18-100 Introduction to Electrical and Computer Engineering

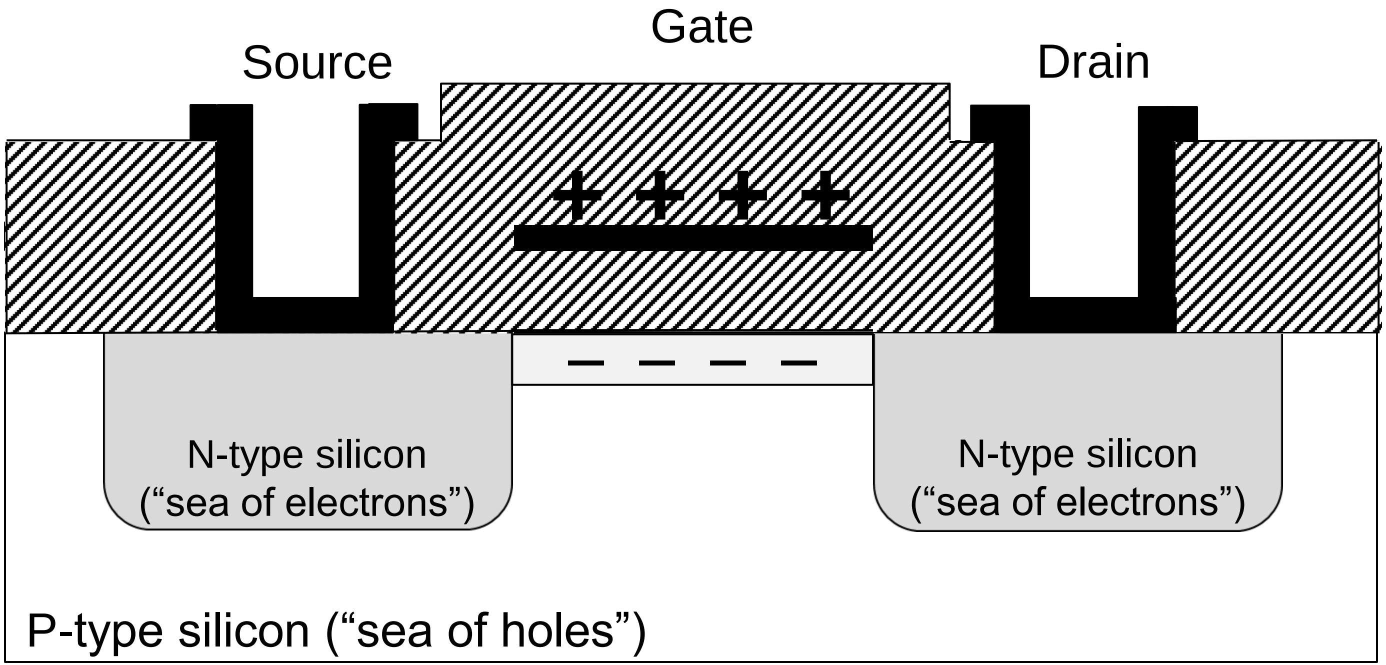
Lecture 07
Capacitors

Week	Date	Day	Lecture Topic	Lab Out		Lecturer
1	13-Jan	M	L01: Intro, Physics, EM, Leveling Students			Greg and Mark
	15-Jan	W	L02: Circuits Basics	Lab1 : Circuits		Mark
2	20-Jan	M	Martin Luther King Celebration (No Lecture)			
	22-Jan	W	L03: Equivalent Circuits	Pause for MLK		Mark
3	27-Jan	M	L04: Semiconductors, Diodes, LEDs			Mark
	29-Jan	W	L05: MOSFETs to Simple Gates	Lab2:Adder		Mark
4	3-Feb	M	L06: Professional Identity, Professional Responsibility, and Ethics			Greg
	5-Feb	**W**	**Exam 1**	Pause for exam		N/A
5	10-Feb	M	L07: Capacitors, RC Time Constants, RC Circuits			Mark
	12-Feb	W	L08: Inductors, RL Time Constants, 555	Lab3 : MOSFETs		Mark
6	17-Feb	M	L09: Binary, Logic Gates, Boolean Logic			Greg
	19-Feb	W	L10: Latches, Registers, RAM, Flip-Flops	Lab4: Timer Lab		Greg
7	24-Feb	M	L11: Computers			Greg
	26-Feb	W	L12: Op Amps	Lab5: Op Amps		Mark
	3-Mar	M	SPRING BREAK			
	5-Mar	W	SPRING BREAK	Pause for break		
8	10-Mar	M	L13: Arduino Programming Case Study			Greg
	12-Mar	W	L14: Serial Communication Protocols	Lab 6: I2C		Greg
9	17-Mar	M	L15: Analog-to-Digital (ADC) and Digital-to-Analog (DAC) Conversion			Greg
	19-Mar	W	L16: Time Varying Signals and Spectra (Trig)	Lab7: ADC		Mark
10	24-Mar	M	L17: Wireless Communication: Modulation to Protocols			Mark
	26-Mar	W	L18: Review/Exam Preview	Pause for exam		Greg
11	**31-Mar**	**M**	**Exam 2**			

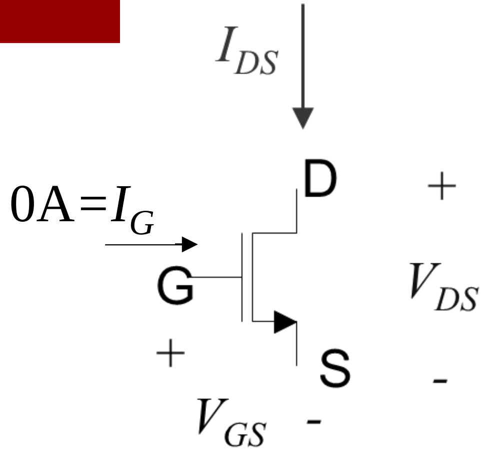
Objectives of this Lecture

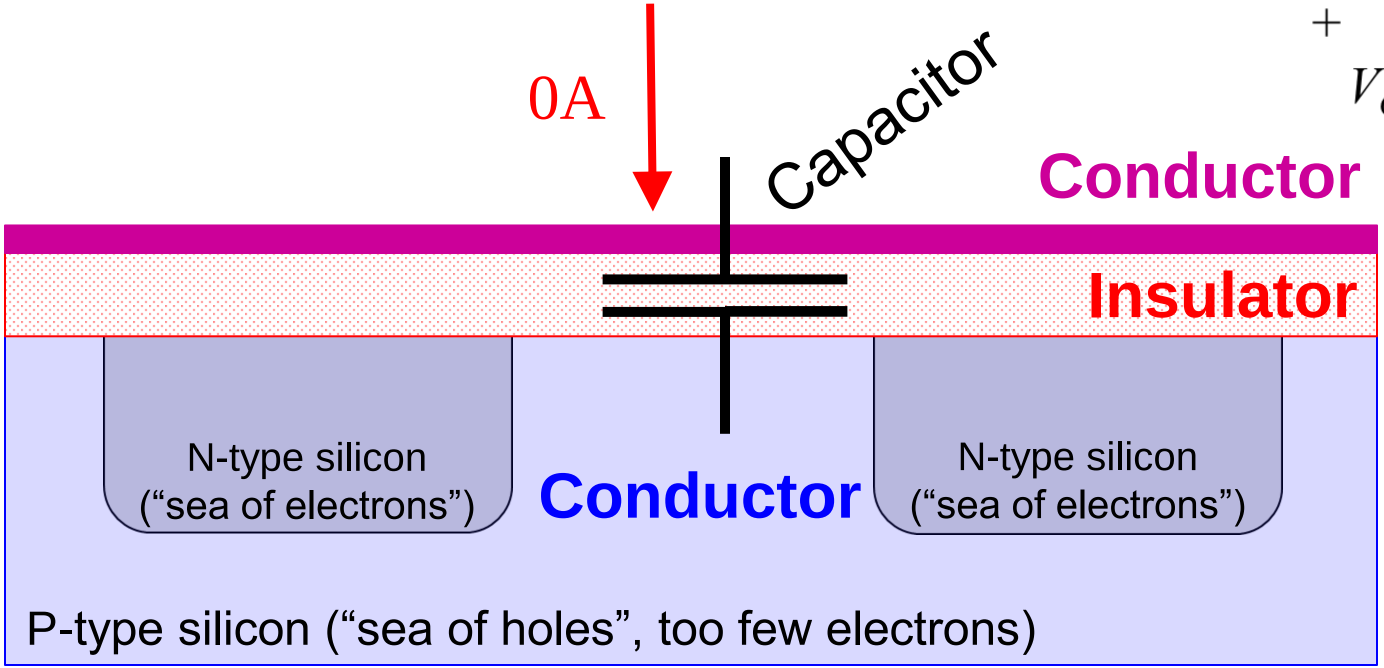
- MOSFETs Have Capacitance
- What Is Capacitance?
- Charging and Discharging Capacitors
- Implications of Output Capacitance on Logic Gates

NMOSFET: Conducts Current for $V_{GS} > V_{THN}$



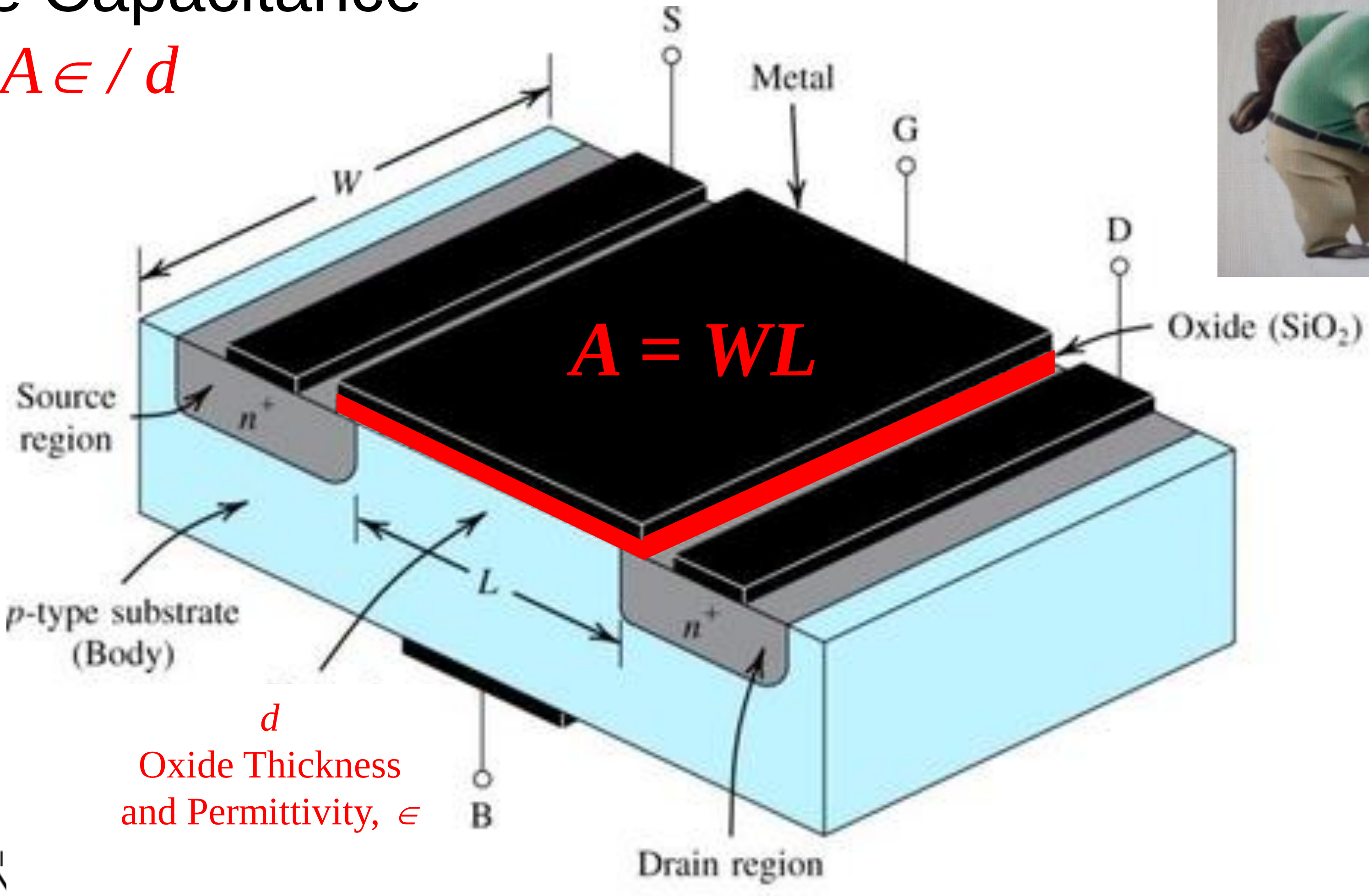
MOSFET: "Metal" Oxide Semiconductor Field Effect Transistor

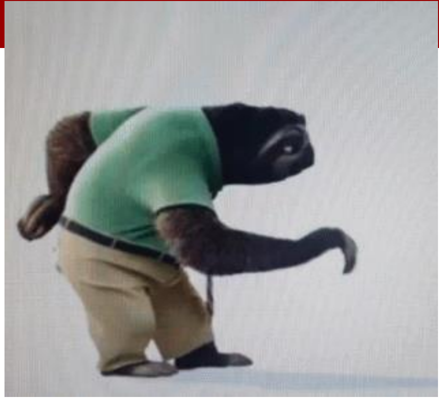




Gate Capacitance

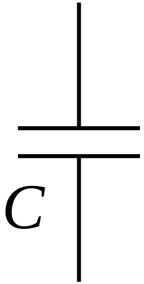
$$C = A \in / d$$



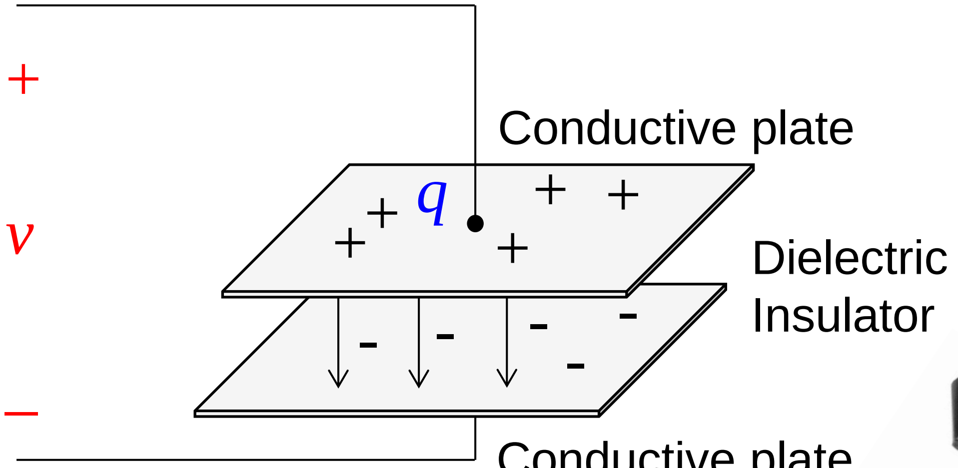


d
Oxide Thickness
and Permittivity, $\in$

Capacitor: $C = A\in/d$ Stores Energy as Electrostatic Potential



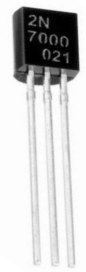
$q(t) = Cv(t)$ As $q\uparrow$, $v\uparrow$



$C_1 = 1\text{F}$

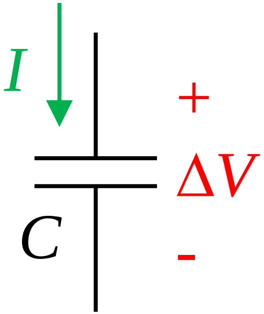
- If I put 1C of charge on C_1, $v_1 = 1\text{V}$
- If I want $V_{Gate} = 5\text{V}$, how much charge do I need on C_1? (5C)

2N7000 MOSFET, $C_{Gate} = 60\text{pF}$



- How much charge do I need to raise V_{Gate} to 5V?
- $q = (60\text{pF})(5\text{V}) = 300\text{pC}$

Capacitor: $C = A\epsilon / d$ Stores Energy as Electrostatic Potential



$$q(t) = Cv(t)$$ As $q\uparrow$, $v\uparrow$

- So, how do we put charge on a capacitor?
- Current! (1A = 1Coulomb/second)

- 300pC = 300×10^{-12} Coulombs
- Charge with 1nA = 1×10^{-9} Amperes
- time = 300pC / 1nA = 300ms!

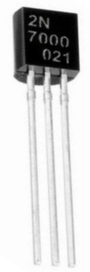
- Cannot charge / discharge instantly!!!

C_1 = 1F

- If I put 1C of charge on C_1, v_1 = 1V
- If I want V_{Gate} = 5V, how much charge do I need on C_1? (5C)

2N7000 MOSFET, C_{Gate} = 60pF

- How much charge do I need to raise V_{Gate} to 5V?
- q = (60pF)(5V) = 300pC



18-001: Capacitors for ECEs

- The voltage across a capacitor cannot change instantly:
  $v_C(t = 0^-) = v_C(t = 0^+)$

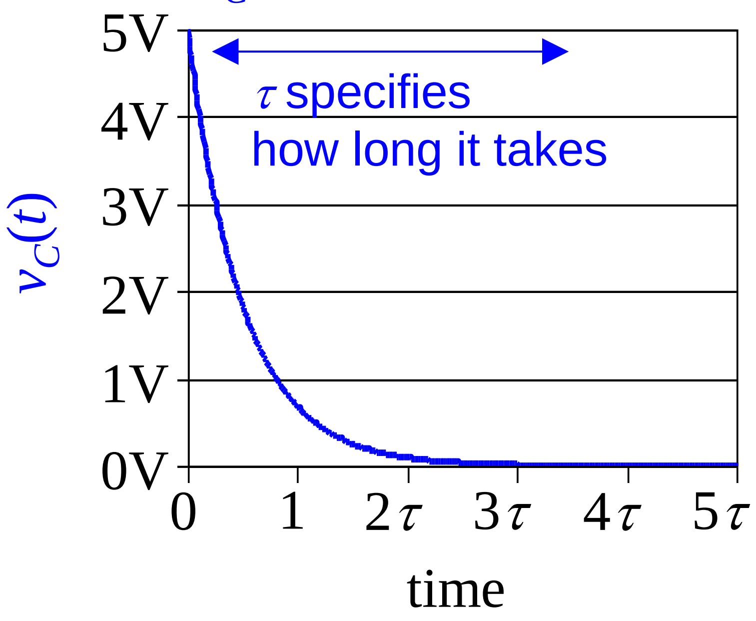
- When current first starts to flow through a capacitor, it acts like a short circuit

- If nothing is changing in your circuit, after a long time (steady state), the capacitor will act like an open circuit

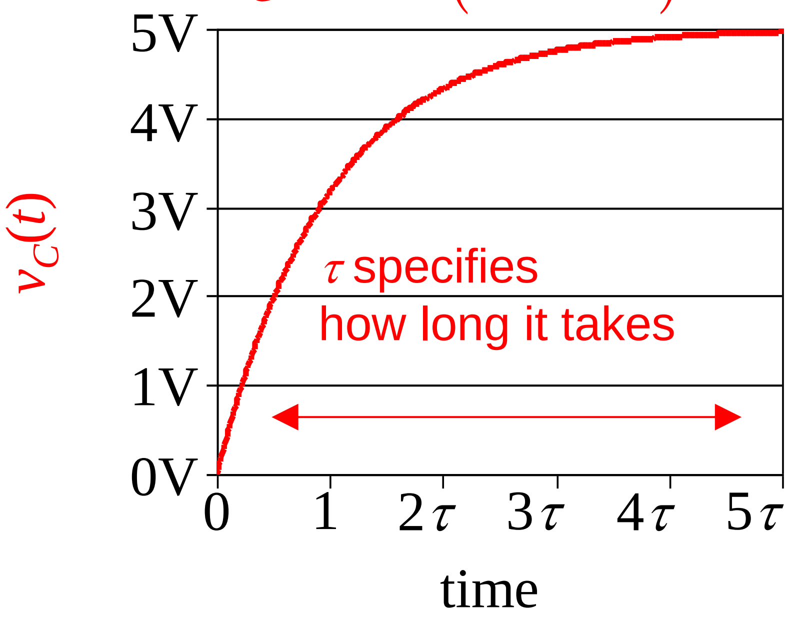
Capacitors Cannot Be Charged (or Discharged) Instantly

(Voltage Across a Capacitor Cannot Change Instantly)

$$v_C(t) = 5e^{-t/\tau} \text{ V}$$

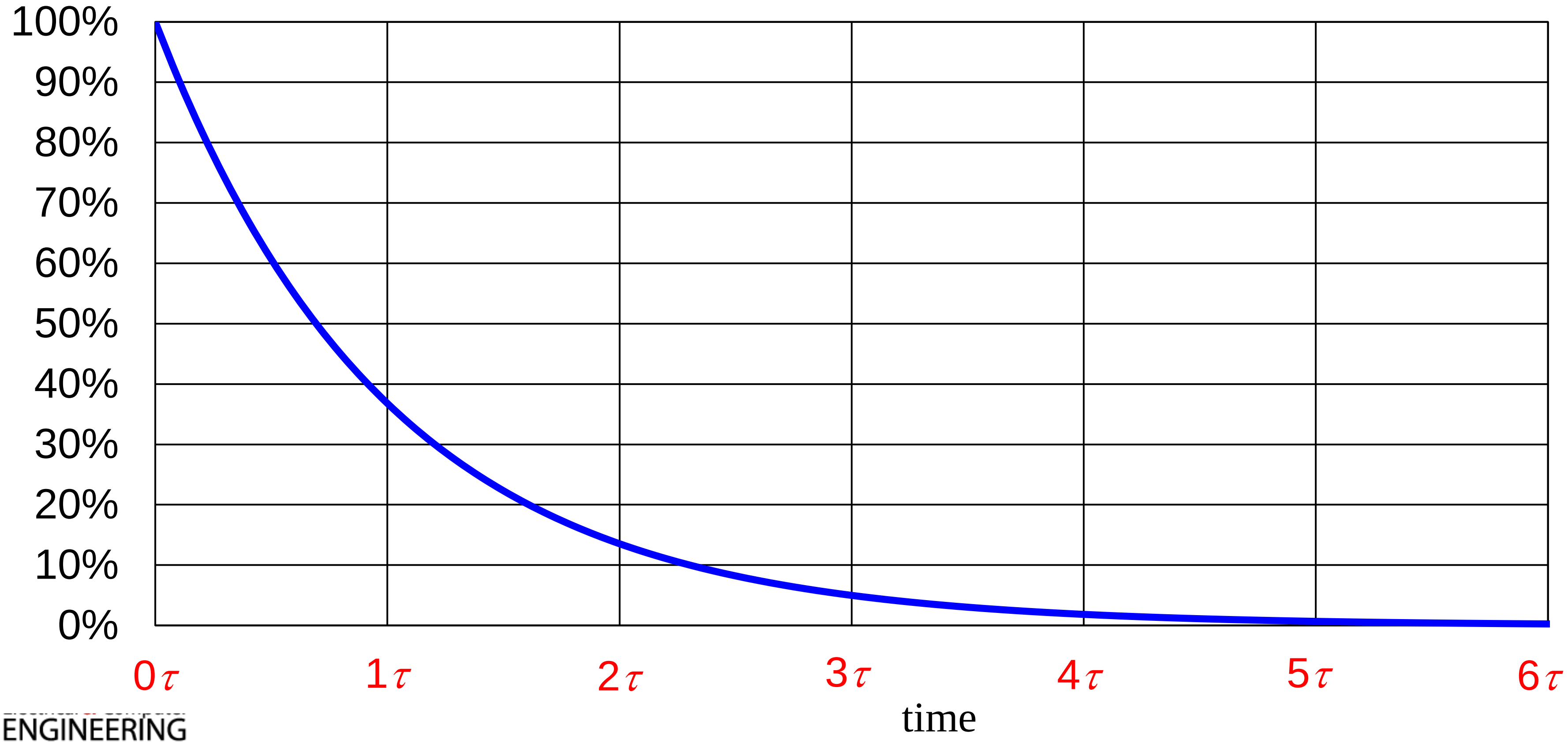


$$v_C(t) = 5\left(1 - e^{-t/\tau}\right) \text{ V}$$



$\tau = RC$ Is Called the **Time Constant**

$$v_C(t) = V_0 e^{-t/\tau}$$



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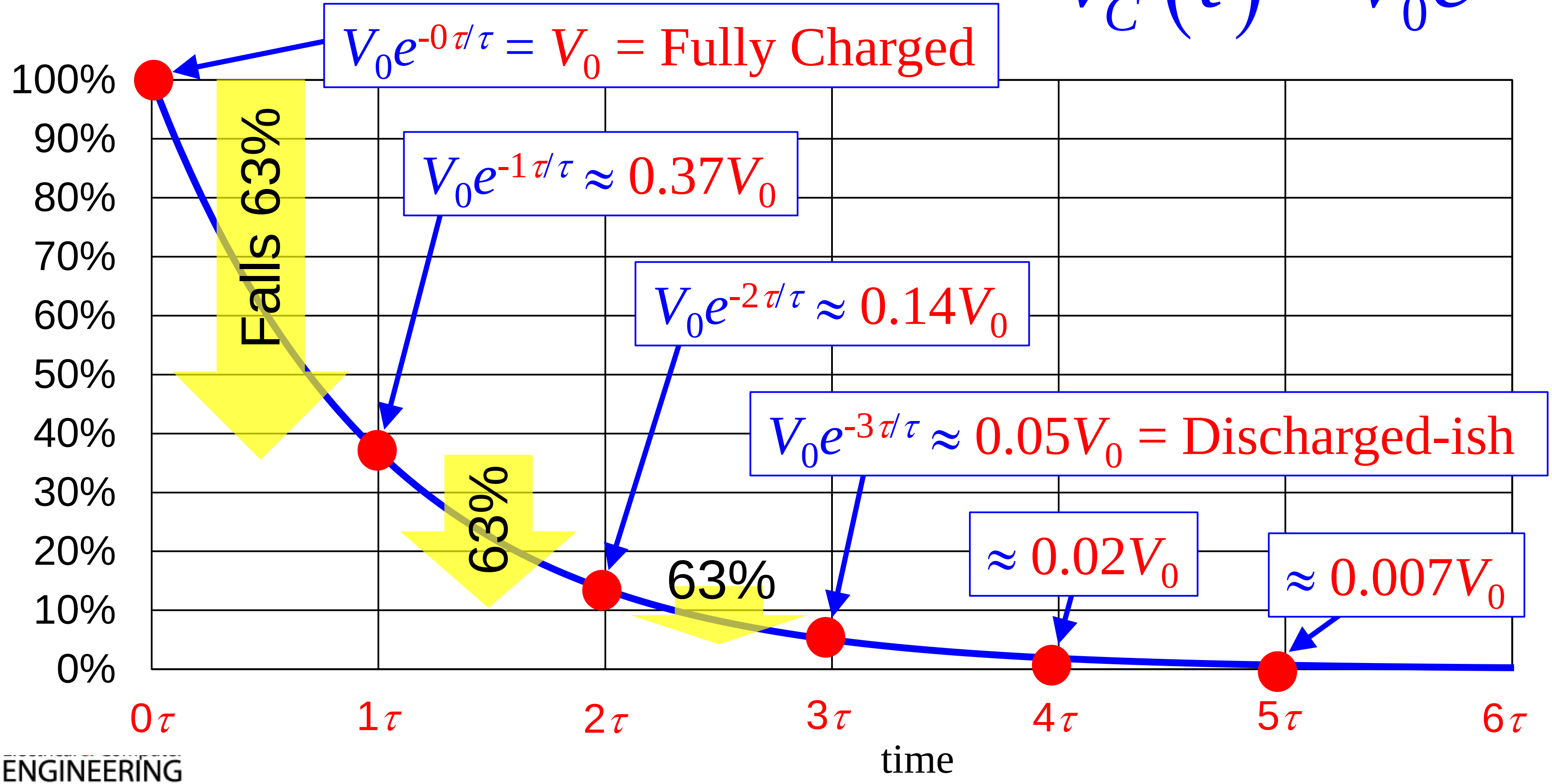

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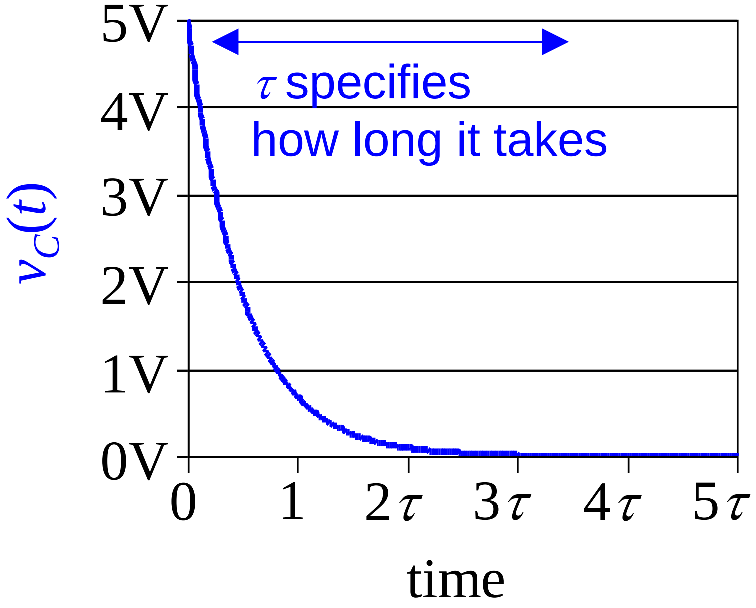

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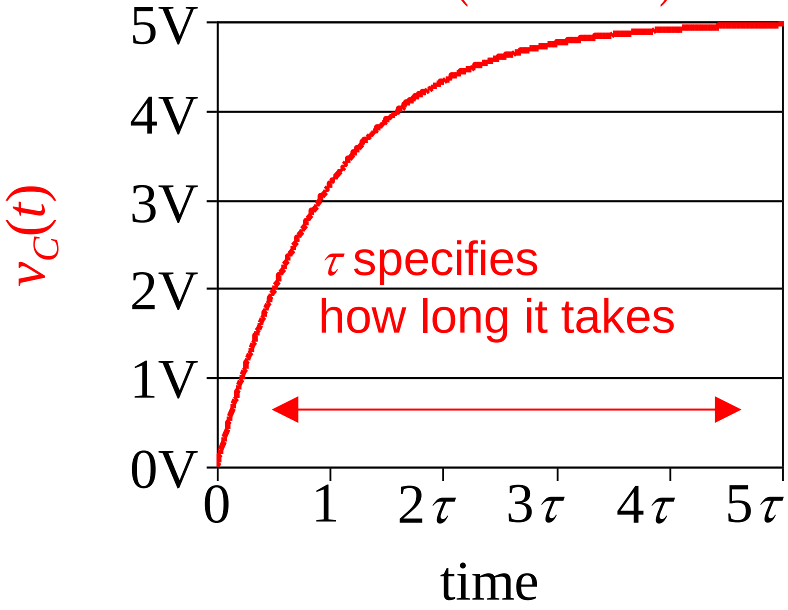

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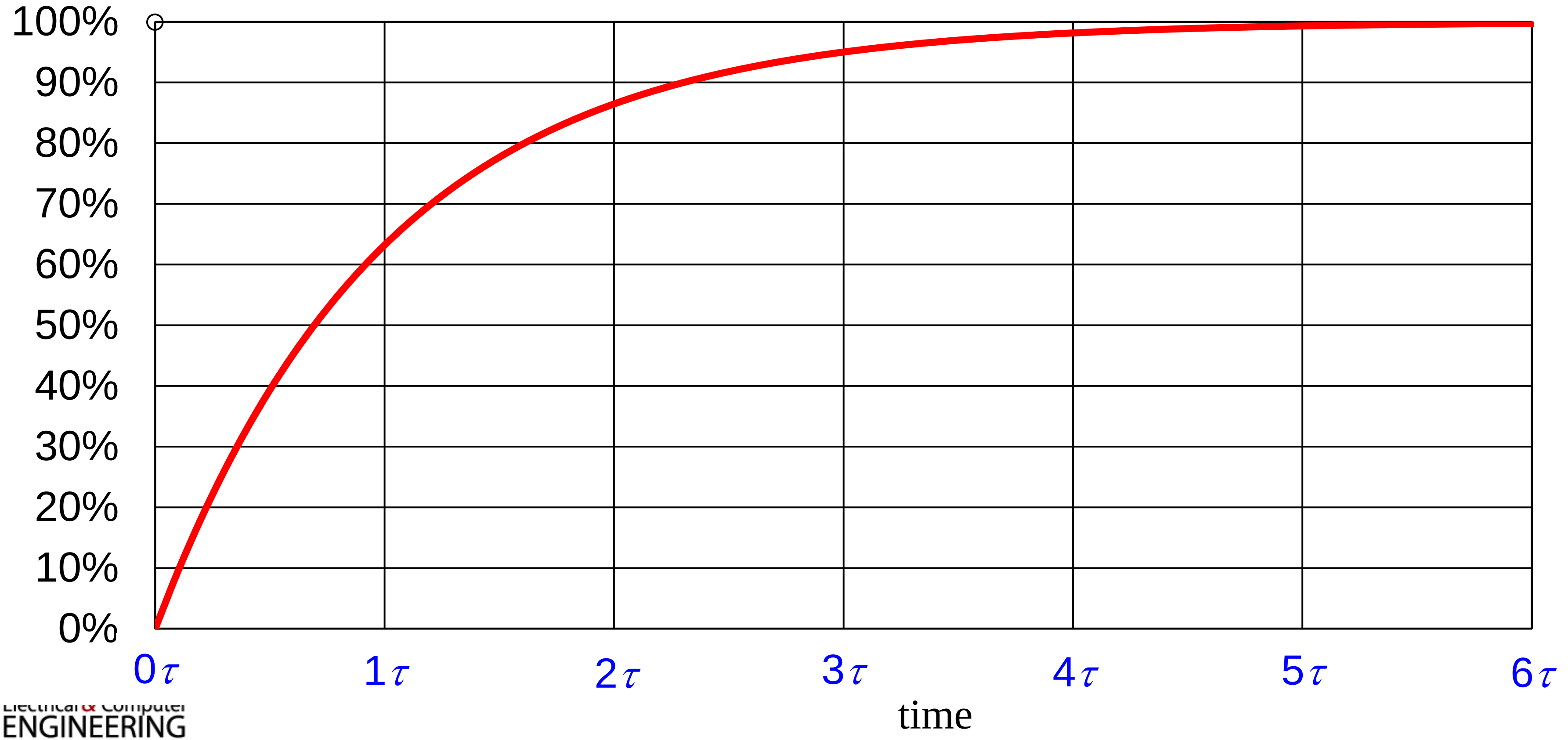

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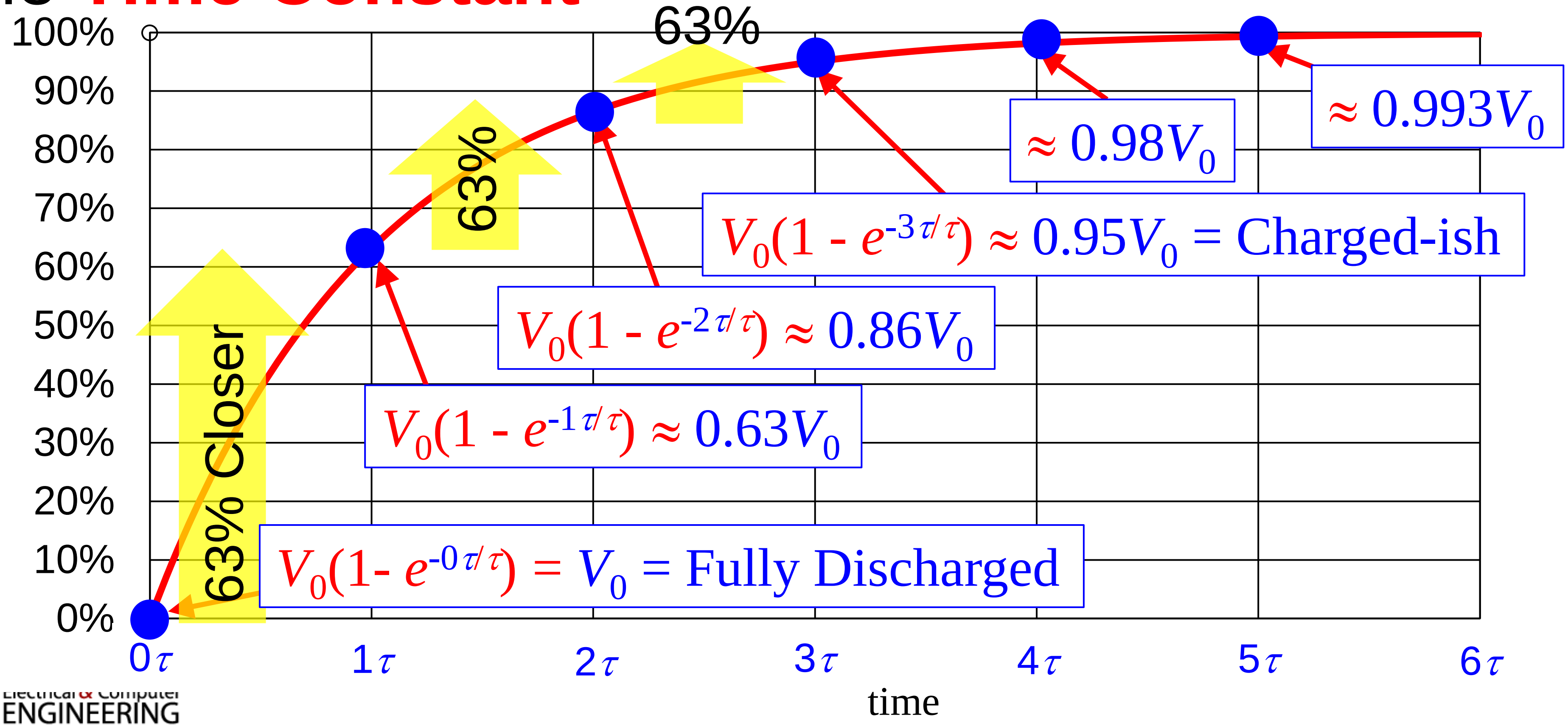

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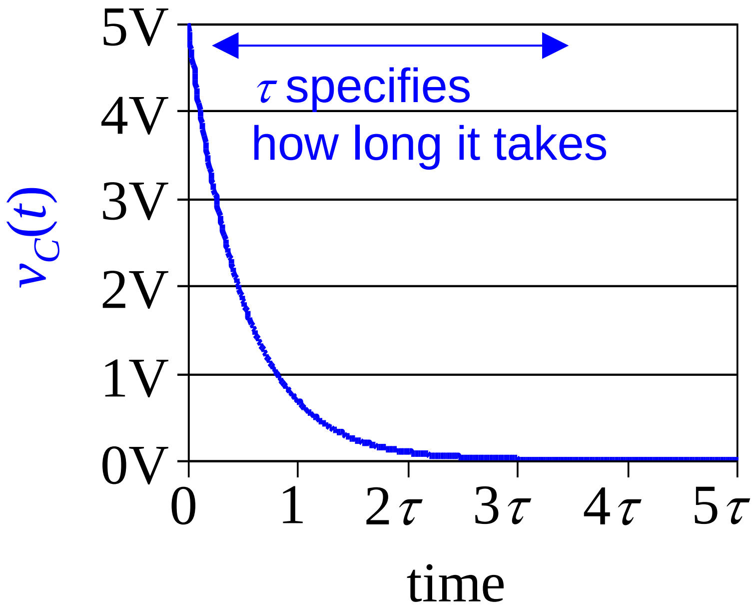

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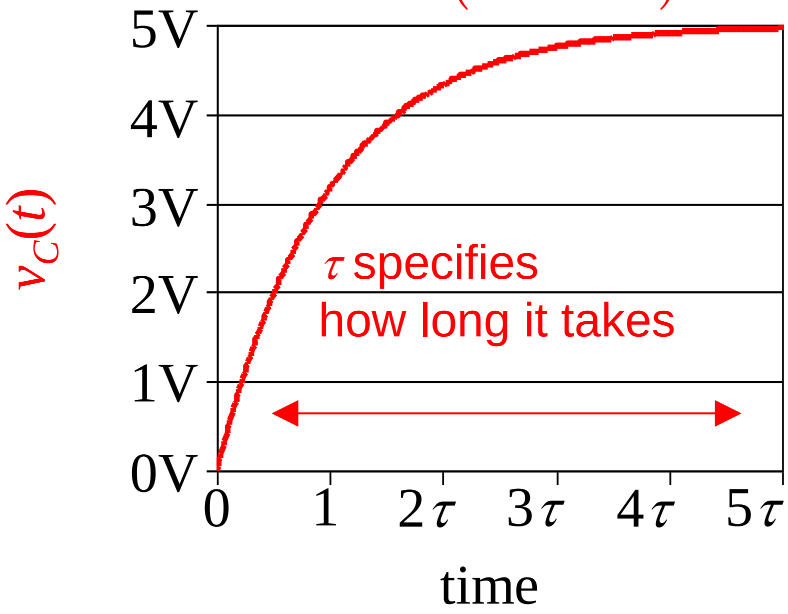
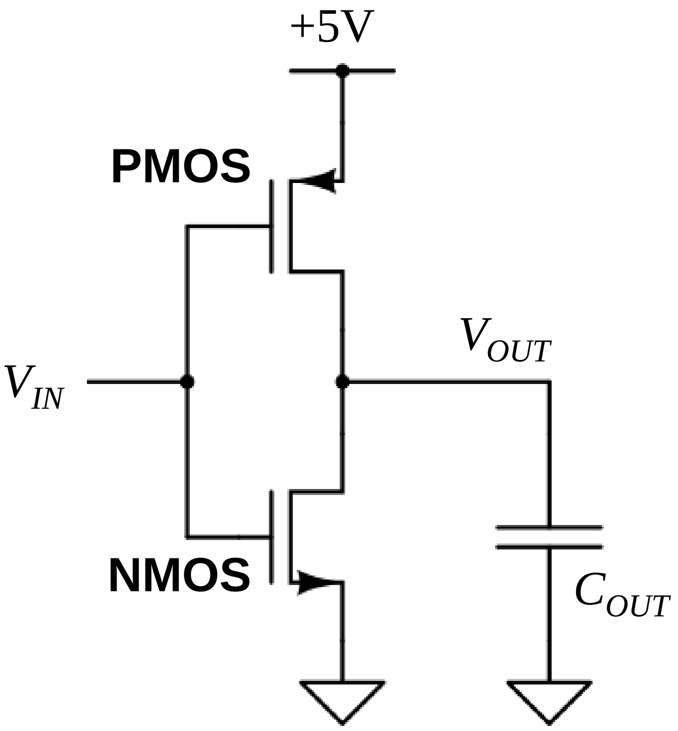


Remember MOSFETs?

Model	V_{Gate} = **HI**	V_{Gate} = **LO**
NMOS	Transistor is on. Acts like a short circuit.	Transistor is off. Acts like an open circuit.
PMOS	Transistor is off. Acts like an open circuit.	Transistor is on. Acts like a short circuit.



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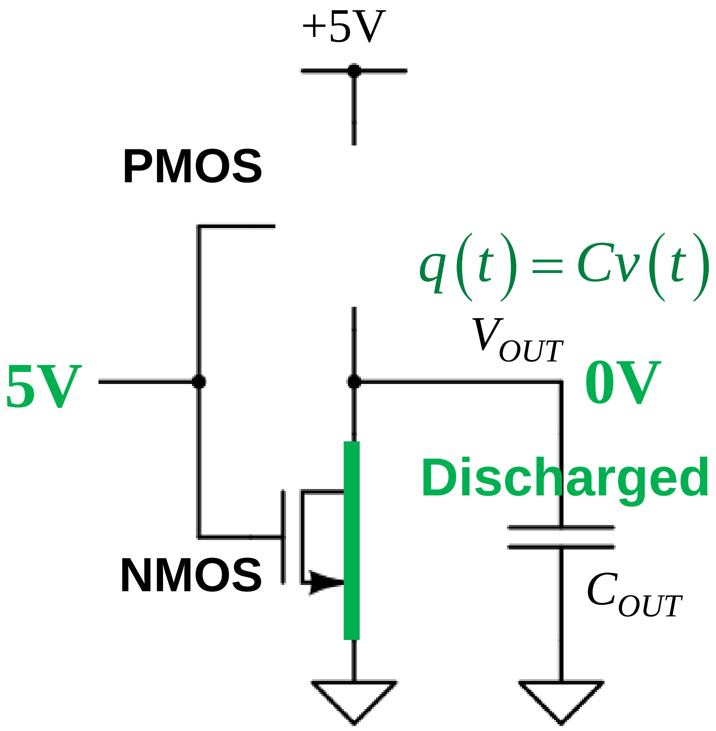

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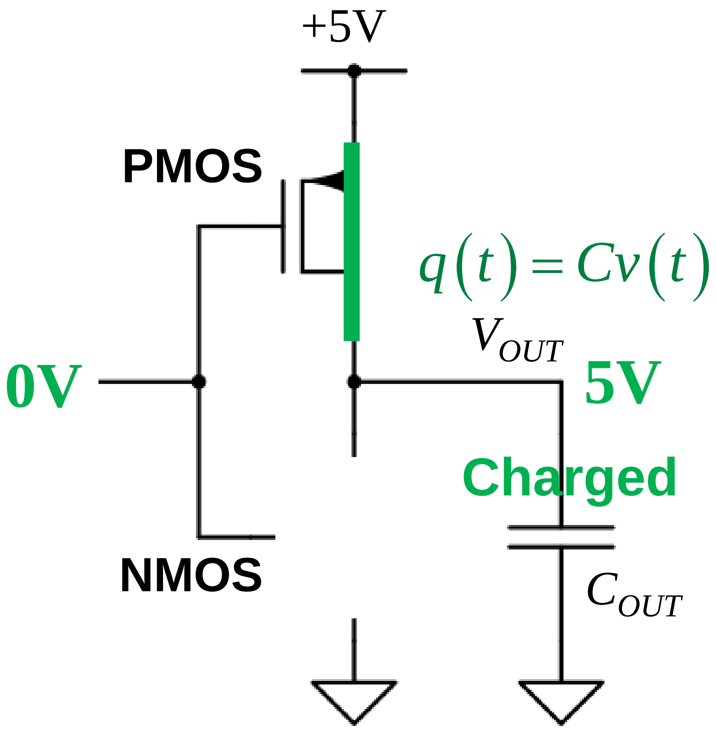
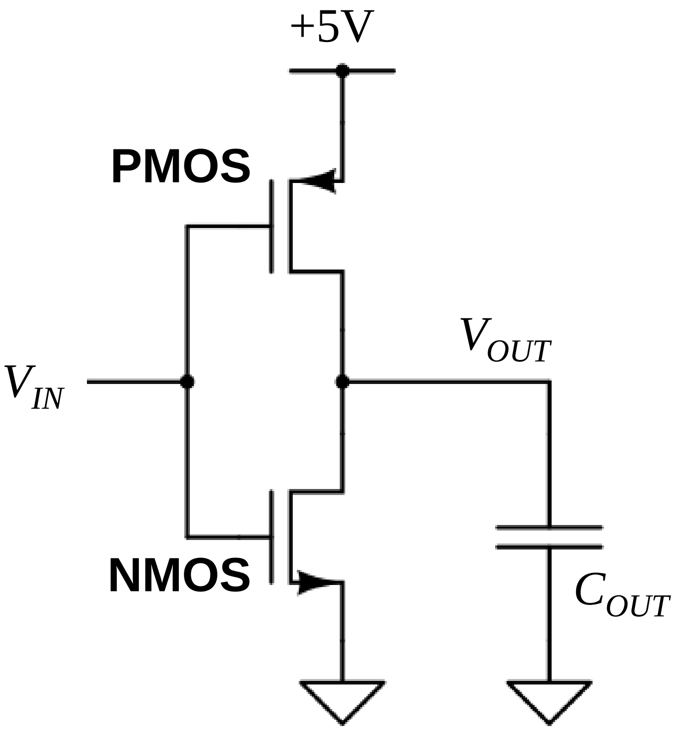


MOSFET (New) Switched Resistor Model

Switched	V_{Gate} = **HI**	V_{Gate} = **LO**
NMOS	Transistor is on. Acts like a **small resistance.**	Transistor is off. Acts like an open circuit.
PMOS	Transistor is off. Acts like an open circuit.	Transistor is on. Acts like a **small resistance.**



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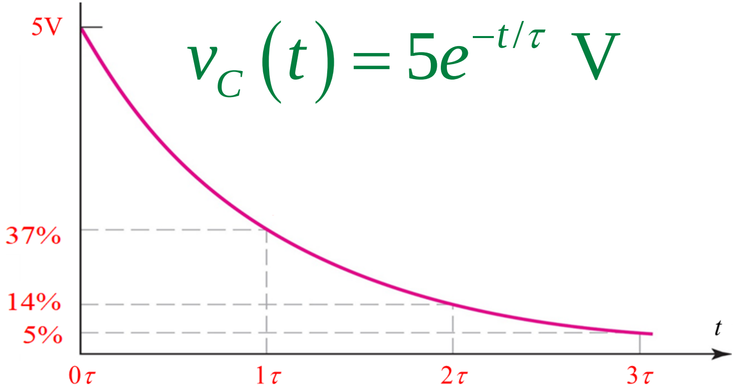
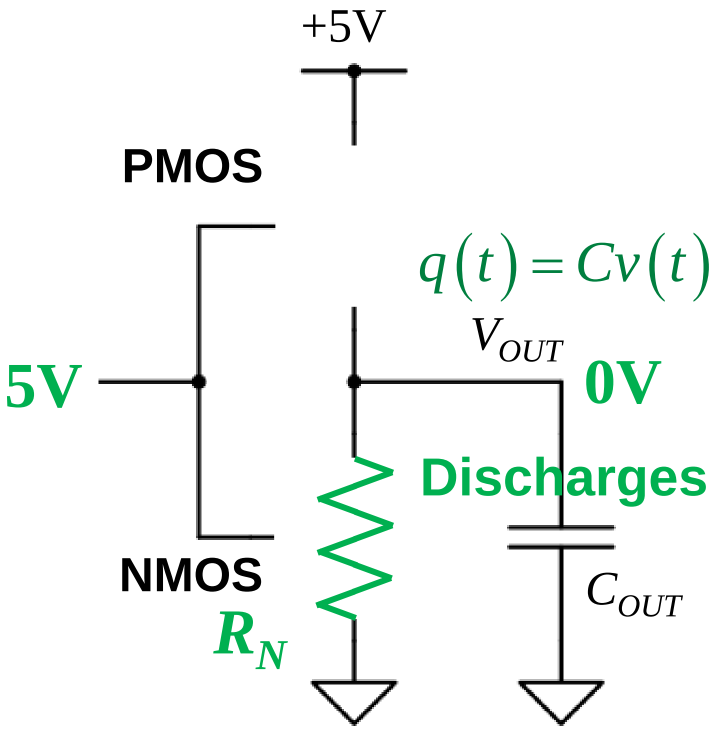


$$v_C(t) = 5e^{-t/\tau}\ \text{V}$$



$$\tau = R_N C_{OUT}$$

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NMOS	Transistor is on. Acts like a **small resistance.**	Transistor is off. Acts like an open circuit.
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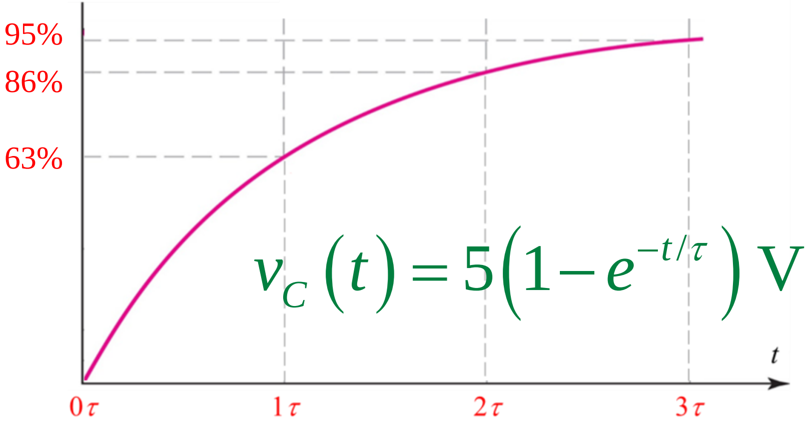
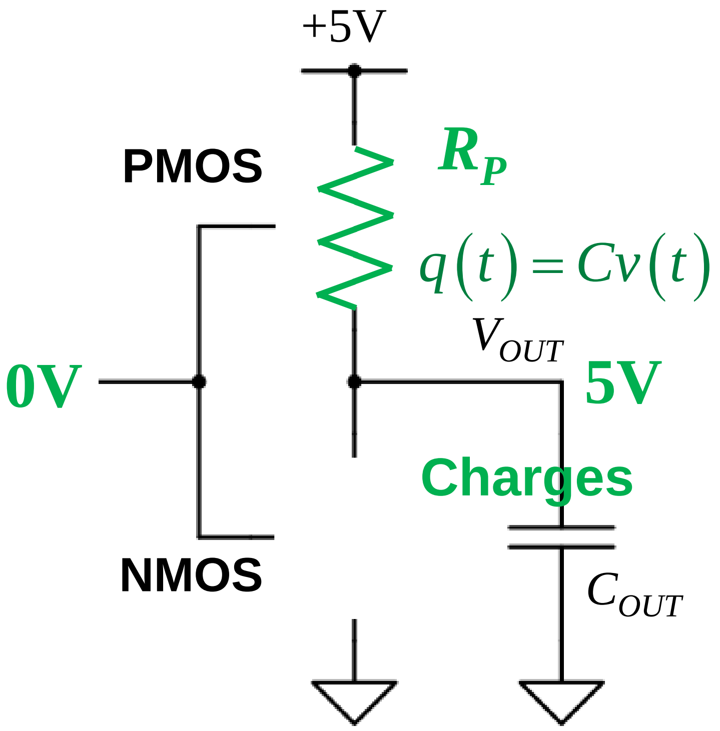
$$\tau = R_P C_{OUT}$$



$$v_C(t) = 5\left(1 - e^{-t/\tau}\right)\text{ V}$$



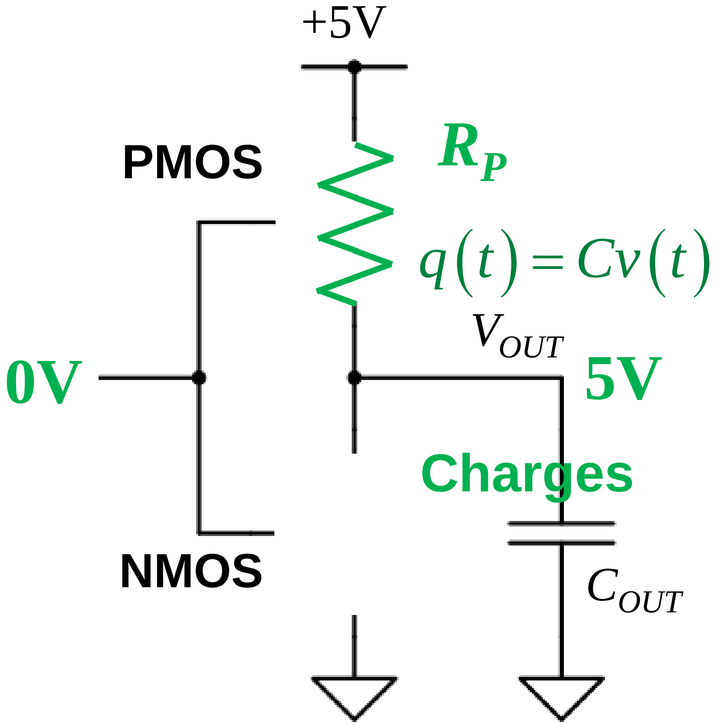
Example: Use the Switched Resistor Model to Estimate the Switching Time of a CMOS Gate

$\tau = R_P C_{OUT}$

$R_N = 10\Omega$ (proportional to cost)
$R_P = 20\Omega$ (usually bigger than R_N)

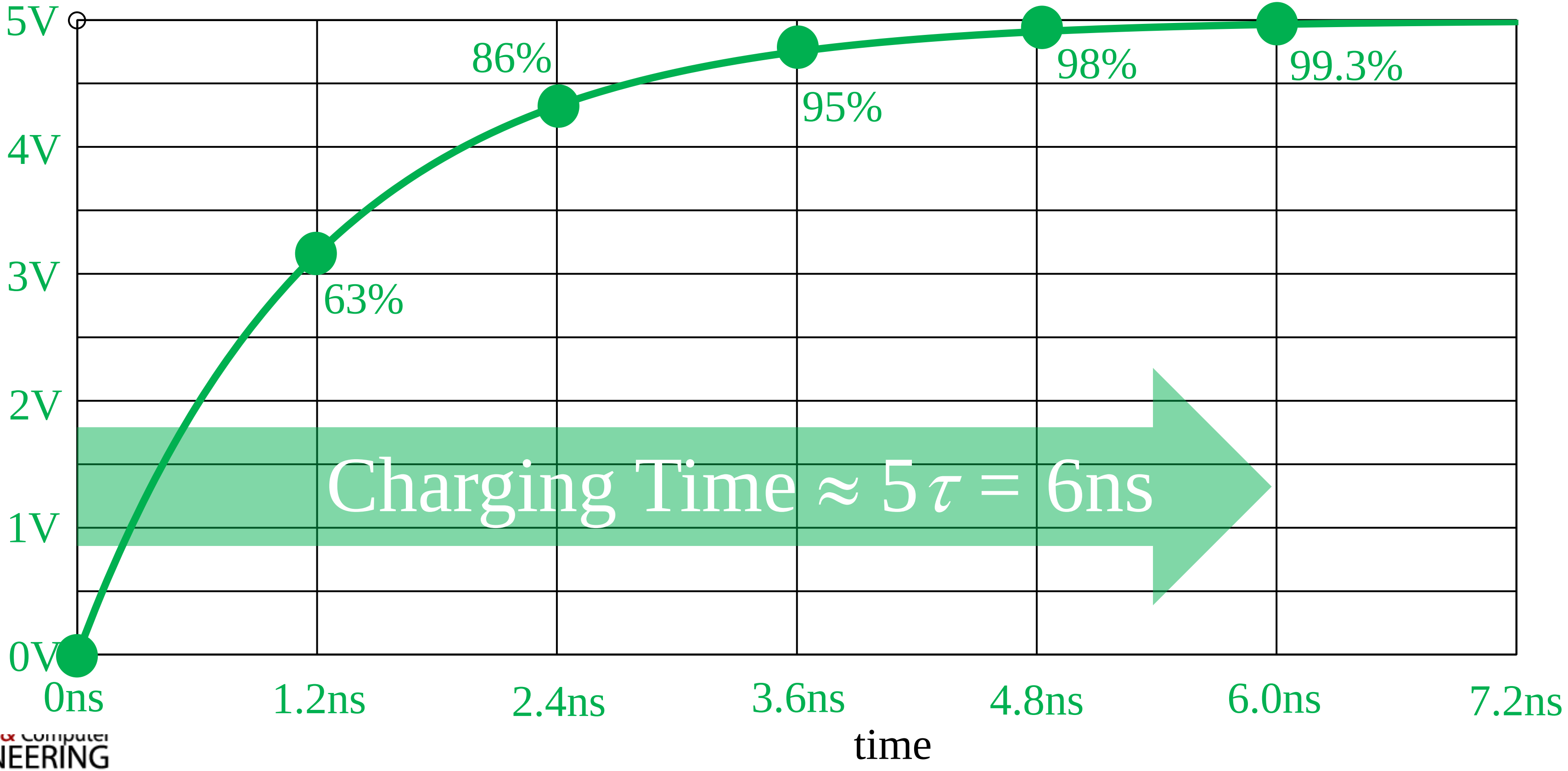
$C_{OUT} = 60\text{pF} = 60\text{x}10^{-12}$ Farads

Charging: $\tau = R_P C_{OUT} = (20\Omega)(60\text{ps}) = 1.2\text{ns}$
Charging Time $\approx 5\tau = 6\text{ns}$



$\tau = RC$ Is Called the **Time Constant**

$$v_C(t) = V_0\left(1 - e^{-t/\tau}\right)$$

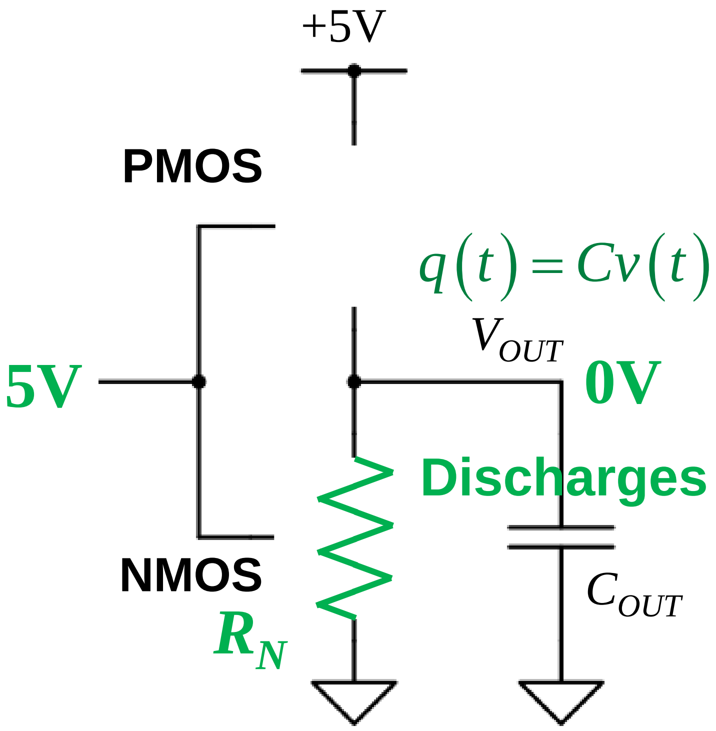


Example: Use the Switched Resistor Model to Estimate the Switching Time of a CMOS Gate

$R_N = 10\Omega$ (proportional to cost)
$R_P = 20\Omega$ (usually bigger than R_N)

$C_{OUT} = 60\text{pF} = 60\text{x}10^{-12}$ Farads

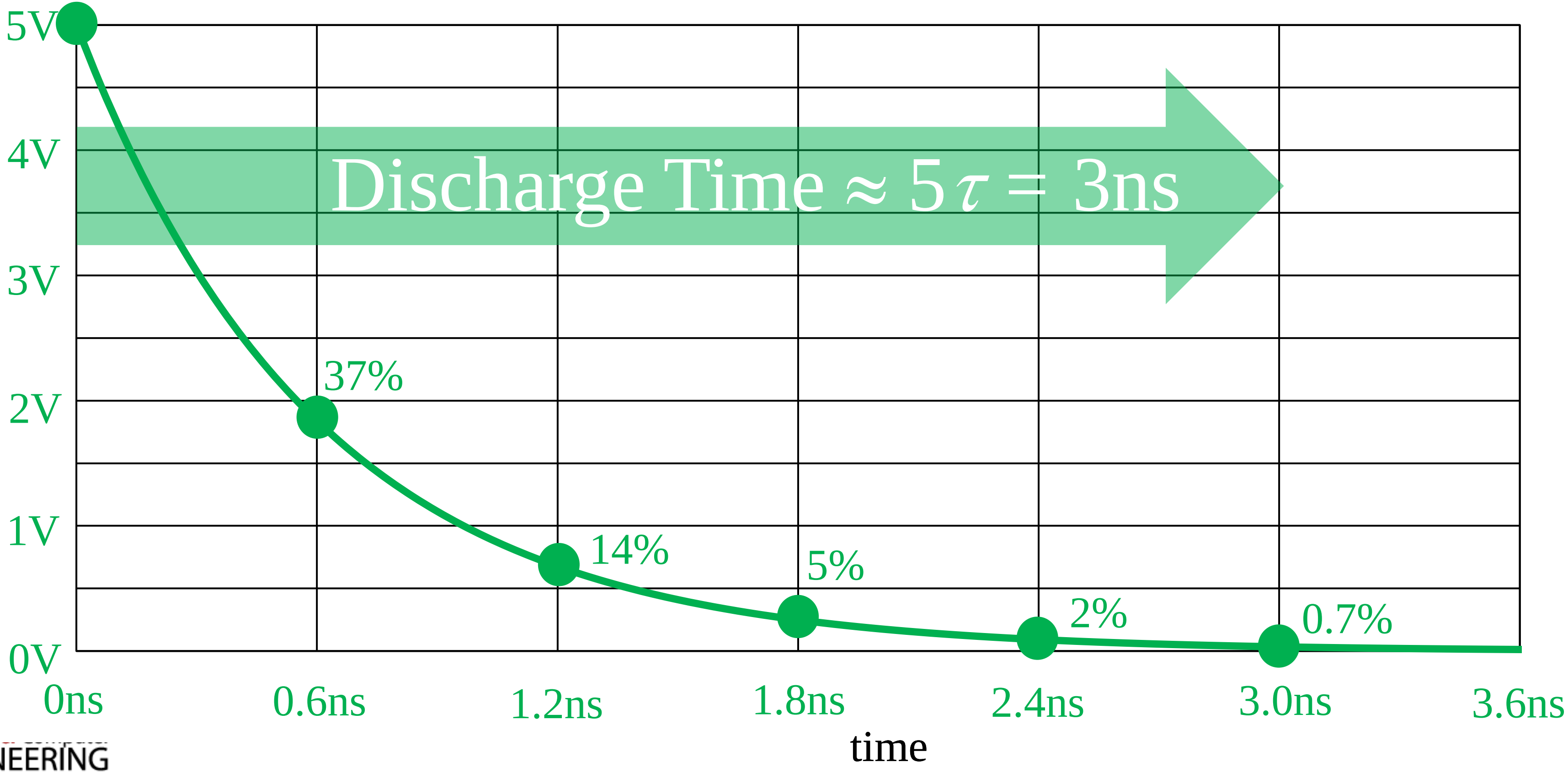
Discharging: $\tau = R_N C_{OUT} = (10\Omega)(60\text{ps}) = 0.6\text{ns}$
Discharging Time $\approx 5\tau = 3\text{ns}$



$\tau = R_N C_{OUT}$

$\tau = RC$ Is Called the **Time Constant**

$$v_C(t) = V_0 e^{-t/\tau}$$

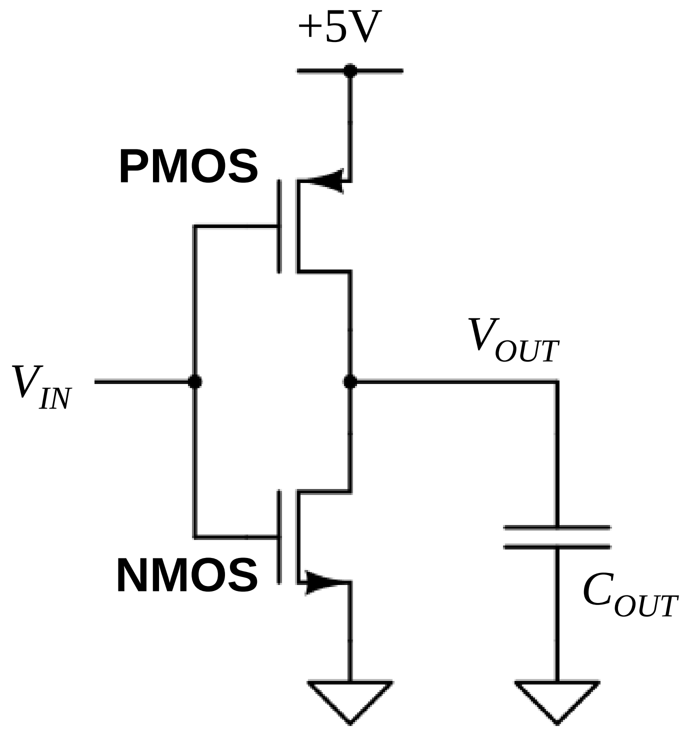


Example: Use the Switched Resistor Model to Estimate the Switching Time of a CMOS Gate and Maximum f

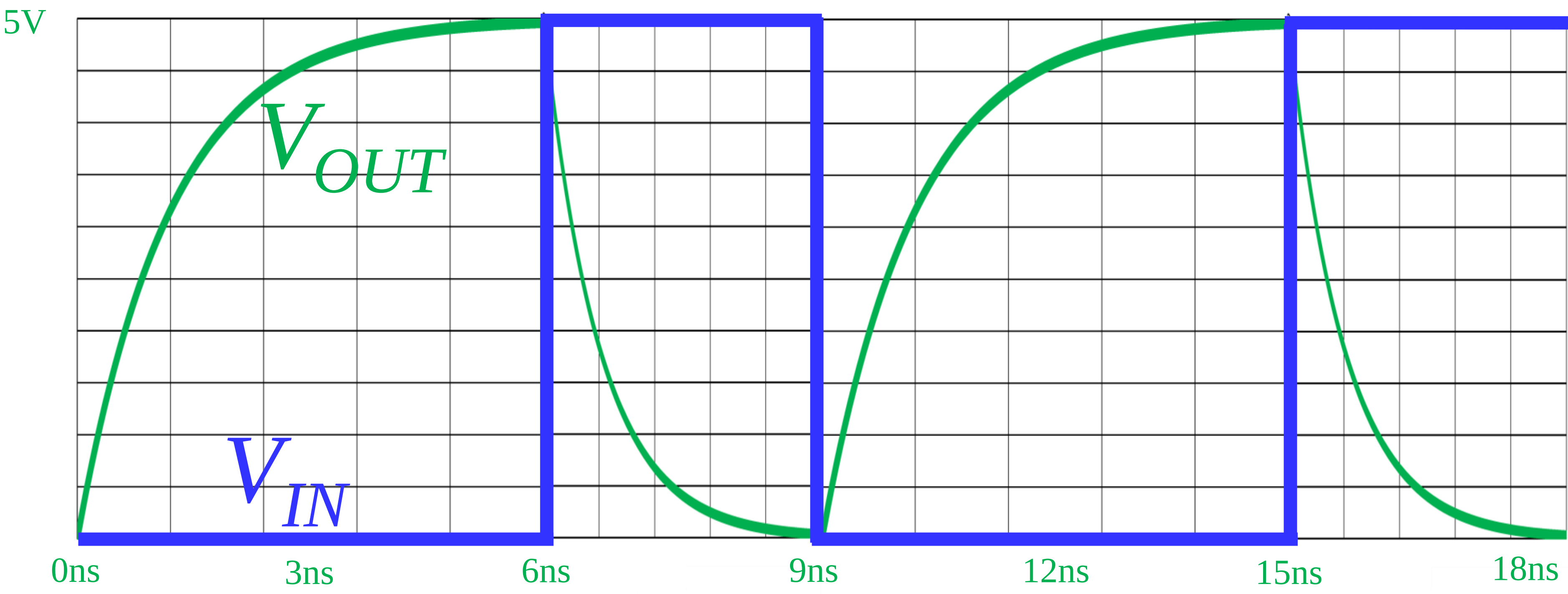
$R_N = 10\Omega$ (proportional to cost)
$R_P = 20\Omega$ (usually bigger than R_N)

$C_{OUT} = 60\text{pF} = 60\text{x}10^{-12}$ Farads

Charging Time $\approx 5\tau = 6\text{ns}$
Discharging Time $\approx 5\tau = 3\text{ns}$



Example: Use the Switched Resistor Model to Estimate the Switching Time of a CMOS Gate and Maximum *f*



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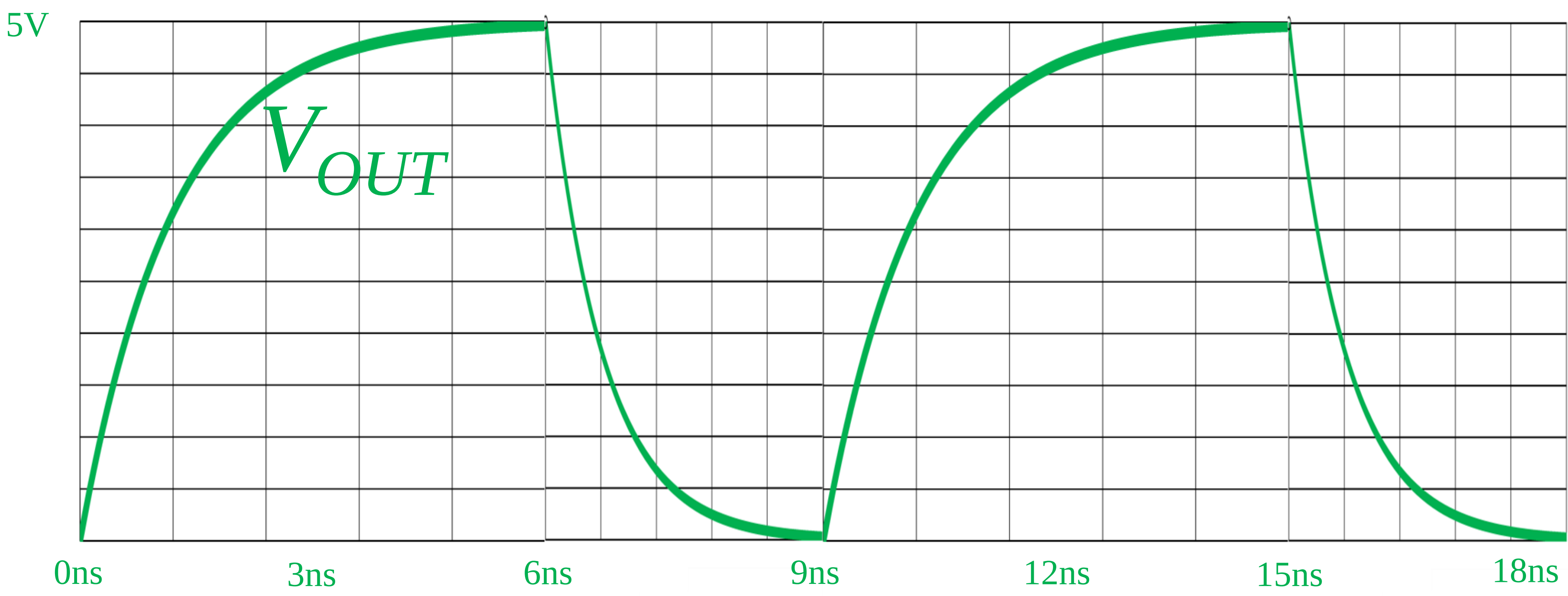

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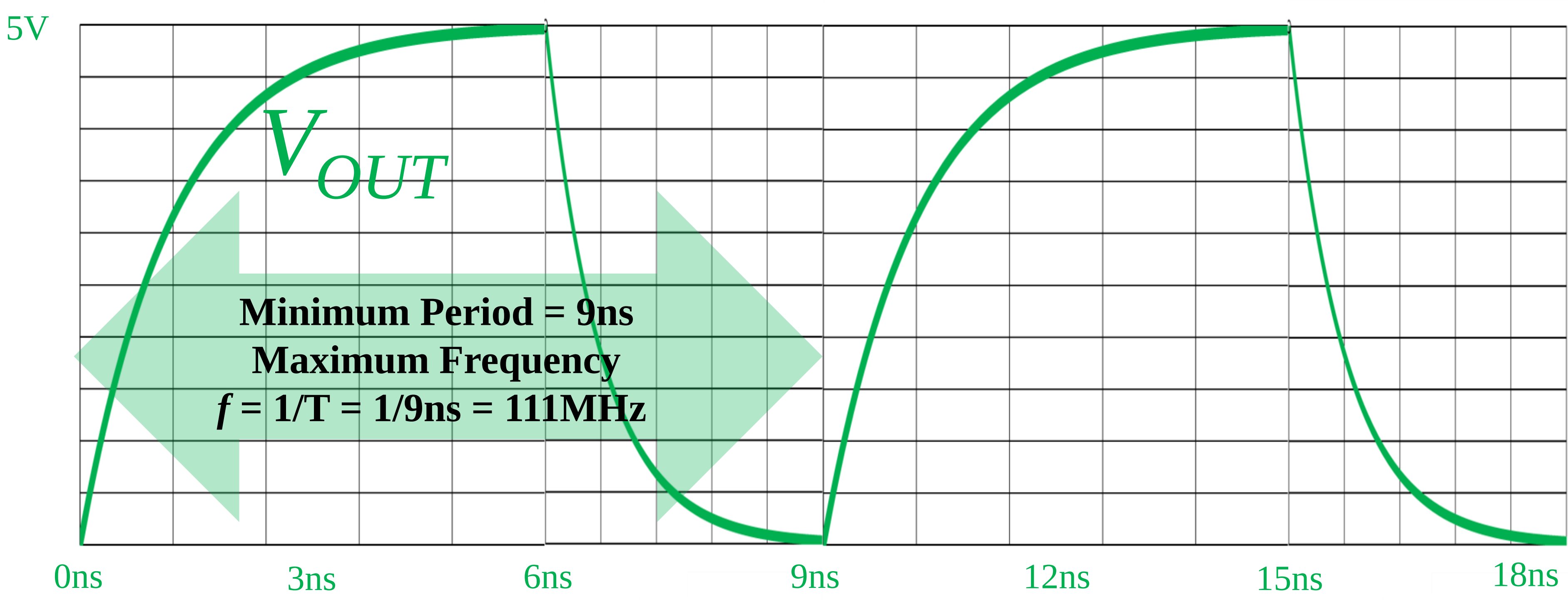


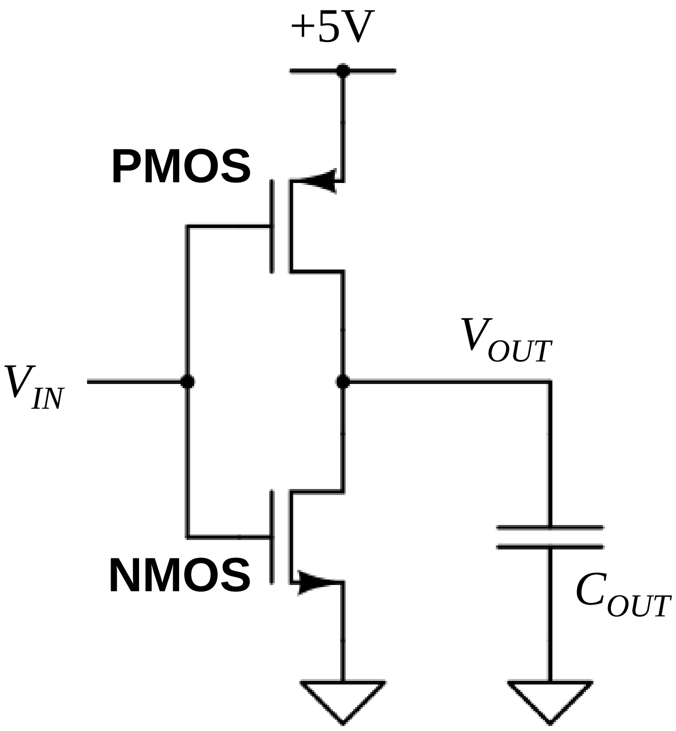
Example: Use the Switched Resistor Model to Estimate the Switching Time of a CMOS Gate and Maximum *f*

R_N = 10Ω (proportional to cost)
R_P = 20Ω (usually bigger than R_N)

C_{OUT} = 60pF = 60×10^{-12} Farads

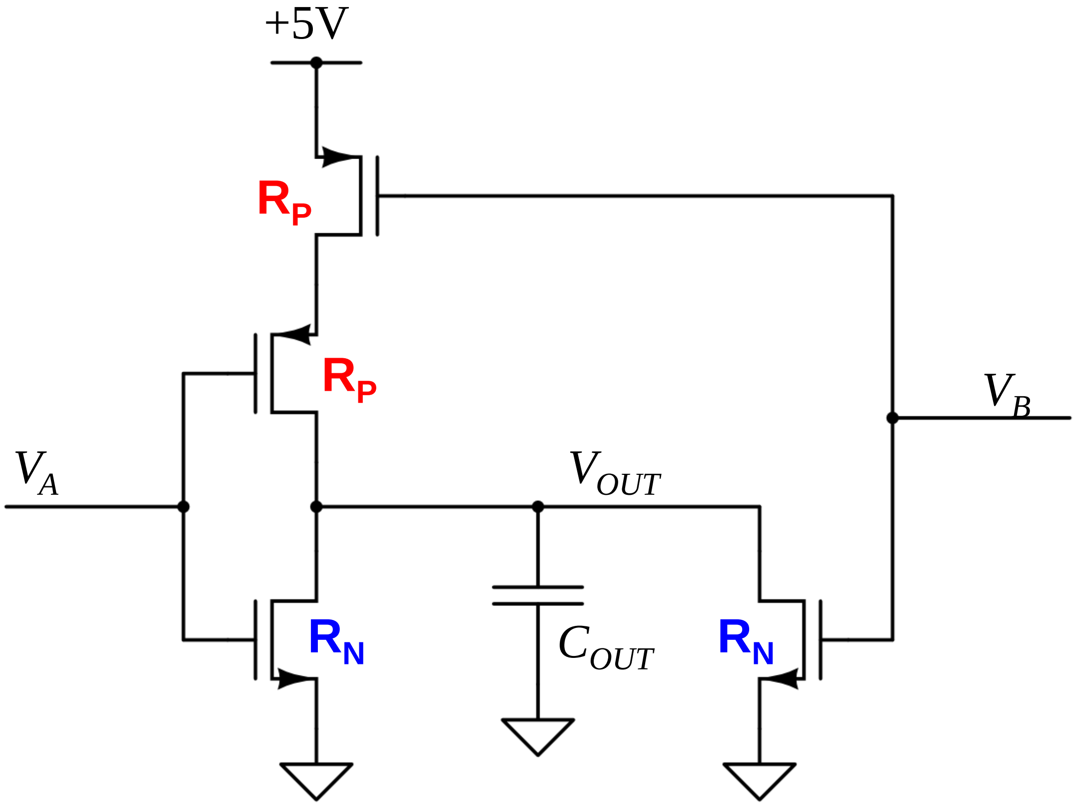
Charging Time ≈ 5τ = 6ns
Discharging Time ≈ 5τ = 3ns

f_{Max} = 1 / T = 1 / (6ns + 3ns)
= 111MHz



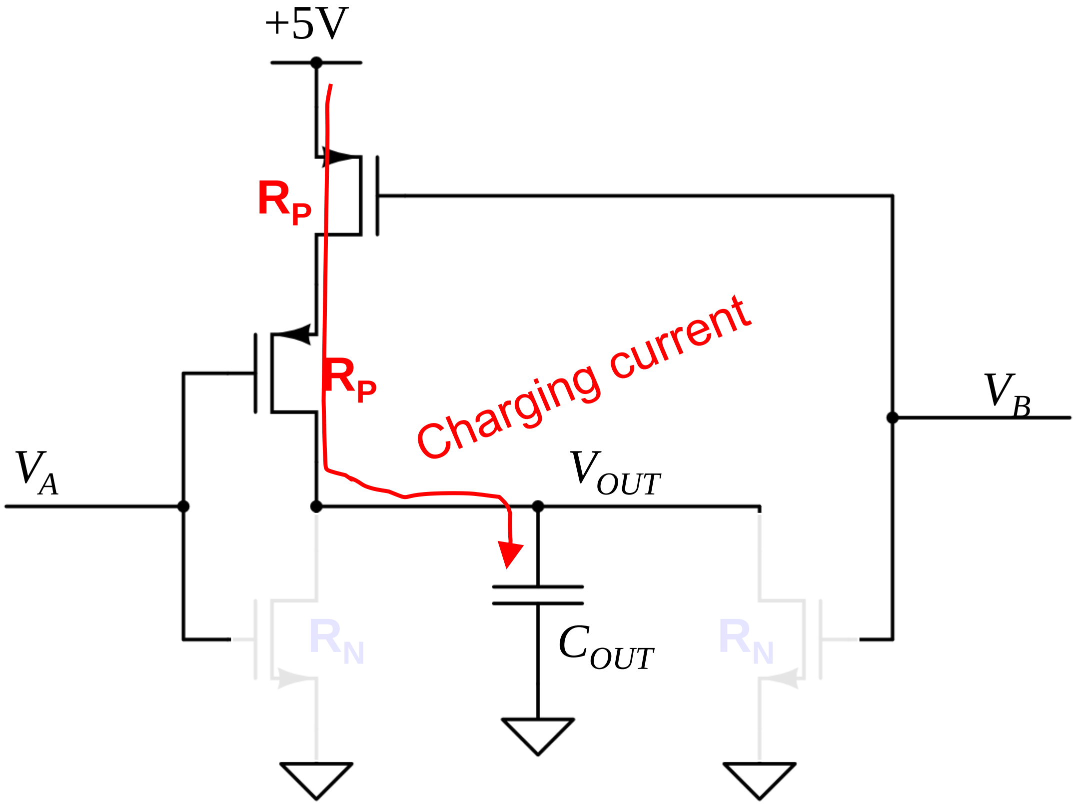
What About Using the Switched Resistor Model with Other Logic Gates? (NOR)

V_A	V_B	V_{OUT}	Time Constant, τ
0	0	1	
0	1	0	
1	0	0	
1	1	0	



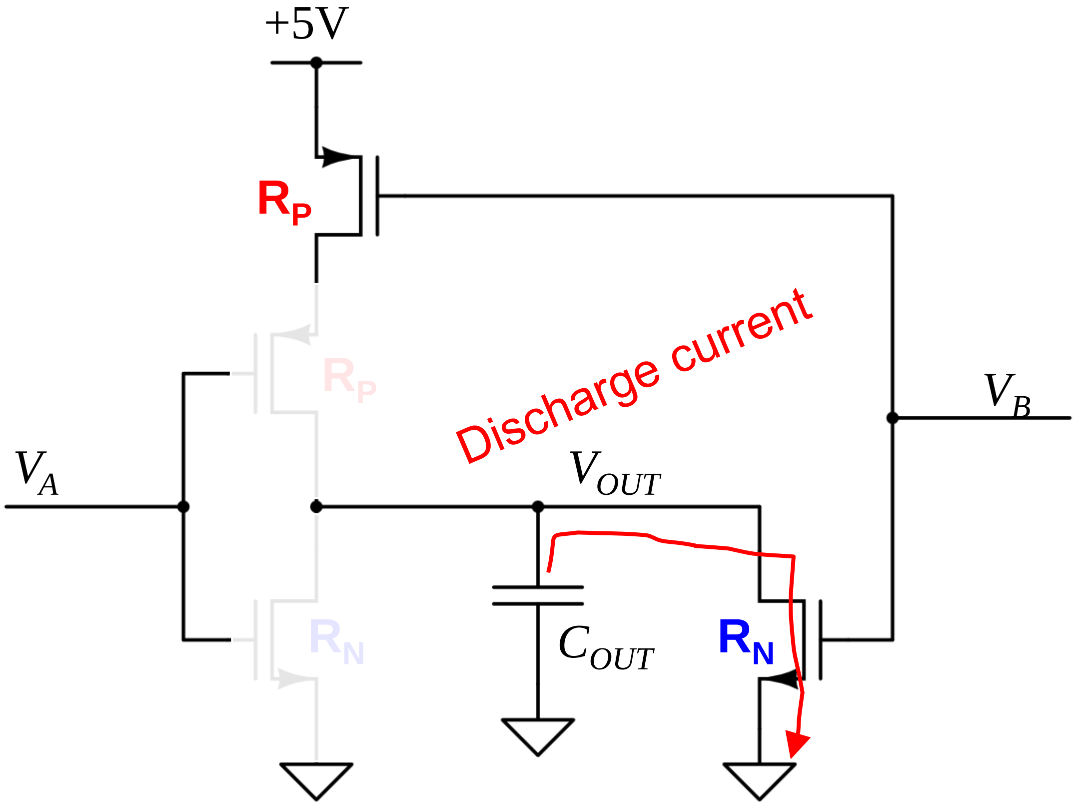
What About Using the Switched Resistor Model with Other Logic Gates? (NOR)

V_A	V_B	V_{OUT}	Time Constant, τ
0	0	1	$(R_P + R_P)(C_{OUT})$
0	1	0	
1	0	0	
1	1	0	



What About Using the Switched Resistor Model with Other Logic Gates? (NOR)

V_A	V_B	V_{OUT}	Time Constant, τ
0	0	1	$(R_P + R_P)(C_{OUT})$
0	1	0	$(R_N)(C_{OUT})$
1	0	0	
1	1	0	



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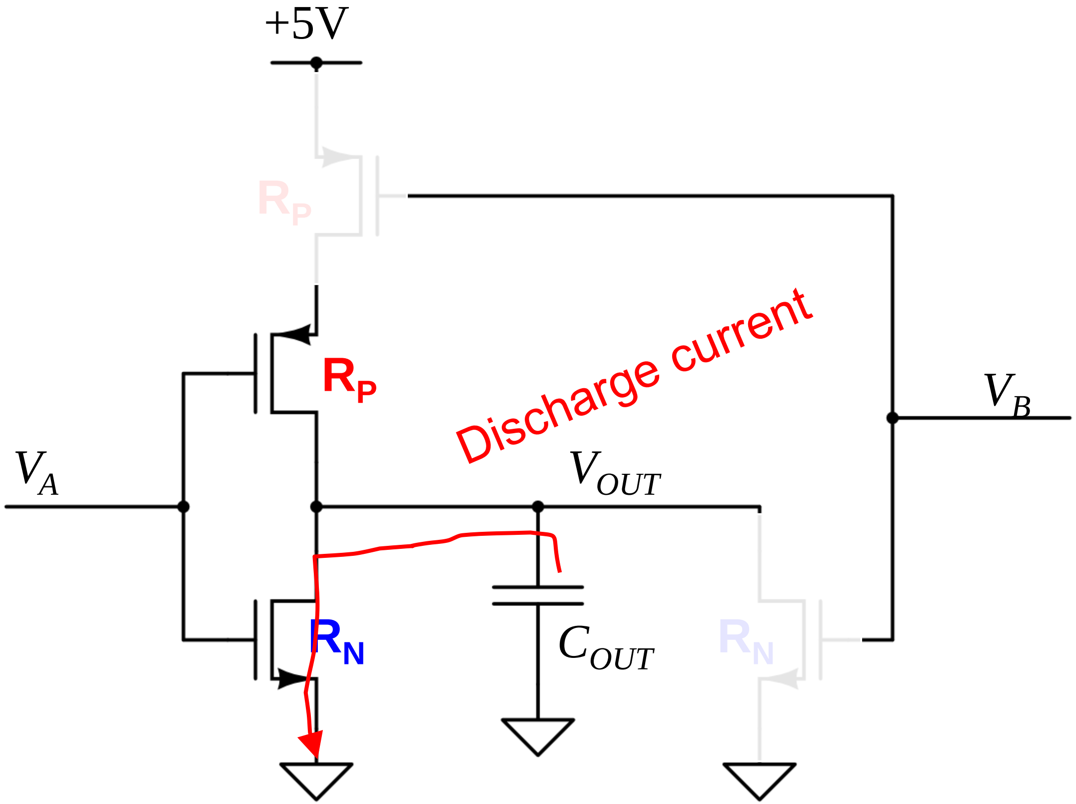
V_A	V_B	V_{OUT}	Time Constant, τ
0	0	1	$(R_P + R_P)(C_{OUT})$
0	1	0	$(R_N)(C_{OUT})$
1	0	0	$(R_N)(C_{OUT})$
1	1	0	



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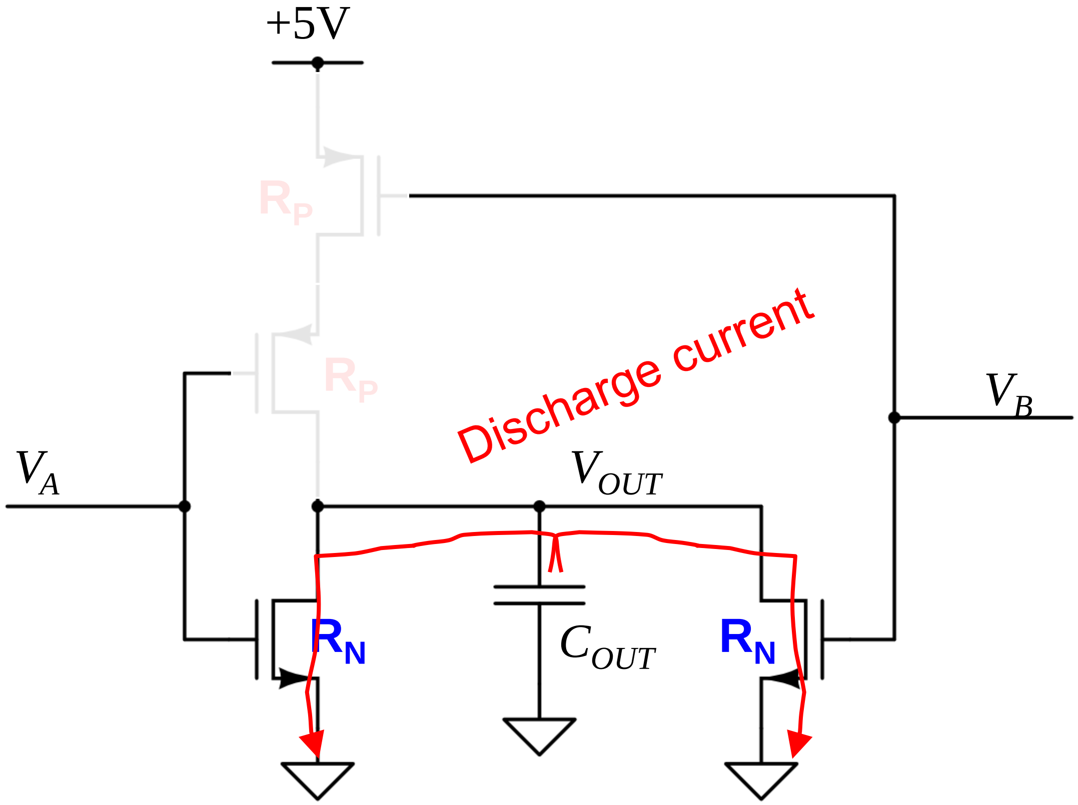
V_A	V_B	V_{OUT}	Time Constant, τ
0	0	1	$(R_P + R_P)(C_{OUT})$
0	1	0	$(R_N)(C_{OUT})$
1	0	0	$(R_N)(C_{OUT})$
1	1	0	$(R_N \parallel R_N)(C_{OUT})$



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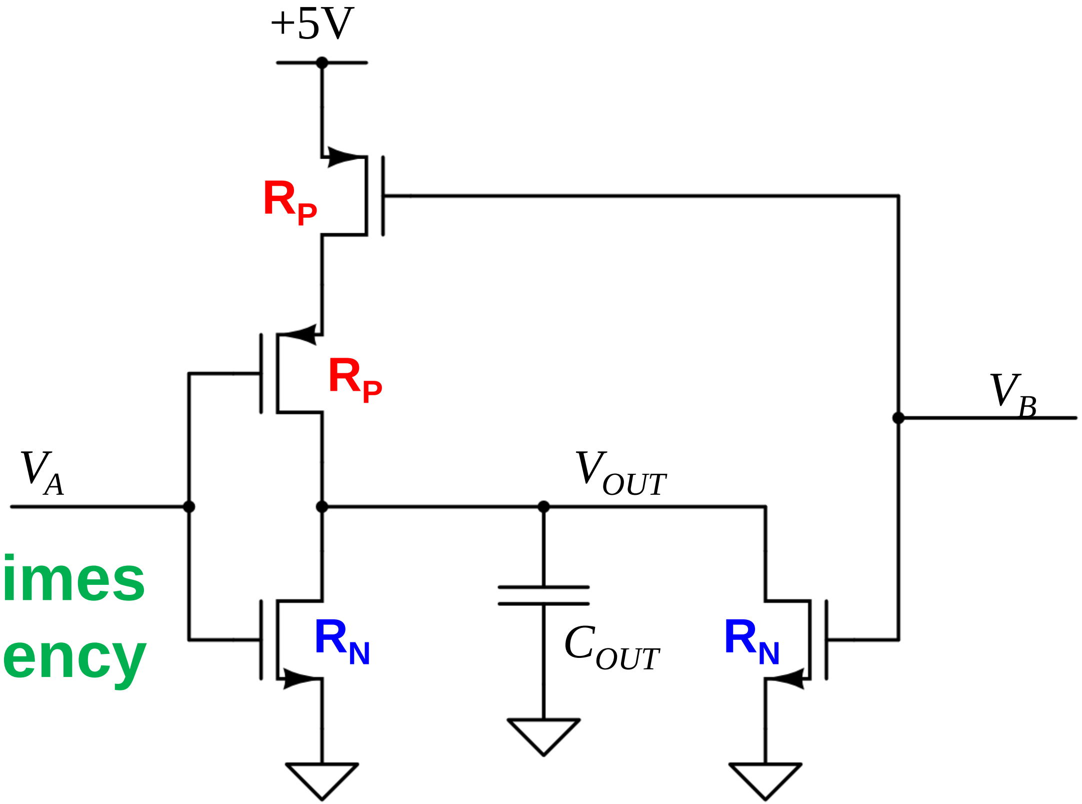
1	1	0	$(R_N \parallel R_N)(C_{OUT})$



Asymmetric switching times

Reduced maximum frequency

Budnik's Home Cooking Recipes for Solving *RC* Circuits

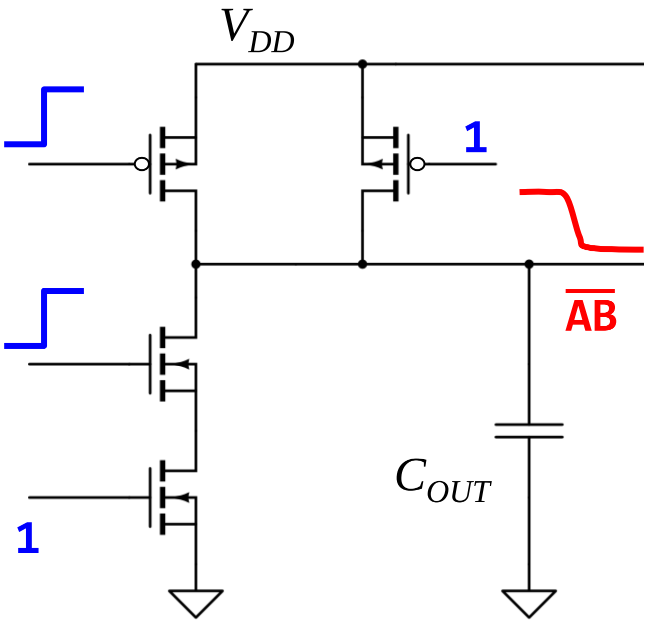
RC Circuits with No Sources

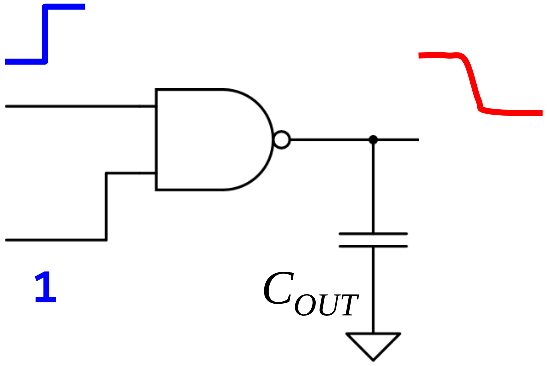
1. You are given $v_C(t=0^-)$
2. Capacitor voltage cannot change instantly: $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. If necessary, find the current $i_C(t=0^+)$ based on V_0
4. Without an external source: $i_C(t=\infty) = 0\text{A}$ and $v_C(t=\infty) = 0\text{V}$
5. Find time constant: $\tau = RC$
6. Both the current and voltage equations will have the form of: $i_C(t)=I_0e^{-t/\tau}$ and $v_C(t)=V_0e^{-t/\tau}$

RC Circuits with Constant Sources

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly: $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. Eventually, capacitor looks like an open again, find $v_C(t=\infty)$
4. If necessary, find the current $i_C(t=0^+)$ and $i_C(t=\infty)$
5. To find the time constant, zero out all sources (0V and 0A): $\tau = RC$
6. Current and voltage each could be the form of: $v_C(t)=V_0e^{-t/\tau}$ or $v_C(t) = (V_{HI}-V_{LO})(1 - e^{-t/\tau}) + V_{LO}$

Switched Resistor Model NAND Gate

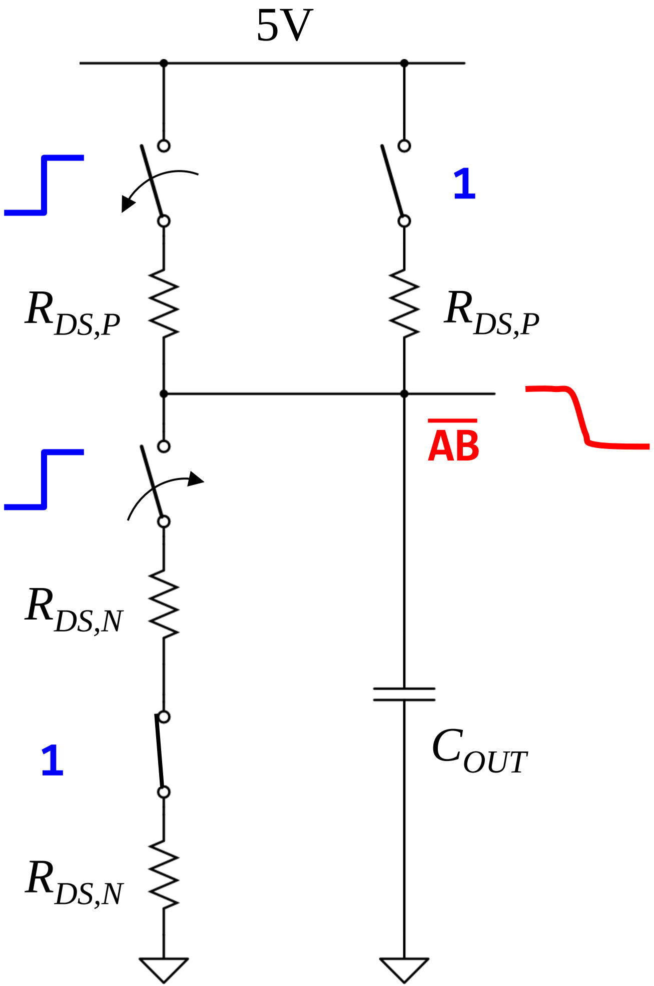




Switched Resistor Model NAND Gate

RC Circuits with Constant Sources

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly:
   $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. Eventually, capacitor acts like an open, find $v_C(t=\infty)$
4. If necessary, find the current $i_C(t=0^+)$ and $i_C(t=\infty)$
5. To find the time constant, zero out all sources (0V and 0A): $\tau = RC$
6. Current and voltage each could be the form of:
   $v_C(t)=V_0e^{-t/\tau}$ or $v_C(t) = V_0(1 - e^{-t/\tau})$

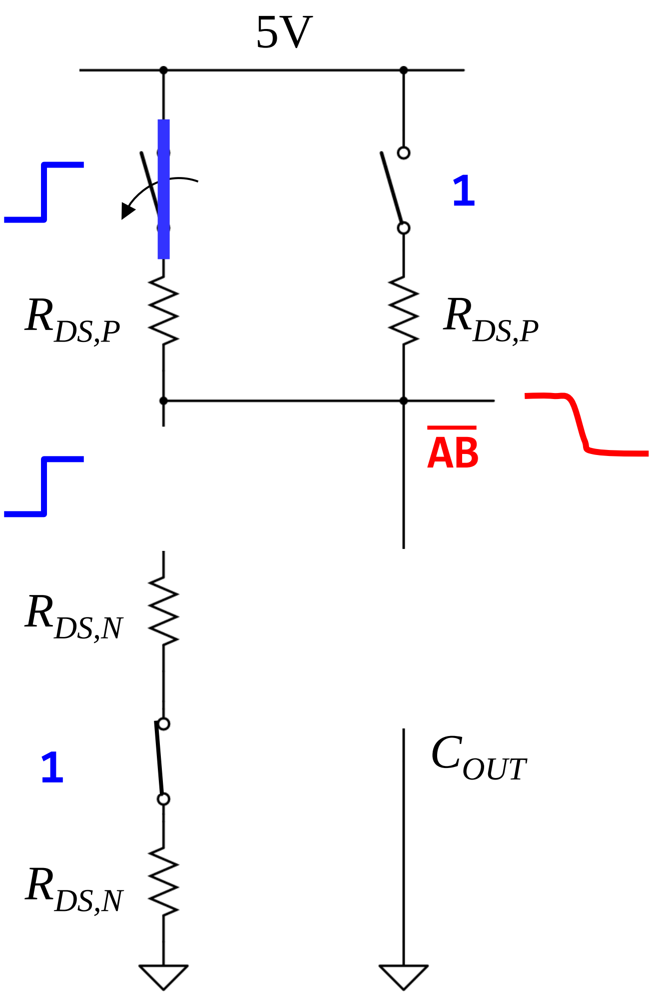


Switched Resistor Model NAND Gate

RC Circuits with Constant Sources

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit

$$v_C(t=0^-) = 5V$$

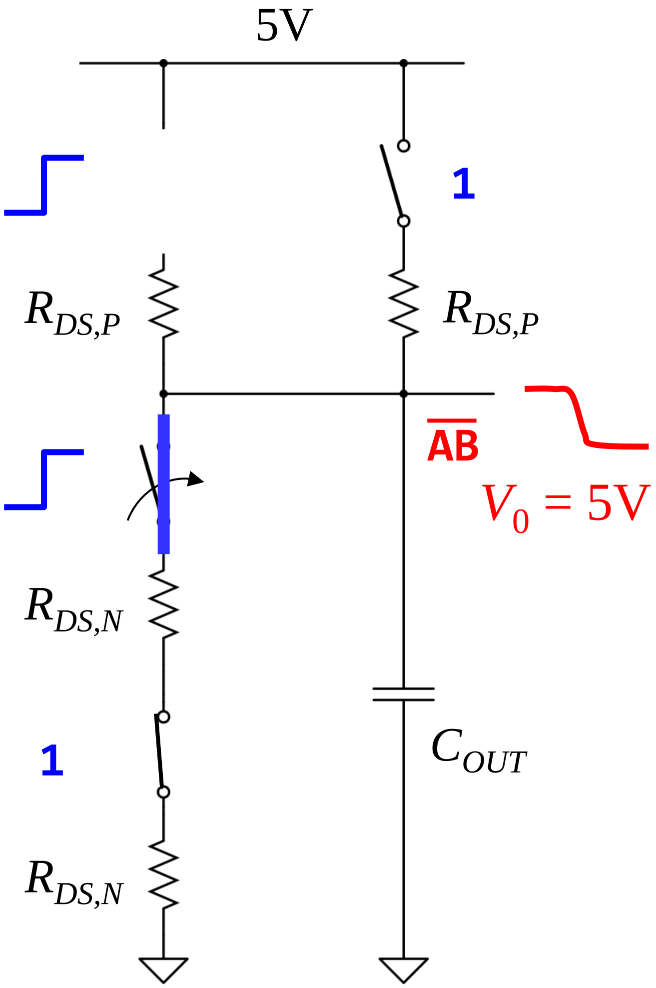


Switched Resistor Model
NAND Gate

<u>RC Circuits with Constant Sources</u>

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly:
   $v_C(t=0^+) = v_C(t=0^-) = V_0$

$$v_C(t=0^-) = 5\text{V} = v_C(t=0^+) = V_0$$

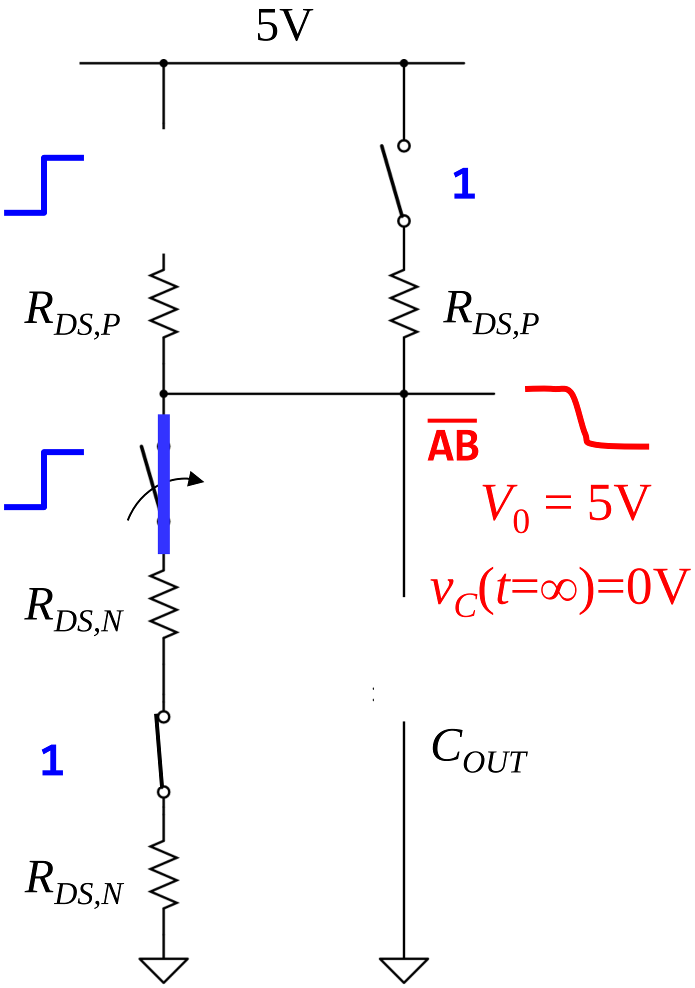


Switched Resistor Model
NAND Gate

<u>RC Circuits with Constant Sources</u>

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly:
   $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. Eventually, capacitor acts like an open, find $v_C(t=\infty)$

$v_C(t = \infty) = 0V$



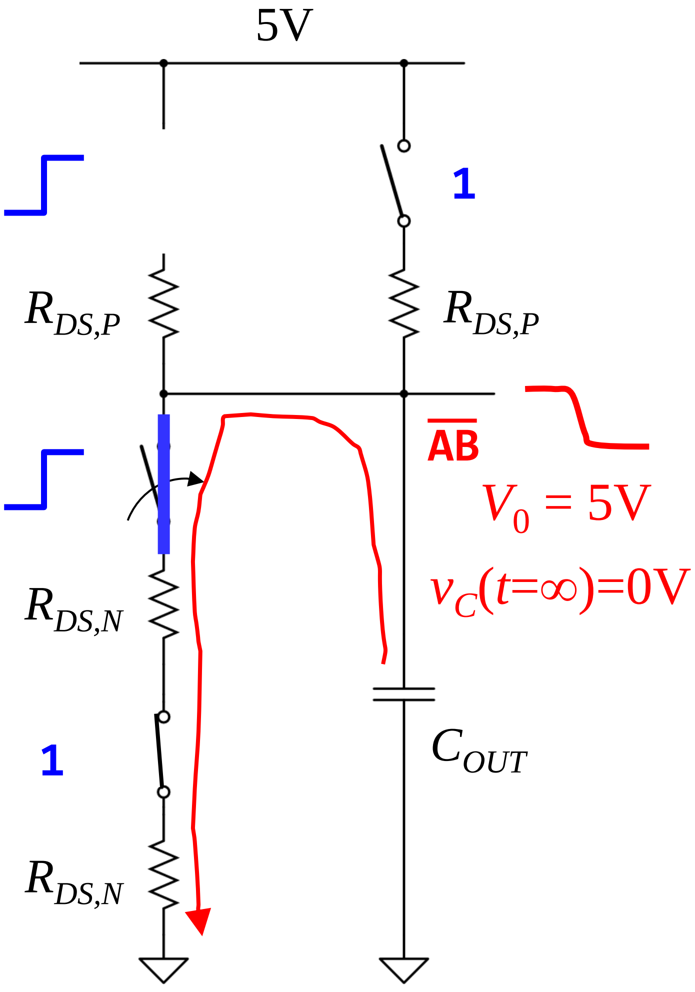
Switched Resistor Model
NAND Gate

<u>RC Circuits with Constant Sources</u>

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly:
   $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. Eventually, capacitor acts like an open, find $v_C(t=\infty)$
4. If necessary, find the current $i_C(t=0^+)$ and $i_C(t=\infty)$

$i_C(t=0^+) = 5V / 2R_{DS,N}$

$i_C(t=\infty) = 0V / 2R_{DS,N} = 0A$

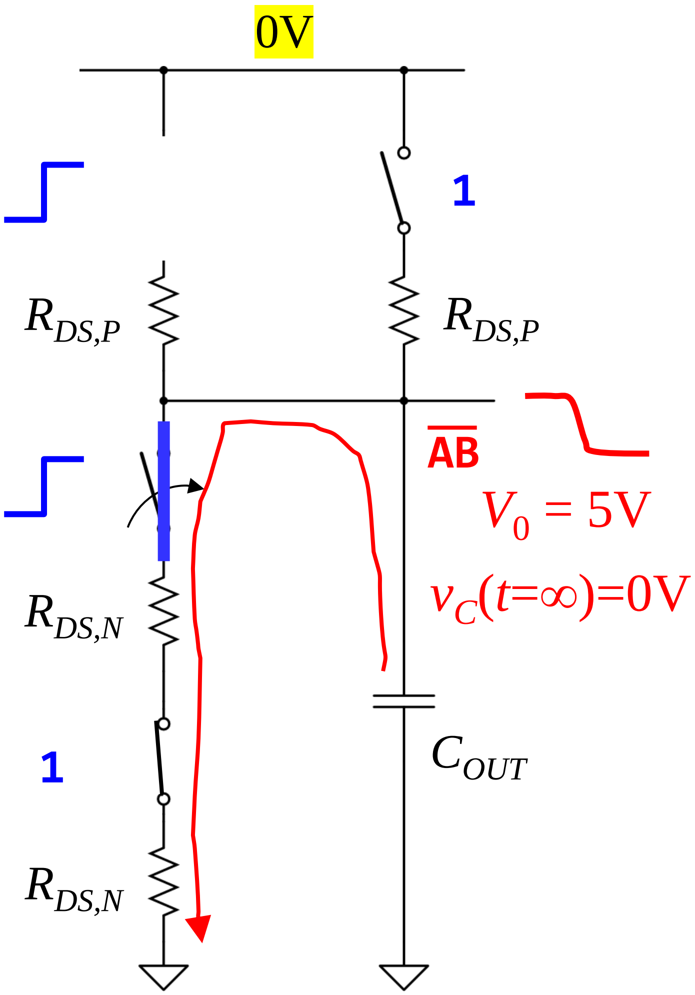


Switched Resistor Model
NAND Gate

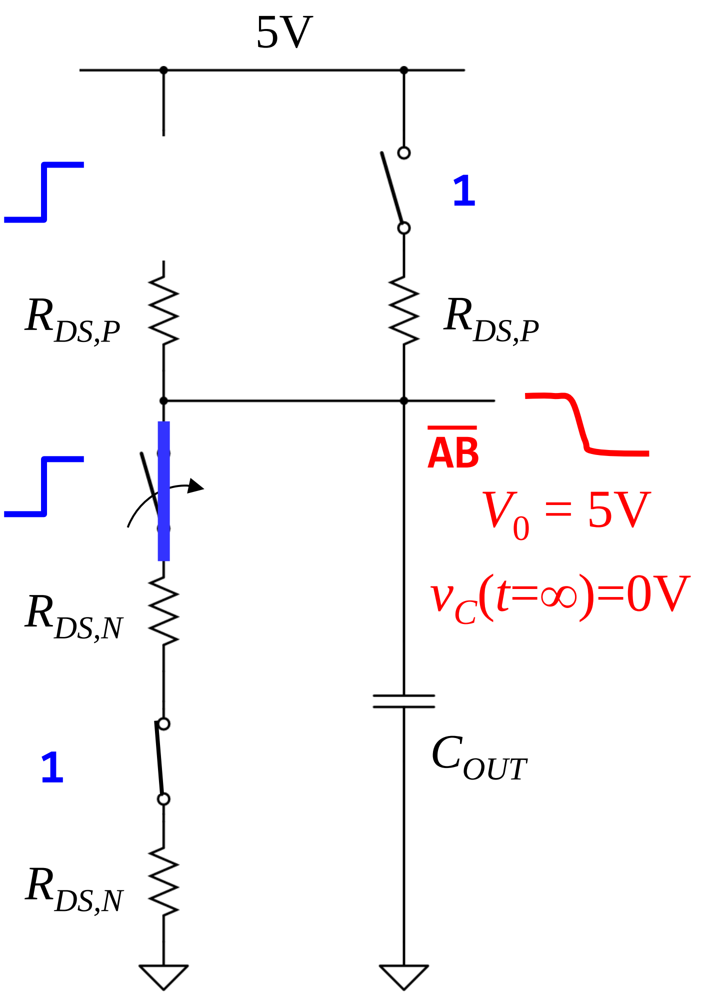
<u>RC Circuits with Constant Sources</u>

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly:
   $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. Eventually, capacitor acts like an open, find $v_C(t=\infty)$
4. If necessary, find the current $i_C(t=0^+)$ and $i_C(t=\infty)$
5. To find the time constant, zero out all sources (0V and 0A): $\tau = RC$

$\tau = 2R_{DS,N}C_{OUT}$



Switched Resistor Model NAND Gate

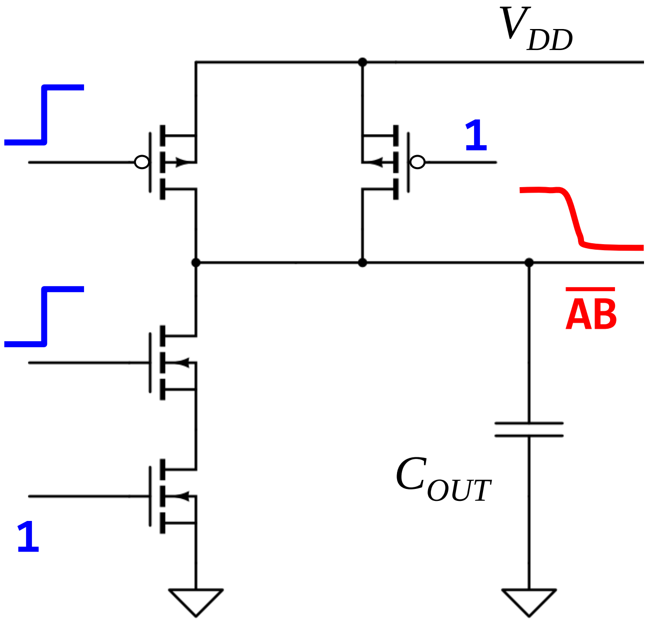


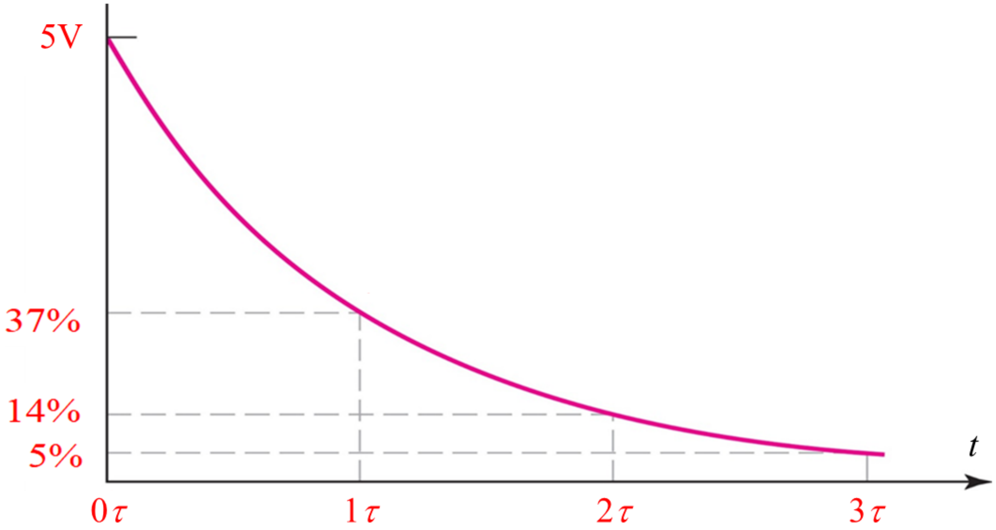
<u>RC Circuits with Constant Sources</u>

1. Calculate $v_C(t=0^-)$ as if capacitor is an open circuit
2. Capacitor voltage cannot change instantly:
   $v_C(t=0^+) = v_C(t=0^-) = V_0$
3. Eventually, capacitor acts like an open, find $v_C(t=\infty)$
4. If necessary, find the current $i_C(t=0^+)$ and $i_C(t=\infty)$
5. To find the time constant, zero out all sources (0V and 0A): $\tau = RC$
6. Current and voltage each could be the form of:
   $v_C(t)=V_0e^{-t/\tau}$ or $v_C(t) = V_0\,(1 - e^{-t/\tau})$

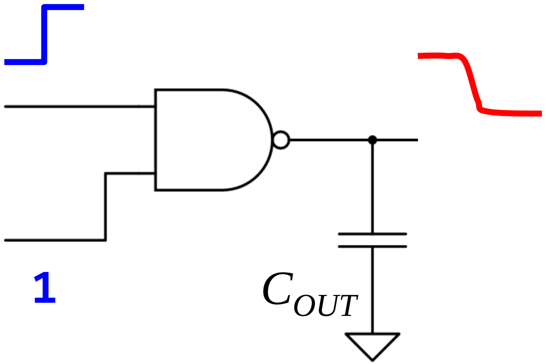
$$v_C(t \geq 0) = 5e^{-t/\left(2R_{DS,N}C_{OUT}\right)}\ V$$

Switched Resistor Model
NAND Gate

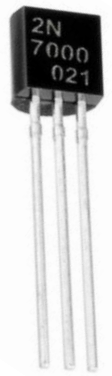




$$v_C(t \geq 0) = 5e^{-t/\left(2R_{DS,N}C_{OUT}\right)}\ V$$



Capacitors Can Store Energy:



- Previous Example (2N7000): $E_C = \frac{1}{2}(60\,pF)(5V)^2 = 750\,pJ$

$$E_C = \frac{1}{2}CV^2$$

- Super Capacitor Example:

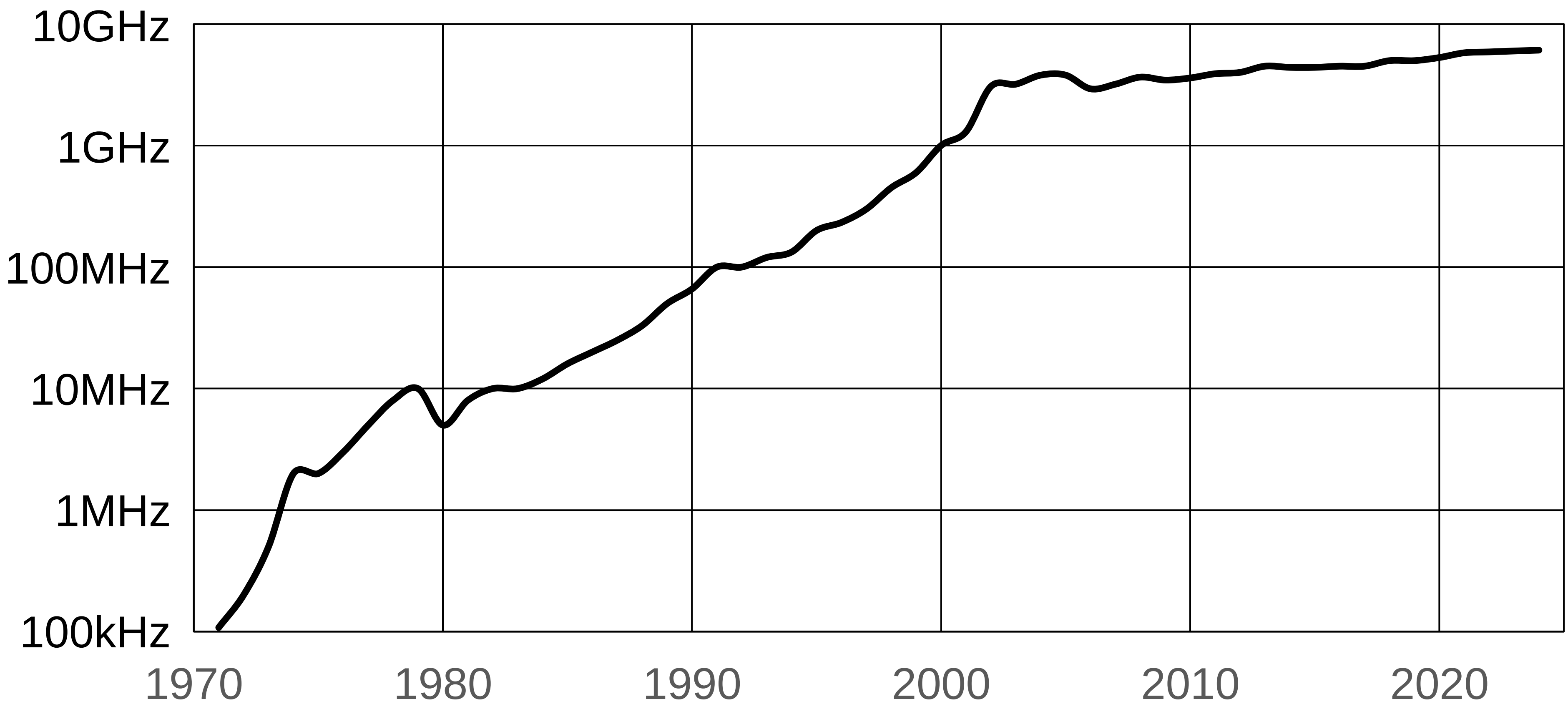
$$E_C = \frac{1}{2}(220mF)(5V)^2 = 2.75J$$

Charging and Discharging Logic Gates Takes Power

$$P = CV^2 f = (0.5)(60\,pF)(5V)^2(100MHz) = 75mW$$

$$P_{1\mathrm{M}\;Gates} = (75mW)(1{,}000{,}000) = 75{,}000W$$

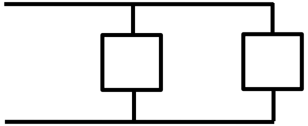
Maximum Processor Frequency by Year



Inductors and Capacitors Are Opposites….

	Inductors	Capacitors
		
Cannot Change Instantly	Current $i_L\left(t=0^-\right)=i_L\left(t=0^+\right)$	Voltage $v_C\left(t=0^-\right)=v_C\left(t=0^+\right)$
Initially, When Current First Starts to Flow, It Looks Like a ….	Open 	Short 
Eventually, It Gets Fully Charged Up, and It Looks Like a ….	Short 	Open 

Inductors and Capacitors Are Opposites….

	Inductors	Capacitors
		
Series Equivalence 	$L_{eq} = L_1 + L_2 + \ldots. + L_N$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots. + \frac{1}{C_N}$
Parallel Equivalence 	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots. + \frac{1}{L_N}$	$C_{eq} = C_1 + C_2 + \ldots. + C_N$
Time Constant	$\tau = L/R$	$\tau = RC$

What Do You Need to Do Next?

1. Take the **Lecture 7 Quiz** on canvas!
2. Check out Piazza and Gradescope

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