Computing Integrals Monte Carlo Way

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Introduction

Let us we have θ with posterior density $\pi(\theta|x)$,then the posterior mean is given by

$$\mu(\theta) = \int_{-\infty}^{\infty} \pi(\theta|x)\mathrm{d}\theta$$

And the posterior of a particular function $h(\theta)$ is given by

$$E(h(\theta)|x) = \int_{-\infty}^{\infty} \pi(\theta|x)h(\theta)\mathrm{d}\theta$$

Then through the Monte Carlo method we can estimate the posterior mean of $E(h(\theta)|x)$ by by taking N samples from the posterior distribution $\pi(\theta|x)$. And the posterior mean can be estimated by

$$\bar{h} = \frac{1}{N}\sum_{i=1}^{N} h(\theta_i)$$

The Monte Carlo Method

Suppose we need to compute the integral

$$\int_0^1 12x^3(1-x)^2\mathrm{d}x$$

If we do integral manually we will get 0.25. But if we use the Monte Carlo method, we can get the same answer by taking N samples from the posterior distribution If we look closely its a beta distibution with parameter $\alpha = 2$ and $\beta = 3$. and it is a second moment of the beta distribution. that mean $h(x) = x^2$. so we have to take N samples from the beta distribution with parameter $\alpha = 2$ and $\beta = 3$.and square them. and at last take mean of them

```
sample = rbeta(n = 100000, 2 , 3) # sample from beta distribution with parameters 2 and 3
sample_squared = sample^2
mean = mean(sample_squared)
standard_error = sd(sample_squared) / sqrt(100000)
cat("Estimate of the integral is ", mean, " with standard error ", standard_error, sep = "")
```

```
## Estimate of the integral is 0.1990839 with standard error 0.000561259
```