Hamming code: EC strategy

Hamming dist (d) = no. of bits by which two codewords differ
->calculated by counting the no. of 1's in the XoR result of the codewords.

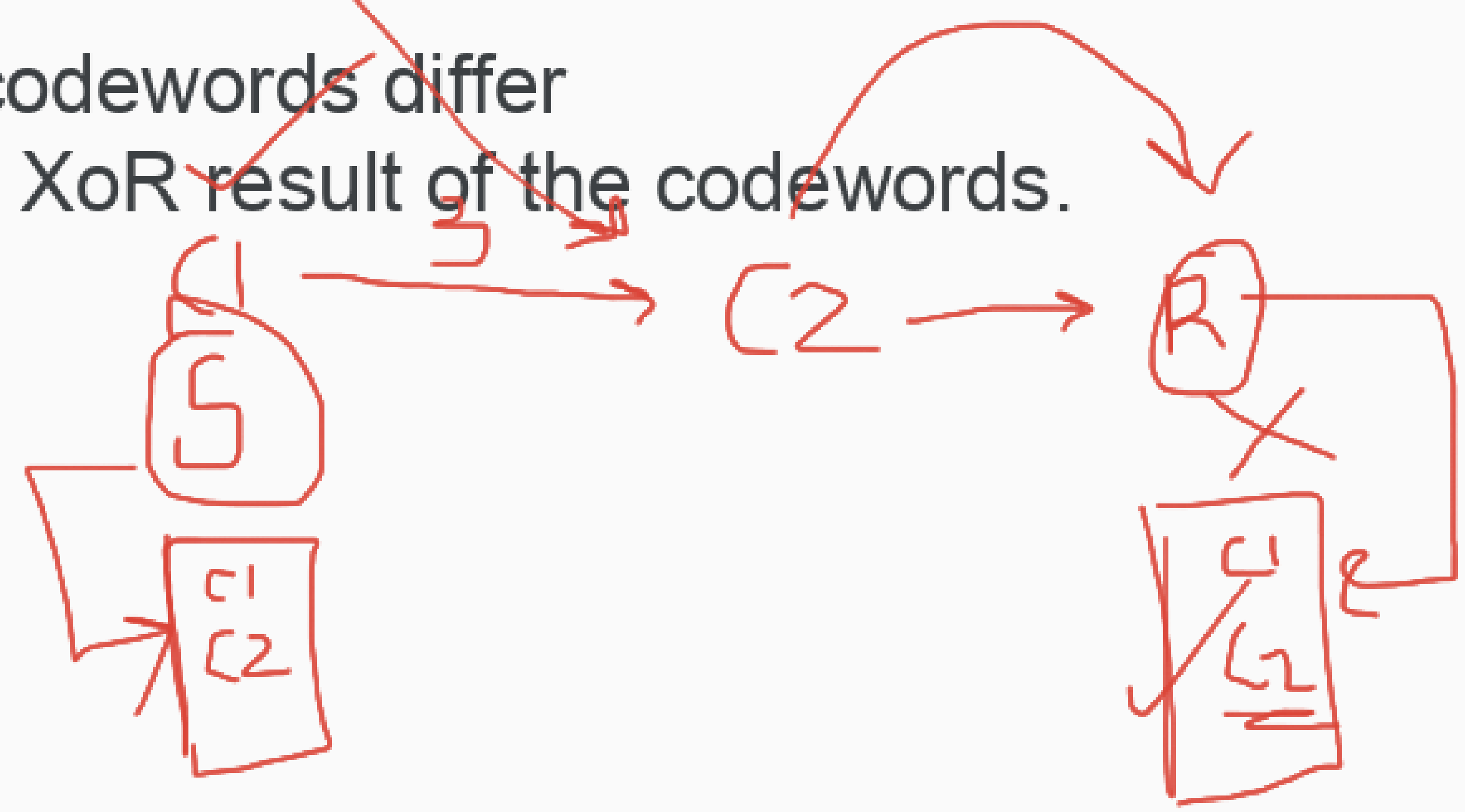
10110001

for ex,
c1= 10001001
c2= 10110001

h.dist between c1 and c2 ?

$$
\begin{array}{r}
10001001 \\
\oplus\ 10110001 \\
\hline
00111000
\end{array}
$$

→ Since, no. of bits that are 1 in the result is 3. thus, h.dist is 3



1. To detect d errors, a minimum distance of d+1 is required.
2. To correct d errors, a minimum distance of 2d+1 is required.

1

$C_1 \xrightarrow{d} C_2$

X

$C_1 \xrightarrow{d+1}$ error

2

$d+1+d$

Hamming code is a set of error-correction codes that can be used to detect and correct the errors that can occur when the data is moved or stored from the sender to the receiver.

Redundant bits – These are extra binary bits that are generated and added to the information-carrying bits of data transfer to ensure that no bits were lost during the data transfer.

The number of redundant bits can be calculated using the following formula:

$2^r \geq m + r + 1$

where, r = redundant bit, m = data bit

Suppose the number of data bits is 7, then the number of redundant bits can be calculated using:

$= 2^4 \geq 7 + 4 + 1$

Thus, the number of redundant bits= 4

Given: m bits msg [m1m2m3m4m5m6m7]
find: n bits codeword

1. find the number of redundant/check bits.
(m+r+1) <= 2^r

for eg, if m=7, r=?
=>7+r+1 <=2^r
=> 8+r <=2^r
=> [r=4]

2. place the check bits at positions of powers of 2, i.e. 2^0, 2^1, 2^2 and so on
c1c2m1c3m2m3m4c4m5m6m7

we know, n=m+r = 7+4 =11

2^0	2^1		2^2				2^3			
1	2	3	4	5	6	7	8	9	10	11
c1	c2	m1	c3	m2	m3		C4			

3. Express data/msg bit positions as the sum of check bits

msg bit positions: 3,5,6,7,9,10,11

3=1+2
5=1+4
6=2+4
7=1+2+4
9=1+8
10=2+8
11=1+2+8

4. Note down the contributing data bits for a check bit

c1= 3,5,7,9,11
c2= 3,6,7,10,11
c4= 5,6,7
c8= 9,10,11

5. Calculation of check bits
-concept of odd/even parity is used for this calculations

-odd parity
no.of 1's even => parity bit =1
else parity bit =0

Example

given m=1001000
find hamming codeword for it.

1. m+r+1 <= 2^r
r=4

2. n=m+r = 11bits

3.

1	2	3	4	5	6	7	8	9	10	11
c1	c2	1	c4	0	0	1	c8	0	0	0

4.
3=1+2
5=1+4
6=2+4
7=1+2+4
9=1+8
10=2+8
11=1+2+8

5. c1-> 3,5,7,9,11
1 0 1 0 0 => even parity
c1=0

c2-> 3,6,7,10,11
1 0 1 0 0
c2=0

c4-> 5,6,7
0 0 1
c4=1

c8-> 9,10,11
0 0 0
c8=0

codeword

00110010000

Q2

m= 1100101