Practical 1

Calculate the dominant eigenvalue and corresponding eigenvector of following matrix using Power method

$$\begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

Method

We will use power method , in which first of all we take a matrix with entry 1,1,1 as initial eigeenvector then further multiply this with the matrix and we will get an another 3×1 matrix then we will divide each element of the obtained matrix by maximum of that matrix , then we will repeat same till we get a reliable estimate , and we can further calculate dominant eigenvector by

$$\lambda = \frac{A \times x.x}{x.x}$$

Workout

First of all we need to put the matrix in R

```
rm(list=ls())
B<-matrix(c(1,2,0,-2,1,2,1,3,1),ncol=3,byrow=1)
a<-matrix((rep(1,3)),ncol=1)
print(B)</pre>
```

```
## [,1] [,2] [,3]
## [1,] 1 2 0
## [2,] -2 1 2
## [3,] 1 3 1
```

```
print(a)
```

```
## [,1]
## [1,] 1
## [2,] 1
## [3,] 1
```

Define two datatype as following

```
temp2<-a #To store the number of iterations
```

Now doing iterations to calculate eigenvalue

```
repeat
{
  temp1 <- B %*% a
  temp1 <- temp1/max(temp1)</pre>
                                          #dividing by the maximum
  temp1<- round(temp1,4)</pre>
                                          #rounding upto 4 decimal places
  i<- i+1
                                          #Increasing i to count iteration
  cat( "\t",i," Iteration"," \t ")
  cat(temp1 ,"\n ")
  a <- temp1
  if(all(temp2[1:3] == temp1[1:3]))
                                         #checking condition to stop iteration
    break
  temp2<-temp1
}
```

```
0.6 0.2 1
##
       Iteration
       2 Iteration 0.4545 0.4545 1
##
                       0.4839 0.5484 1
##
       3 Iteration
##
       4 Iteration
                      0.5052 0.5051 1
##
       5 Iteration
                      0.5017 0.4949 1
       6 Iteration
                      0.4994 0.4994 1
##
       7 Iteration
                      0.4998 0.5006 1
##
##
       8 Iteration
                      0.5001 0.5001 1
       9 Iteration
##
                      0.5 0.4999 1
       10 Iteration
##
                      0.5 0.5 1
       11 Iteration
##
                       0.5 0.5 1
##
```

Now calculating and printing eigenvalue of the eigenvector we got in 11 iteration

```
cat("\t Dominant eigenvalue is ", sum((B %*% temp1) * temp1)/sum( temp1 * temp1))
```

```
## Dominant eigenvalue is 3
```

Conclusion

After 11 iteration we got eigenvalue and eigenvector

$$eigenvector = \begin{bmatrix} 0.5, 0.5, 1 \end{bmatrix}^T \qquad eigenvalue = 3$$