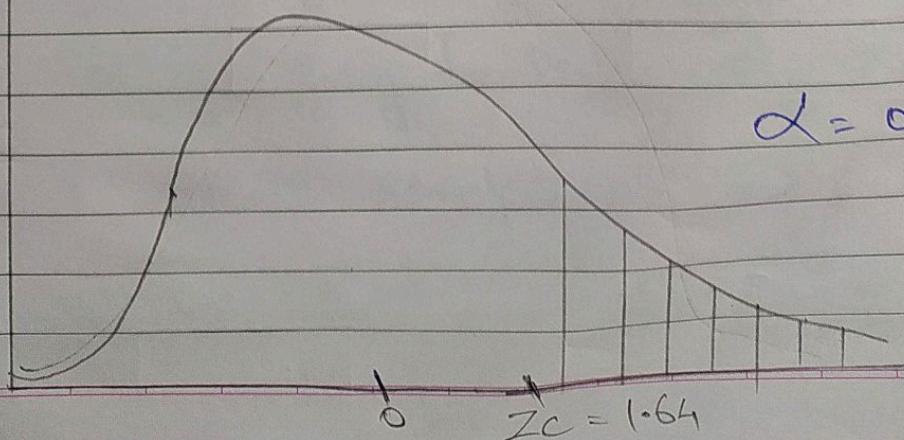
$$q = 0.95$$

$$Z\text{-Score} = \frac{\hat{P} - P}{\sqrt{\frac{P \cdot q}{n}}}$$

$$= \frac{0.11 - 0.05}{\sqrt{\frac{0.05 \times 0.95}{384}}}$$

$$Z\text{-Score} = 5.39.$$

$$\alpha = 0.05$$



(4)

Step-7 : Take Statistical Action
 using Z-table
 we have calculated

$$\begin{aligned} Z\text{-Critical} &= 1.64 \\ Z\text{-Score} &= 5.39 \end{aligned}$$

on the basis of decision rule,

$$Z\text{-Critical} < Z\text{-Score}$$

$$1.64 < 5.39$$

* we will reject Null hypothesis

So,

The More than 5% had autism.

The Increase in certain chemical in the Environment in autism.

(5)

- Q.2) A Company with a fleet of 150 cars found that the Emission Systems of 7 out of the 22 cars tested failed to meet pollution guidelines.
- a) write a hypothesis to test if more than 20% of the entire fleet might be out of compliance.
- b) Test the hypothesis based on the binomial distribution and report P-value.
- c) Is the test significant at the 10%, 5%, 1% level?

* Step - 1 : Establish Null & Alternate hypothesis.

Null hypothesis :- 20% of the car failed to meet population guidelines

Alternate hypothesis : More than 20% of the failed to meet population guidelines.

$$H_0 : P = 0.20$$

$$H_A : P > 0.20$$

It is a one tailed test since we are going to check only one end of experiment.

Step - 2 :- Determine the test

We are going to perform the Z-test

Step - 3 :- Set the Significance Value.

$$\text{Significance Value} = 10\% \\ \alpha = 0.10$$

⑥

Step-4 : Establish Decision rule.

For Z-critical,

$Z_{\text{critical}} < Z_{\text{Score}}$ (test score)

we will reject null hypothesis.

For p-value,

If,

$p_{\text{value}} <$ Significance level.

we will reject our null hypothesis.

Step-5 : Collecting data.

A company with a fleet of 150 cars found that the emissions type systems of 7 out of the 22 cars tested failed to meet pollution guidelines.

Step-6 : Analysis of data.

For Z-score

$$Z_{\text{Score}} = \frac{\hat{P} - P}{\sqrt{\frac{P \cdot Q}{n}}}$$

We know that,

$$Q = 0.20$$

$$n = 22$$

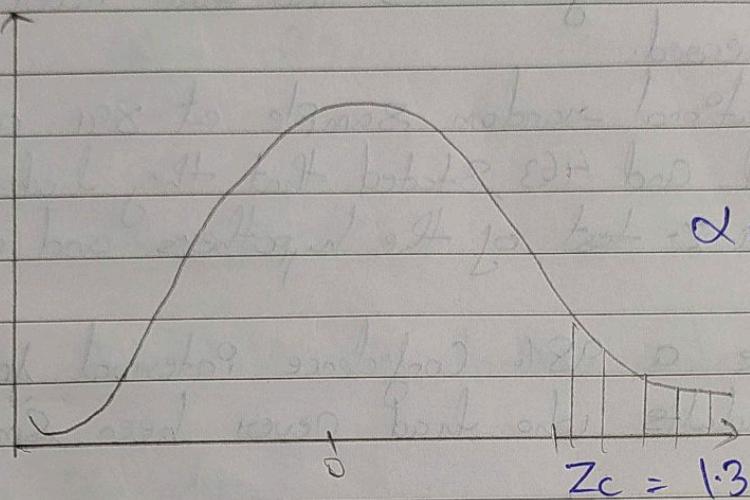
To calculate \hat{P} ,

$$\hat{P} = \frac{7}{22}$$

$$Q = 1 - P \\ Q = 0.80$$

$$\hat{P} = 0.31$$

$$\text{Z-Score} = \frac{0.31 - 0.20}{\sqrt{\frac{0.20 \times 0.80}{22}}} = \boxed{\text{Z-Score} = 1.28}$$



By Z-table,

$$Z_c = 1.3$$

$$P\text{-Value} = 1.003$$

Step-7 : Take Statistical action.

Given Z-critical,

$$Z_{\text{Critical}} > Z_{\text{Score}}$$

$$1.3 > 1.28$$

We will not reject the null hypothesis
We will accept the null hypothesis.

Therefore,

20% of the entire meat of the cows are failed to meet the population guidelines.

Step-8 : Determine business Implications.

So, the 20% of the cows are failed to meet the population guidelines, that's why we should improve the cows.

- Q.3) (8)
- National Data in the 1960 showed that about 44% of the adult population had never smoked.
- a) State a null and alternate to test that the fraction of the 1995 of adults that had never smoked had increased.
- b) A national random sample of 891 adults were interviewed and 463 stated that they had never smoked. Perform a Z-test of the hypothesis and given an appropriate P-value.
- c) Create a 98% Confidence Interval for the proportion of the adults who had never been smoked.

Step 1 :- Establish Null & Alternate Hypothesis.

Null hypothesis : 44% of the adult population had never smoked.

Alternate hypothesis :- More than 44% of the adult population had never smoked.

$$H_0 : P = 0.44$$

$$H_A : P > 0.44$$

It is a one-tailed test, since we are testing only one end of the spectrum.

Step 2 :- Determine the test

We are going to perform the Z-test.

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Step 3 : Set the value of significance level.

Since, confidence level is 98%
then,

$$\alpha = 2\%$$

$$\alpha = 0.02$$

$$\alpha = 0.02$$

Step 4 : Establish the decision rule.

For Z-critical,

If,
Z-critical < Z-test (Test score)
we will reject null hypothesis.

For P-values,

If,
P-values < significance level value
we will reject the null hypotheses.

Step 5 : Gathering the data.

A national random sample of 891 adults were interviewed and 463 stated that they had never smoked.

Step 6 : Analysis of data.

For Z-score,

$$Z\text{-score} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

(10)

We know that

$$P = 0.44$$

$$n = 891$$

$$\begin{aligned}q &= 1-P \\&= 1-0.44\end{aligned}$$

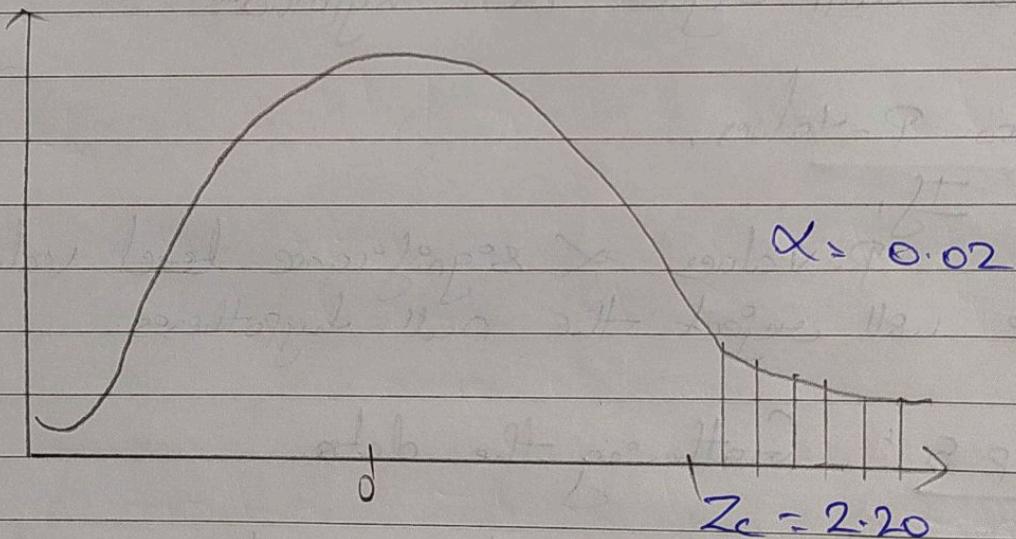
$$P = \frac{463}{891}$$

$$q = 0.56$$

$$\hat{P} = 0.519.$$

$$Z\text{-Score} = \frac{0.519 - 0.44}{\sqrt{\frac{0.44 \times 0.56}{891}}}$$

$$Z\text{-Score} = 4.75$$



By using Z-table

$$Z\text{-critical} = 2.20$$

Step 7 : The Statistical Action.

We have calculated,

$$Z\text{-Score} = 4.75$$

$$Z\text{-Critical} = 2.20$$

On the basis of decision rule,

$$Z\text{-Critical} < Z\text{-Score}$$

$$2.20 < 4.75$$

We will reject the null hypothesis.

So,

The more than 44% of the adult population had never smoked.

Step 8

So,

The more than 44% of the adult population never smoked.

(12)

- Q.4) one of the lenses in your supply is suspected to have a focal length of 9.1 cm. Rather than 9 cm claimed by the manufacturer.
- Here's s_1 is the distance from the lens to the object and s_2 is the distance from the lens to the real image of the object
- the distances s_1 and s_2 are each independently measured 25 times. The sample mean of the measurement is $\bar{s}_1 = 26.6$ centimetre and $\bar{s}_2 = 13.8$ cm respectively. The standard deviation of the measurement is 0.1 cm for s_1 and 0.5 cm for s_2
- Write the appropriate test hypotheses test for this situation.
 - use this to devise a Z-test for the hypotheses and report a P-value for the test.

Step 1 : Established Null & Alternate hypothesis.

Null hypothesis : The distance from the lens to the object and distance from the lens to real image is same.

Alternate hypothesis : The distance from the lens to the object and distance from the lens to real image is not same.

$$H_0 : M_A \underset{=}{\sim} M_B$$

$$H_a : M_A \neq M_B$$

Step 2 Determine the test

we are going to perform the Z-test.

Step 3 : Set the significance value

As the significance value is not given in the problem we are going to take the default significance value.

$$\text{i.e. } \alpha = 0.05$$

Since, it is a two tailed test and we are going to check left & right end of the experiment

So,

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

Step 4 : Establish the decision rule.

Critical Value

$$\text{If } Z_{\text{calculated}} < Z_{\text{base}}$$

we will reject the null hypothesis.

For P-value.

If P-value < Significance value.
we will reject the null hypothesis.

(1)

Step 5 : Collecting the data.

The distance each S_1 & S_2 are measured independently 25 times. The Sample mean of the measurement is $\bar{S}_1 = 26.6$ cm & $\bar{S}_2 = 13.8$ cm respectively. The Standard deviation of the measurement is 0.7 cm for S_1 , 0.5 cm for S_2 .

Step 6 : Analysis of data

Since we are going to perform the Z-test
for two sample:

$$\alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

For distance from the lens to object S_1 ,

$$\begin{aligned}\bar{S}_1 &= 26.6 \text{ cm} \\ \sigma_1 &= 0.1 \text{ cm} \\ n_1 &= 25\end{aligned}$$

For distance from lens to real image S_2 ,

$$\begin{aligned}\bar{S}_2 &= 13.8 \text{ cm} \\ \sigma_2 &= 0.5 \text{ cm} \\ n_2 &= 25\end{aligned}$$

For Z-score,

$$Z \cdot \text{Score} = \frac{\bar{S}_1 - \bar{S}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{26.6 - 13.8}{\sqrt{\frac{(0.5)^2 + (0.5)^2}{25}}}$$

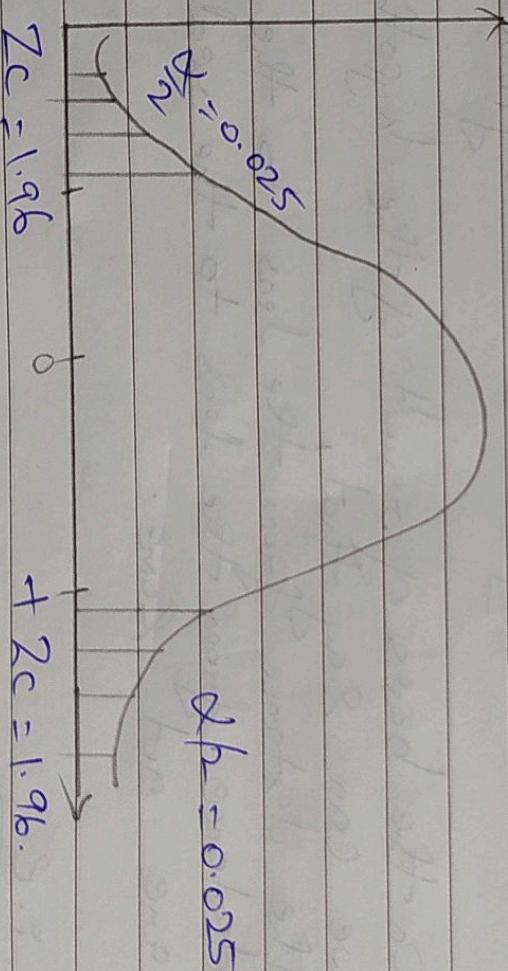
$$= \frac{12.8}{0.102}$$

$$Z\text{-Score} = 125.51$$

Now using Z-Table

$$Z\text{-selected} = 1.96$$

$$P\text{-Value} = 0.0000$$



$$- Z_c = 1.96 \quad + Z_c = 1.96$$

Step 7: Taking Statistical action.

on the basis of decision Rule,

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Our Critical Value,

$$\text{If } Z_{\text{critical}} < Z\text{-score.}$$

$$1.96 < 125.51$$

we will reject the null hypothesis.

If our Critical value,

$$\text{If } P\text{-value} < \text{Significance value} \\ 0.0000 < 0.05$$

we will reject the null hypothesis.

On the basis of result of the hypothesis test we can say that
The distance of the lens to the object and distance from the lens to the real image are not same.

i.e.

$$M_A \neq M_B$$

Q.5 The body temperature in degrees Fahrenheit of 52 randomly chosen healthy adults is measured with the following summary of data.

$$n = 52, \bar{x} = 98.2846, S = 0.6824.$$

- Find the necessary conditions for constructing a valid interval & defined
- Find a 98% Confidence interval for the mean body temperature at 98.6°C Fahrenheit and use the information above to the evaluate a test with significance level of $\alpha = 0.02$

Step 1 : Establish Null & Alternate Hypotheses.

Null Hypothesis : Mean Body Temperature is 98.6

Alternate Hypothesis - Mean Body Temperature is not equal to 98.6 .

$$H_0 : \mu = 98.6$$

$$H_A : \mu \neq 98.6$$

It is two tailed test since we are going to test is on both ends or left & right end of the test

$$\frac{\alpha}{2} = \frac{0.02}{2} = 0.01$$

(2)

Step 2 : Determine the test

we are going to perform the t -test

Step 3 : Set the significance level.

$$\alpha = 0.02 \quad \text{Given.}$$

Step 4 : Establish decision rule.

For critical value.

$|t_{\text{critical}}| < |t_{\text{score}}|$
we will reject the null hypothesis.

Our P -value.

$|t_{\text{score}}| < |t_{\text{significance level}}|$
we will reject the null hypothesis.

Step 5 : collecting the data

The body temperature in degrees Fahrenheit of 52 random chosen healthy adults is measured with the following Summary of data

$$n = 52, \bar{x} = 98.2846, S = 0.6824$$

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Step-6 :- Analysis of data.

Since,
we testing only one random variable and its
Standard deviation is given.

$$t\text{-score} = \frac{\bar{X} - M}{\frac{S}{\sqrt{n}}}$$

where $n = 52$ $DF = n-1$
 $\bar{X} = 98.2846$ $= 52-1$

$$S = 0.6824$$
 $DF = 51$
 $n = 98.6$

$$\therefore t\text{-score} = \frac{98.2846 - 98.6}{\frac{0.6824}{\sqrt{52}}}$$

$$t\text{-score} = 3.333$$

Since it is two tailed test

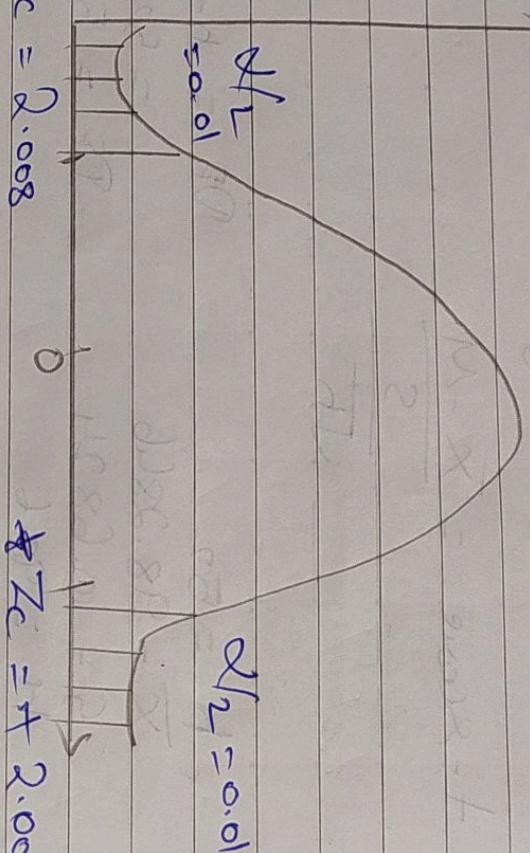
$$\alpha = \alpha_2 = \frac{0.02}{2}$$

$$\alpha = 0.01, DF = 51$$

(20)

From using t -table,

$$\begin{array}{|l|l|} \hline \text{t-critical} & = 2.008 \\ \text{P-value} & = 0.0016 \\ \hline \end{array}$$



Step 7: Take Statistical test.

Based on decision rule,
for critical values.

If
 t -critical $<$ t score
 $2.008 < 2.333$
 we will reject null hypothesis.

For P-value

If P-value $<$ significance level
 $0.0016 < 0.02$
 we will reject null hypotheses

So,
 The mean body temperature is not equal to 98.6°F .

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Q.6. Prices of cars calling for regular gas sometimes premium in the hopes that it will improve gas mileage. Here a rental car company takes 10 randomly chosen car in the lot meet and run a tank of gas accordingly to coin toss, run a tank of gas of each type.

Car	1	2	3	4	5	6	7	8	9	10
Regular	16	20	21	22	23	22	27	25	27	28
Premium	19	22	24	24	25	26	26	28	32	

- a) write an appropriate hypothesis test for this situation and state the testing procedure. appropriate to this circumstances.
- b) complete the necessary Summary Statistics. for the test in part (A)
- c) Perform t-test and report the p-value.
- d) Compare your result to that of a two sample t - test.

Step 1: Establish null & Alternate Hypotheses.

Hull hypothesis: There is no difference between the Premium & Regular gas tank in term of milage.

Alt-hypothesis: There is difference between the milage of regular and Premium gas tank.

(22)

$$\begin{aligned} H_0 : \quad M_A &= M_B \\ H_A : \quad M_A &\neq M_B \end{aligned}$$

It is a two tailed test since we are going to check it on two ends of the experiment.

Step 2: Determine the test.

Since we are going to perform the t-test.

Step 3: Set the Significance Value.

As the significance value is not given so we take the default value of significance level.

$$\alpha = 0.05$$

Since,

It is a two tailed test

$$\alpha/2 = 0.025$$

Step 4: Establish the decision rule.

Our critical value.

T_t-Critical t-t test

We will reject the null hypothesis

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For P-value.

If P-value < Significance value.
we will reject the null hypothesis.

Step 5 : Collecting the data.

The Rental car company takes 10 randomly chosen cars in the fleet and runs a tank of gas accordingly to a coin toss Runs a tank of gas of each type.

Step 6 : Analysis of data.

Since it is a t-test for two sample Random Variable

$$DF = \left[\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right]^{-2}$$

$$\left[\frac{(S_1)^2}{n_1} + \frac{(S_2)^2}{n_2} \right]^{-2}$$

$$\left[\frac{n_1}{n_1 - 1} + \frac{n_2}{n_2 - 1} \right]$$

$$t - test = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

For Sample of regular tank,

$$M_A = \bar{x}_1 = 023.1$$

$$n = 10$$

$$S_1 = 3.72 \quad (\bar{x}_1 - M_A) = 23.1$$

(2)

for Sample of Premium tank.

$$\bar{X}_2 = h_2 = 25.1$$

$$n_2 = 10.$$

$$S_2 = 3.44.$$

$$D.F. = \frac{\left[\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} \right] n_1 n_2}{\left[\frac{(S_1^2)^2}{n_1^2} + \frac{(S_2^2)^2}{n_2^2} \right]}$$

$$= \frac{(1.38 + 1.18)^2}{(1.38)^2 + (1.18)^2}$$

$$= \frac{0.55}{0.36}$$

$$= 18$$

$\therefore D.F. = 18.$

$$t\text{-test} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

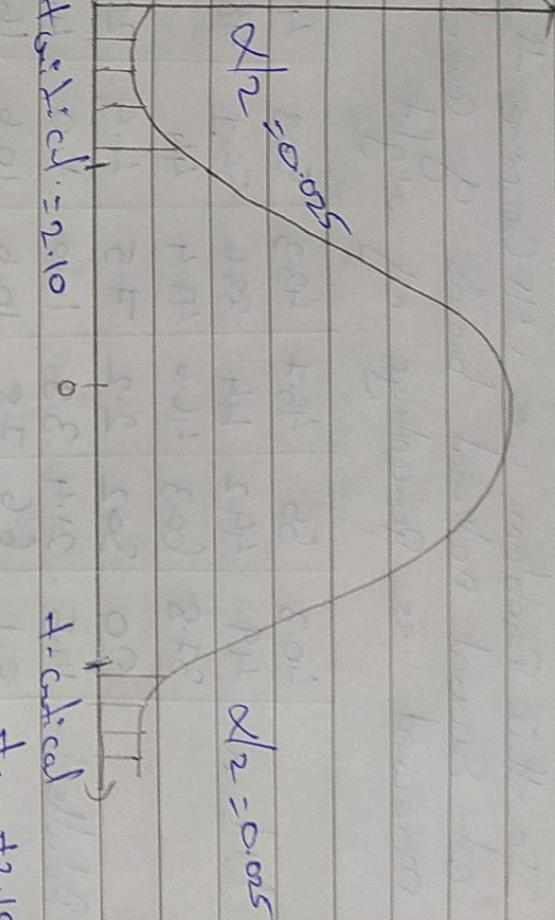
$$= \frac{23.1 - 25.1}{\sqrt{\frac{(3.72)^2 + (3.44)^2}{10 + 10}}}$$

$$t\text{-test} = 1.24$$

$$t\text{-test} = 1.24$$

(5)

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$$\begin{aligned} \alpha_1 &= 0.05 \\ \text{Actual} &= 2.10 \\ \alpha_2 &= 0.05 \end{aligned}$$

By using t -Table,

$$t_{\text{critical}} = 2.10.$$

$$P = 0.22$$

Step 7 :- Taking statistical action.

On the basis of decision rule

From t -critical

$$t_{\text{critical}} < t\text{-Score}$$

Reject null hypothesis

$$t_{\text{critical}} > t\text{-Score}$$

$$2.10 > 1.24$$

we will accept the null hypothesis.

So, on the basis of decision rule we can conclude that there is no difference between the mileage of Regular and Premium tank of the car.

$$\therefore \mu_A = \mu_B$$

(Q.7) In this Problem we will examine the sugar content at several national brands of cereals, here measured as percentage of weight

	40.3	55	45.7	43.3	50.3	45.9	53.5
children	49	44.2	44	33.6	55.1	48.8	50.4
	37.8	60.3	46.6	47.4	44		
	20	30.2	2.2	7.5	4.4	22.2	16.6
Adults	14.5	21.4	3.3	10.0	1.0	4.4	1.3
	8.1	6.6	7.8	10.6	10.6	16.2	14.5
	4.1	15.8	4.1	2.4	3.5	8.5	4.7

- a) Give a summary of two data set.
- b) Create side by side boxplots and interpret what you see.
- c) Use R to create a 95% confidence interval for the difference in mean Sugar Content and explain your result.

Step 1 : Establish Null & Alternate hypothesis.

Null hypothesis : Sugar content of brand of cereal for children & adult are same.

Alternate hypothesis : Sugar content of brand of cereals for children and adult are not same.

$$H_0 : \mu_A = \mu_B$$

$$H_a : \mu_A \neq \mu_B$$

(27)

Since we are going to check it on left and right both ends of experiment it is a two tailed test.

Step 2 :- Determine the test.

Since we are comparing the mean of two samples.

we are going to perform the t-test

Step 3 :- Set the Significance Value.

As the confidence level is given 95%.

∴ the significance level is 5%.

$$\alpha = 5\%$$

$$\alpha = 0.05$$

Since, it is a two tailed test

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

Step 4 :- Establish the determine rule.

For critical value,

If, $t_{critical} < t_{test}$
we will reject null hypothesis.

for p-value

If,
p-value < significance level
we will reject null hypothesis

Step 5: collecting the data.

Sugar Content at several national
brand of cereals (here measured as a
percentage of weight in question).

Step 6:-

	40.3	55.0	45.7	43.3	50.3	45.9	53.5
children	43.0	44.2	44.0	33.6	55.1	48.8	50.4
	37.8	60.3	46.6	47.4	44.0		
	20.0	30.2	2.2	7.5	4.4	22.2	16.6
Adults	14.5	21.4	3.3	10.0	1.0	4.4	1.3
	15.8	6.6	10.6	10.6	16.2	14.5	4.1
	15.8	4.1	2.4	3.5	8.5	7.8	4.7
	18.4						

Step 6: Analysis of data.

Since it is a t-test for two sample
random variables.

$$DF = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right]^2}{\left[\frac{\left(\frac{s_1^2}{n_1} \right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2} \right)^2}{n_2 - 1} \right]}$$

(Q)

$$t\text{-test} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

For sample of children,

$$\bar{X}_1 = M_A = 46.8$$

$$n_1 = 19$$

$$\bar{s}_1 = 6.41$$

For sample of adult,

$$\bar{X}_2 = M_B = 10.16$$

$$n_2 = 23$$

$$\bar{s}_2 = 7.47$$

$$DF = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right]^2}{\left[\frac{\left(\frac{s_1^2}{n_1} \right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2} \right)^2}{n_2 - 1} \right]}$$

$$= \frac{(2.16 + 1.92)^2}{(2.16)^2 + (1.92)^2} \times 28$$

$$= \frac{16.64}{0.38}$$

$$\boxed{DF = 43}$$

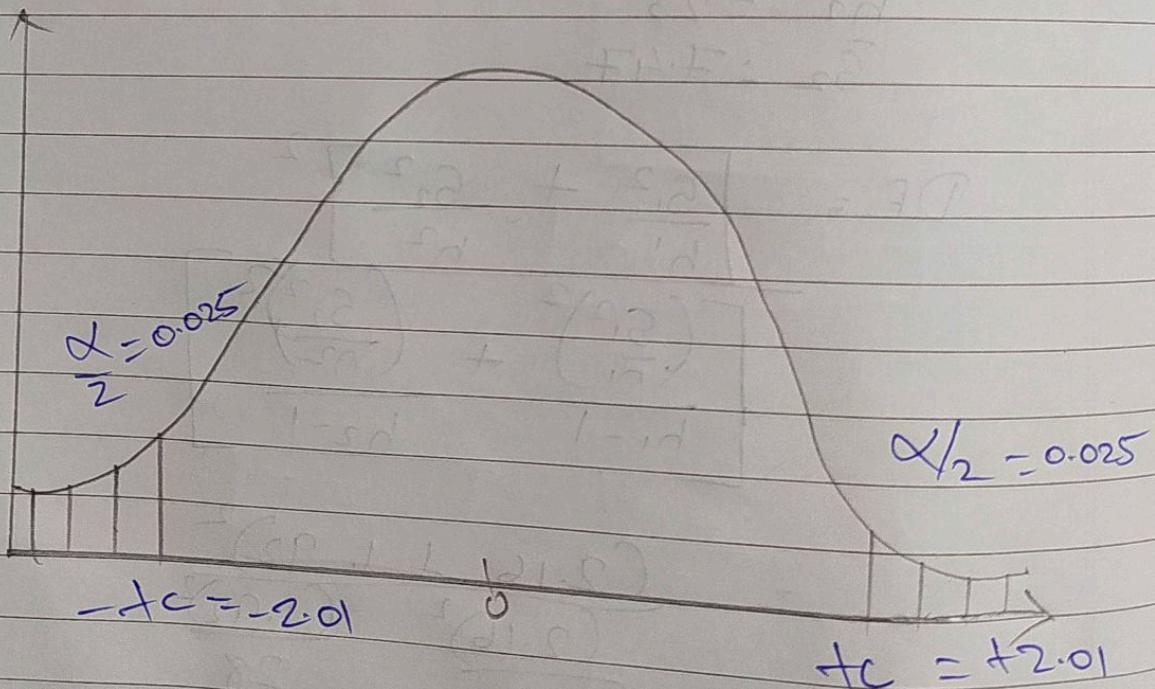
(80)

Then,
 $t\text{-test} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

$$= \frac{6.41 - 7.47}{\sqrt{\frac{(6.41)^2}{19} + \frac{(7.47)^2}{19}}}$$

$$= \frac{36.38}{1.96}$$

$$t\text{-test} = 18.10$$



By using $t\text{-table}$

$$t_c = 2.01$$

$$P\text{-Value} = 0.0001$$

(31)

Step 7 : Taking statistical action
on the basis of decision rule.

For critical value.

$$t_{\text{critical}} < t_{\text{test}} \\ 2.01 < 18.10$$

we will reject the null hypothesis

For P-values

$$P\text{-Value} < \text{significance value.} \\ 0.0001 < 0.05$$

we will reject the null hypothesis.

So, the sugar content in different brand
of cereals for children and adult are not
same or equal.