18-100 Introduction to Electrical and Computer Engineering

Lecture 03
Equivalent Circuits

Schedule (Subject to Change)

Week	Date	Day	Lecture Topic
1	13-Jan	M	L01: Intro, Physics, EM, Leveling Students
	15-Jan	W	L02: Circuits Basics
2	20-Jan	M	Martin Luther King Celebration (No Lecture)
	22-Jan	W	L03: Equivalent Circuits
3	27-Jan	M	L04: Semiconductors, Diodes, LEDs
	29-Jan	W	L05: MOSFETs to Simple Gates
4	3-Feb	M	L06: Professional Identity, Professional Responsibility, and Ethics
	5-Feb	**W**	**Exam 1**
5	10-Feb	M	L07: Capacitors, RC Time Constants, RC Circuits
	12-Feb	W	L08: Inductors, RL Time Constants, 555
6	17-Feb	M	L09: Binary, Logic Gates, Boolean Logic
	19-Feb	W	L10: Latches, Registers, RAM, Flip-Flops
7	24-Feb	M	L11: Computers
	26-Feb	W	L12: Op Amps
	3-Mar	M	SPRING BREAK
	5-Mar	W	SPRING BREAK

Objectives of this Lecture

- Review
- Power
- Equivalent Circuits
- Voltage Division and Current Division

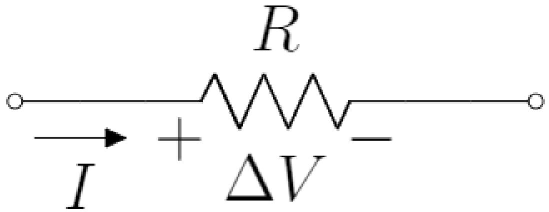
Electrical and Computer Engineering Models

$$\Delta V = IR$$

$$\sum i_{IN} = \sum i_{OUT}$$

$$\sum v_{UP} = \sum v_{DOWN}$$

Ohm's Law



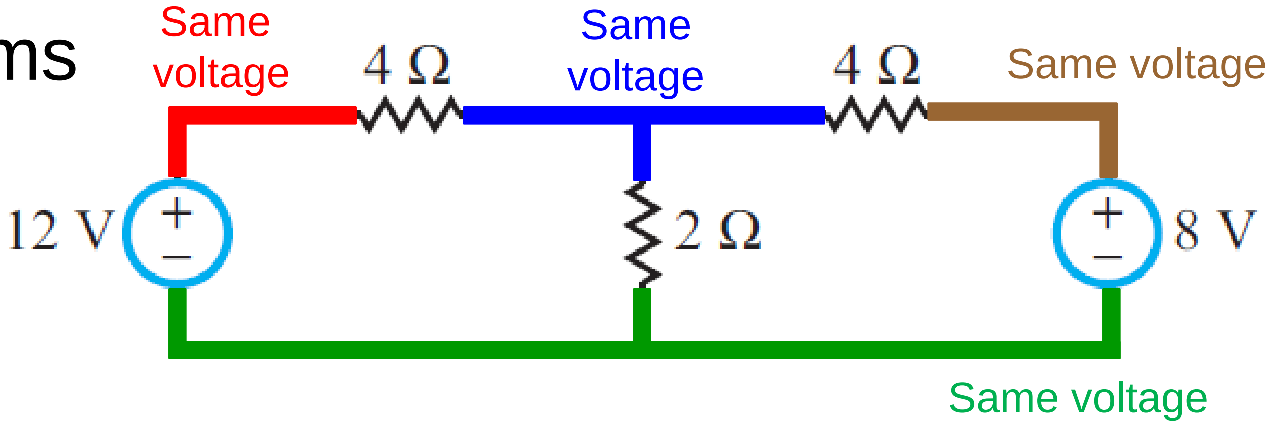
- Voltage drop (ΔV) across a **resistive** (R) material is proportional to the flowing current I

$$\Delta V = IR = V_{POS} - V_{NEG} = V_{FROM} - V_{TO}$$

- The current, I, flows **into** the positive terminal of the voltage drop
- Positive current will flow **from** a higher voltage **to** a lower voltage

Schematic Diagrams

Always

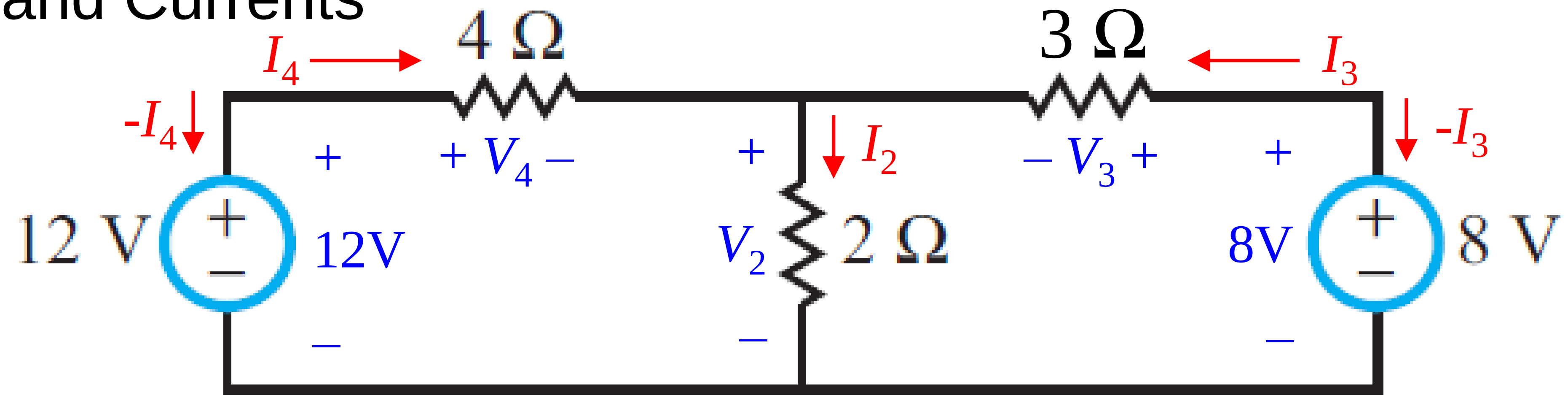


- Schematics represent **connections between** elements, not necessarily their relative or actual physical locations on a circuit board

- **MODEL**: Remember, every part of a node is at the same potential

- **MODEL**: Voltage does not drop across a node, only across circuit elements

Labeling Voltages and Currents



- When analyzing circuits, every element generally has a **voltage difference across** it and a **current through** it
- For resistors, **current** enters the positive terminal
- Not sure which way to draw ΔV or I ? Guess! It will work out either way!

Always

Kirchhoff's Current Law (KCL) and Voltage Law (KVL)

$$\sum I_{in} = \sum I_{out}$$

What goes in,
must come out

KCL

$$\sum V_{up} = \sum V_{down}$$

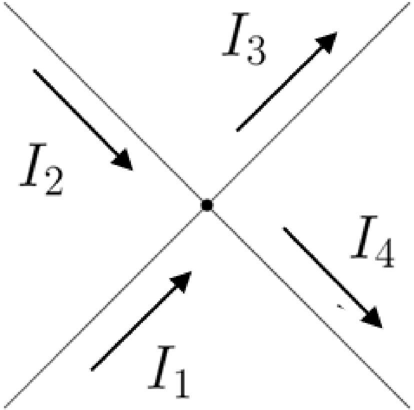
What goes up,
must come down

KVL

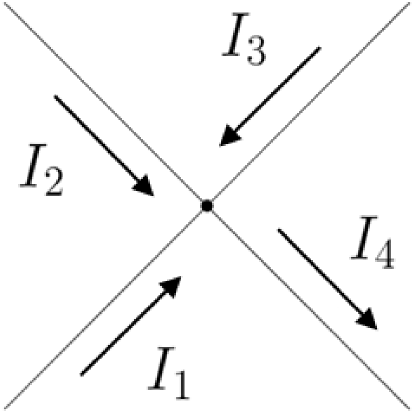
Always

Kirchhoff’s Current Law (KCL)

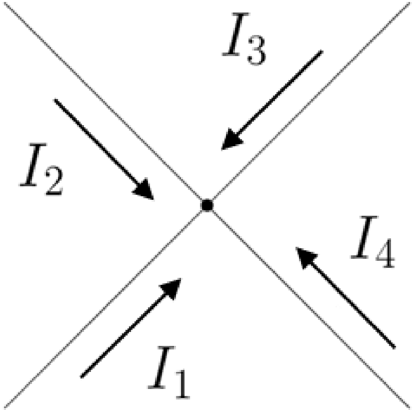
$$\sum I_{in} = \sum I_{out}$$



$$I_1 + I_2 = I_3 + I_4$$



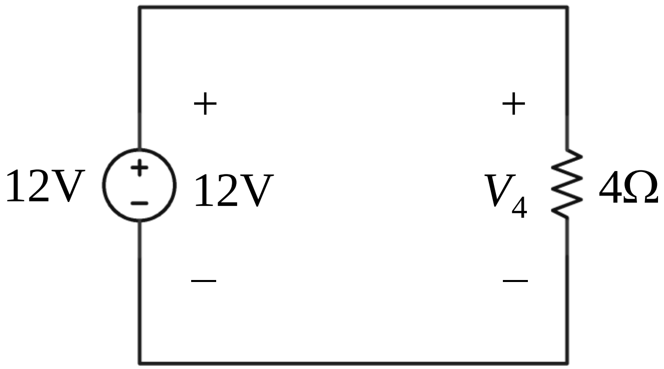
$$I_1 + I_2 + I_3 = I_4$$



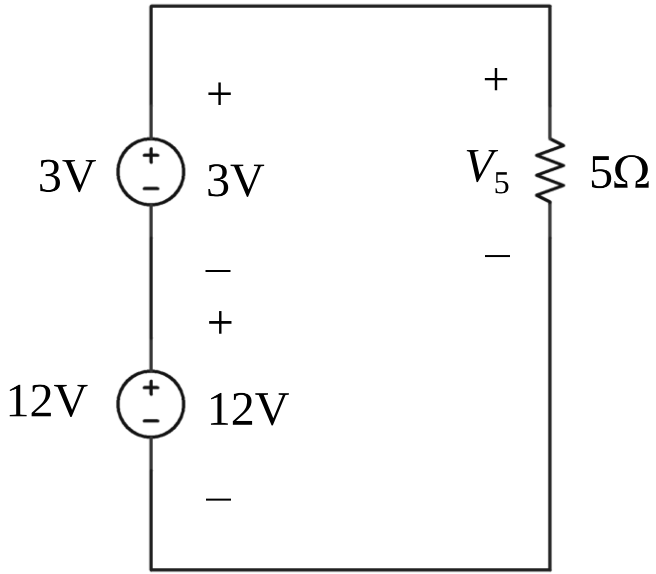
$$I_1 + I_2 + I_3 + I_4 = 0\text{A}$$

Kirchhoff's Voltage Law (KVL)

$$\sum V_{up} = \sum V_{down}$$



$12\text{V} = V_4$

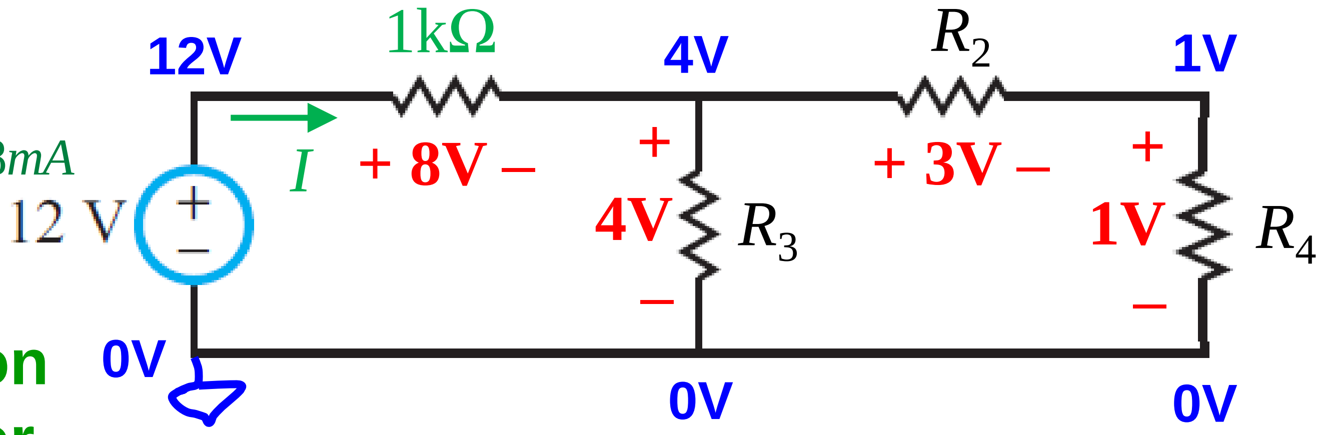


$12\text{V} + 3\text{V} = V_5$

Independent of ground!

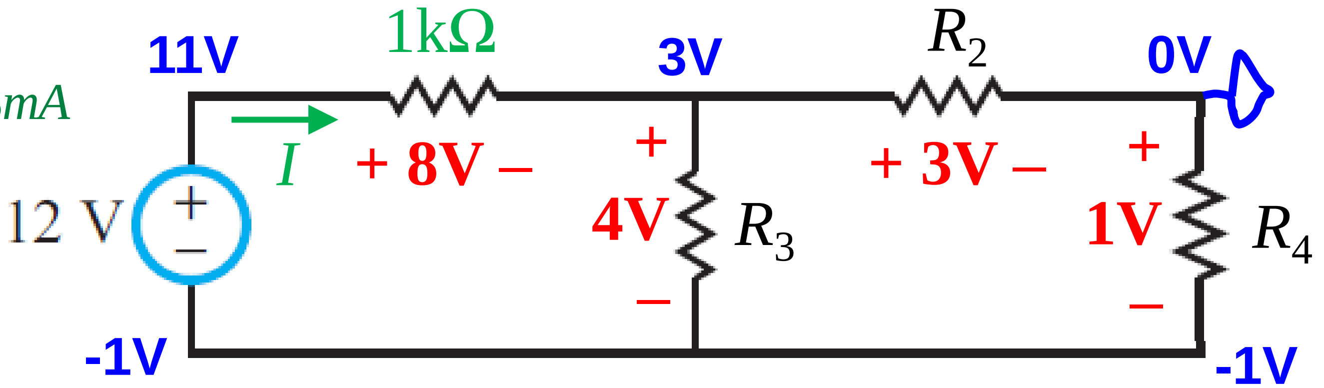
Reference Voltage (Ground)

$$I = \frac{\Delta V}{R} = \frac{12V - 4V}{1k\Omega} = 8mA$$

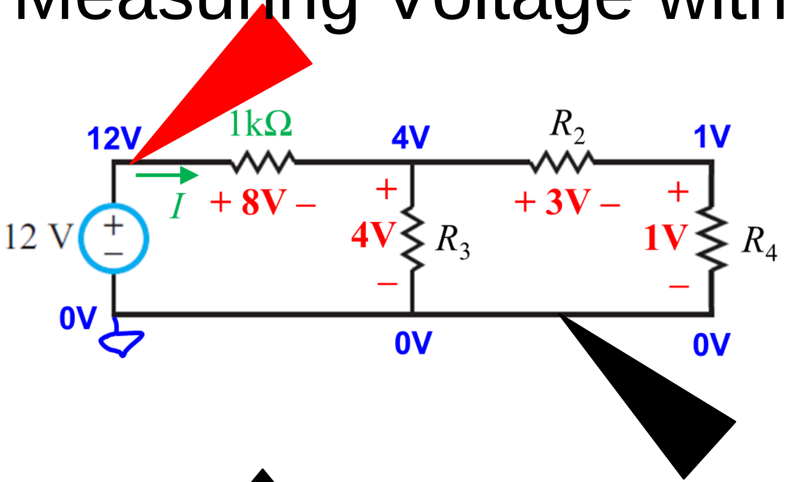


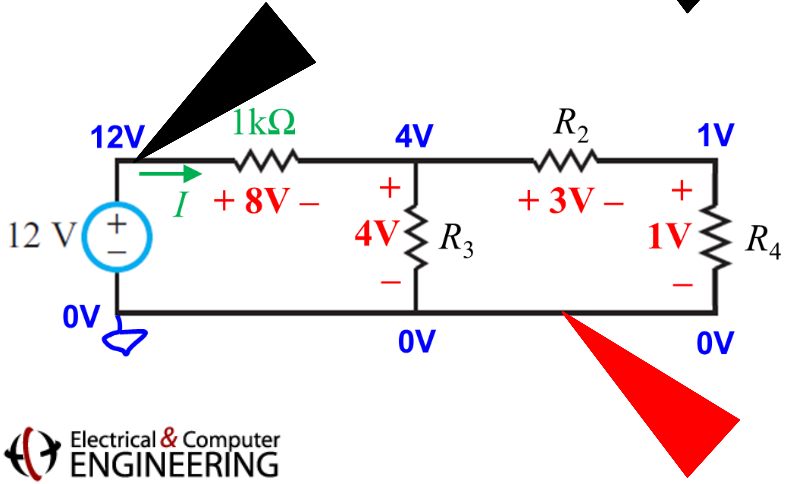
Ground location does not matter

$$I = \frac{\Delta V}{R} = \frac{11V - 3V}{1k\Omega} = 8mA$$



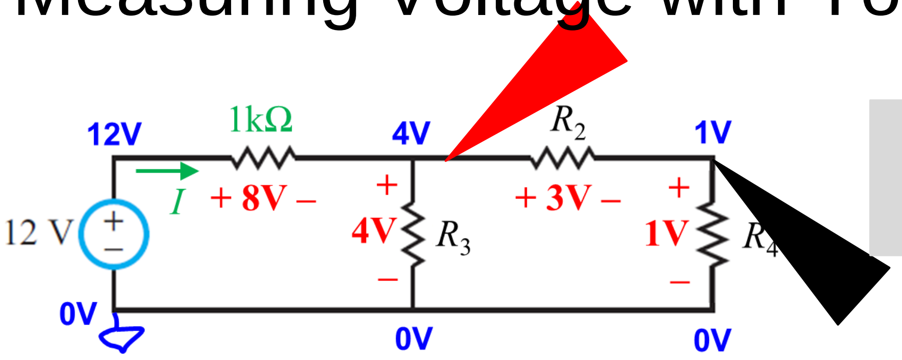
Measuring Voltage with Your Digital Multimeter (DMM)

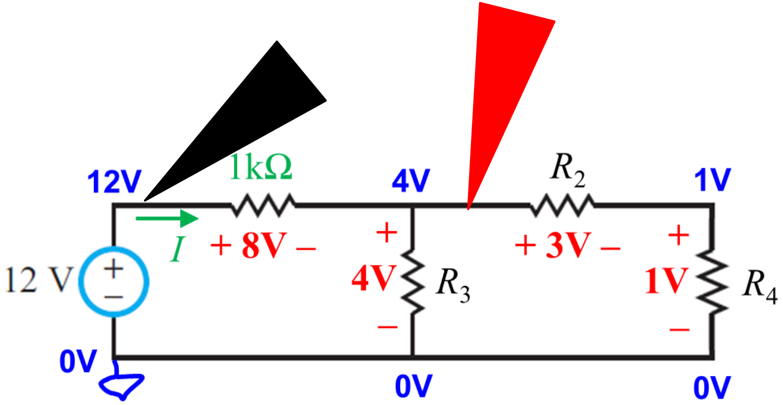




$V_{RED} - V_{BLACK}$

Measuring Voltage with Your Digital Multimeter (DMM)



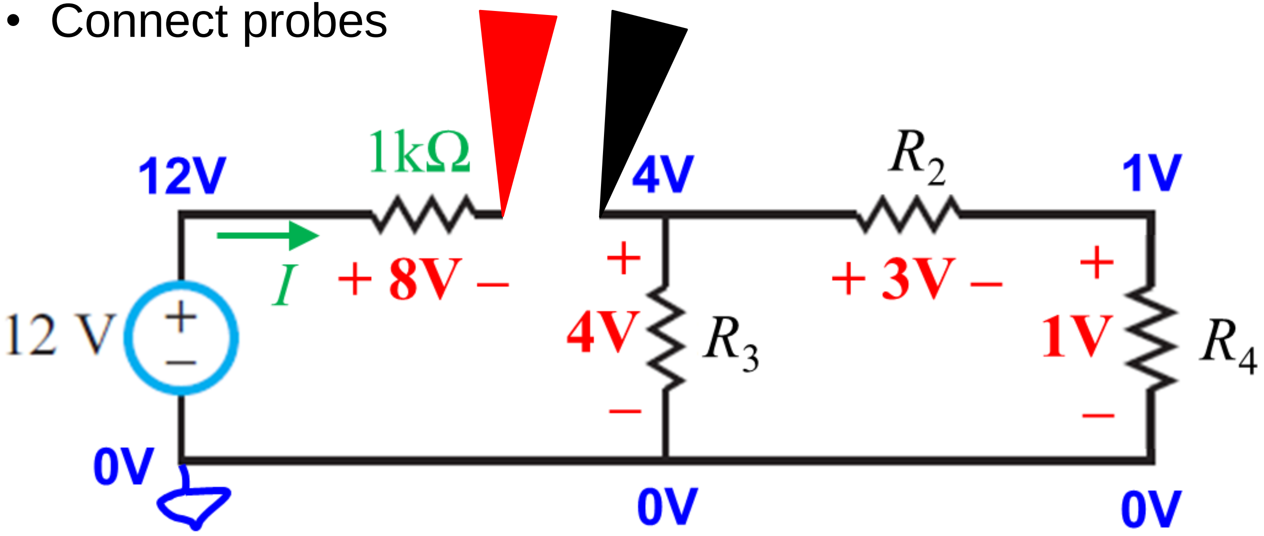


$V_{RED} - V_{BLACK}$

Measuring Current with Your Digital Multimeter (DMM)

- Current must flow THROUGH the DMM for it to be measured!
- Break open the circuit
- Connect probes

mA 8.00



$$I_{Into\ Red} = I_{Out\ of\ Black}$$

0Ω

Measuring Current with Your Digital Multimeter (DMM)

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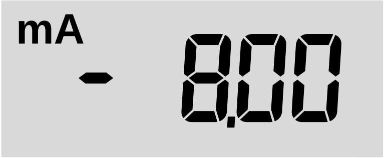

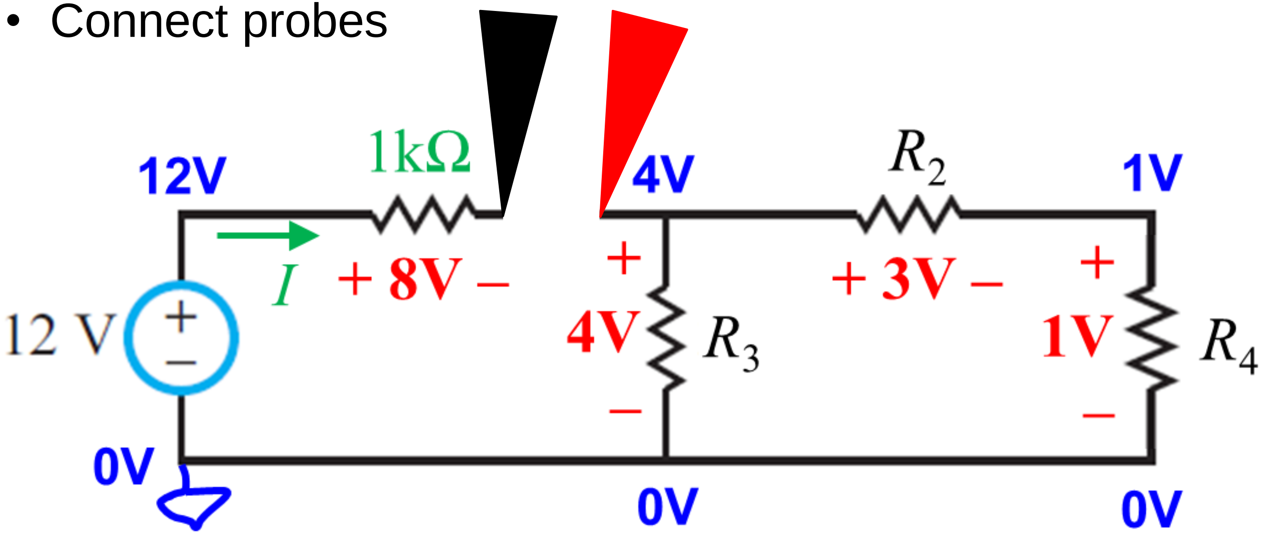

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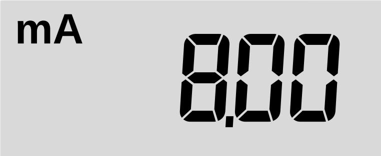
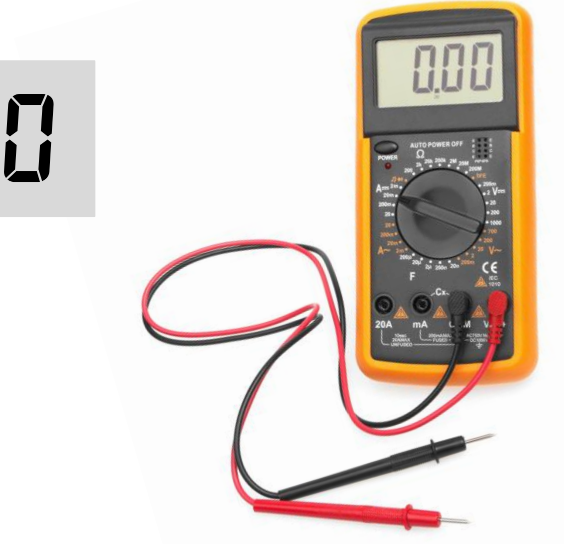
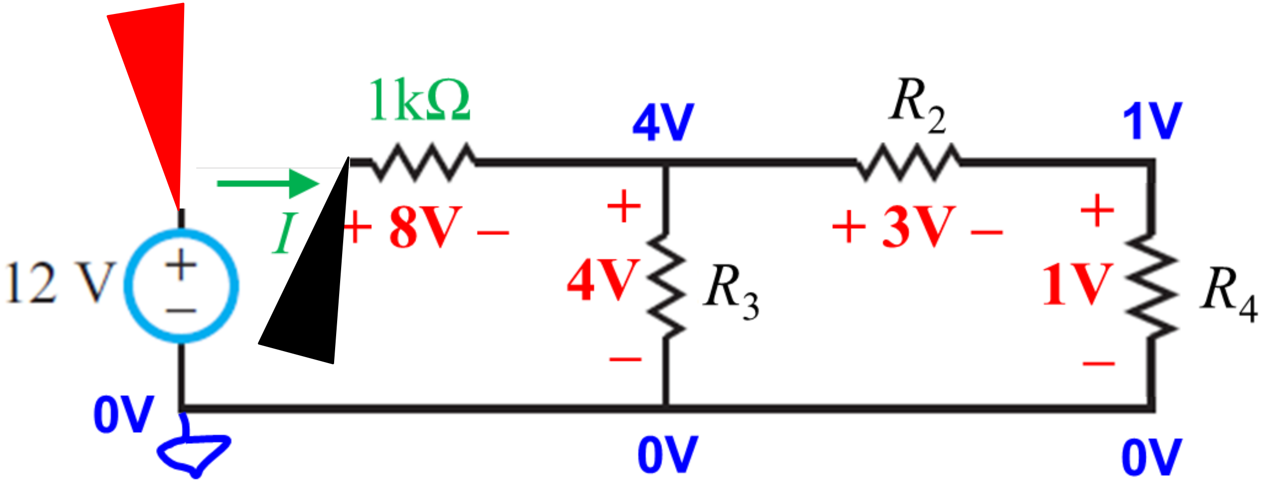
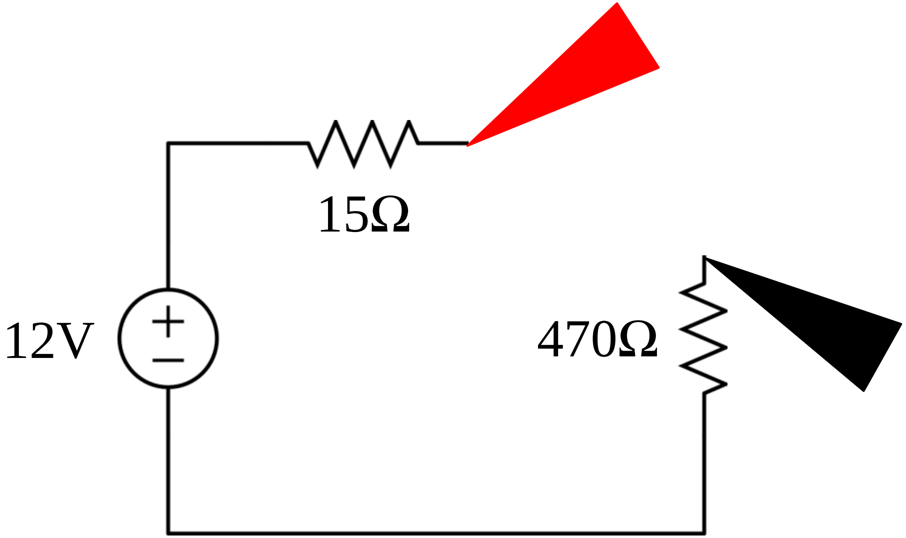






$$I_{Into\ Red} = I_{Out\ of\ Black}$$

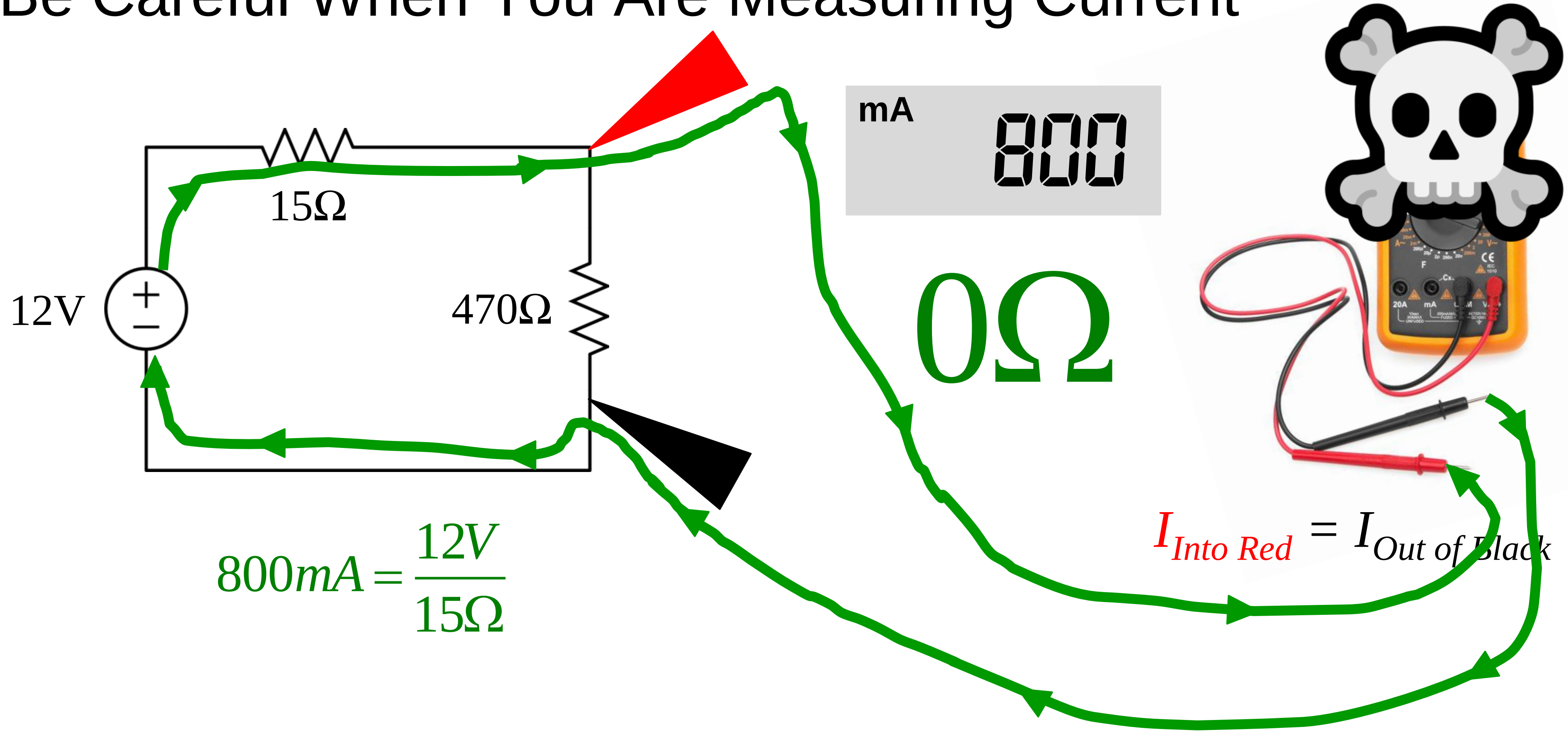
Be Careful When You Are Measuring Current



$$24.74mA = \frac{12V}{15\Omega + 470\Omega}$$

$I_{\text{Into Red}} = I_{\text{Out of Black}}$

Be Careful When You Are Measuring Current



$$800mA = \frac{12V}{15\Omega}$$

Be Careful When You Are Measuring Current

- Because current travels through the sensitive DMM, excess current can destroy it! :(

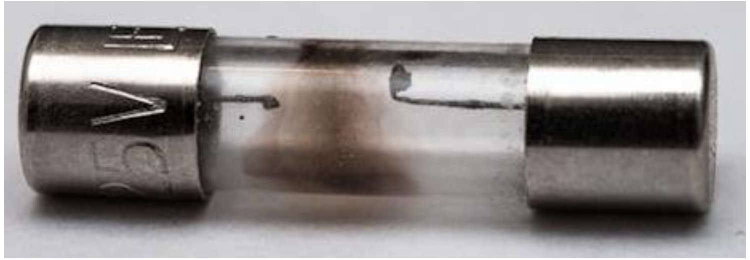
- DMMs include a fuse that "blows" or self-destructs when excess current starts to flow

- By giving its life, the $0.25 fuse can protect the $10 - $1,000 DMM

Fuse

Schematic Symbol

Damaged Fuse



Objectives of this Lecture

- Review
- **Power**
- Equivalent Circuits
- Voltage Division and Current Division

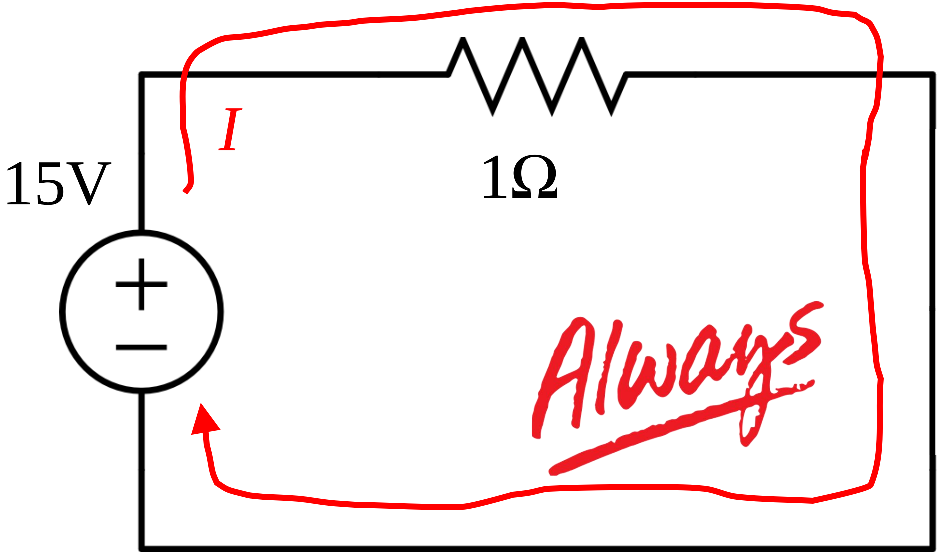
Power

$$P_{absorbed} = \left(\Delta V_{pos\ to\ neg}\right)\left(I_{into\ pos}\right)$$

Electrical Power

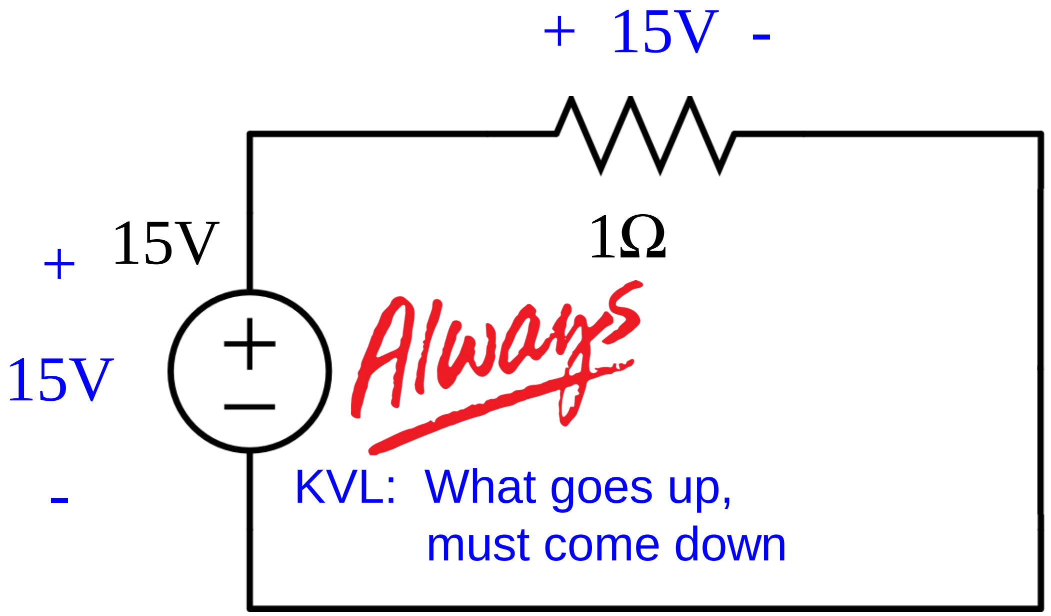
$$P_{absorbed} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ positive\ terminal}\right)$$

KCL: What goes in,
must come out



Electrical Power

$$P_{absorbed} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ positive\ terminal}\right)$$

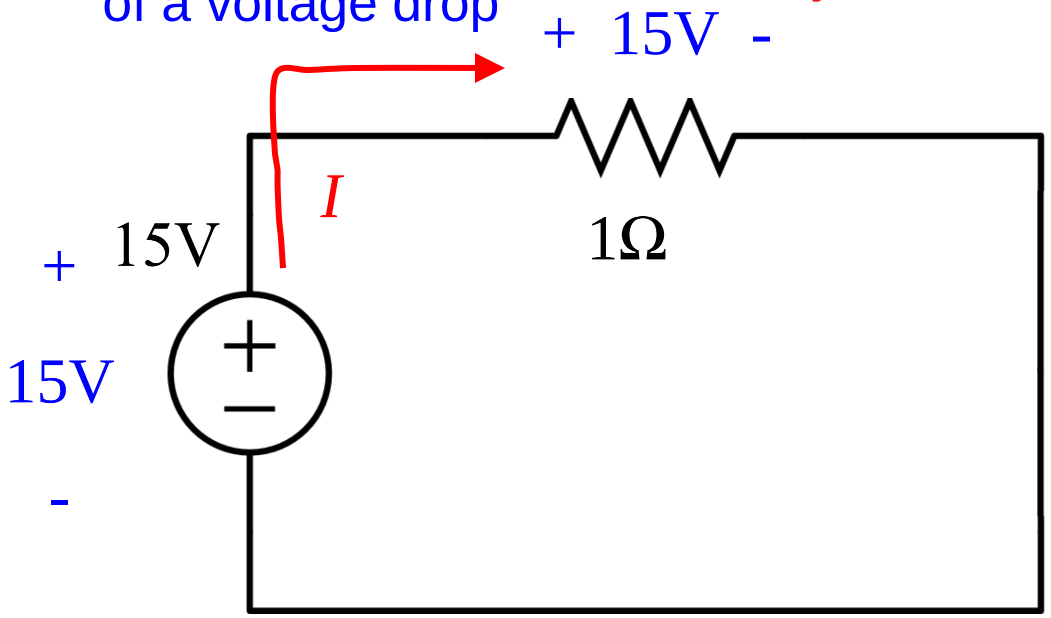


Electrical Power

$$P_{absorbed} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ \ positive\ terminal}\right)$$

Always draw current flowing into the positive terminal of a voltage drop

Always



Electrical Power

$$P_{absorbed} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ positive\ terminal}\right)$$

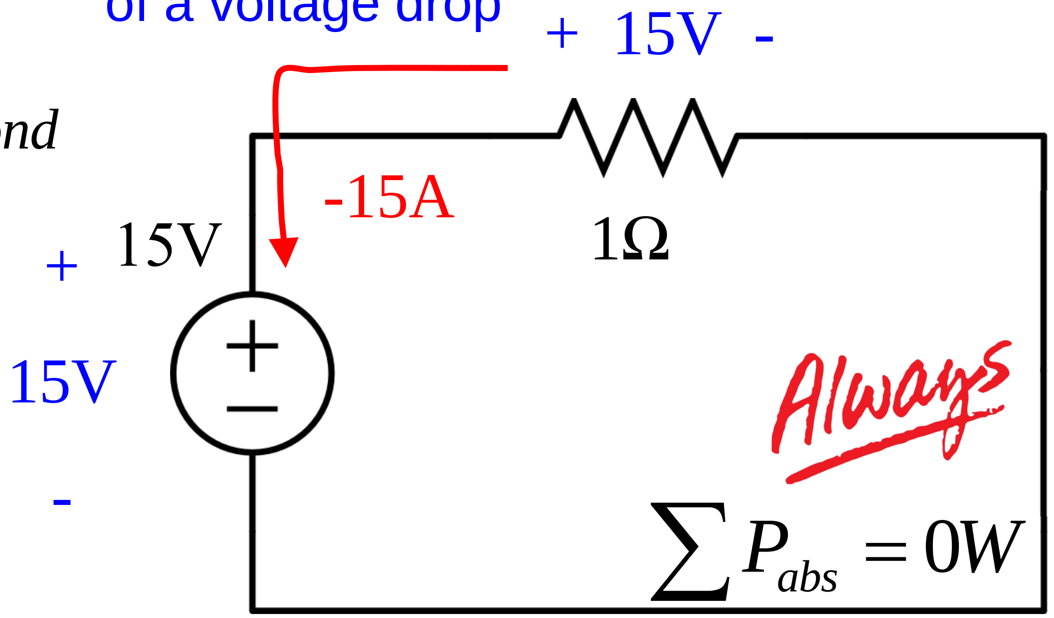
$$I = \frac{+15V}{1\Omega} = +15A$$

Always draw current flowing into the positive terminal of a voltage drop

$$P_{abs,1\Omega} = (+15V)(+15A)$$
$$= +225W = +225\ Joules / second$$

$$P_{abs,15V} = (+15V)(-15A)$$
$$= -225W\ J/s$$

$$P_{gen,15V} = -P_{abs,15V} = +225W$$



Electrical Power

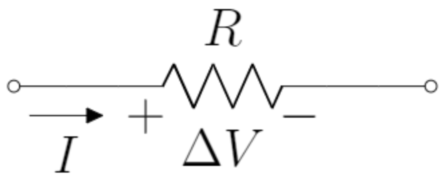
$$P_{absorbed} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ positive\ terminal}\right) = -P_{generated}$$

Always

$$\sum P_{abs} = 0W$$

For Resistors

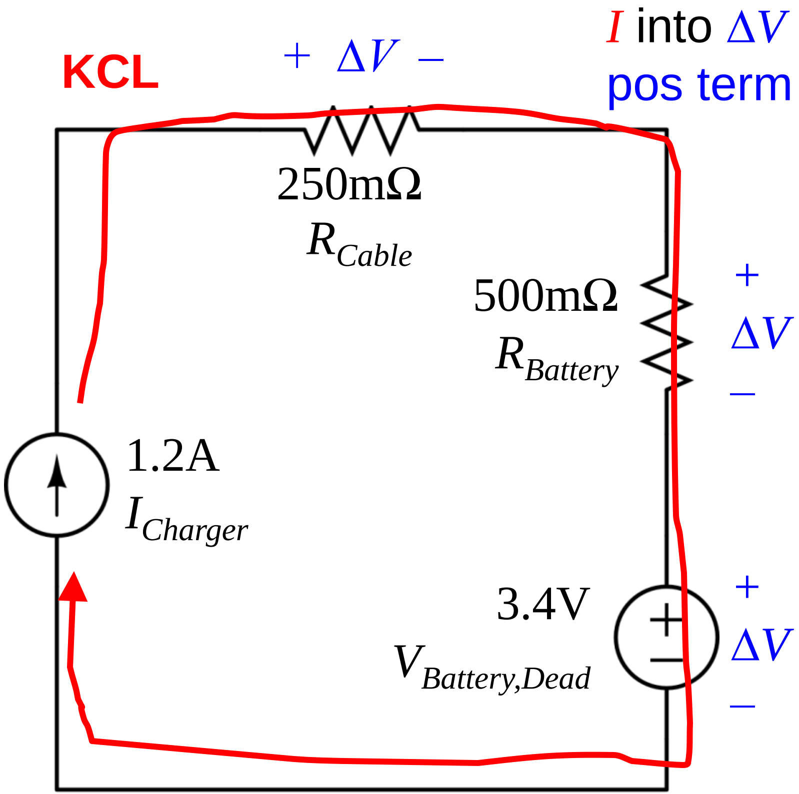
$$\Delta V_{pos\ to\ neg\ term} = \left(I_{into\ pos\ term}\right)(R) \qquad \left(I_{into\ pos\ term}\right) = \frac{\Delta V_{pos\ to\ neg\ term}}{R}$$



$$P_{abs,R} = \left(I_{into\ pos\ term}\right)^2(R)$$

$$P_{abs,R} = \frac{\left(\Delta V_{pos\ to\ neg\ term}\right)^2}{R}$$

Example: Charging a Cell Phone



$$P_{abs,Cable} = \left(I_{into\ pos\ term}\right)^2 (R) = (1.2A)^2 (0.25\Omega)$$
$$= 0.36W = 360mW$$

$$\Delta V_{R,Battery} = (1.2A)(0.5\Omega) = 0.6V = 600mV$$

$$P_{abs,R,Bat} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ pos\ term}\right) = (0.6V)(1.2A)$$
$$= 720mW$$

$$P_{abs,Battery} = \left(\Delta V_{pos\ to\ neg\ term}\right)\left(I_{into\ pos\ term}\right) = (3.4V)(1.2A)$$
$$= 4.08W$$

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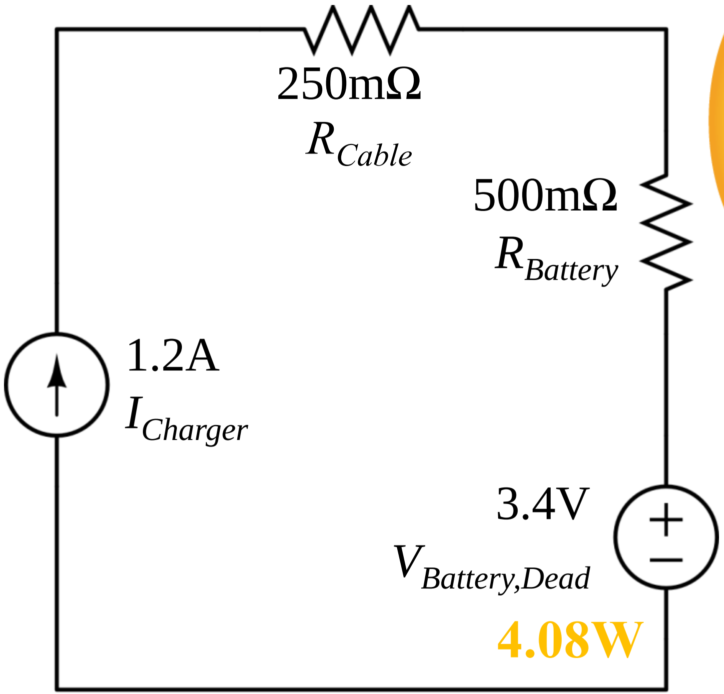
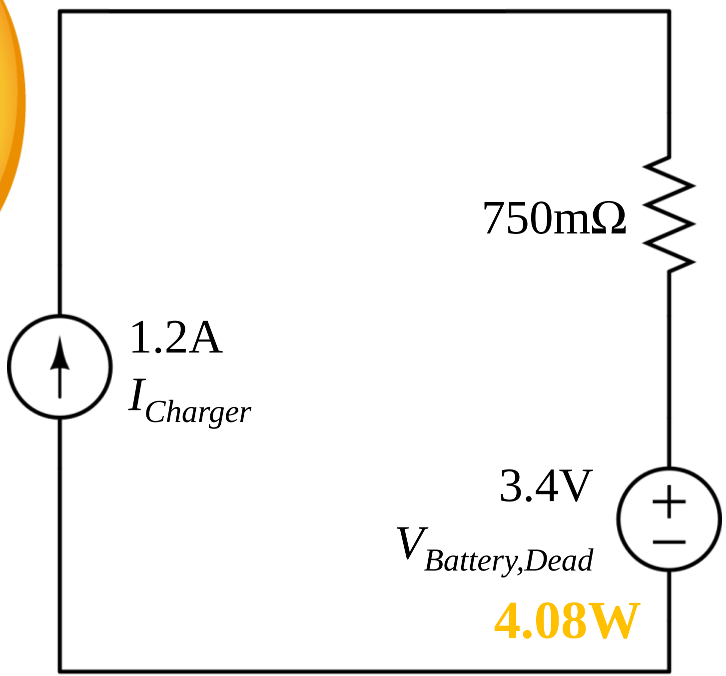
Is this Circuit? The Same as this Circuit?



Technically, no.
But, they are equivalent…



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Objectives of this Lecture

- Review
- Power
- **Equivalent Circuits**
- Voltage Division and Current Division

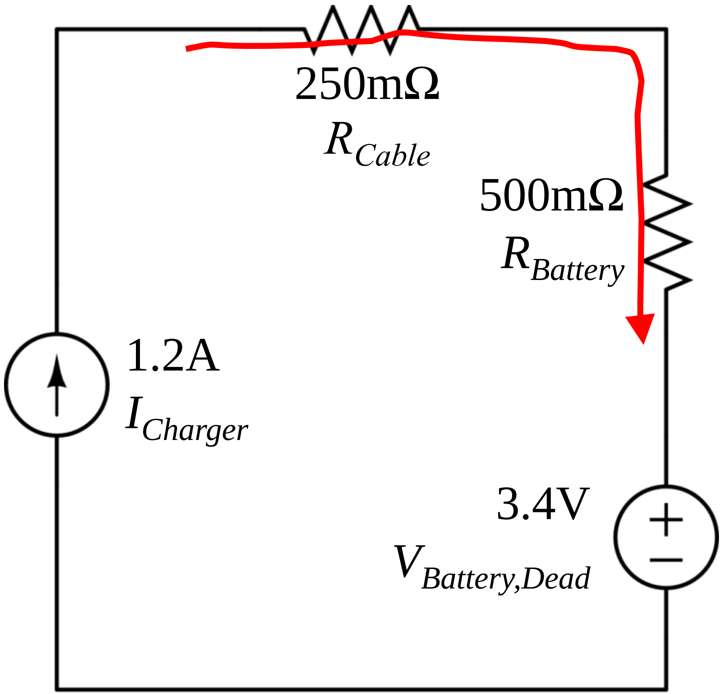
Equivalent Circuits

Series Resistors: Same Current Flows Through

- Part of a circuit is replaced with a simpler part

- Resistors in Series: $R_{Equivalent} = R_1 + R_2$

$R_{Equivalent} = 0.25\Omega + 0.5\Omega = 750\text{m}\Omega$



Equivalent Circuits

Series Resistors: Same Current Flows Through

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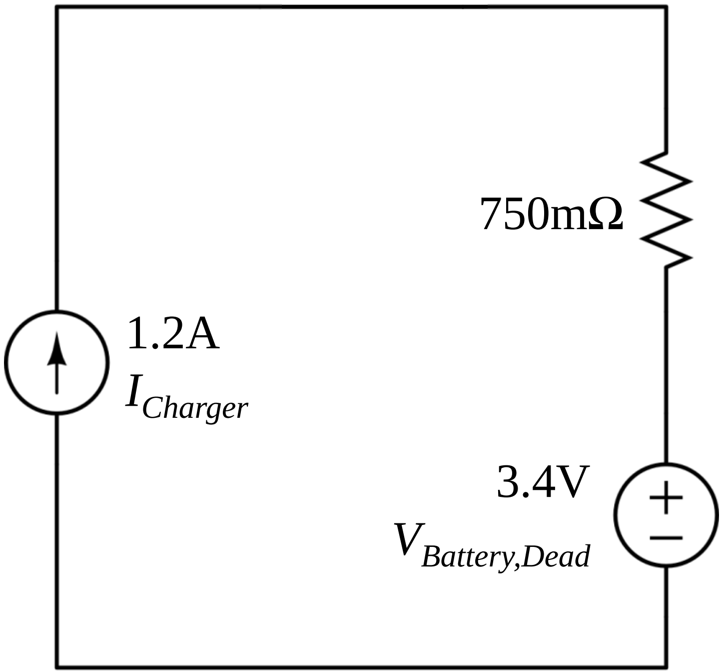
- Resistors in Series: $R_{Equivalent} = R_1 + R_2$

  $R_{Equivalent} = 0.25\Omega + 0.5\Omega = 750\text{m}\Omega$

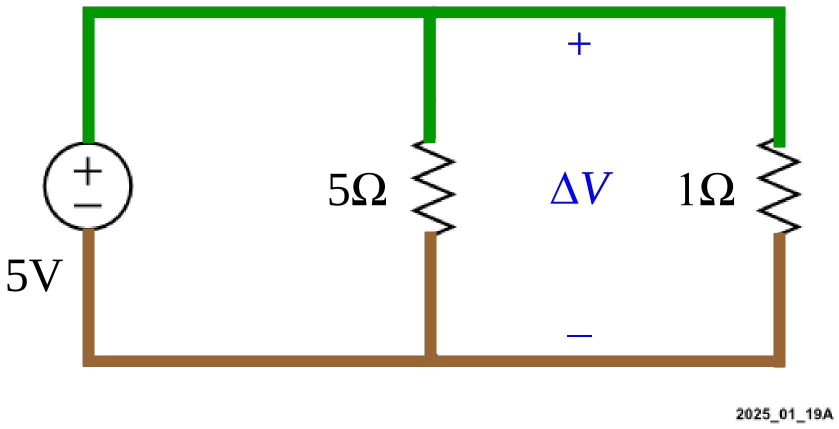
- In general

  $R_{Series,Equivalent} = R_1 + R_2 + R_3 + R_4 + R_5 + \ldots$



Equivalent Circuits (Parallel Resistors)

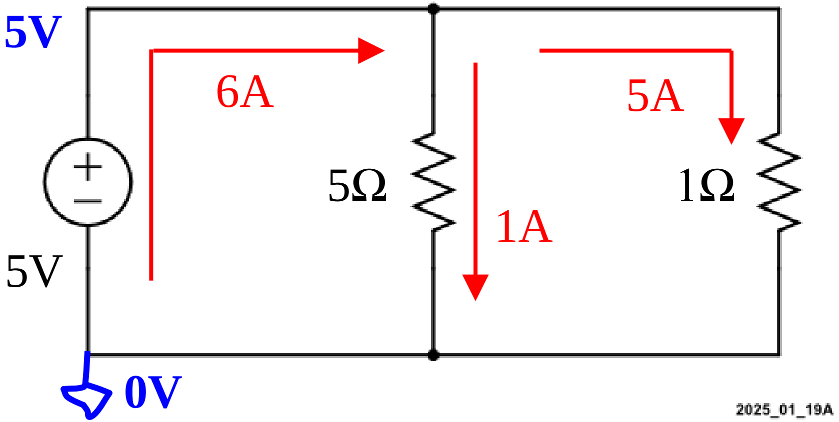
Part of a Circuit Is Replaced with a Simpler Part

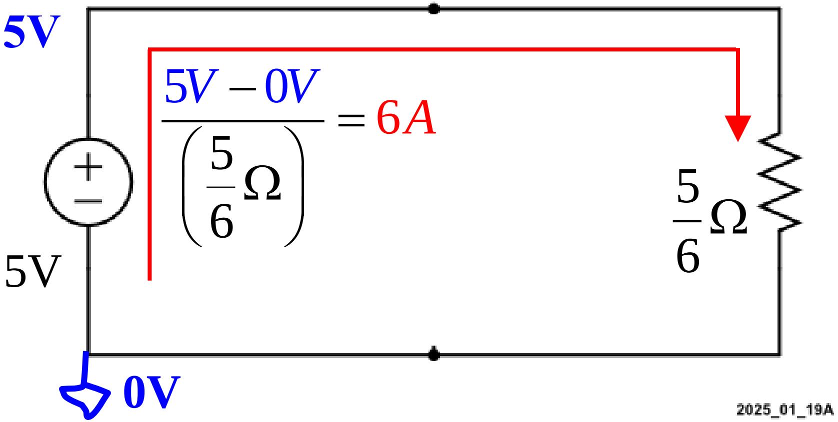


- Parallel resistors have the same voltage across them
- No voltage drop across a node
- Same voltage drop, ΔV, across parallel resistors

Equivalent Circuits (Parallel Resistors)

Part of a Circuit Is Replaced with a Simpler Part





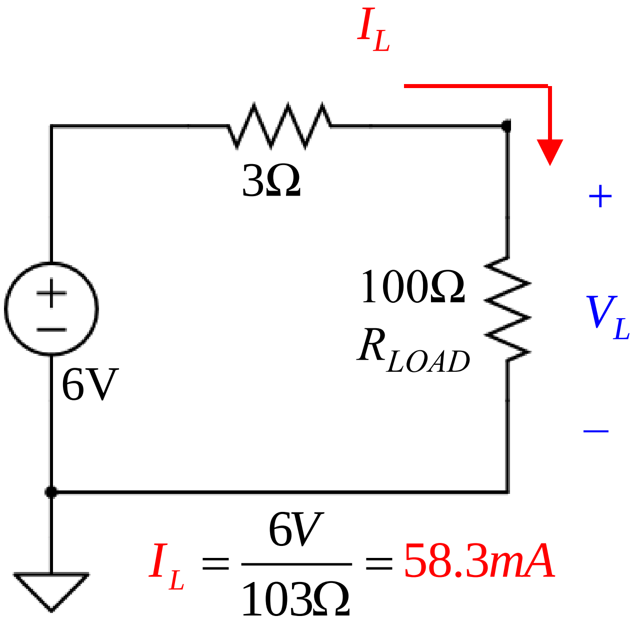
$$\frac{1}{R_{\parallel,Equivalence}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}+\ldots$$

$$\frac{1}{R_{\parallel,Equivalence}}=\frac{1}{5\Omega}+\frac{1}{1\Omega}=\frac{0.2}{\Omega}+\frac{1}{\Omega}=\frac{1.2}{\Omega}$$

$$R_{\parallel,Equivalence}\ \frac{1}{\left(1.2/\Omega\right)}=\frac{5}{6}\Omega\approx 0.833\Omega=833m\Omega$$

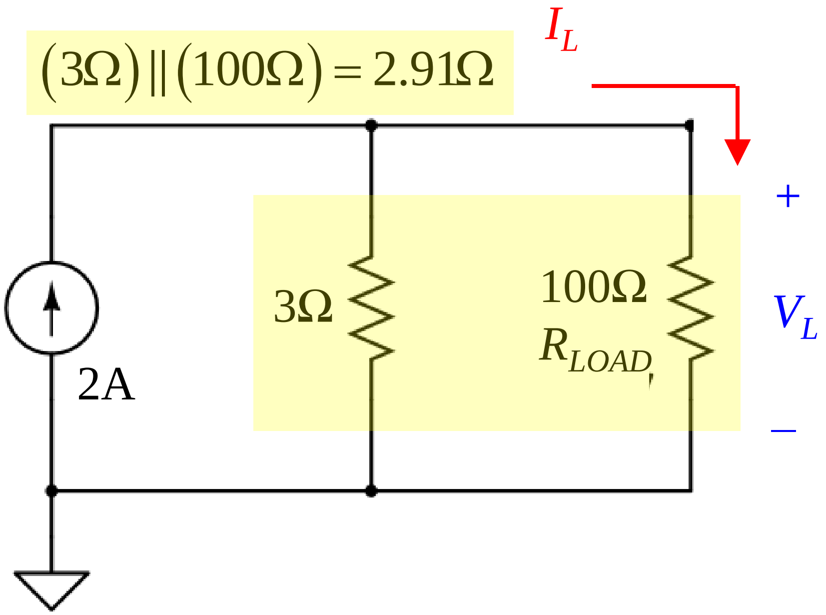
Are These Two Circuits Equivalent from the Perspective of the Load Resistor (R_L)?

Same $P_{abs,L} = \Delta V_L I_L$



$$I_L = \frac{6V}{103\Omega} = 58.3mA$$

$$V_L = (58.3mA)(100\Omega) = 5.83V$$



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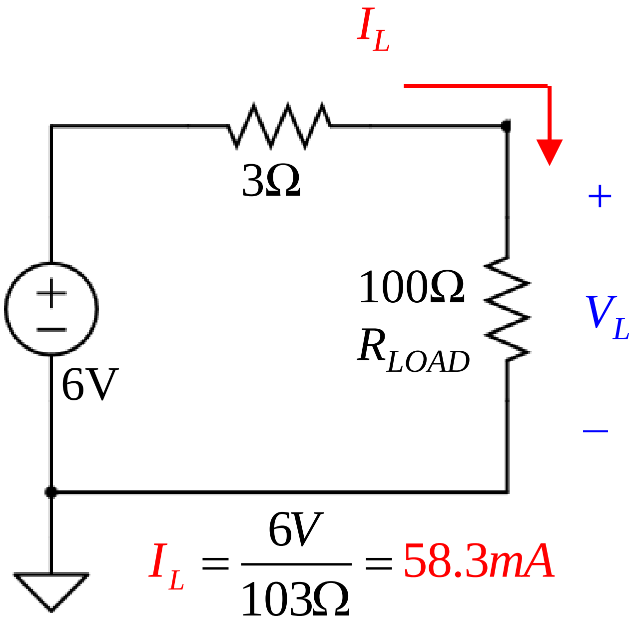

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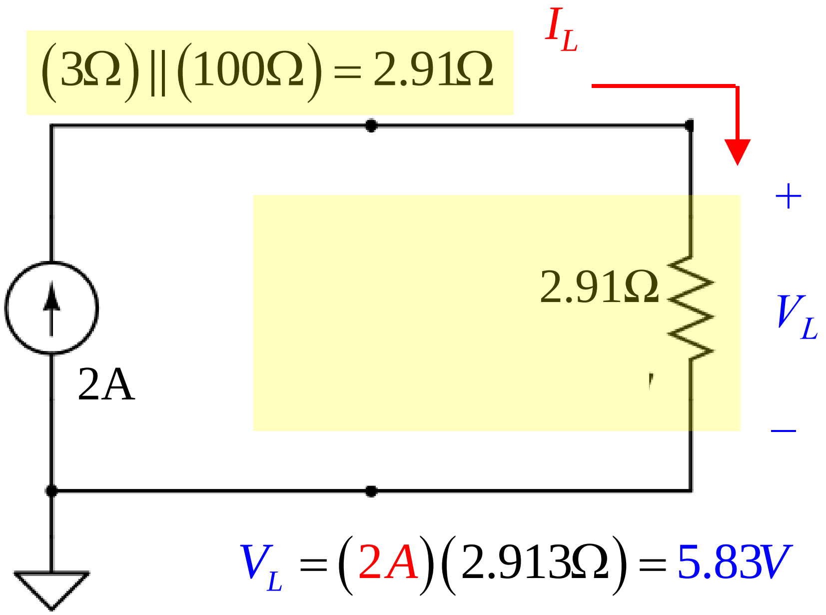
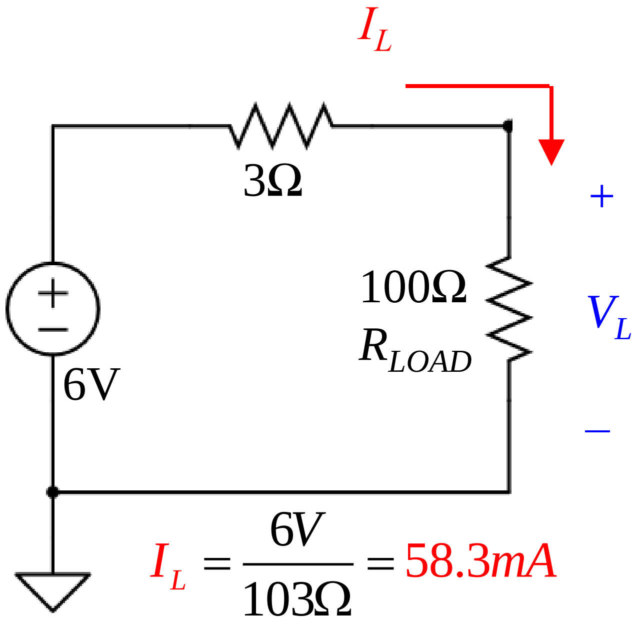


$$V_L = (2A)(2.913\Omega) = 5.83V$$

Are These Two Circuits Equivalent from the Perspective of the Load Resistor (R_L)?

Same $P_{abs,L} = \Delta V_L I_L$

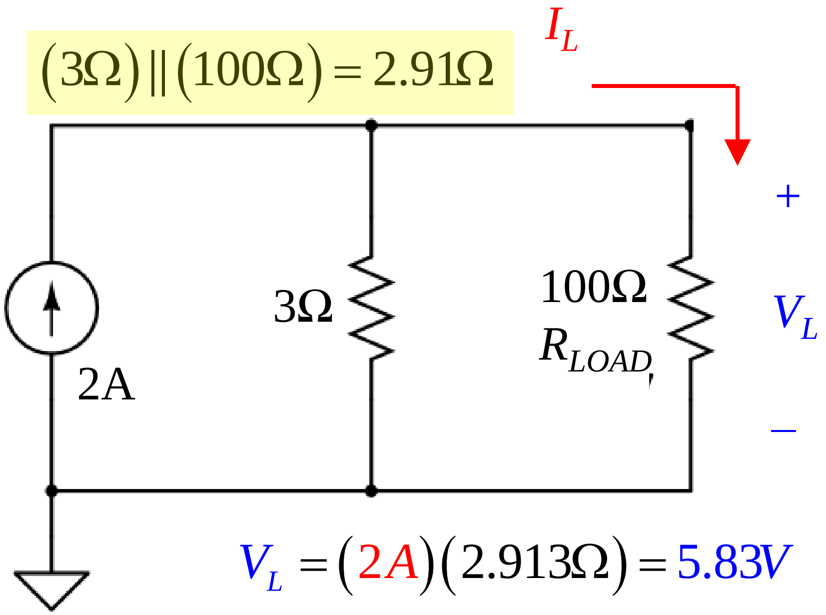
YES



$$I_L = \frac{6V}{103\Omega} = 58.3mA$$

$$V_L = (58.3mA)(100\Omega) = 5.83V$$

$$(3\Omega)\|(100\Omega) = 2.91\Omega$$

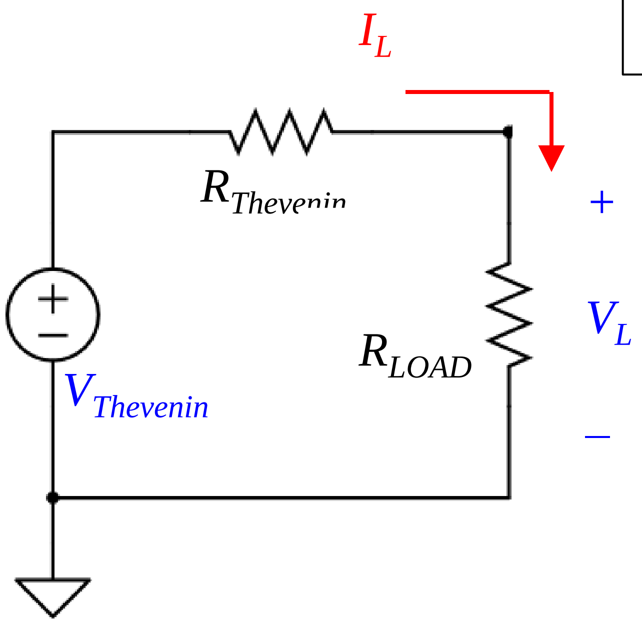


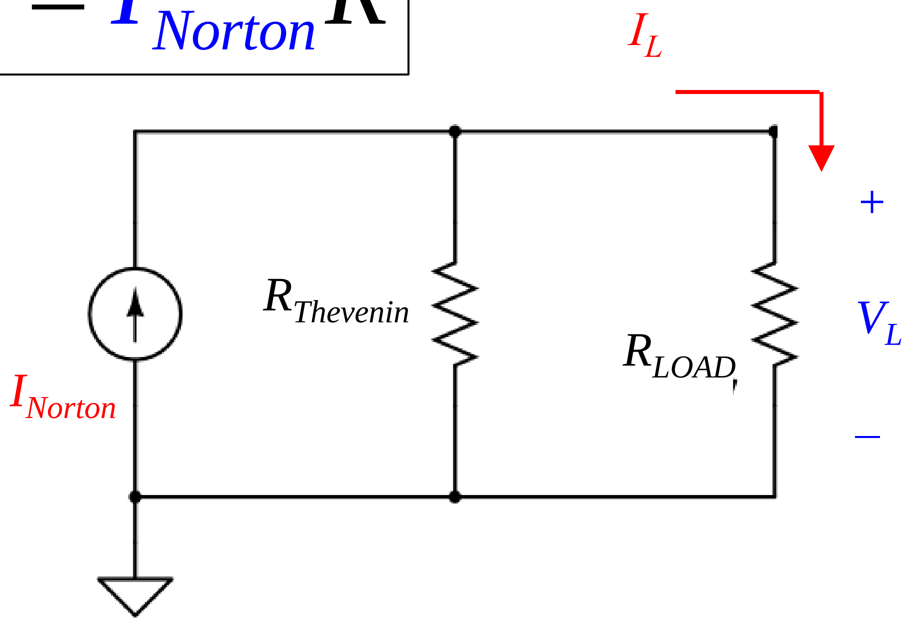
$$V_L = (2A)(2.913\Omega) = 5.83V$$

$$I_L = \frac{5.83V}{100\Omega} = 58.3mA$$

Thevenin and Norton Equivalent Circuits

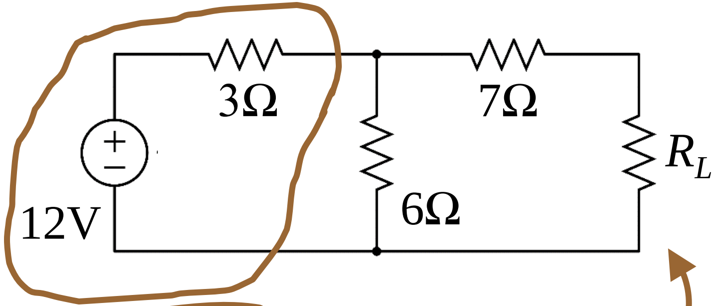
$$V_{Thevenin} = I_{Norton} R$$





$$P_{abs,LOAD} = P_{abs,LOAD}$$

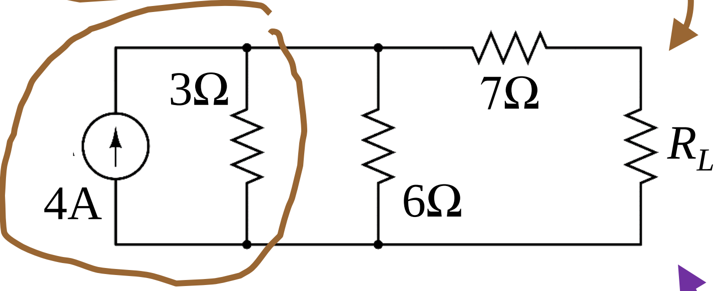
Finding the Thevenin Equivalent for R_L



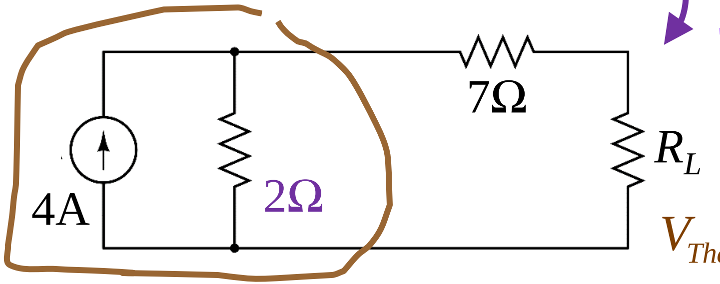
Transform Thevenin equivalent into Norton equivalent

$$I_{Norton} = 12V \,/\, 3\Omega = 4A$$

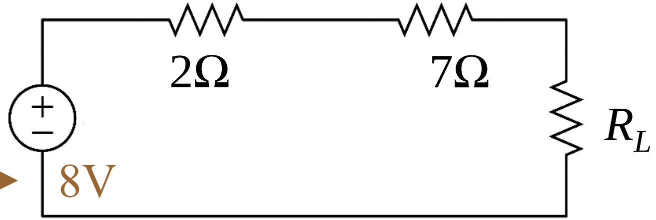
Equivalent circuits

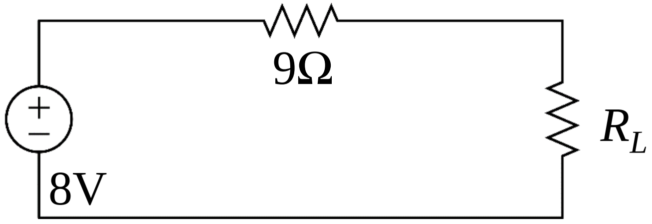


$$\frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{2\Omega}$$

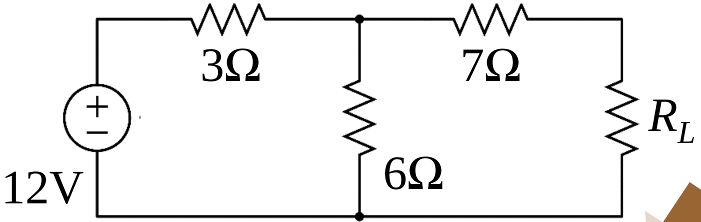


$$V_{Thevenin} = (4A)(2\Omega)$$



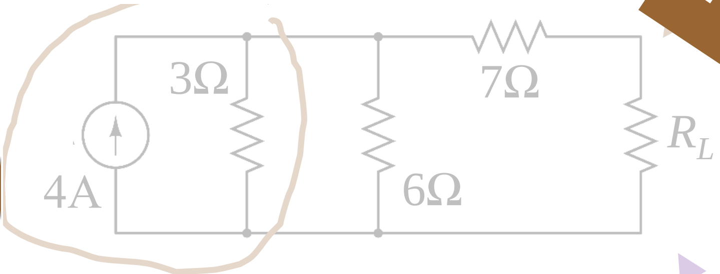


Finding the Thevenin Equivalent for R_L

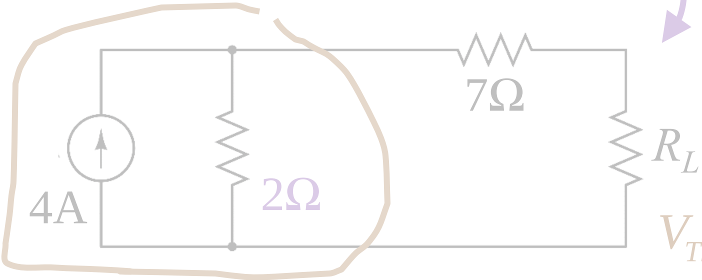


Transform Thevenin equivalent into Norton equivalent

$$I_{Norton} = 12V / 3\Omega = 4A$$



$$\frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{2\Omega}$$



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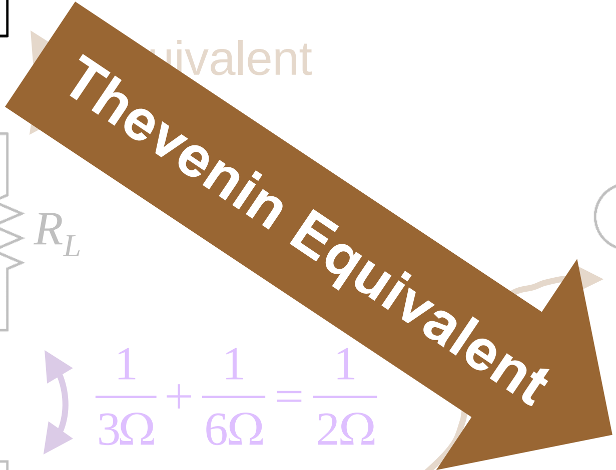
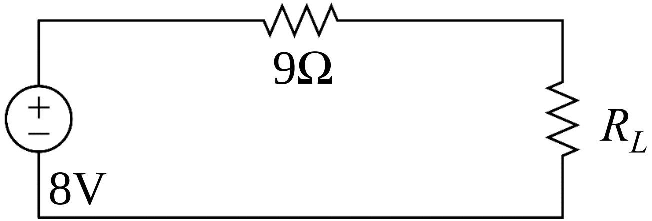
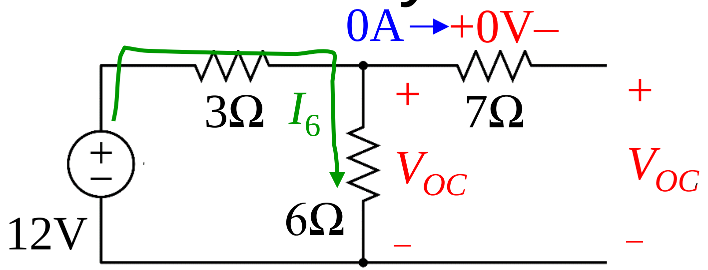


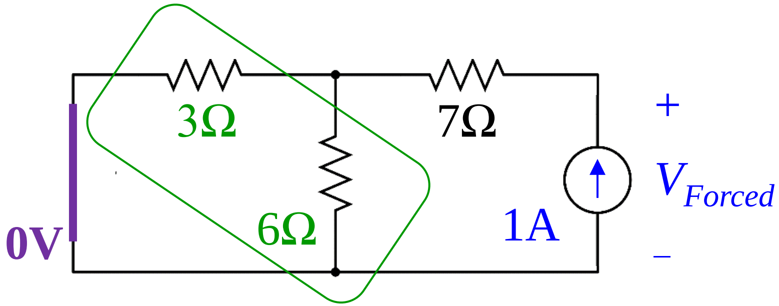


Another Way to Find the Thevenin Equivalent for R_L



$$I_6 = \frac{12V}{3\Omega + 6\Omega} = 1.33A$$

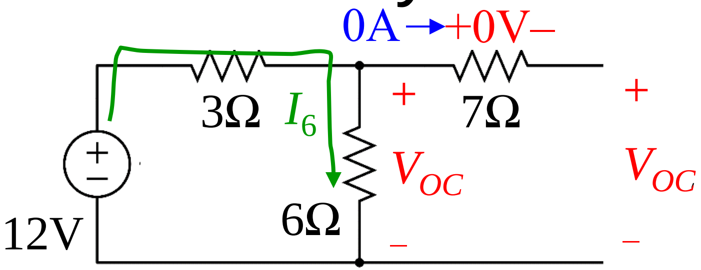
$$V_{OC} = (1.33A)(6\Omega) = 8.00V$$



$$\frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{2\Omega}$$

1) Disconnect the R_{LOAD}
2) Find the **O**pen **C**ircuit voltage, V_{OC} , across where R_{LOAD} was ($V_{Thevenin} = V_{OC}$)
3) Zero **independent** sources (0V short, 0A open)
4) Use a 1A source to calculate $R_{Thevenin}$:
   $R_{Thevenin} = V_{Forced}$ / 1A

Another Way to Find the Thevenin Equivalent for R_L



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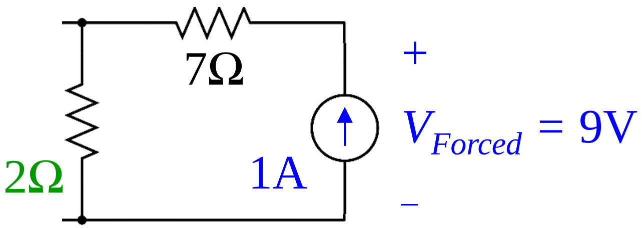
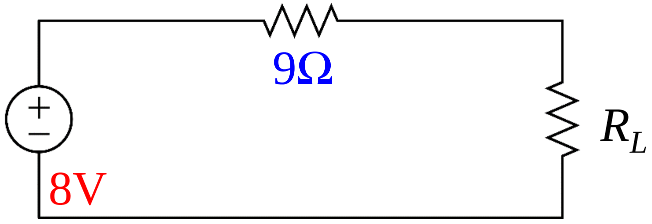


$$\frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{2\Omega}$$

$R_{Thevenin}$ = 9Ω

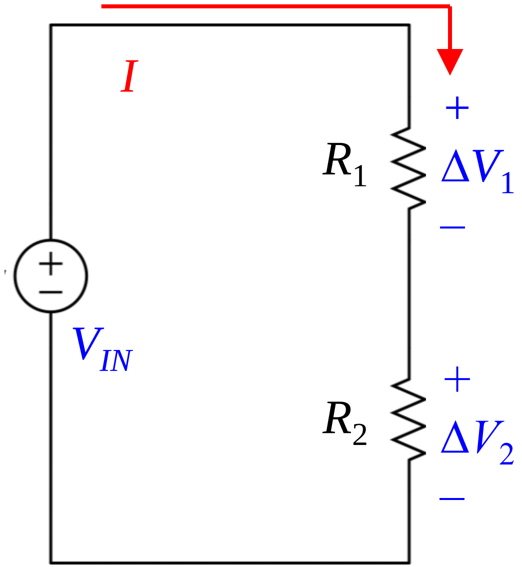


Objectives of this Lecture

- Review
- Power
- Equivalent Circuits
- **Voltage Division and Current Division**

Voltage Division

- **CAN** (never need to) combine Ohm's Law and KCL to simplify **SOME** calculations
- Works for one voltage drop across two resistors in series (same current flows through)



- KCL: What goes in, has to go out
- Series: Same current goes through components
- Current flows into ΔV positive terminal

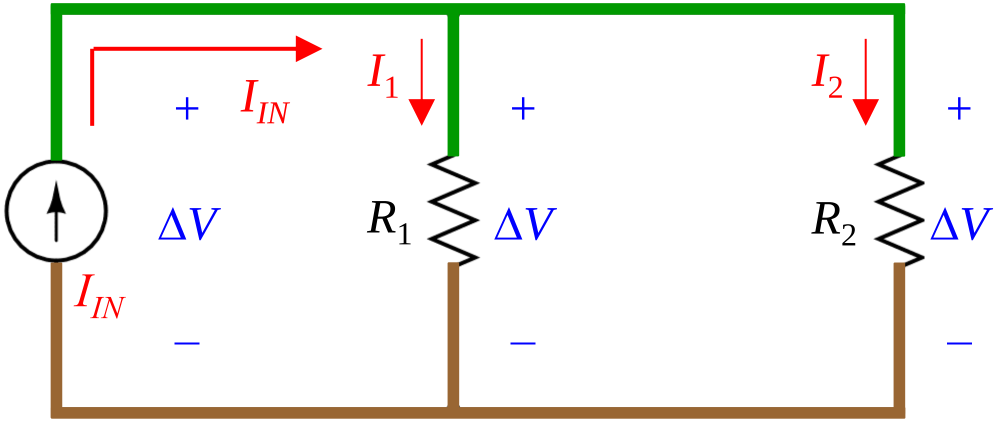
- Ohm's Law: $I = \dfrac{V_{IN}}{R_1 + R_2} = \dfrac{\Delta V_1}{R_1}$

$$I = \frac{V_{IN}}{R_1 + R_2} = \frac{\Delta V_2}{R_2}$$

$$\boxed{\Delta V_1 = V_{IN}\left(\frac{R_1}{R_1 + R_2}\right) \qquad \Delta V_2 = V_{IN}\left(\frac{R_2}{R_1 + R_2}\right)}$$

Current Division

- **CAN** (never need to) combine Ohm's Law and KVL to simplify **SOME** calculations
- Works for one current flowing into two resistors in parallel (same voltage drop across)



$$I_1 = I_{IN}\left(\frac{R_2}{R_1 + R_2}\right) \qquad I_2 = I_{IN}\left(\frac{R_1}{R_1 + R_2}\right)$$

- No voltage drop across each node
- Same voltage drop, $\Delta V_{top\text{-}to\text{-}bottom}$
- Current flows into positive terminal
- KCL: $I_{IN} = I_1 + I_2$
- Ohm's Law

$$\Delta V = I_1 R_1 = I_2 R_2 = I_{IN}\left(R_1 \parallel R_2\right)$$

$$\Delta V = I_1 R_1 = I_2 R_2 = I_{IN}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Objectives of this Lecture

- Review
- Power
- Equivalent Circuits
- Voltage Division and Current Division