Assignment

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Problem 1

Generate 100 random from a Bivariate Generalized exponential, Bivariate Weibull distribution.

Bivariate Generalized Exponential

To generate Random Variables From Bivariate Generalized Exponential Distribution We can use following Steps

```
1. Generate U_1 \sim GE(\alpha_1, \lambda), U_2 \sim GE(\alpha_2, \lambda), U_3 \sim GE(\alpha_3, \lambda)
2. Assign X_1 = max\{U_1, U_3\} and X_2 = max\{U_2, U_3\}
3. Return (X_1, X_2)
```

Now to Generate Random Numbers from Generalized Exponential Distribution we can use Inverse Transform Method as follows

CDF of Generealized Exponential is given by $F(x,\alpha,\lambda) = (1-e^{-\lambda x})^{\alpha}$ $U = (1-e^{-\lambda x})^{\alpha}$ $x = -\frac{1}{\lambda}ln\left(1-U^{\frac{1}{\alpha}}\right)$

Creating a function to generate Random Variable from Generalized Exponential

```
rge <- function(alpha , lambda){
  u = runif(1)
  x = -log(1 - u^(1/alpha))/lambda
  return(x)
}</pre>
```

Creating a function to generate Random Variable from Bivariate Generalized Exponential

```
rbge <- function(alpha1,alpha2,alpha3,lambda){
    u1 = rge(alpha1,lambda)
    u2 = rge(alpha2,lambda)
    u3 = rge(alpha3,lambda)
    x1 = max(u1,u3)
    x2 = max(u2,u3)
    x = c(x1,x2)
    return(x)
}</pre>
```

Code

```
a = c()
for(i in 1:100){
    x = rbge(20,24,32,0.1)
    a = rbind(a,x)
}
print(a)
## [,1] [,2]
```

```
## x 32.98167
              38.28575
## x 50.90362 51.30821
## x 69.21246 35.88713
## x 30.77216 66.00130
## x 44.74599 44.74599
## x 32.86428
              30.96410
## x 34.85648 35.55930
## x 39.72127
              28.95534
## x 44.11453 44.11453
## x 30.38555
              27.84260
## x 44.35035 38.35938
## x 57.96818 57.96818
## x 49.90315 43.73717
## x 49.50310 49.50310
## x 36.84536 36.84536
## x 37.15952 36.81199
## x 25.05961 48.99083
## x 45.83839 45.83839
## x 63.31067
              50.87229
## x 33.64776 33.64776
## x 32.35209
              64.49875
## x 32.23705 32.23705
## x 34.61133 28.99005
## x 33.13287 33.13287
## x 48.76130 48.76130
## x 45.58545 41.37105
## x 35.09156 62.60688
## x 46.89798 46.89798
## x 42.77921 42.77921
## x 44.69805 76.04416
## x 48.57341 31.83144
              27.56855
## x 53.40027
## x 40.26329 40.26329
## x 65.93334 66.20889
## x 78.88973 78.88973
## x 45.12570
              62.62375
## x 47.16711 50.81783
## x 44.07527 44.07527
## x 43.16186 43.16186
## x 55.43809
              44.28747
## x 48.17708 37.72436
## x 46.59593 46.59593
```

x 43.18800 43.18800

```
## x 27.27219 34.09563
## x 35.98047 35.95372
## x 32.08790 32.08790
## x 27.76454 35.57212
## x 36.78308
              42.54361
## x 55.75893 30.29091
## x 42.97378
              42.97378
              35.83629
## x 33.36168
## x 48.10890
              65.37916
## x 51.30314 51.30314
## x 34.17851 32.59459
## x 37.47508 30.49478
## x 47.64433 47.64433
## x 44.63731
              33.22850
## x 47.09776
              26.36653
## x 35.75864
              65.18044
## x 42.71531
              42.71531
## x 38.45816 38.45816
## x 33.72060 108.89004
## x 40.42622 40.42622
## x 33.29870 55.38058
## x 31.41928 31.35238
## x 35.97464 43.49078
## x 61.08670
              61.08670
## x 72.92120 72.92120
## x 36.71710 36.71710
## x 82.40466 82.40466
## x 64.07384 43.20974
## x 50.87984
              44.64367
## x 45.41981
              46.11304
              70.23372
## x 36.82850
## x 38.34960
              78.05238
## x 29.56275
              50.17854
## x 36.52012 36.93764
## x 47.11547
              53.72321
## x 56.68117
              41.55774
## x 38.32497
              49.85134
## x 48.49802 64.73851
## x 42.88559
              42.88559
## x 38.04049 37.56887
## x 34.87315 39.12049
## x 32.55250 30.00176
## x 48.62505
              33.10007
## x 46.95719
              46.95719
## x 26.94416
              28.58011
## x 54.59511
              54.59511
## x 79.35683
              35.62576
## x 40.90355
              46.56381
## x 34.08399
             54.58121
## x 43.12482
              43.12482
## x 51.24623
              45.39755
## x 39.64014 39.64014
## x 40.87362 38.83317
## x 45.80782 45.80782
```

```
## x 37.14929 37.14929
## x 35.09541 41.19232
## x 55.71332 55.71332
```

Bivariate Weibull Distribution

To generate Random Variables From Bivariate Weibull Distribution We can use following Steps

```
1. Generate U_1 \sim WE(\alpha, \lambda_1), U_2 \sim WE(\alpha, \lambda_2), U_3 \sim WE(\alpha, \lambda_3)
2. Assign X_1 = min\{U_1, U_2\} and X_2 = min\{U_1, U_3\}
3. Return (X_1, X_2)
```

Code

```
rbwd <- function(n,alpha,lambda1,lambda2,lambda3){
    #u1 = rweibull(n,alpha,1/lambda1)
    #u2 = rweibull(n,alpha,1/lambda2)
    #u3 = rweibull(n,alpha,1/lambda3)
    u1 = (-log(1-runif(n))/lambda1)^(1/alpha)
    u2 = (-log(1-runif(n))/lambda2)^(1/alpha)
    u3 = (-log(1-runif(n))/lambda3)^(1/alpha)
    x1 = pmin(u1,u2)
    x2 = pmin(u1,u3)
    x = cbind(x1,x2)
    return(x)
}</pre>
```

```
b = rbwd(100,10,22,3,7)
print(b)
```

```
##
                 x1
                           x2
##
     [1,] 0.4376432 0.4376432
##
     [2,] 0.7683868 0.7435689
##
     [3,] 0.7580092 0.6828425
##
     [4,] 0.6168975 0.6168975
     [5,] 0.6346159 0.6346159
##
     [6,] 0.7683515 0.7683515
##
     [7,] 0.7848283 0.5333769
##
##
     [8,] 0.7998586 0.8568250
##
     [9,] 0.5049943 0.5049943
## [10,] 0.6813877 0.6813877
   [11,] 0.7034721 0.7034721
##
##
   [12,] 0.7320021 0.7320021
  [13,] 0.7388841 0.7388841
## [14,] 0.5813444 0.6657414
##
   [15,] 0.5827276 0.5827276
## [16,] 0.7195141 0.7195141
## [17,] 0.6149591 0.6149591
## [18,] 0.7579327 0.7579327
##
   [19,] 0.6286172 0.6286172
## [20,] 0.7116264 0.7725224
## [21,] 0.7007504 0.6302454
## [22,] 0.5941464 0.5941464
```

```
[23,] 0.7066732 0.4106476
    [24,] 0.7591608 0.7530800
    [25,] 0.7011768 0.7011768
   [26,] 0.6943094 0.6943094
    [27,] 0.7511172 0.7511172
##
   [28,] 0.6737623 0.6737623
    [29,] 0.7287660 0.7438368
##
    [30,] 0.7241101 0.7241101
    [31,] 0.7419701 0.7419701
##
    [32,] 0.6884241 0.6884241
    [33,] 0.6232324 0.6232324
##
    [34,] 0.6742368 0.6742368
    [35,] 0.8013463 0.8013463
##
    [36,] 0.7736404 0.7202582
    [37,] 0.7529167 0.7529167
##
    [38,] 0.5994939 0.5994939
##
    [39,] 0.5960961 0.5960961
    [40,] 0.6385933 0.6385933
   [41,] 0.7116130 0.7116130
    [42,] 0.5792930 0.5792930
##
    [43,] 0.8271750 0.7944939
    [44,] 0.5038562 0.5038562
##
    [45,] 0.6399992 0.6399992
    [46.] 0.6420831 0.6420831
##
    [47,] 0.5994543 0.5994543
   [48,] 0.6373942 0.6373942
##
   [49,] 0.6770474 0.6770474
    [50,] 0.6578934 0.6578934
   [51,] 0.6231399 0.6231399
    [52,] 0.7560412 0.7560412
    [53,] 0.7970370 0.7970370
##
    [54,] 0.6548575 0.8154473
    [55,] 0.7240459 0.7240459
   [56,] 0.7438969 0.7438969
##
    [57,] 0.6132058 0.6132058
    [58,] 0.7160790 0.7160790
    [59,] 0.7145191 0.7145191
##
    [60,] 0.7335338 0.7335338
    [61,] 0.7297332 0.7297332
##
    [62,] 0.6572135 0.6572135
    [63,] 0.7506818 0.7506818
##
    [64,] 0.7105426 0.6863287
    [65,] 0.7024244 0.7707047
##
    [66,] 0.7090964 0.7090964
    [67,] 0.6593639 0.6593639
##
    [68,] 0.6838953 0.6838953
    [69,] 0.4944332 0.4944332
    [70,] 0.7958726 0.7958726
   [71,] 0.6779947 0.6779947
##
   [72,] 0.6256031 0.7912275
##
   [73,] 0.6797429 0.6797429
##
   [74,] 0.7178073 0.7178073
  [75,] 0.6309862 0.6309862
## [76,] 0.6797688 0.6650911
```

```
##
    [77,] 0.6907832 0.6907832
##
   [78,] 0.5211621 0.5211621
##
   [79,] 0.7328634 0.7107630
   [80,] 0.6173766 0.6173766
##
##
    [81,] 0.5996414 0.5996414
   [82,] 0.8052665 0.8052665
##
   [83,] 0.5968458 0.5968458
##
##
    [84,] 0.6059696 0.6059696
##
    [85,] 0.6214558 0.6214558
##
   [86,] 0.6232360 0.6232360
   [87,] 0.7251119 0.7251119
   [88,] 0.7214239 0.7214239
##
##
   [89,] 0.7576003 0.7543790
   [90,] 0.7375451 0.7375451
##
##
   [91,] 0.5135439 0.5135439
##
    [92,] 0.6744597 0.6808289
##
   [93,] 0.7274869 0.7274869
##
   [94,] 0.6151761 0.6151761
   [95,] 0.7769037 0.5140910
##
##
   [96,] 0.7425012 0.7425012
##
  [97,] 0.6778900 0.5321962
  [98,] 0.6644253 0.6644253
## [99,] 0.7499110 0.7499110
## [100,] 0.6966879 0.6966879
```

Problem 2

Generate 100 random from an absolutely continuous bivariate Generalized exponential, absolutely continuous Bivariate Weibull distribution

Absolutely Continuous Bivariate Generalized Exponential

To Get Random Number with absolutely continuous bivariate Generalized exponential we can use the same procedure as we have done in Problem 1. We just have to take care of singular points so we will reject the sample when $X_1 = X_2$

Code

```
c = c()
while(length(c)<200){
    x = rbge(1,2,3,4)
    if(x[1] == x[2]){
        next
    }
    c = rbind(c,x)
}
print(c)</pre>
```

```
## [,1] [,2]

## x 0.52331296 0.4896304

## x 0.77477916 0.8553549

## x 0.24920121 0.4316822

## x 0.27774605 0.3679842

## x 0.09057178 0.3389646
```

```
## x 0.48417283 0.7214691
```

- ## x 0.28335816 0.6838526
- ## x 0.43024667 0.6295717
- ## x 0.52444719 0.5733910
- ## x 1.03150534 0.7887037
- ## x 0.47287342 0.6019730
- ## x 0.22105280 0.1740717
- ## x 0.12090370 0.8841873
- ## x 0.76467017 0.5927732
- ## x 0.55598486 0.5090078
- ## x 0.27908759 0.2530989
- "" A 0.27000700 0.2000000
- ## x 0.47689990 1.1105562
- ## x 0.24618189 0.4150565
- ## x 0.19765002 1.2813366
- ## x 0.61325402 0.2202113
- ## x 0.48236704 0.2142941
- ## x 0.64692456 0.4932252
- ## x 0.37447688 1.5141102
- ## x 0.34685416 0.2510132
- ## x 0.42943685 0.3621314
- ## x 0.30518875 0.6119012
- ## x 0.13362094 0.1484226
- ## X 0.13302094 0.1404220
- ## x 0.53240594 0.3478859
- ## x 0.07586211 0.3474518
- ## x 0.13559948 0.2956995
- ## x 1.15636703 0.2677929
- ## x 0.21157397 0.7762685
- ## x 0.37535201 0.2456984
- ## x 0.48454519 0.7885115
- ## x 0.22467831 0.4460881
- ## x 0.70972116 1.1615541
- ## x 0.64908383 0.3776358
- ## x 0.13846001 0.1838414
- ## x 0.26897790 0.2991643
- ## x 0.72034905 0.6747098
- ## x 0.73947849 0.1367004
- ## x 0.36594795 0.3256849
- ## x 0.43579579 0.4788031
- ## x 0.49686939 0.7309911
- ## x 0.44525361 0.5338485
- ## x 0.44926099 0.6951170
- ## x 0.33313032 0.8747752
- ## x 0.44406750 0.5660730
- ## x 0.50736720 0.3582661
- ## x 0.38802957 0.4071692
- ## x 0.32357418 0.3808982
- ## x 0.18550558 0.6532238
- ## x 0.44657081 0.2139014 ## x 0.27305533 0.4876360
- ## x 0.28847711 0.2298697
- ## x 0.34605716 0.5503920
- ## x 0.41296778 0.4564674
- ## x 0.69643012 0.3244819
- ## x 0.80761023 0.3138947

```
## x 0.47793877 0.4324659
## x 0.13200287 0.1149219
## x 0.24992324 0.7429885
## x 1.02324001 0.2188300
## x 0.29151285 1.2074265
## x 0.47591818 0.5676950
## x 0.15229243 0.5025883
## x 0.32284322 0.3549467
## x 0.82479677 1.6775123
## x 0.20869499 0.3123936
## x 0.43252102 0.6892547
## x 0.69697074 0.3510380
## x 0.40219059 0.5030779
## x 0.22617401 0.6275296
## x 0.71358617 0.5362875
## x 0.80371069 0.7245815
## x 0.29207499 0.6925190
## x 0.36238450 0.5841089
## x 0.72404001 0.3877413
## x 0.30115375 0.2884593
## x 0.15937137 0.3321843
## x 0.66234178 0.4339677
## x 0.50576058 0.3921808
## x 0.15143163 0.4503879
## x 0.19649532 0.3689760
## x 0.07113336 0.1386867
## x 0.16845684 0.1996286
## x 0.54596817 1.5002297
## x 0.62469369 0.2753200
## x 0.22647475 0.4259819
## x 0.27363840 0.2784844
## x 0.06067439 0.4523126
## x 0.14180286 0.3090951
## x 0.21673187 0.9564048
## x 0.85771525 0.8875275
## x 0.70104101 0.7236980
## x 0.10591453 0.1258520
## x 0.71947941 0.3373176
## x 0.28119655 0.3146871
## x 0.36099244 0.3644119
## x 0.32046737 0.4866876
```

Absolutely Continuous Bivariate Weibull distribution

Similarly as above we can write

```
d = c()
while(length(d)<200){
    x = rbwd(1,1,2,3,4)
    if(x[1] == x[2]){
        next
    }
    d = rbind(d,x)
}
print(d)</pre>
```

```
##
                                x2
                   x1
##
     [1,] 0.258290415 0.021660739
##
     [2,] 0.206460744 0.048642293
     [3,] 0.006894769 0.054326843
##
##
     [4,] 0.079892589 0.122466582
     [5,] 0.160722733 0.520115132
##
##
     [6,] 0.336406017 0.049621636
##
     [7,] 0.515837756 0.087877238
##
     [8,] 0.486375358 0.198934681
##
     [9,] 0.369776379 0.668619929
##
    [10,] 0.140754605 0.124989917
##
    [11,] 0.177093389 0.121225979
    [12,] 0.296024350 0.087797239
    [13,] 0.312592836 0.039389532
##
    [14,] 0.553326001 0.171610154
##
    [15,] 1.057984320 0.095461673
    [16,] 0.011207560 0.030626680
##
    [17,] 0.333017242 0.029200470
##
    [18,] 0.302978167 0.091572250
##
    [19,] 0.525073689 0.333080117
    [20,] 0.123376890 0.156852279
##
    [21,] 0.005662339 0.343342640
##
    [22,] 0.128349997 0.197144041
##
    [23,] 0.446115967 0.288074632
    [24,] 0.164576864 0.133737647
    [25,] 0.194593186 0.516076360
##
##
    [26,] 0.771319743 0.459670273
##
    [27,] 0.096000546 0.525408858
##
    [28,] 0.065155146 0.098722273
##
    [29,] 0.545550780 0.028472010
##
    [30,] 0.163207392 0.092637534
##
    [31,] 0.258210520 0.327617144
##
    [32,] 0.015075107 0.310750968
##
    [33,] 0.041781232 0.291257351
##
    [34,] 0.036869919 0.294047265
    [35,] 0.022507748 0.067063790
##
    [36,] 0.034403454 0.008316772
    [37,] 0.553037369 0.101810353
##
    [38,] 0.333412085 0.001306942
    [39,] 0.002733268 0.067859324
##
    [40,] 0.295960139 0.081079222
    [41.] 0.048407514 0.218468898
##
##
    [42,] 0.110651661 0.139692052
    [43,] 0.432234578 0.335533302
##
    [44,] 0.504211791 0.611426209
##
    [45,] 0.072681915 0.273342103
##
    [46,] 0.148101280 0.396687222
    [47,] 0.092610080 0.015126410
##
##
    [48,] 0.221743826 0.157050793
##
    [49,] 0.111932312 0.080961355
    [50,] 0.154538096 0.116095997
##
    [51,] 0.744290497 0.026804982
```

```
[52,] 0.457439086 0.070128385
##
    [53,] 0.105514253 0.093235548
    [54,] 0.156568834 0.430395668
    [55,] 0.404500576 0.006868035
    [56,] 0.146800951 0.149339694
##
    [57,] 0.155939675 0.115763915
    [58,] 0.169189945 0.154498792
##
    [59,] 0.135722903 0.128973373
    [60,] 0.159090112 0.002116385
##
    [61,] 0.110547357 0.241721457
    [62,] 0.153169785 0.097117484
##
    [63,] 0.078066414 0.148543797
    [64,] 0.249655939 0.251920348
##
    [65,] 0.279557527 0.103910144
    [66,] 0.286824958 0.055153575
##
    [67,] 0.117526301 0.457856216
##
    [68,] 0.186271484 0.042445537
##
    [69,] 0.087575774 0.304179454
    [70,] 0.143536774 0.404403695
    [71,] 0.056315369 0.094044390
##
    [72,] 0.135747188 0.208927205
    [73,] 0.385147818 0.111668267
##
    [74,] 0.172266301 0.031940945
    [75.] 0.524794760 0.069575600
##
    [76,] 0.161880754 0.137491898
    [77,] 0.156482643 0.038289395
##
    [78,] 0.053920200 0.033856051
    [79,] 0.411810287 0.171391353
    [80,] 0.066407708 0.095905769
    [81,] 0.145143012 0.012685676
    [82,] 0.329810230 0.067382862
##
    [83,] 0.207322419 0.058855379
##
    [84,] 0.150402570 0.172670780
    [85,] 0.173251902 0.505122873
##
##
    [86,] 0.375654870 0.060933582
    [87,] 0.084888622 0.256407428
##
    [88,] 0.110241856 0.049709887
##
    [89,] 0.042172031 0.177222836
##
    [90,] 0.115822372 0.050404288
##
    [91,] 0.107161822 0.138773458
    [92,] 0.026526026 0.001404156
##
    [93,] 0.288105333 0.026543883
    [94,] 0.191404751 0.302837307
##
   [95,] 0.086986002 0.412728463
   [96,] 0.012758007 0.399465386
##
   [97,] 0.188138288 0.007132840
   [98,] 0.014919704 0.530861644
  [99,] 0.345279610 0.067674348
## [100,] 0.359263131 0.133176376
```