Q. 20th Jan 2026

$\Sigma = \{a, b, c\}$

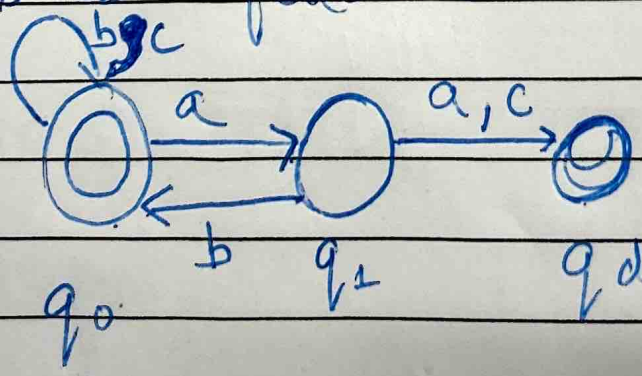
$A(x), B(x), C(x)$: x position contains a, b, c respectively.

$S(x, y)$: y is successor of x.

Now. $\forall x\,(A(x) \rightarrow \exists y(S(x,y) \wedge B(y))$

So every 'a' is followed by a 'b'.

Is this regular? Yes. Finite automata. is as follows:



So it is regular.

Yes, every language defined by such FOL ~~over~~ formula regular.

Because:

1) FOL over position can only talk about local relationship (adjacency, ordering, existence, etc)

2) They can't count unboundedly

Yes, every regular language can be expressed by such an FOL formula.

Sketch of proof:

- Let DFA have states $q_0, \ldots, q_i$
- We introduce many predicates $Q_i(n)$ s.t. at position n, DFA state is q_i
- Now A) Inital state at position 1

2) Transition using successor:

$\forall x \forall y (S(x,y) \rightarrow$ correct transition holds)

3) Last position must be accepting state.