Lab 6

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Problem: Suppose you are playing a game where you roll a fair six-sided die. Let X be the random variable representing the outcome of the die roll.

1. Compute the expectation E[X] of the random variable X.
2. Compute the expectation E[X^2] of the random variable X.
3. Using the formula for variance, calculate the variance of X.

Solution:

Random Variable (X): A random variable is a quantity whose possible values depend on the outcome of a random process, each value having a certain probability.

Expectation $E[X]$: The expectation, denoted as $E[X]$, signifies the average value of the random variable X across multiple trials. It's determined by multiplying each potential value of X by its probability and summing these products.

$$E[X] = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$$

Expectation of Squared Outcome $E[X^2]$: We can find the expectation of the squared outcome of X, indicating its average value squared over numerous trials.

$$E[X^2] = \sum_{i=1}^{n} x_i^2 \cdot P(X = x_i)$$

Variance (Var$[X]$): Variance quantifies the spread of values of a random variable around its mean (expected value). It's computed by averaging the squared deviations of each value from the mean.

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

Example: Consider a fair six-sided die, where each face has an equal probability of $\frac{1}{6}$. Let X be the random variable representing the outcome of rolling the die.

$$E[X] = \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6) = 3.5$$

$$E[X^2] = \frac{1}{6}(1^2) + \frac{1}{6}(2^2) + \frac{1}{6}(3^2) + \frac{1}{6}(4^2) + \frac{1}{6}(5^2) + \frac{1}{6}(6^2) = 15.17$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 = 15.17 - (3.5)^2 = 2.92$$

Implementation in C++

```cpp
#include <iostream>
#include <cmath>
using namespace std;

class Die {
private:
    int sides;

public:
    Die(int sides) : sides(sides) {}

    double expectation() const {
        return (1.0 + sides) / 2.0;
    }

    double expectationSquared() const {
        double sum = 0.0;
        for (int i = 1; i <= sides; ++i)
            sum += pow(i, 2);
        return sum / sides;
    }

    double variance() const {
        double ex = expectation();
        double exSquared = expectationSquared();
        return exSquared - pow(ex, 2);
    }
};

int main() {
    int sides = 10; // Number of sides on the die
    // Create a Die object
    Die die(sides);

    // Calculate and print expectation E[X]
    cout << "Expectation E[X]: " << die.expectation() << endl;
    // Calculate and print expectation E[X^2]
    cout << "Expectation E[X^2]: " << die.expectationSquared() << endl;
    // Calculate and print variance of X
    cout << "Variance of X: " << die.variance() << endl;
    return 0;
}
```

Time Complexity: $O(n)$

Output:

```
Expectation E[X]: 3.5
Expectation E[X^2]: 15.1667
Variance of X: 2.91667
```