Week 11 - ReadMe

We extend the synthesis tool to quadratic invariants. The high level idea is the same as Week 6's tool.

High-level sketch

1. **Parse the Dafny file** (Week 5 parser or JSON summary).
2. **Extract** the target loop, guard condition, variables, and assignments.
3. **Enumerate candidate quadratic templates** by choosing small integer coefficients.
4. **Check each candidate** with Z3:
5. **Return the strongest few invariants**, ranked by structural complexity.

Example

Input

```
method TriangularSum(n: int) returns (k: int, sum: int)
  requires n >= 0
  ensures sum == n * (n - 1) / 2
{
  k := 0;
  sum := 0;

  while k < n
    // manually added invariants
    invariant 0 <= k <= n
    invariant sum == k * (k - 1) / 2

    // synthesized invariants:
    invariant -1*k*k + 2*sum <= 0
    invariant -1*k*k + 2*sum - 1 <= 0
    invariant -1*k*k + 2*sum - 2 <= 0
    invariant -1*k*k + 2*sum - 5 <= 0
```

```
    decreases n - k
  {
    sum := sum + k;
    k := k + 1;
  }
}
```

Terminal Output

```
{
  "file": "week3\\quadratic_triangular.dfy",
  "methods": [
    {
      "method_name": "TriangularSum",
      "preconditions": ["n >= 0"],
      "postconditions": ["sum == n * (n - 1) / 2"],

      "parameters": [
        {"name": "n", "type": "int"}
      ],
      "returns": [
        {"name": "k", "type": "int"},
        {"name": "sum", "type": "int"}
      ],

      "loops": [
        {
          "condition": "k < n",
          "variables": ["k", "sum"],
          "template_variables": ["k", "sum"],
          "template": "a*x^2 + b*y^2 + c*x*y + d*x + e*y + f <= 0",

          "quadratic_invariants": [
            "-1*k*k + 2*sum <= 0",
            "-1*k*k + 2*sum - 1 <= 0",
            "-1*k*k + 2*sum - 2 <= 0",
            "-1*k*k + 2*sum - 3 <= 0",
```

```
        "-1*k*k + 2*sum - 5 <= 0"
       ]
      }
     ]
    }
   ]
}
```