Lecture 15: Analog to Digital Converters (ADCs) and Digital to Analog Converters (DACs)

18-100: Introduction to ECE

SPRING 2025

Electrical & Computer ENGINEERING

Carnegie Mellon University

Agenda

- Analog and Digital Worlds – The Problem with Pizza
- Analog to Digital Converters (ADCs)
- Digital to Analog Converters (DACs)

Digital Inputs and Outputs

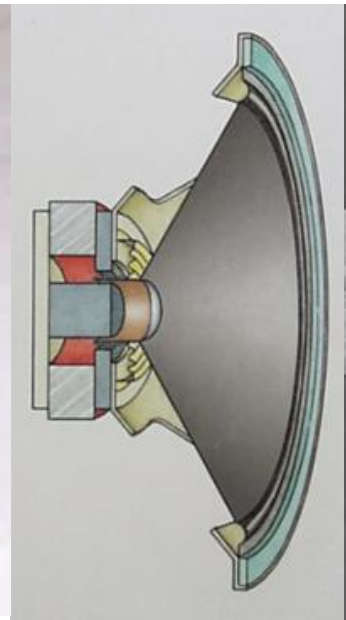
- Digital inputs and outputs (I/O) represent YES/NO or TRUE/FALSE information

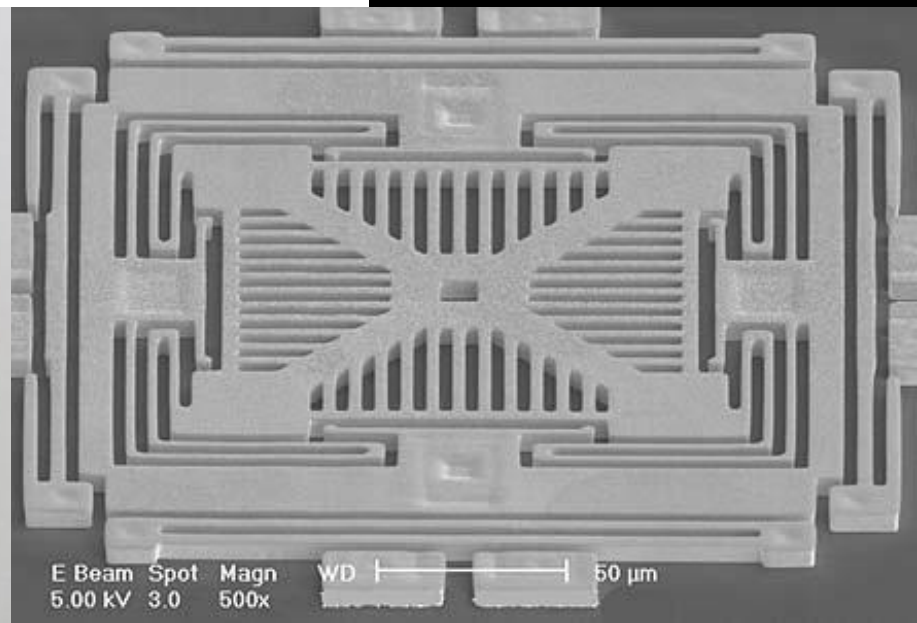
- Examples: The light is on.
  A button is pushed.
  Turn on a bell.
  Turn off a light emitting diode.

We Live in an Analog World

- Everything in the physical world is an analog signal: Sound, light, temperature, pressure….

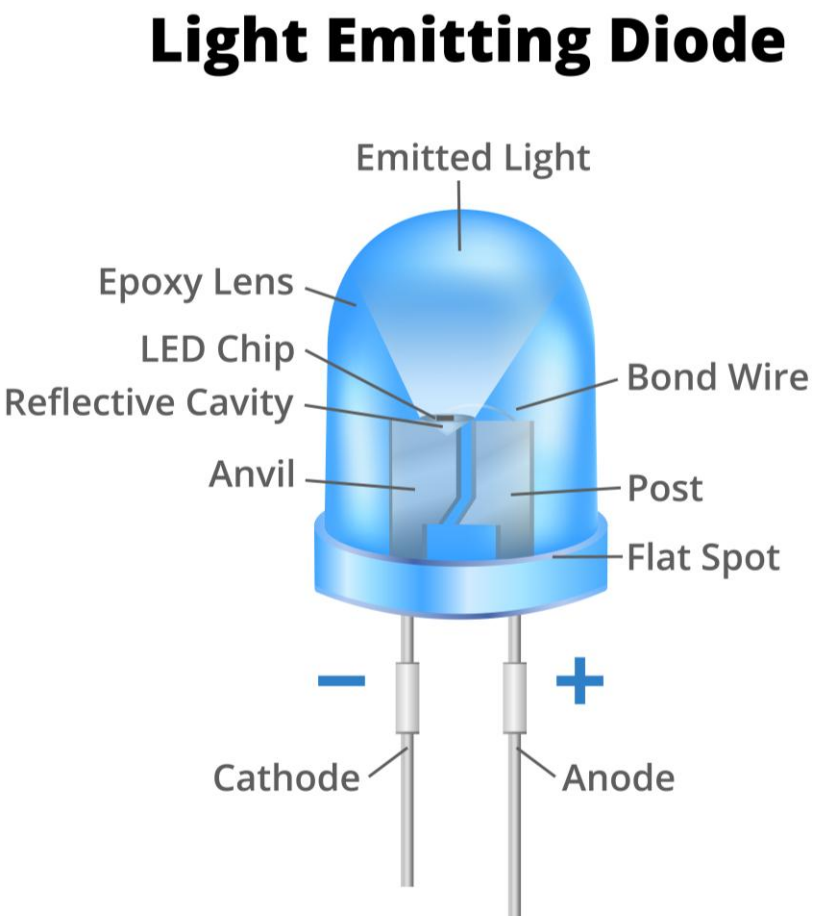
- Need to convert “real world” information into electrical signals
- Transducers: converts one type of energy to another
  - Microphone
  - Speaker
  - Thermocouple
  - Accelerometer
  - Photoresistor
  - LEDs
  - Motors….

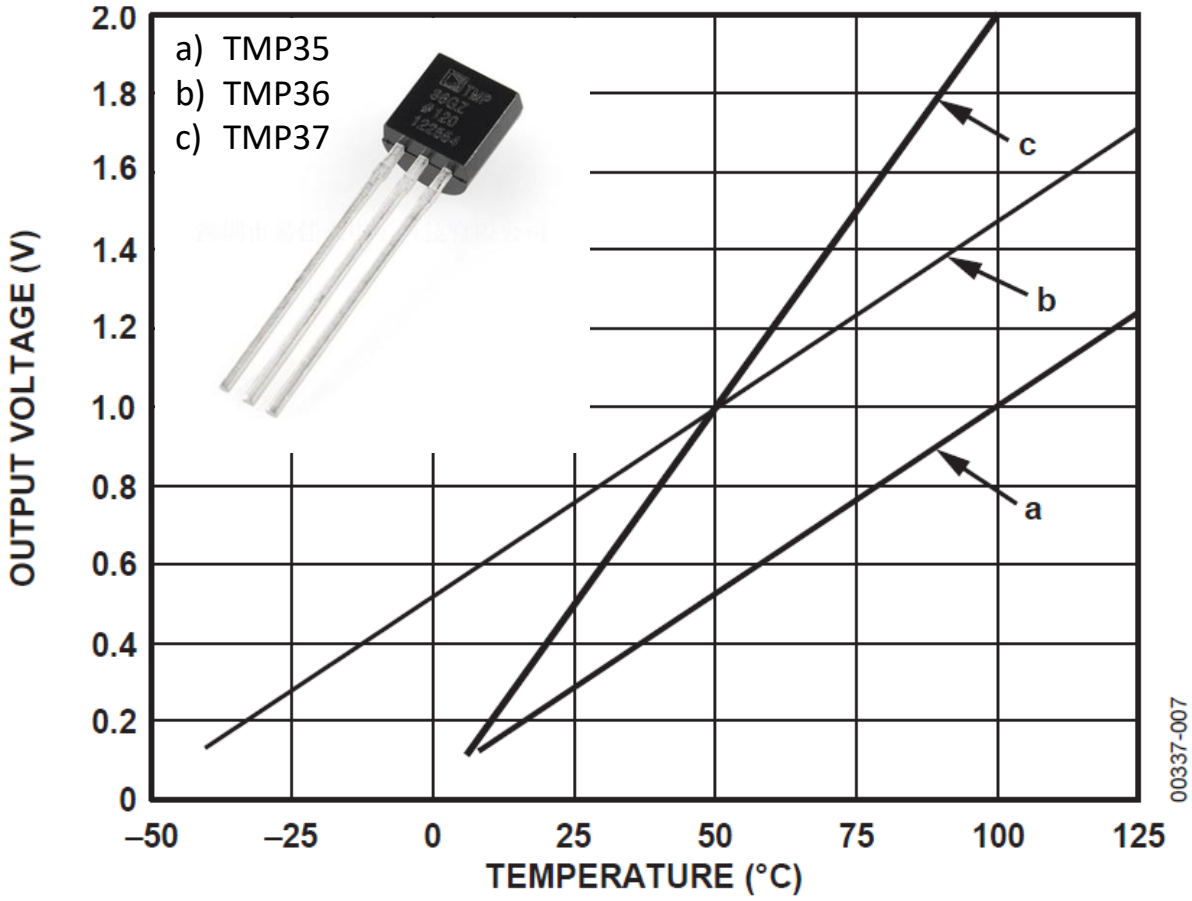




Transducers Convert One Form of Energy into Another

- Sensors convert real-world phenomena into electrical signals (like a temperature sensor)
- Actuators convert electrical signals into real-world phenomena (like an LED)



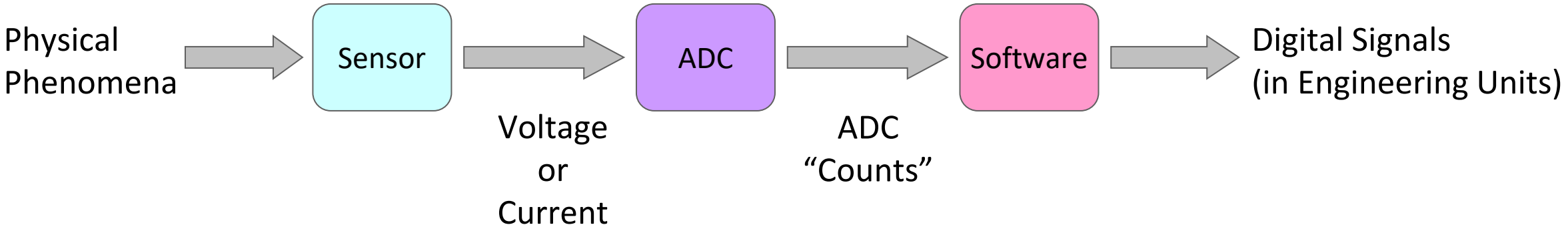


https://www.analog.com/media/en/technical-documentation/data-sheets/TMP35_36_37.pdf

Going from an Analog Real-World Information to Digital Signals

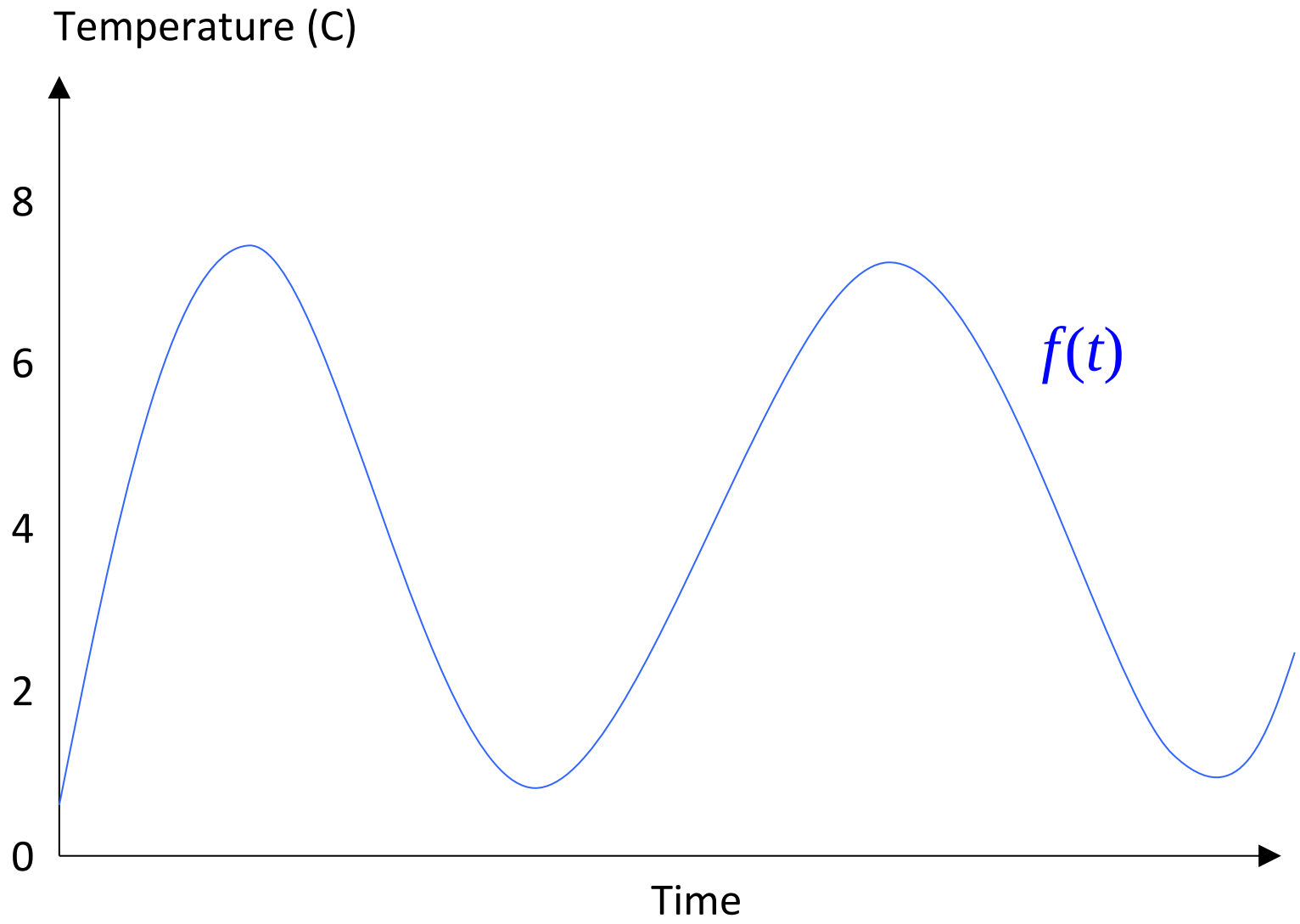
- What we want:

- How we have to get there:



Most Real-World Phenomena: Analog **AND** Continuous-Time

- x-axis is often time
- y-axis is often amplitude
- Both are often analog and continuous signals



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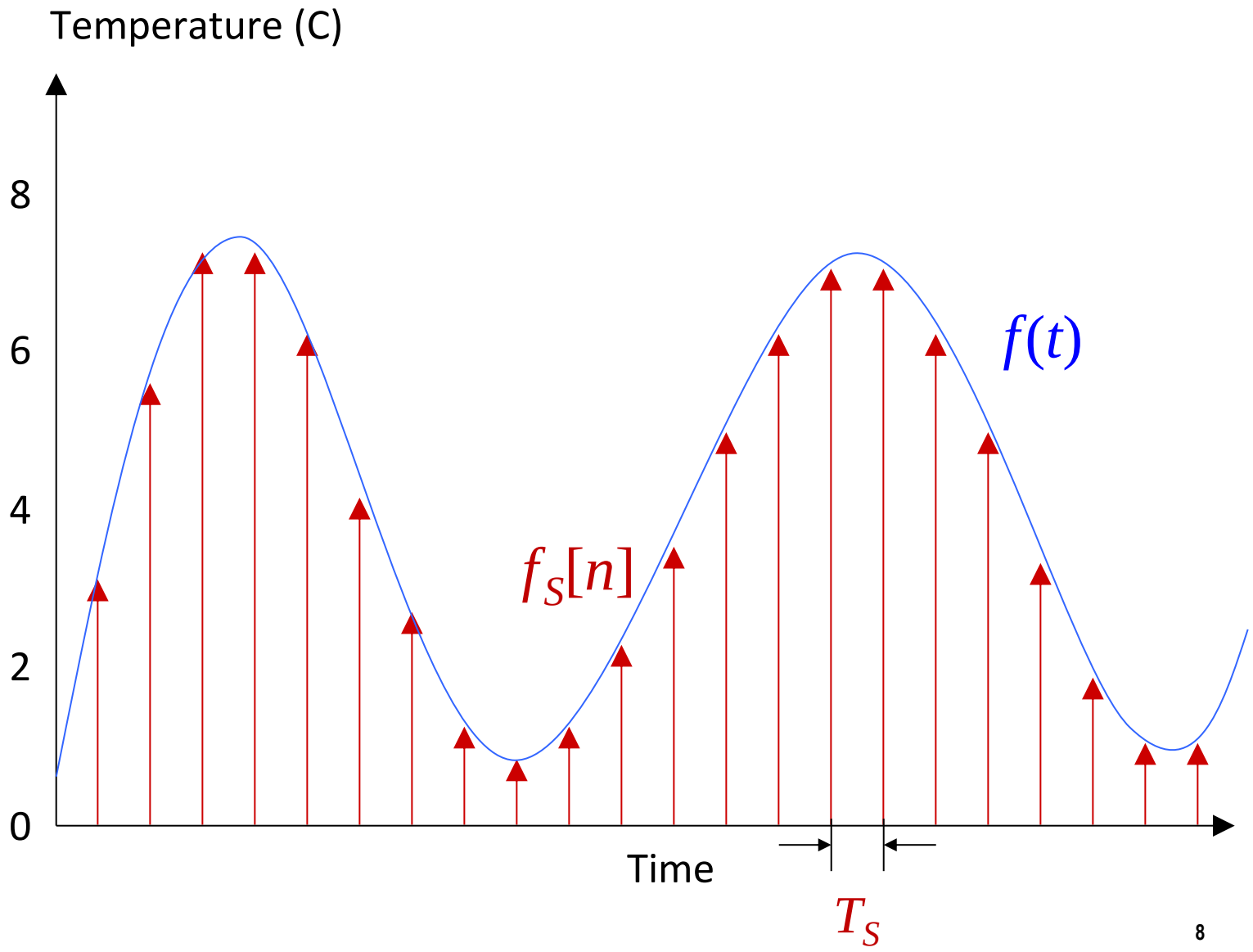

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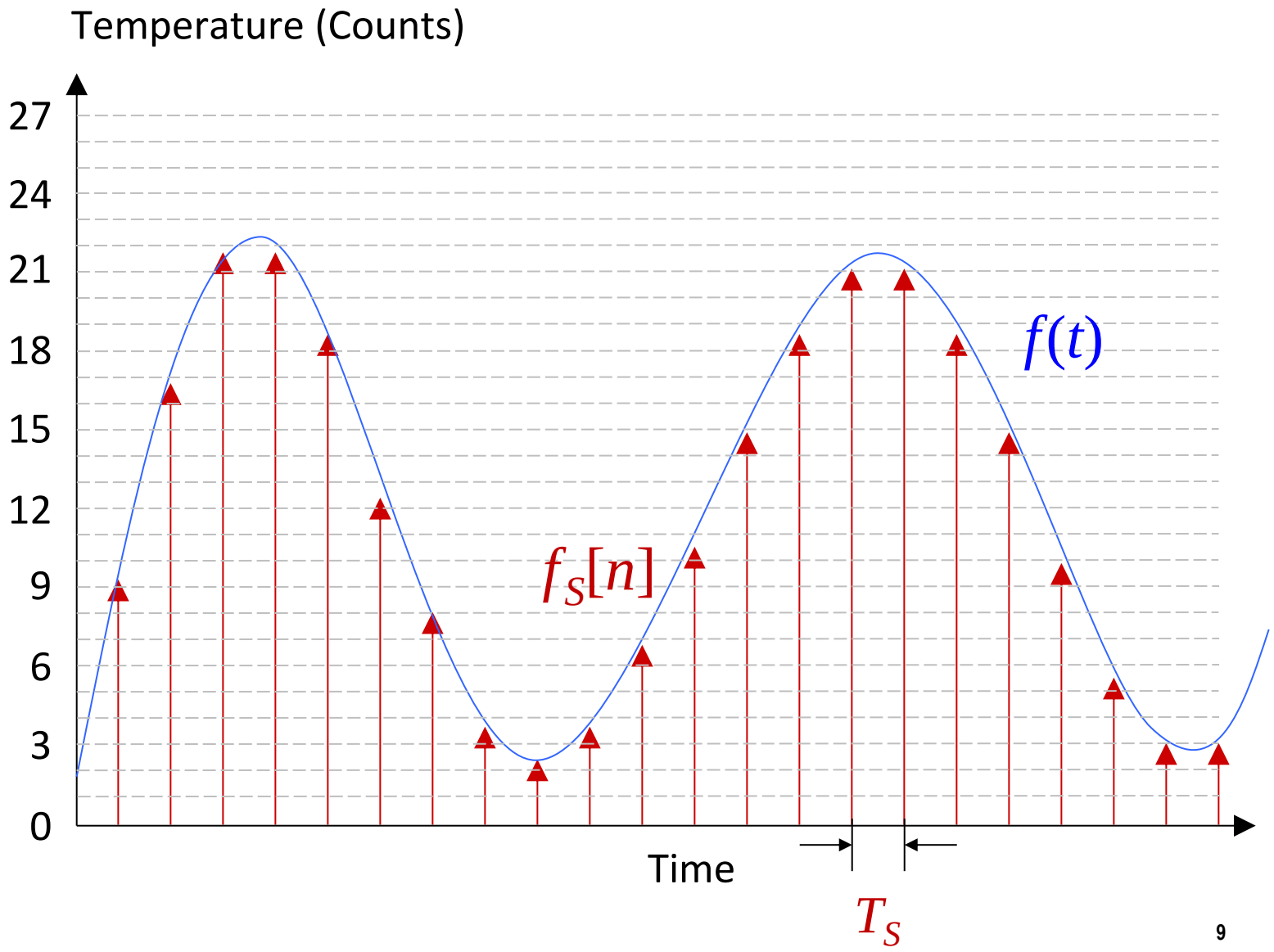
- Sample every T_S seconds to create a discrete-time signal

- Sample amplitude with a finite step-size to create a digital, discrete-value signal



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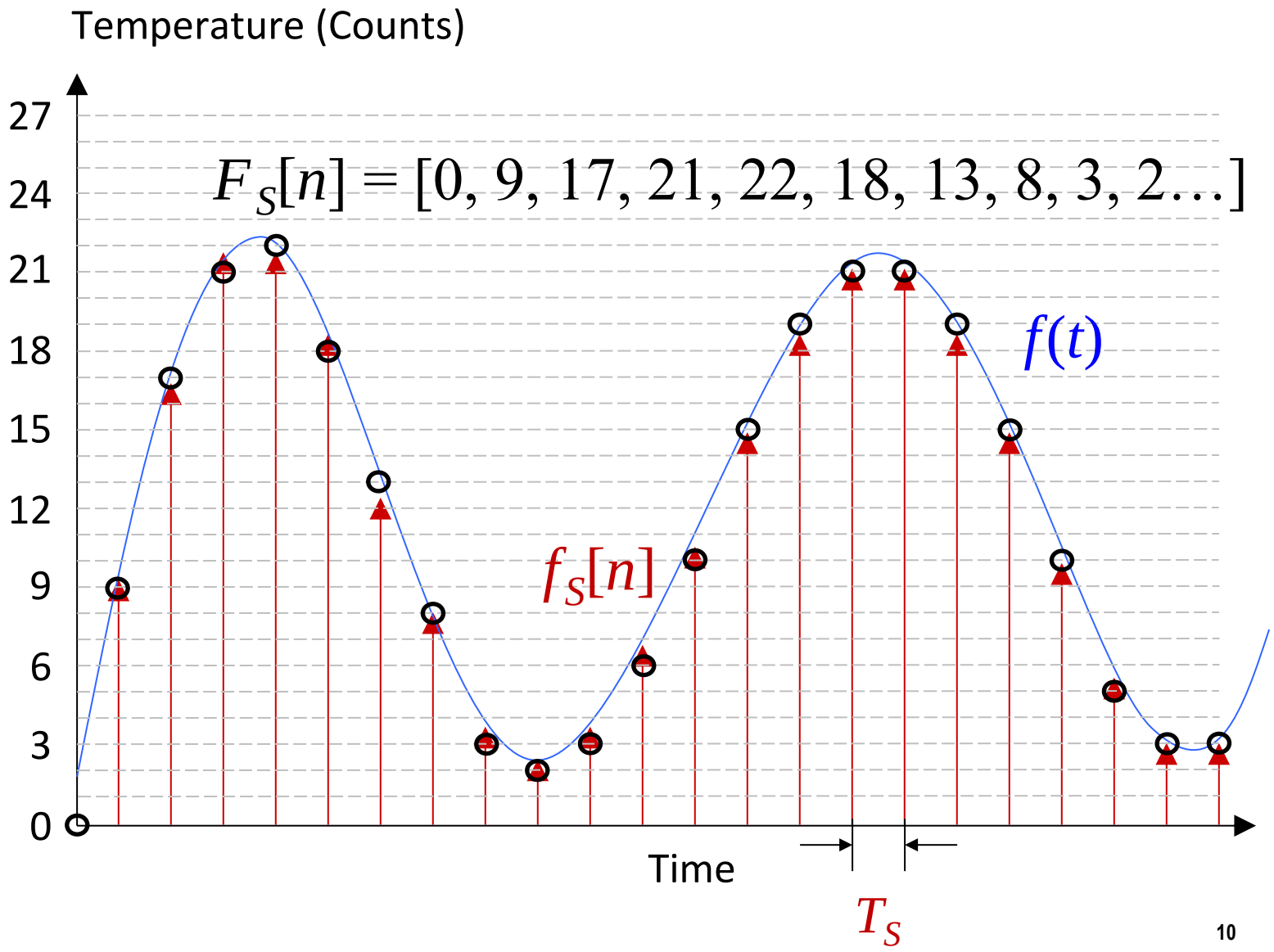
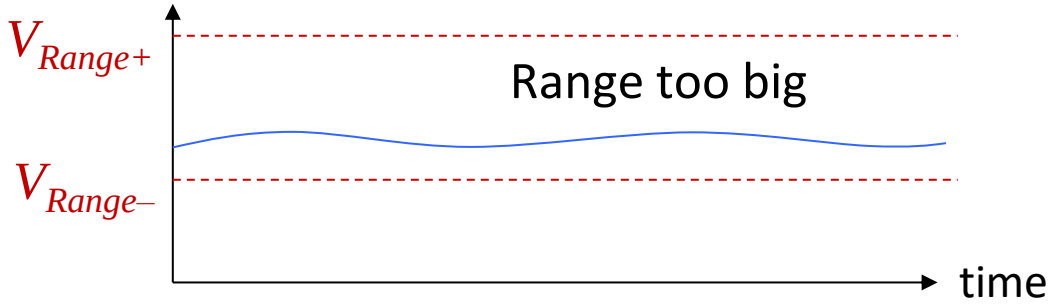
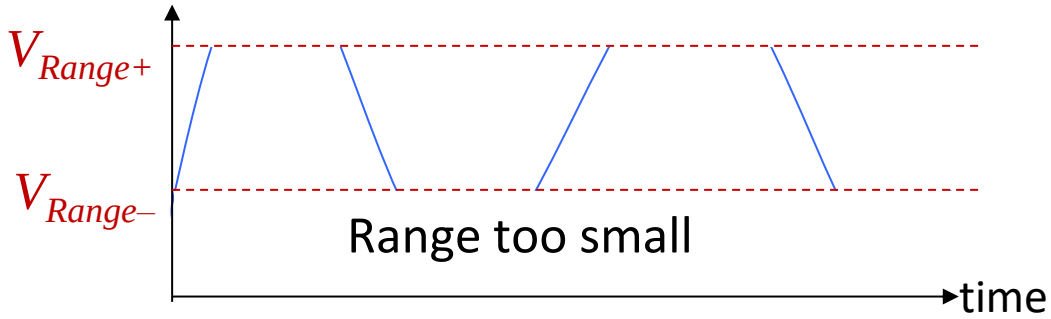
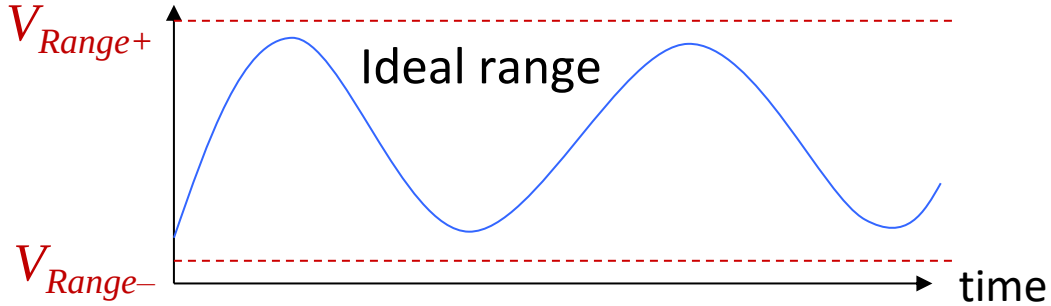


Choosing the Right Sensor Range

- The sensor's (or sensor's circuit) has a specified output voltage range
- The ADC has a specified voltage range for its input V_{Range-} (or V_{r-}) to V_{Range+} (or V_{r+})
  - V_{Range-} is usually 0V, but some ADCs allow this to be changed to other values
  - V_{range+} is usually V_{DD} (like 3.3V), but some ADCs allow this to be changed to other values
- Usually want a little margin



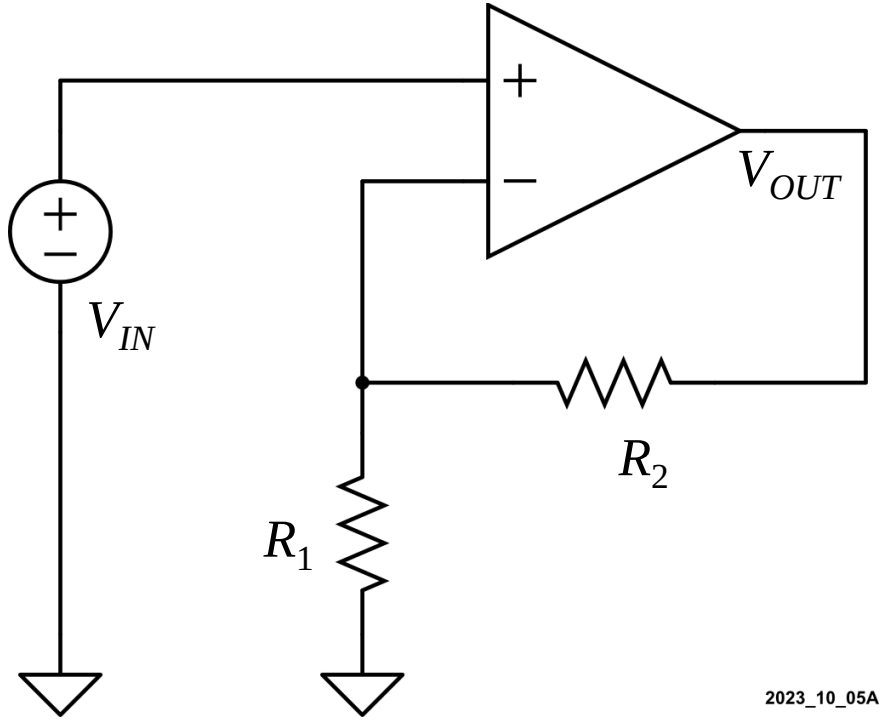




Signal Conditioning with Operational Amplifiers (Op Amps)

- Usually, op amps are modeled as ideal devices with negative feedback:
  - The non-inverting (V_+) and inverting (V_-) inputs will be the same voltage
  - The current flowing into the V_+ and V_- inputs is 0A
  - The output voltage will adjust to make the above two rules true
- Non-inverting amplifiers can make smaller voltages larger
- Other configurations have different transfer functions

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_2}{R_1}$$



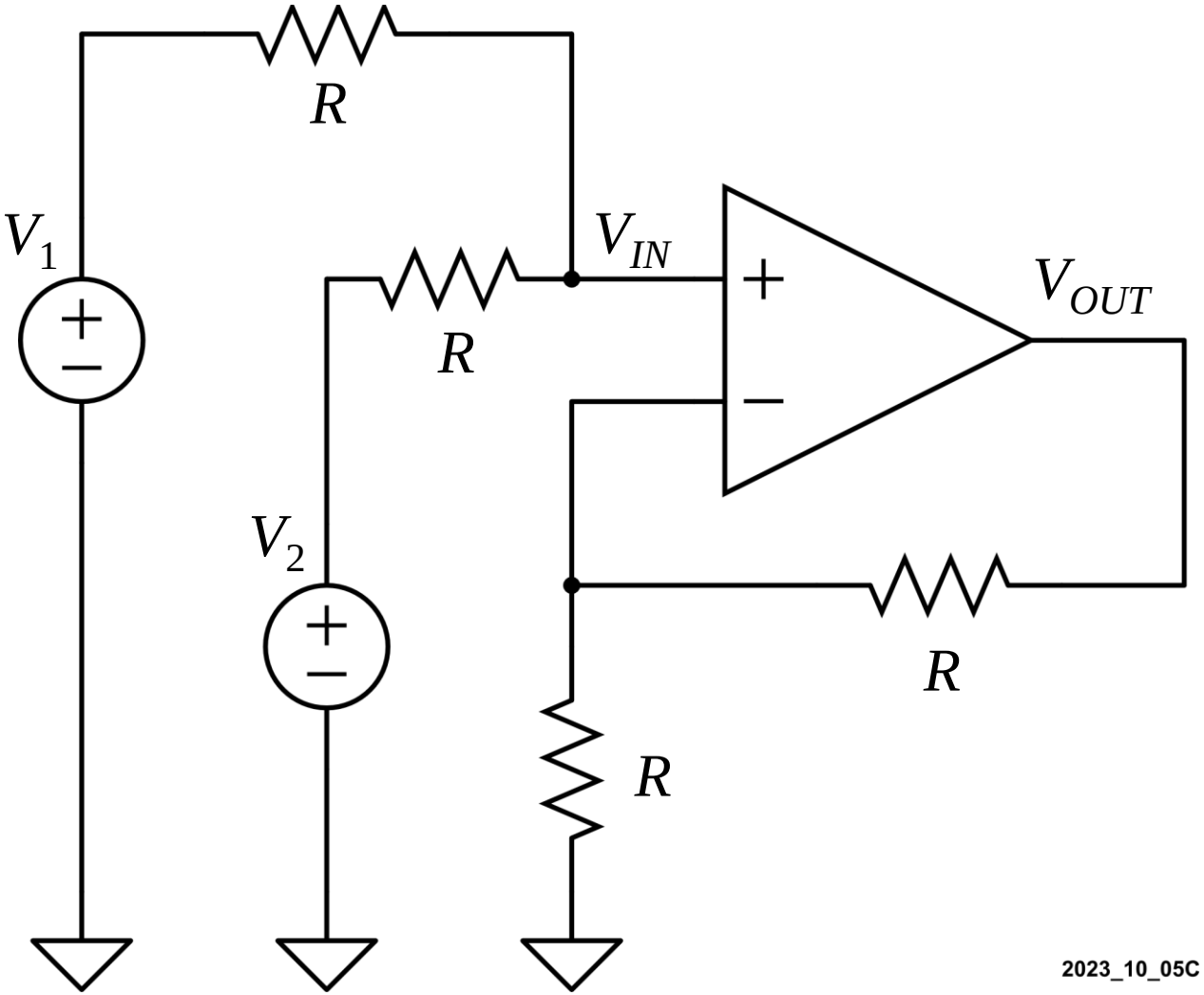
Add a DC Offset to Small Voltages with a Summing Amplifier Op Amp Circuit

- Add two (or more) voltages

$$V_{IN} = \frac{V_1 + V_2}{2}$$

$$V_{OUT} = V_{IN}\left(1 + \frac{R}{R}\right) = 2V_{IN}$$

$$V_{OUT} = 2\left(\frac{V_1 + V_2}{2}\right) = V_1 + V_2$$



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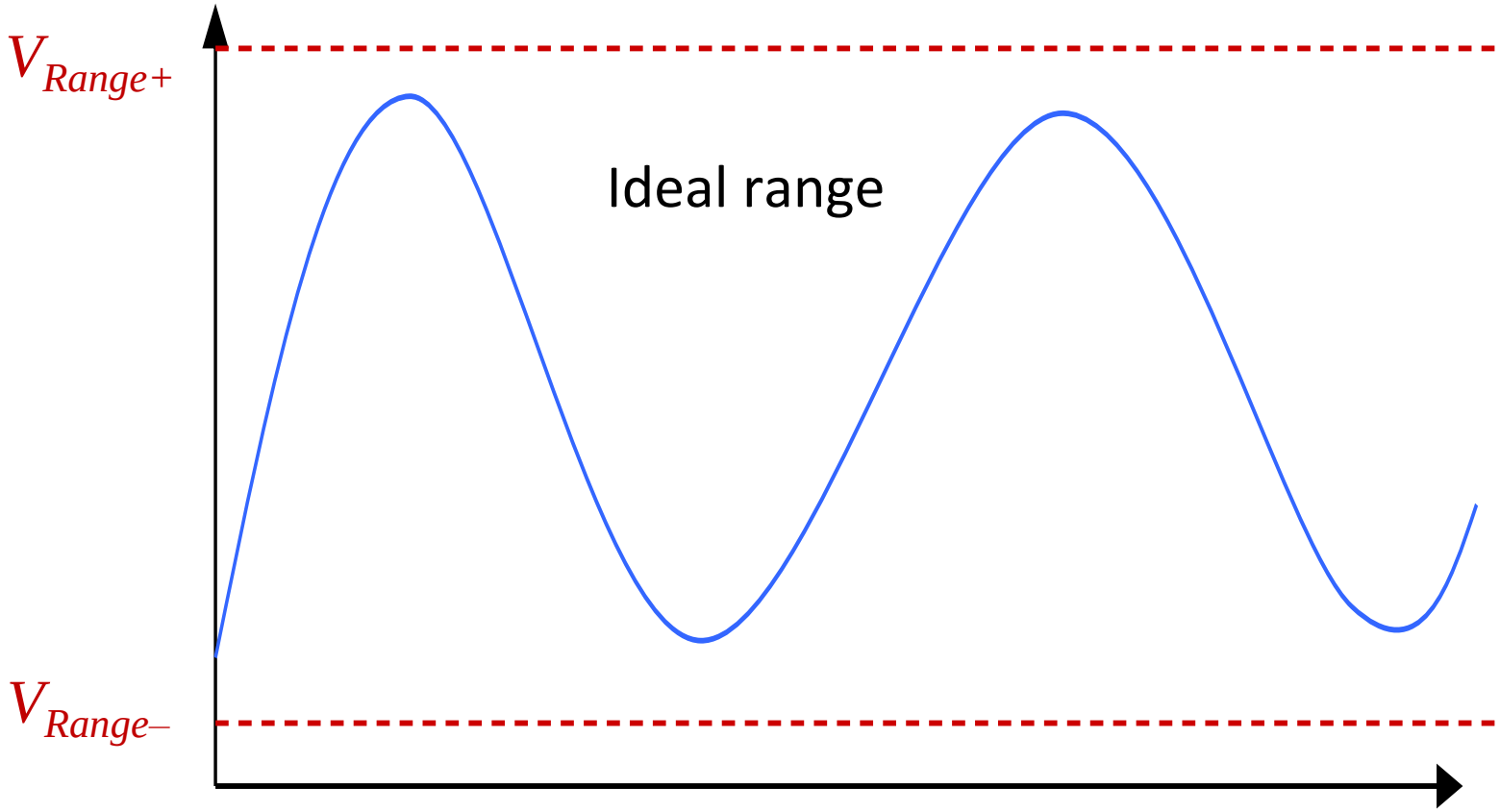
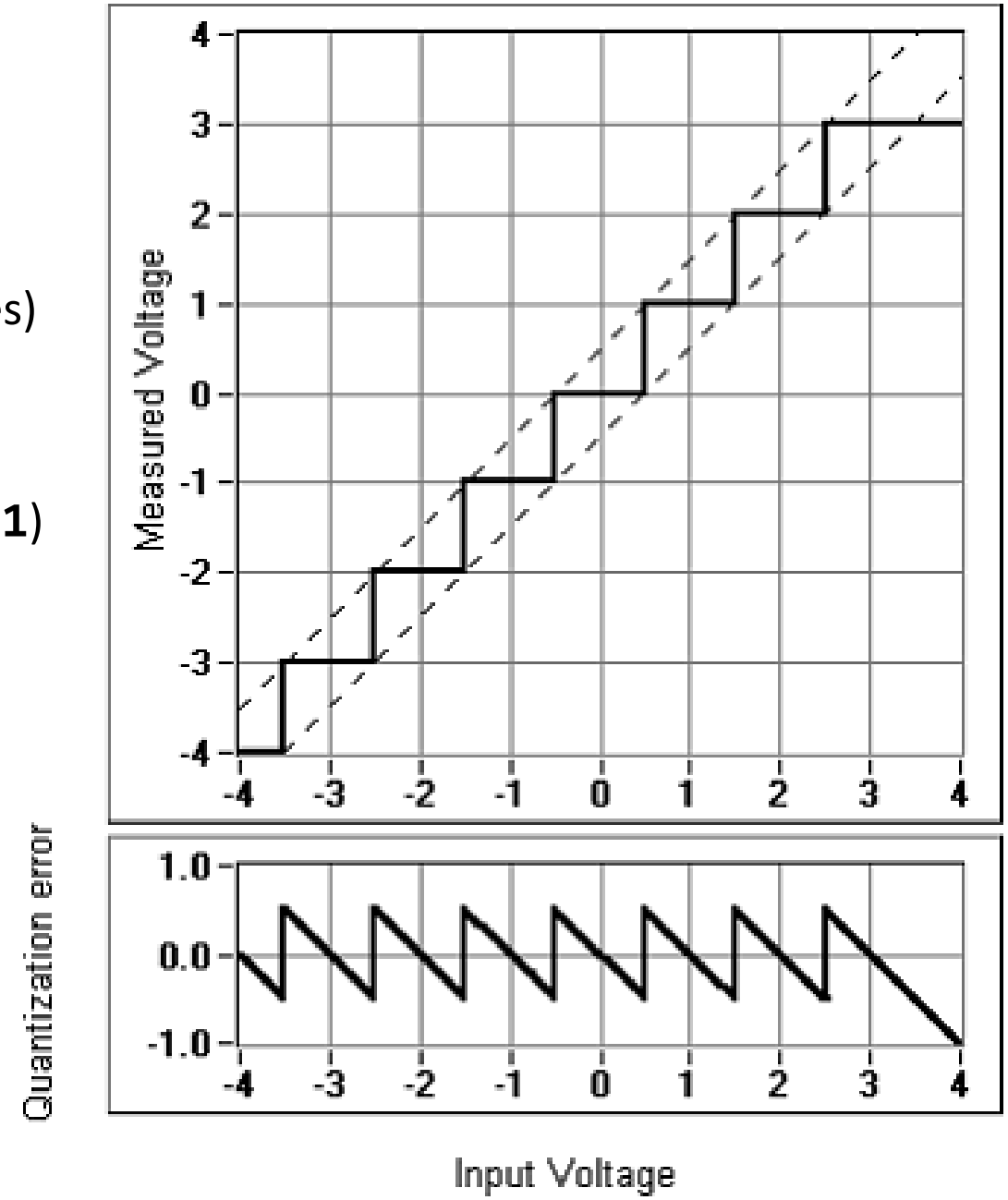
Condition the Sensor Output Voltage by Setting the ADC Range and DC Offset (with a Summing Amplifier)

- Adjust the ADC's range
- Maximize the signal's maximum and minimum values inside the ADC range
- Might require you to try a few different options to optimize



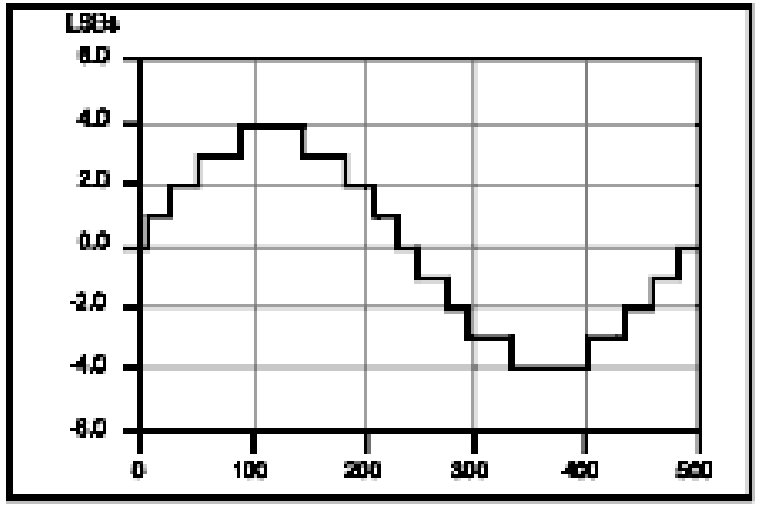
Amplitude Sampling Granularity

- Resolution: the range of analog values represented by a discrete digital value (infinite voltages → finite binary values)
- Consider a 12-bit ADC
  - 2^{12} = 4096 values (**000000000000** through **111111111111**)
  - Each binary value (step) represents $(V_{r+} - V_{r-}) / 4096$
  - For example, $(3.3\text{V} - 0\text{V}) / 4096 \text{ steps} = 806\mu\text{V} / \text{step}$
- Larger range $(V_{r+} - V_{r-})$ ➞ less information
- Larger range $(V_{r+} - V_{r-})$ ➞ more quantization error
- More bits ➞ more information

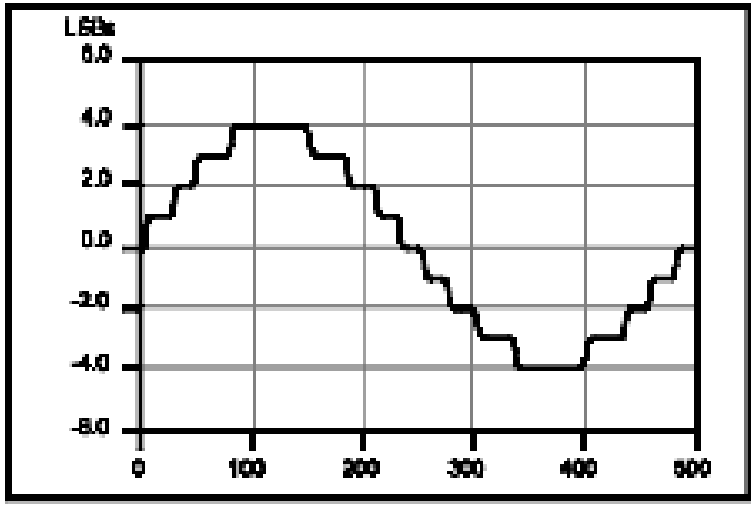


Reduce Quantization Errors with Dithering

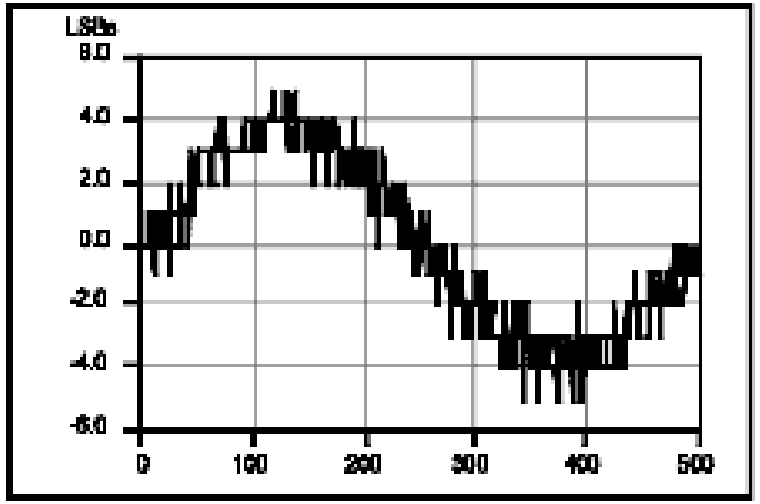
- Quantization errors can result in large-scale patterns that don't accurately describe the analog signal
- Oversample and dither (introduce white noise to randomize the quantization error
- Many general-purpose microcontroller boards have white noise generation capability for this very reason



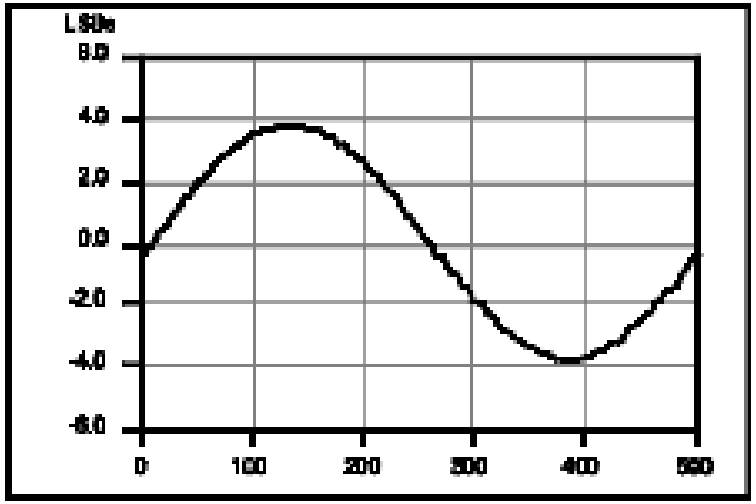
a. Dither disabled; no averaging



b. Dither disabled; average of 50 acquisitions



c. Dither enabled; no averaging



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Reduce Quantization Errors with Dithering

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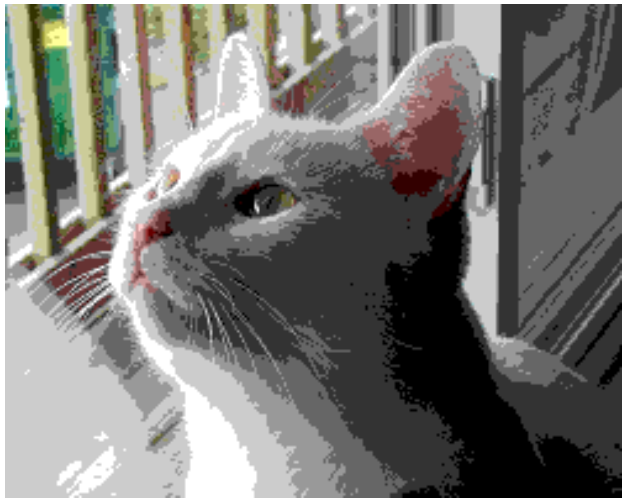

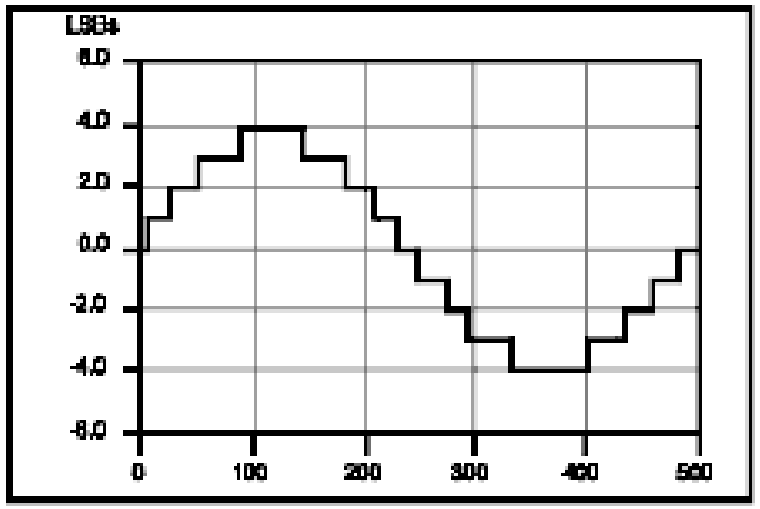

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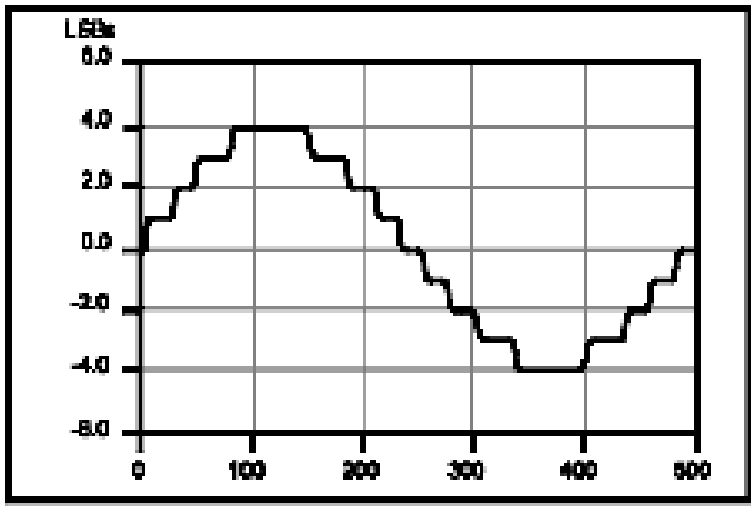

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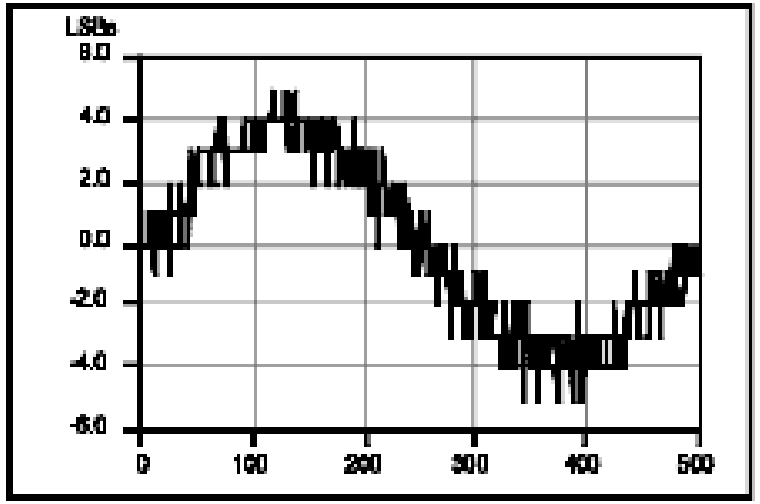
Dithering enabled



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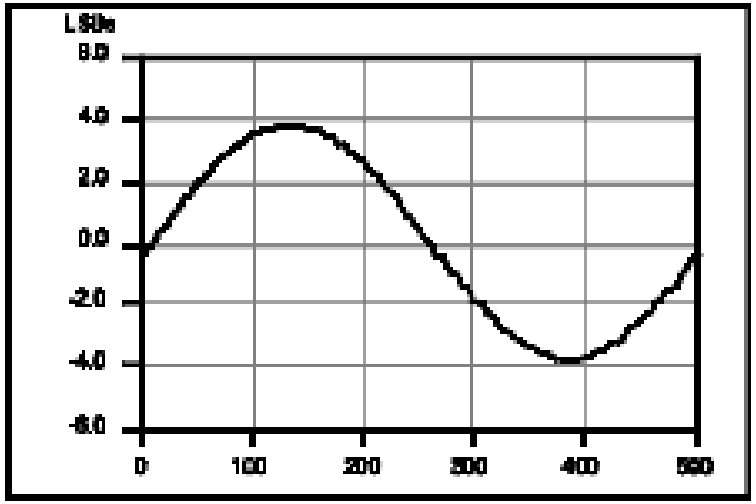

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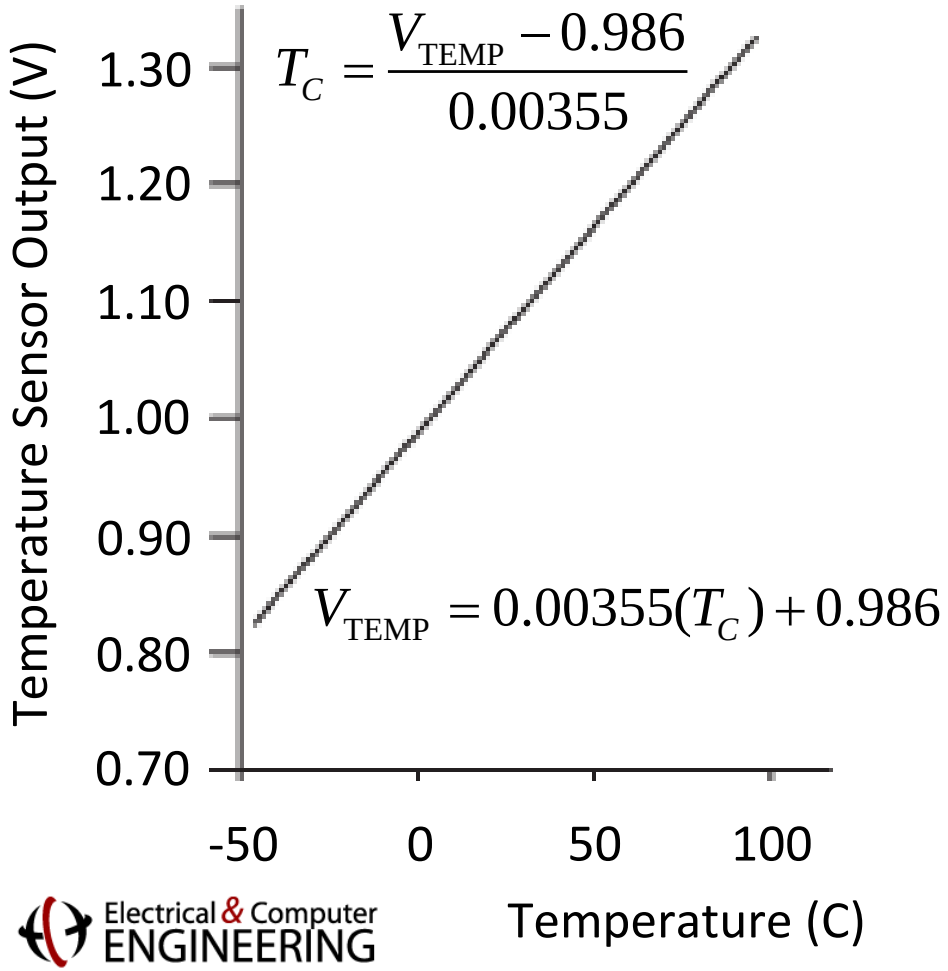
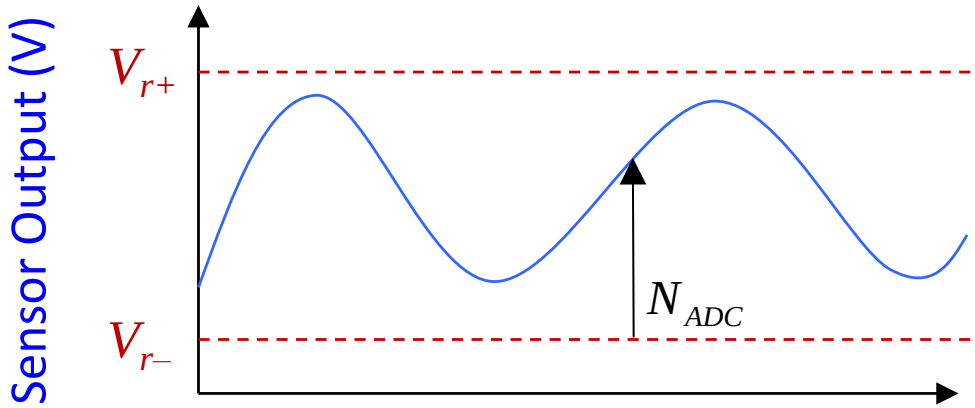
Converting Between Temperature, Voltage, ADC Counts, and Engineering Units

- Converting temperature to voltage



- Converting voltage to ADC counts or steps (N_{ADC})



$$N_{ADC} = \left(2^n - 1\right)\left(\frac{V_{IN} - V_{r-}}{V_{r+} - V_{r-}}\right) = \left(2^{12} - 1\right)\left(\frac{V_{IN} - V_{r-}}{V_{r+} - V_{r-}}\right) = 4095\left(\frac{V_{IN} - V_{r-}}{V_{r+} - V_{r-}}\right)$$

- Converting ADC counts to engineering units
  - Convert V_{TEMP} (between 0 and 4095) back into temperature (-40C and +100C)

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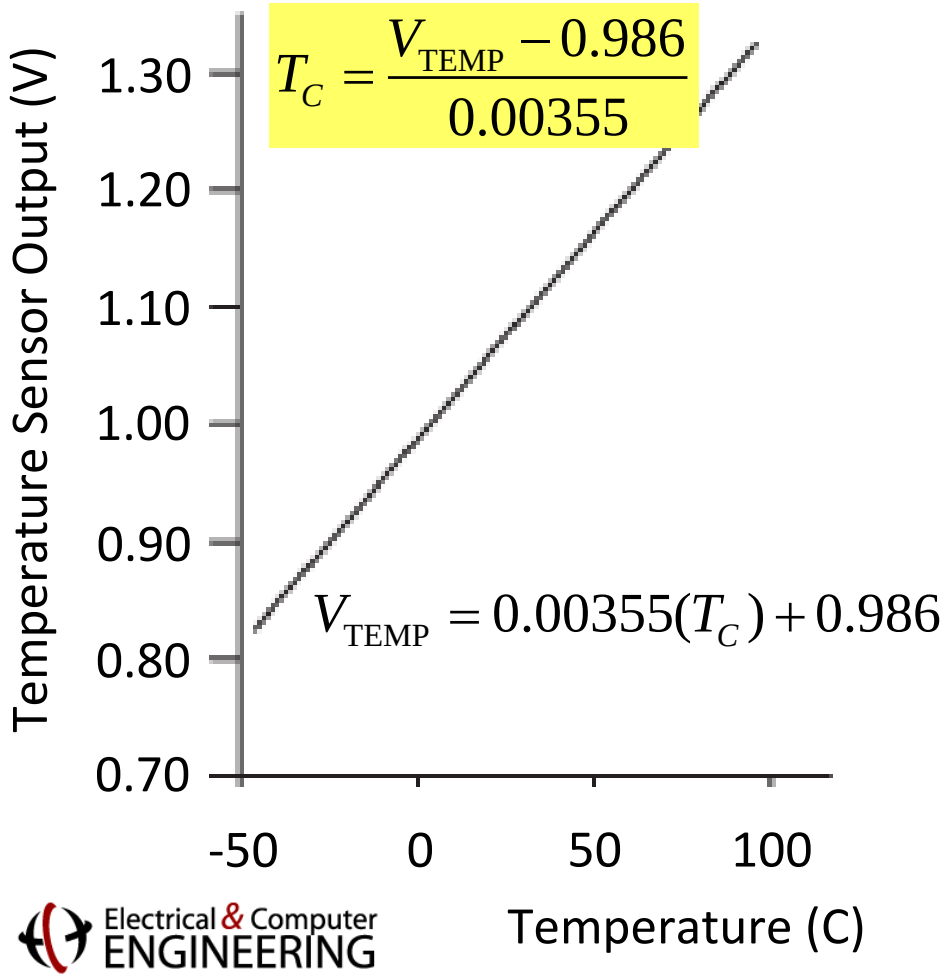
Converting Between Temperature, Voltage, ADC Counts, and Engineering Units

- Converting temperature to voltage



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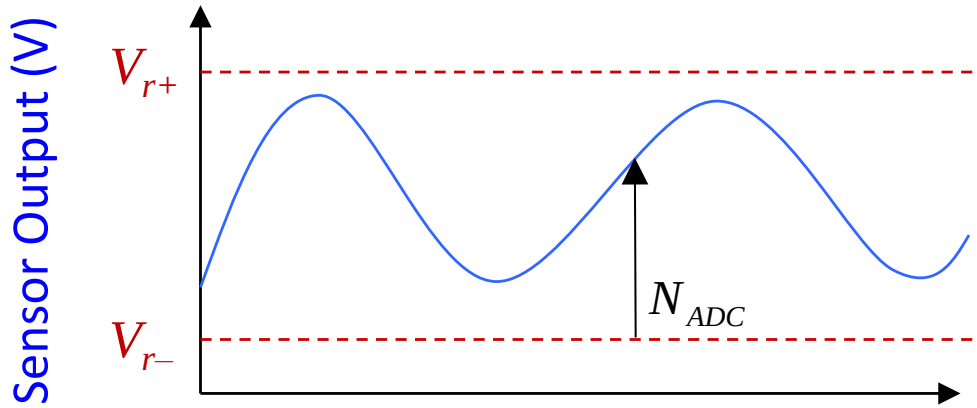

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- Converting ADC counts to engineering units
  - Convert N_{ADC} (between 0 and 4095) back into temperature (-40C and +100C)

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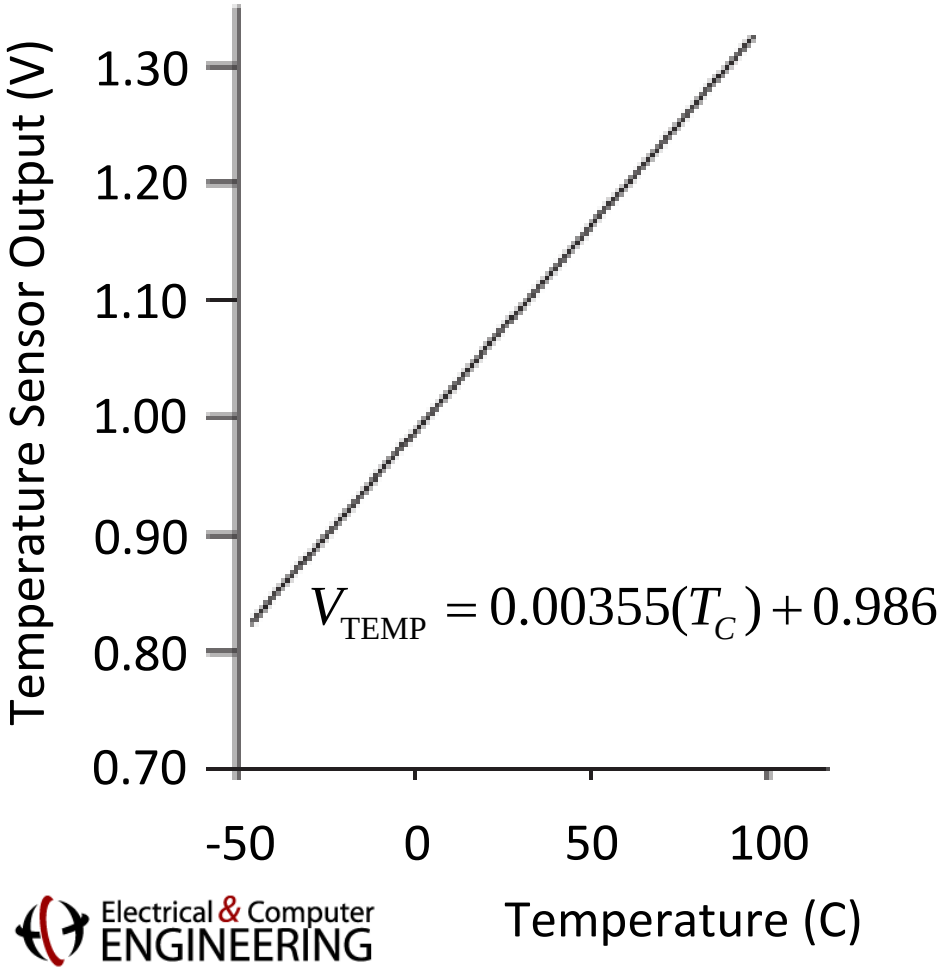
Example: Converting ADC Counts into Engineering Units

- Sensor: $0.8\text{V} < V_{OUT} < 1.35\text{V}$
- Choose $V_{r+} = 1.500\text{V}$ and $V_{r-} = 0.000\text{V}$



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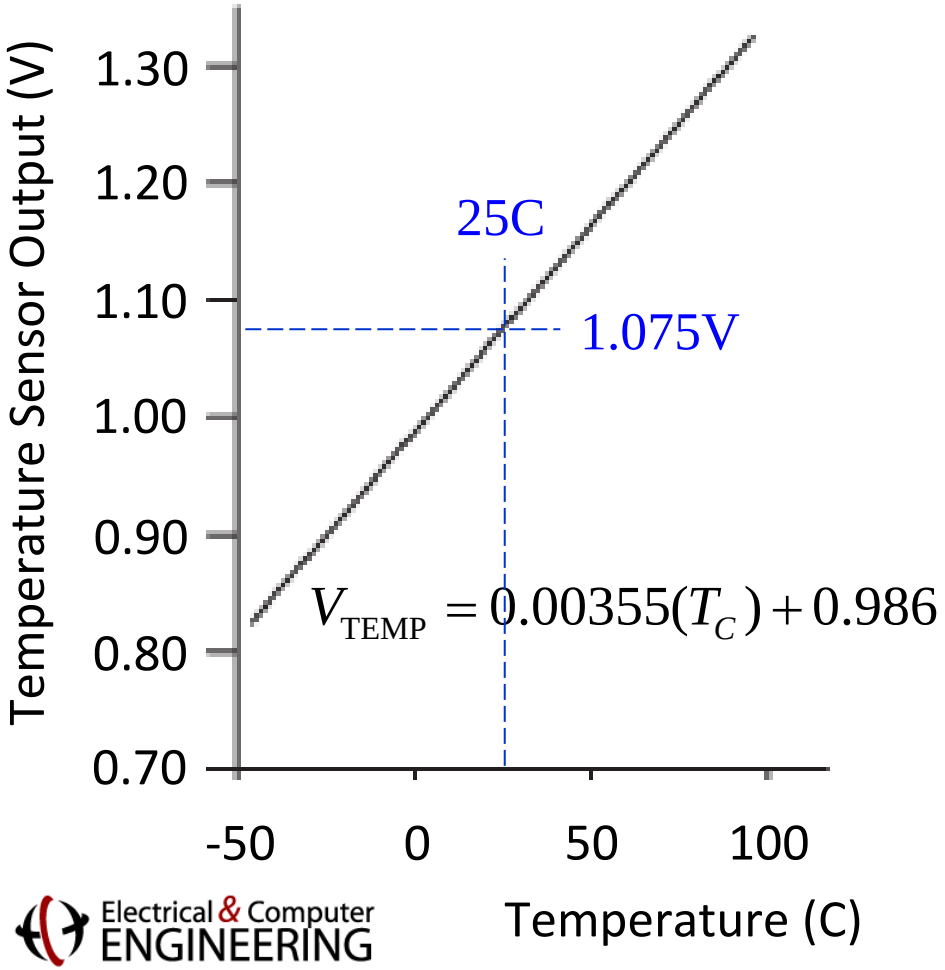

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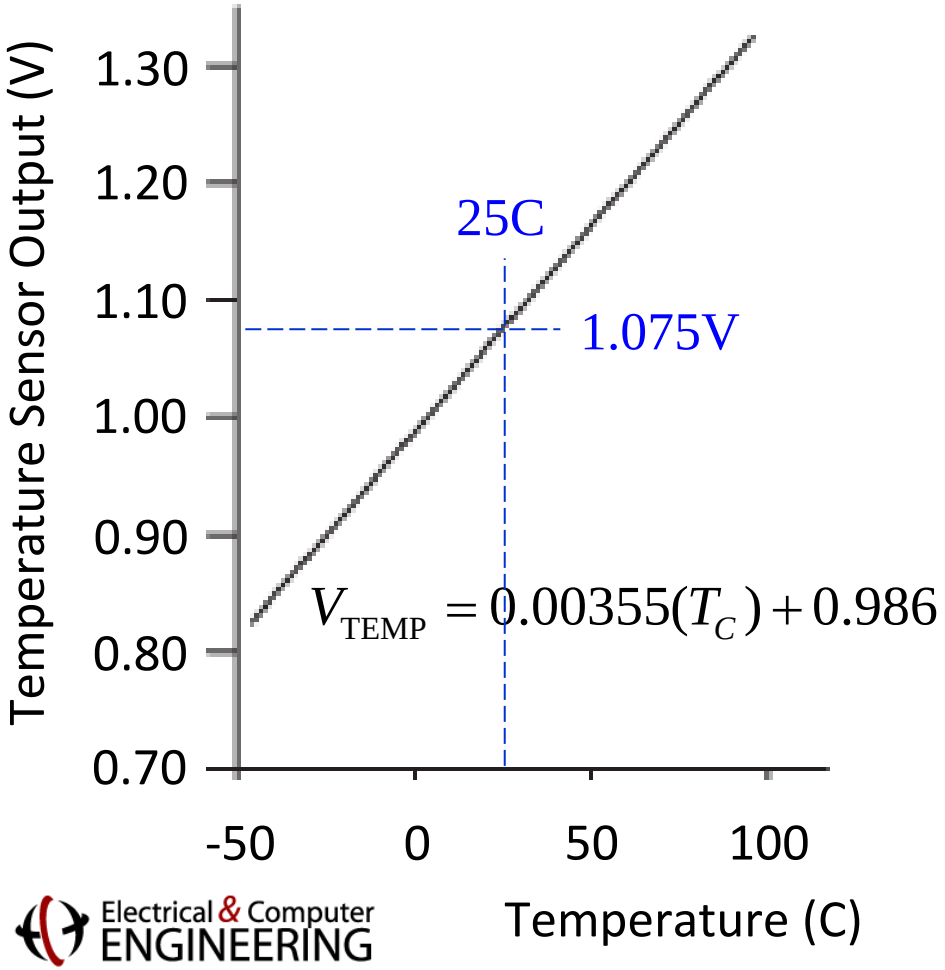

- Converting voltage to ADC counts (N_{ADC})

$$N_{ADC} = 4095\left(\frac{V_{IN} - V_{r-}}{V_{r+} - V_{r-}}\right) = 4095\left(\frac{1.075V - 0V}{1.500V - 0V}\right) = 2,934\ steps$$

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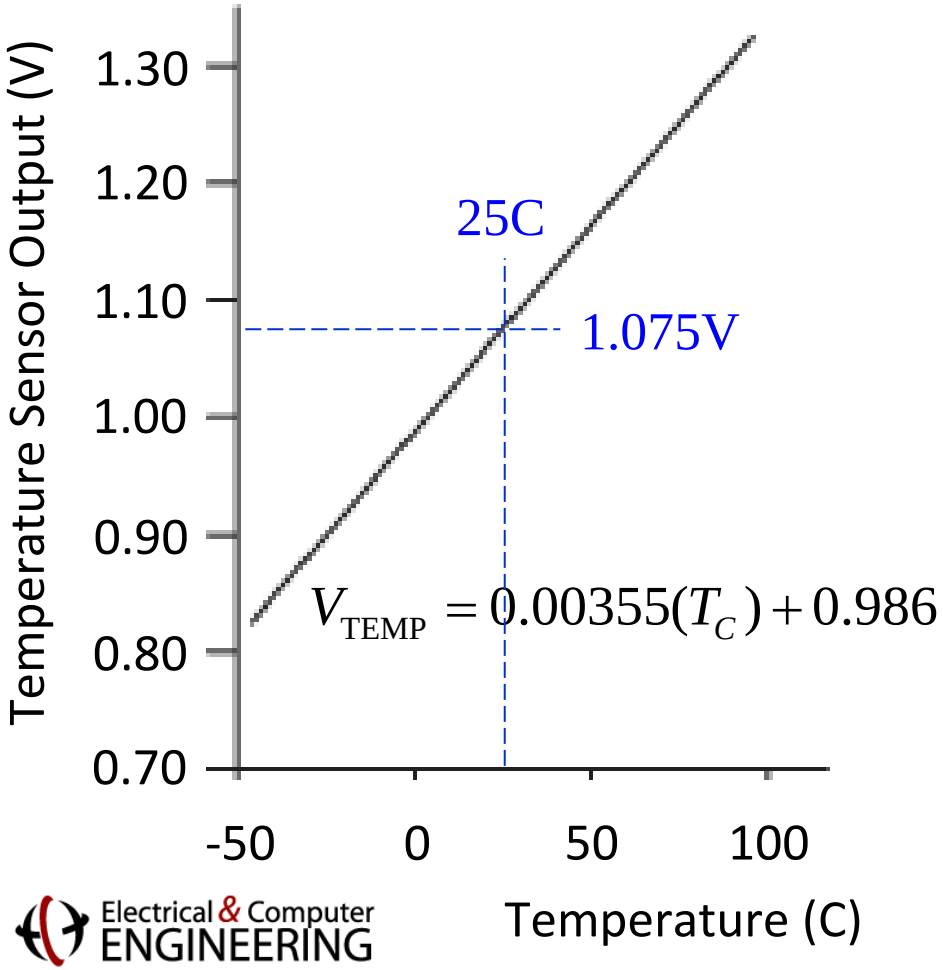

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- Converting ADC counts (N_{ADC}) to temperature

$$V_{IN} = \left(\frac{N_{ADC}}{4095}\right)\left(V_{r+} - V_{r-}\right) + V_{r-}$$

$$V_{IN} = \left(\frac{2934}{4095}\right)\left(1.5V\right) + 0V = 1.075V$$

- Converting voltage back into temperature

$$T_C = \frac{V_{\text{TEMP}} - 0.986}{0.00355} = \frac{1.075 - 0.986}{0.00355} = 25C$$

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Example: Converting ADC Counts into Engineering Units

- Sensor: $0.8\text{V} < V_{OUT} < 1.35\text{V}$
- Choose $V_{r+} = 1.500\text{V}$ and $V_{r-} = 0.000\text{V}$
- $T_C = 25\text{C} \rightarrow V_{OUT} = 1.075\text{V}$
- $V_{OUT} = 1.075\text{V} \rightarrow N_{ADC} = 2{,}934$ steps

- Program converts $2{,}934$ steps back to 25C

$$V_{IN} = \left(\frac{2934}{4095}\right)\left(1.5V\right) + 0V = 1.075V$$

$$T_C = \frac{V_{\text{TEMP}} - 0.986}{0.00355} = \frac{1.075 - 0.986}{0.00355} = 25C$$

```
int main() {

    int   adccount = 2934;
    float range    = 1.5;
    float vtemp;
    float tempc;

    vtemp = adccount/4095 * range;
    tempc = (vtemp-0.986)/0.00355;
    printf("vtemp: %f\n", vtemp);
    printf("tempc: %f\n", tempc);
}
```

Example: Converting ADC Counts into Engineering Units

- Sensor: $0.8\text{V} < V_{OUT} < 1.35\text{V}$
- Choose $V_{r+} = 1.500\text{V}$ and $V_{r-} = 0.000\text{V}$
- $T_C = 25\text{C} \rightarrow V_{OUT} = 1.075\text{V}$
- $V_{OUT} = 1.075\text{V} \rightarrow N_{ADC} = 2{,}934$ steps

- Program converts $2{,}934$ steps back to 25C

$$V_{IN} = \left(\frac{2934}{4095}\right)\left(1.5V\right) + 0V = 1.075V$$

$$T_C = \frac{V_{\text{TEMP}} - 0.986}{0.00355} = \frac{1.075 - 0.986}{0.00355} = 25C$$

```
int main() {

    int   adccount = 2934;
    float range    = 1.5;
    float vtemp;
    float tempc;

    vtemp = adccount/4095 * range;
    tempc = (vtemp-0.986)/0.00355;
    printf("vtemp: %f\n", vtemp);
    printf("tempc: %f\n", tempc);
}

$ gcc arithmetic.c

$ ./a.out
vtemp: 0.000000
tempc: -277.746490
```

Final Thoughts on Amplitude

- Sensor provides a range of voltages for inputs of interest to us

- Use op amps to convert those voltages into a desired ADC range of voltages
- Use the full range of the ADC, perhaps with a small margin

- Choose an ADC with enough bits to provide the resolution required ($Cost\ \alpha\ e^{\#\ of\ bits}$)
- Understand ADC and DAC errors will be a fact-of-life

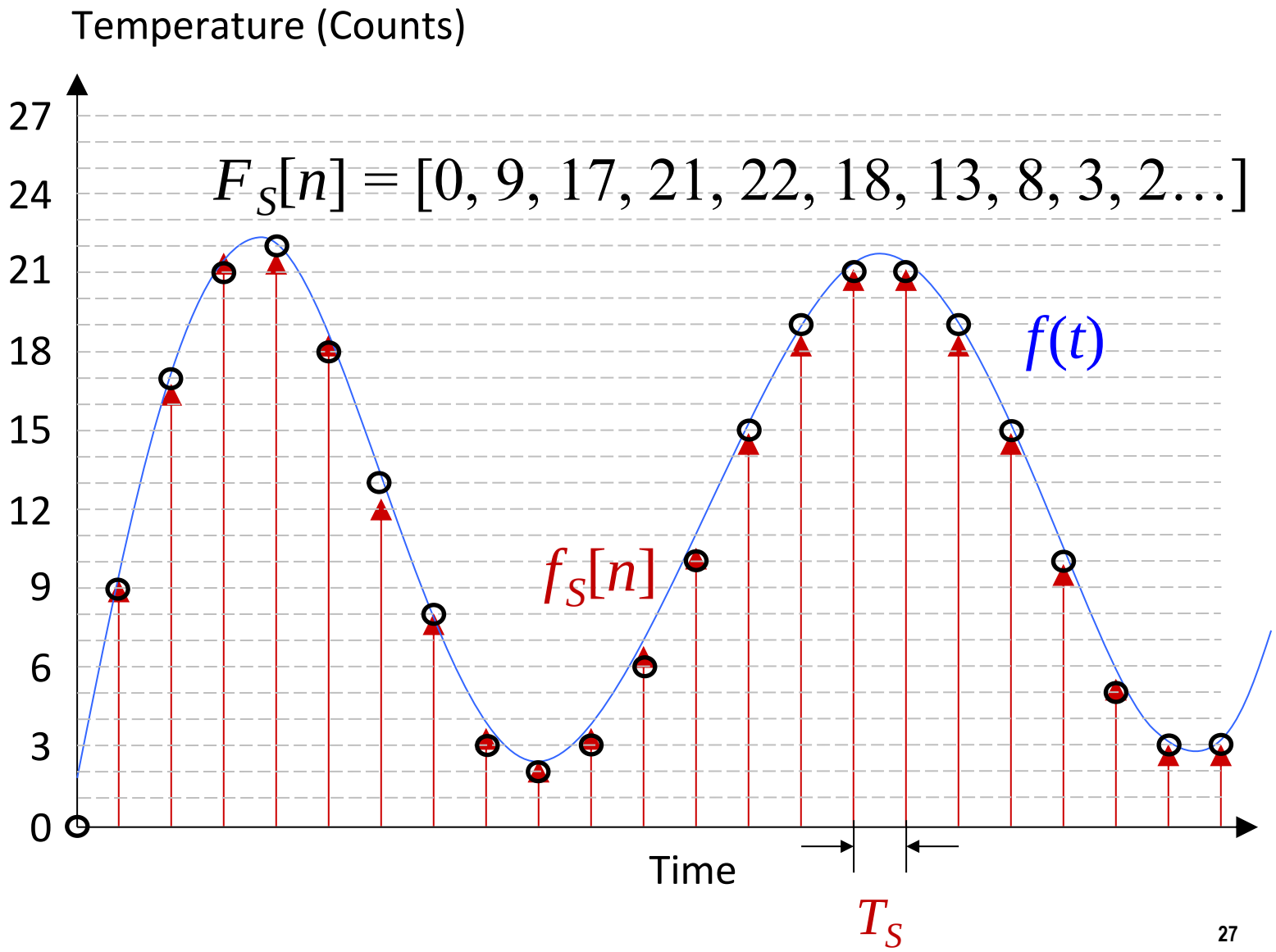
- Carefully write your code to manipulate values without losing resolution of your data
- Be careful about each arithmetic operation
- Be careful about software emulation of floating-point numbers

Continuous-Time Data and Discrete-Time Sampling

- x-axis is often time
- y-axis is often amplitude
- Both are often analog and continuous signals

- Sample every T_S seconds to create a discrete-time signal

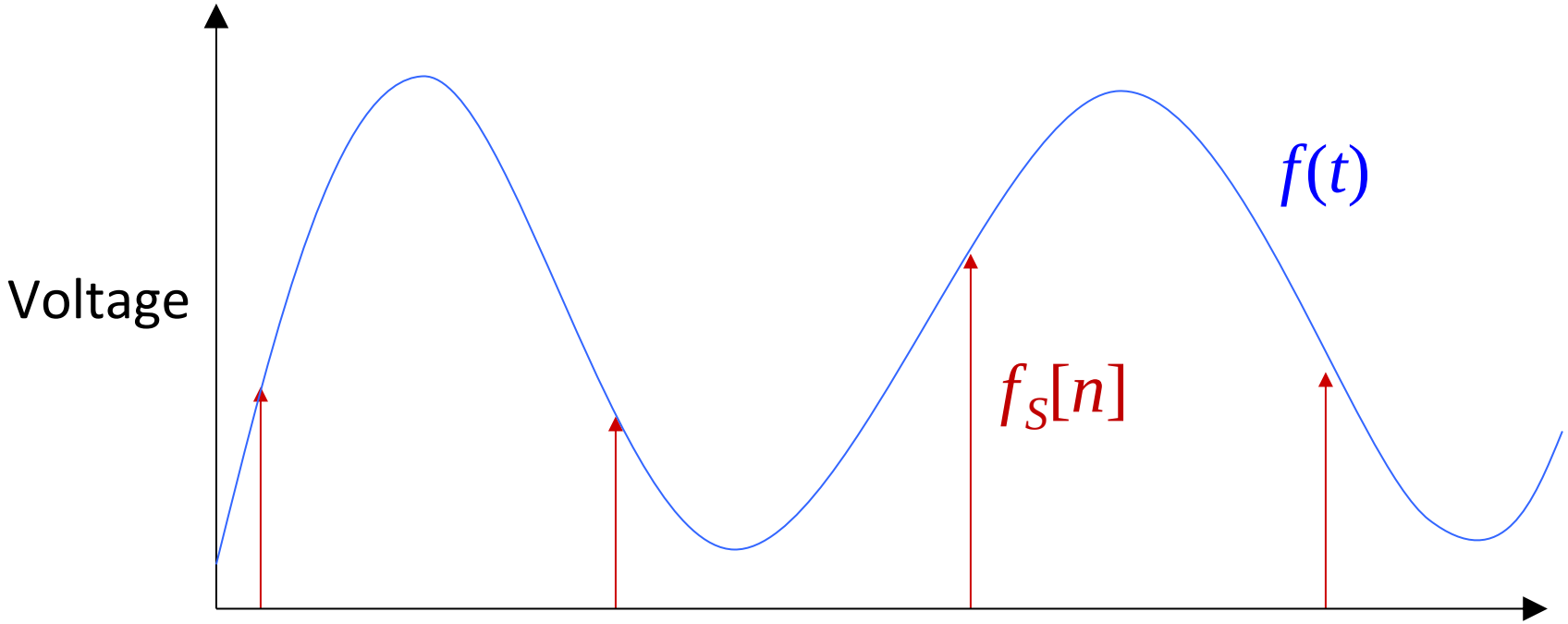
- Sample amplitude with a finite step-size to create a digital, discrete-value signal



Choosing a Sampling Rate

- ADC peripherals have a maximum sampling rate
- In addition, your program may be limited in how often it can read an ADC sample

- Sample too slowly: We can't reconstruct the signal we care about
- Sample too quickly: Waste computation, energy, resources



Nyquist-Shannon Sampling Theorem

- If a continuous-time signal contains no frequencies higher than f_{max}, it can be completely determined by discrete samples taken at a rate of

$$f_{sampling} > 2f_{max}$$

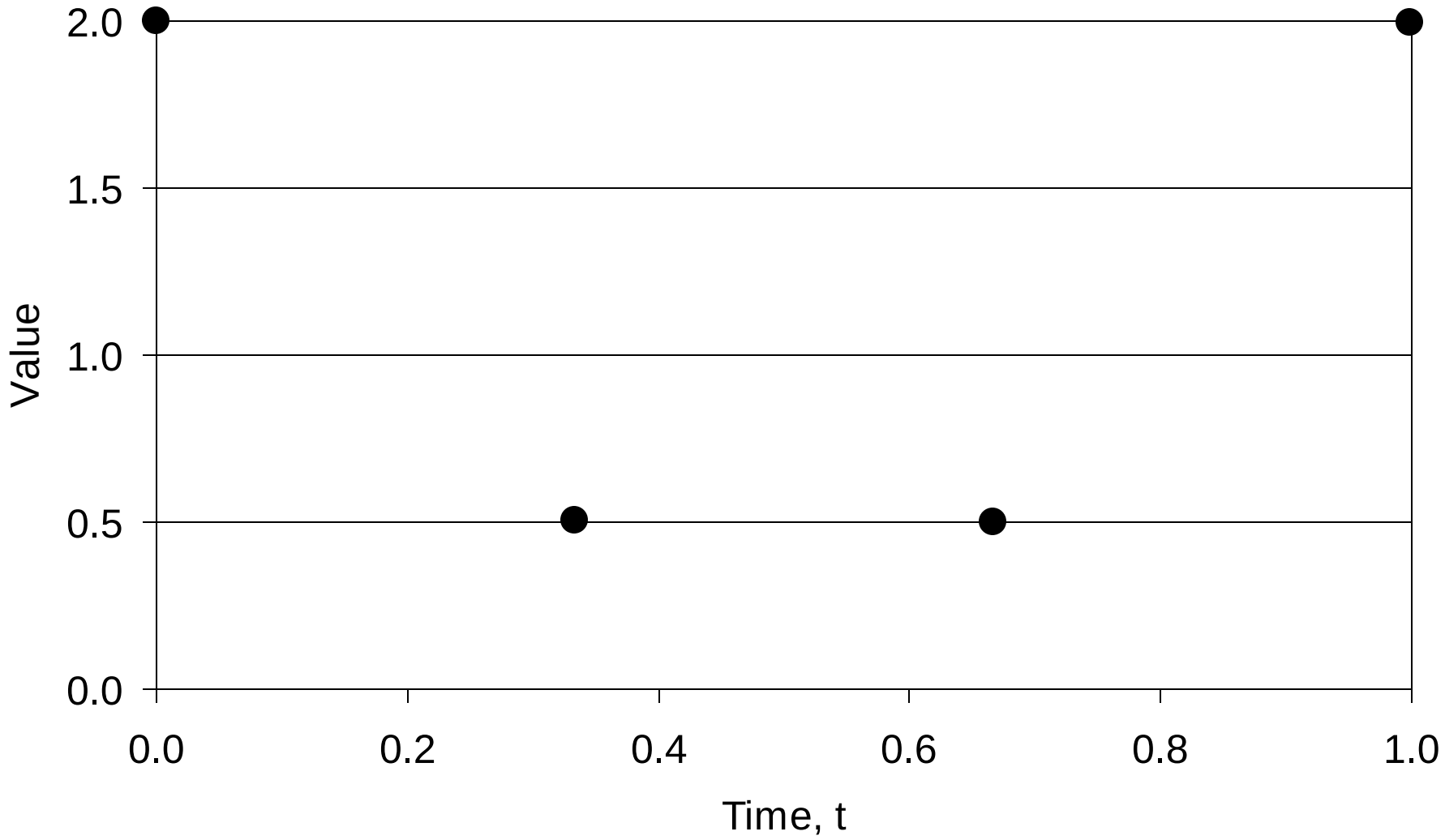
- For example, most people can "hear" audio signals between 20Hz and 20kHz
- Audio CDs: Sampled at 44.1kHz

Song sampled at 44.1kHz

Same song sampled at 1,024Hz (distorts frequencies above 512Hz)

Aliasing

Samples of Different Signals Have the Same Sampled Values



Aliasing

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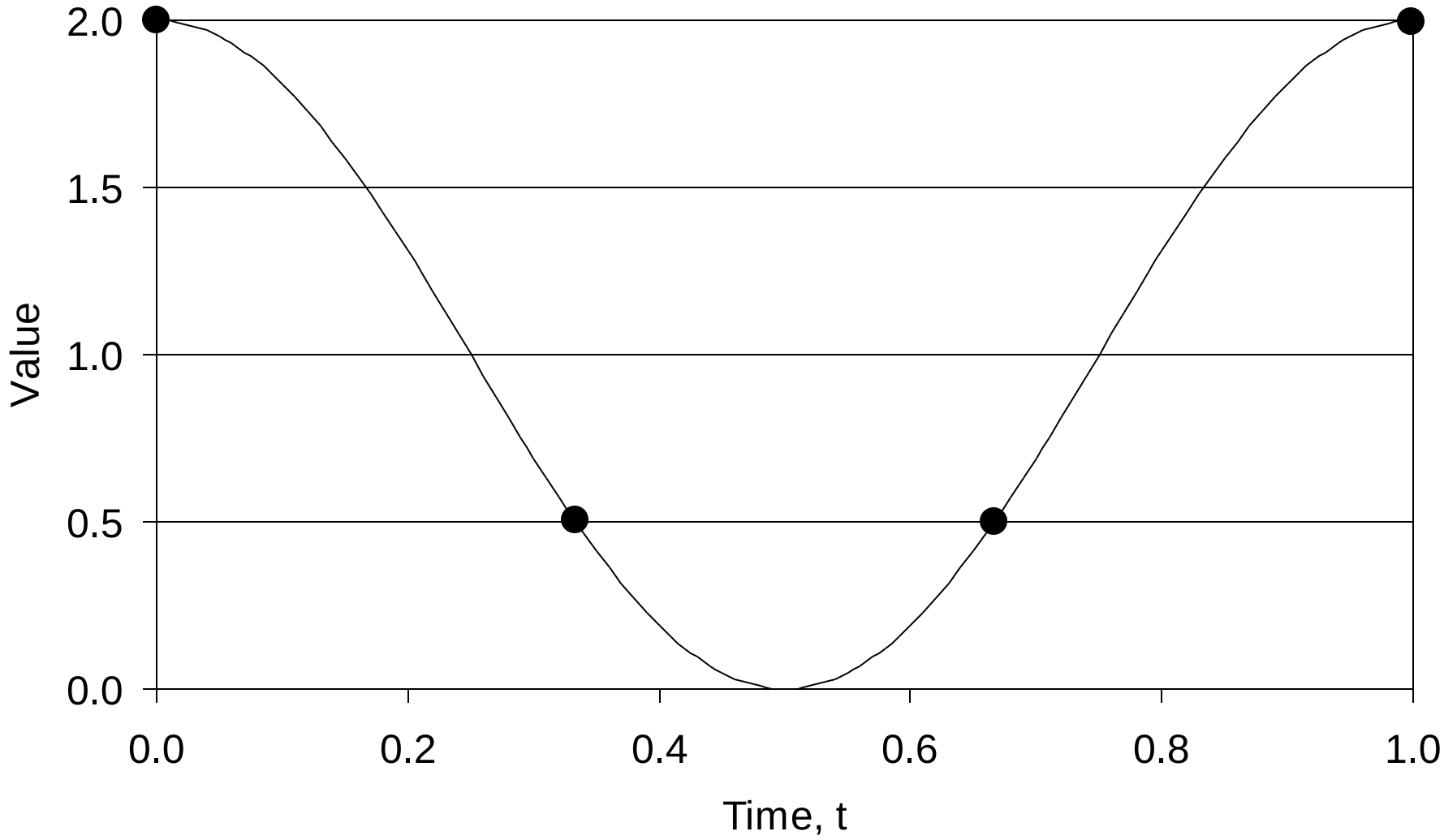

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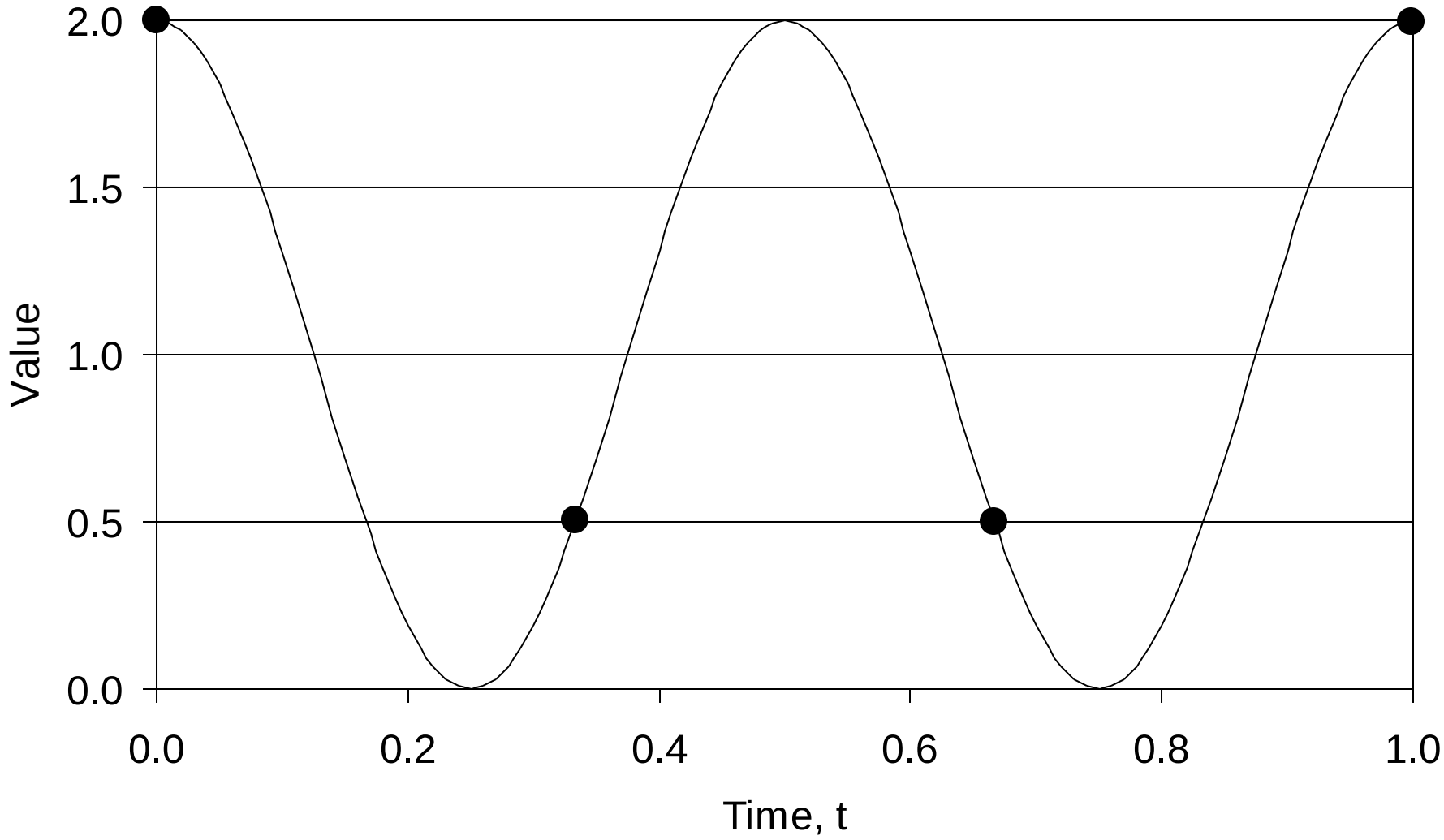

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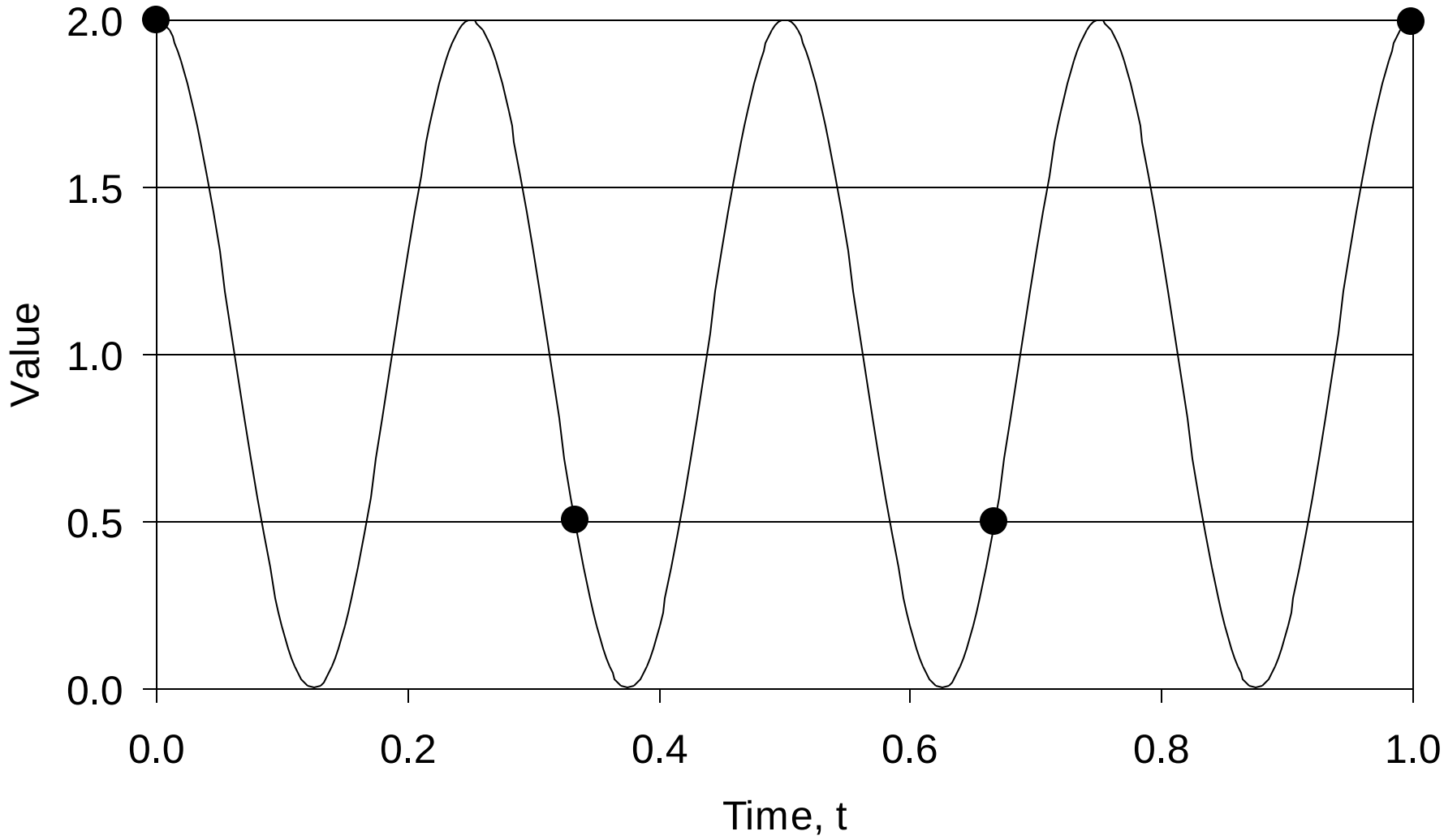

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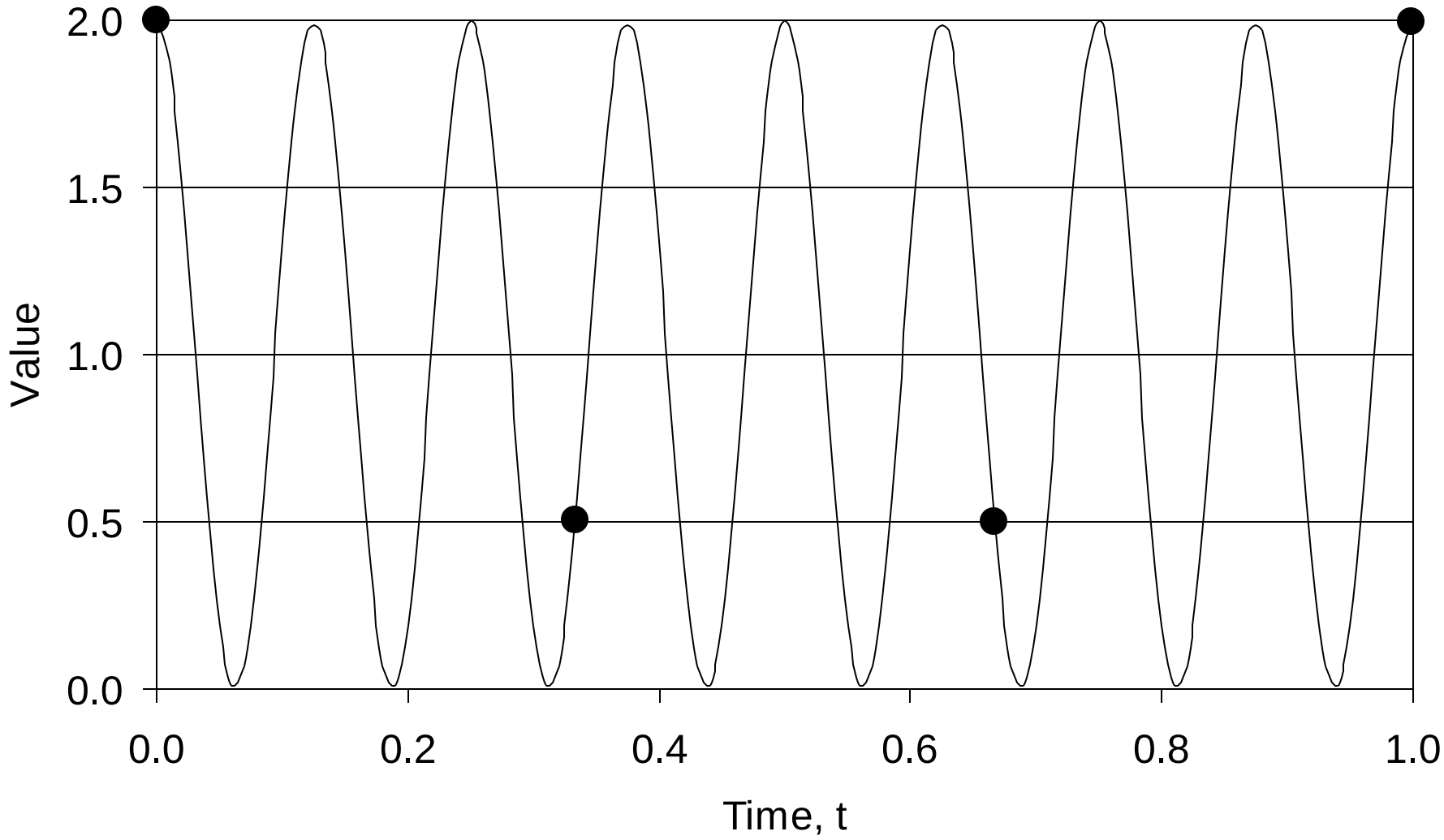

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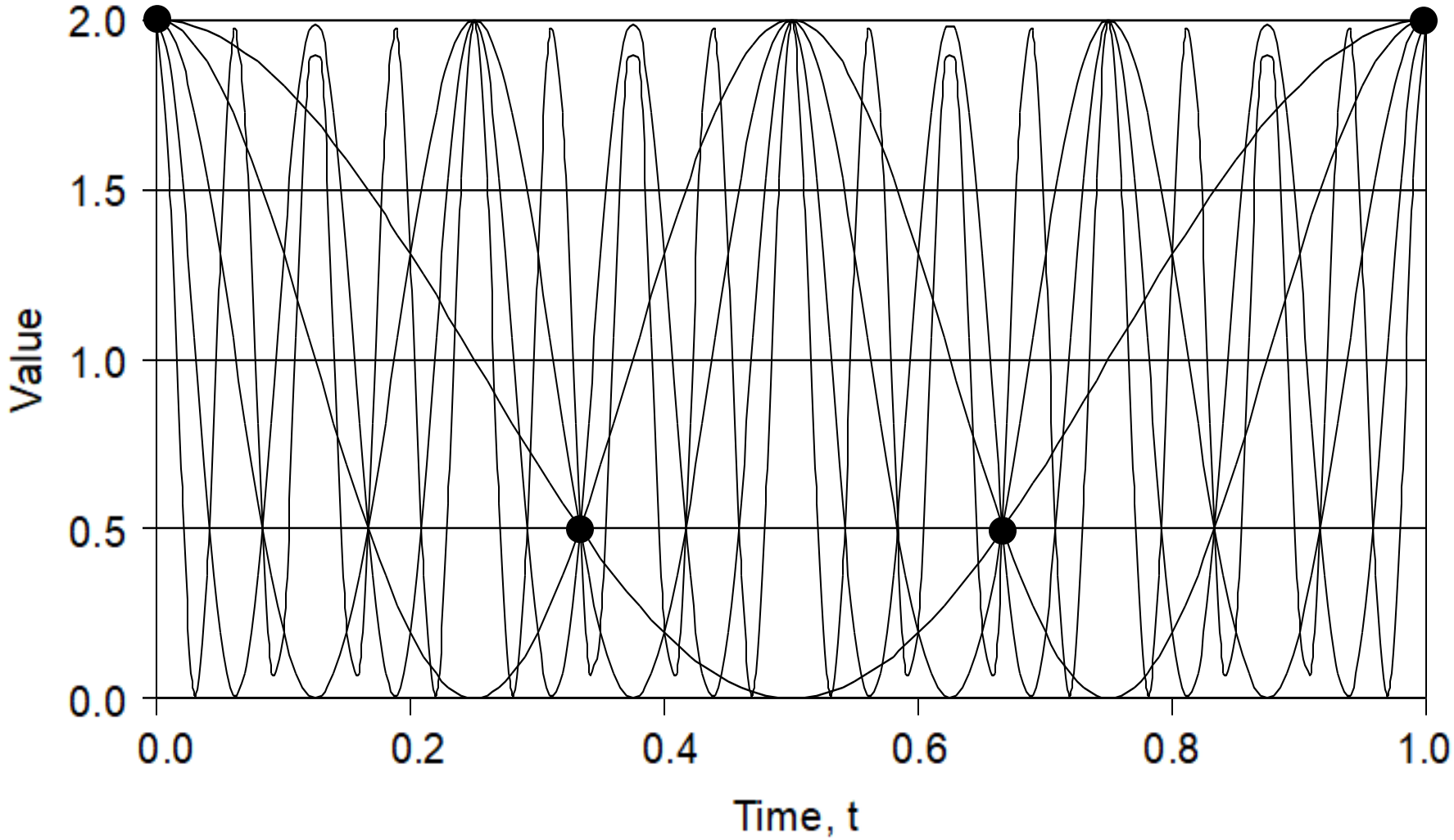
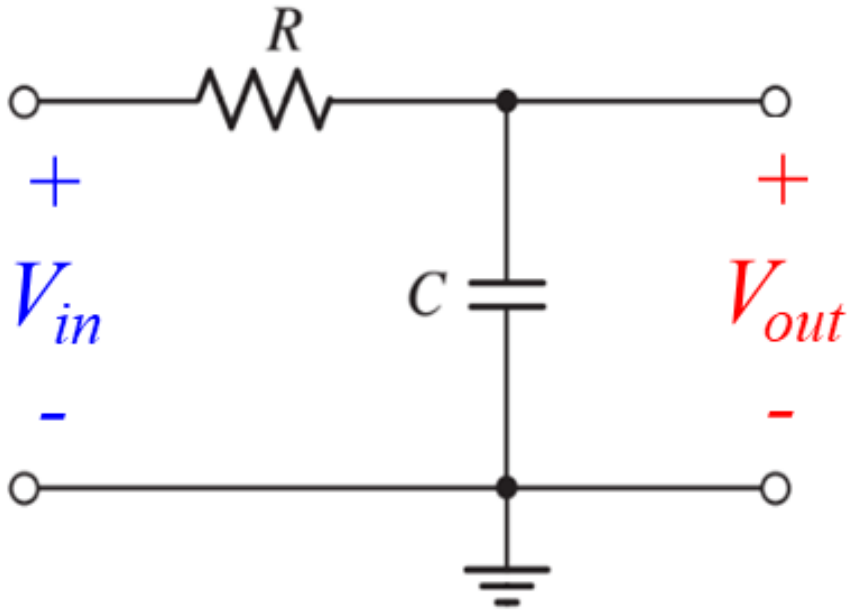
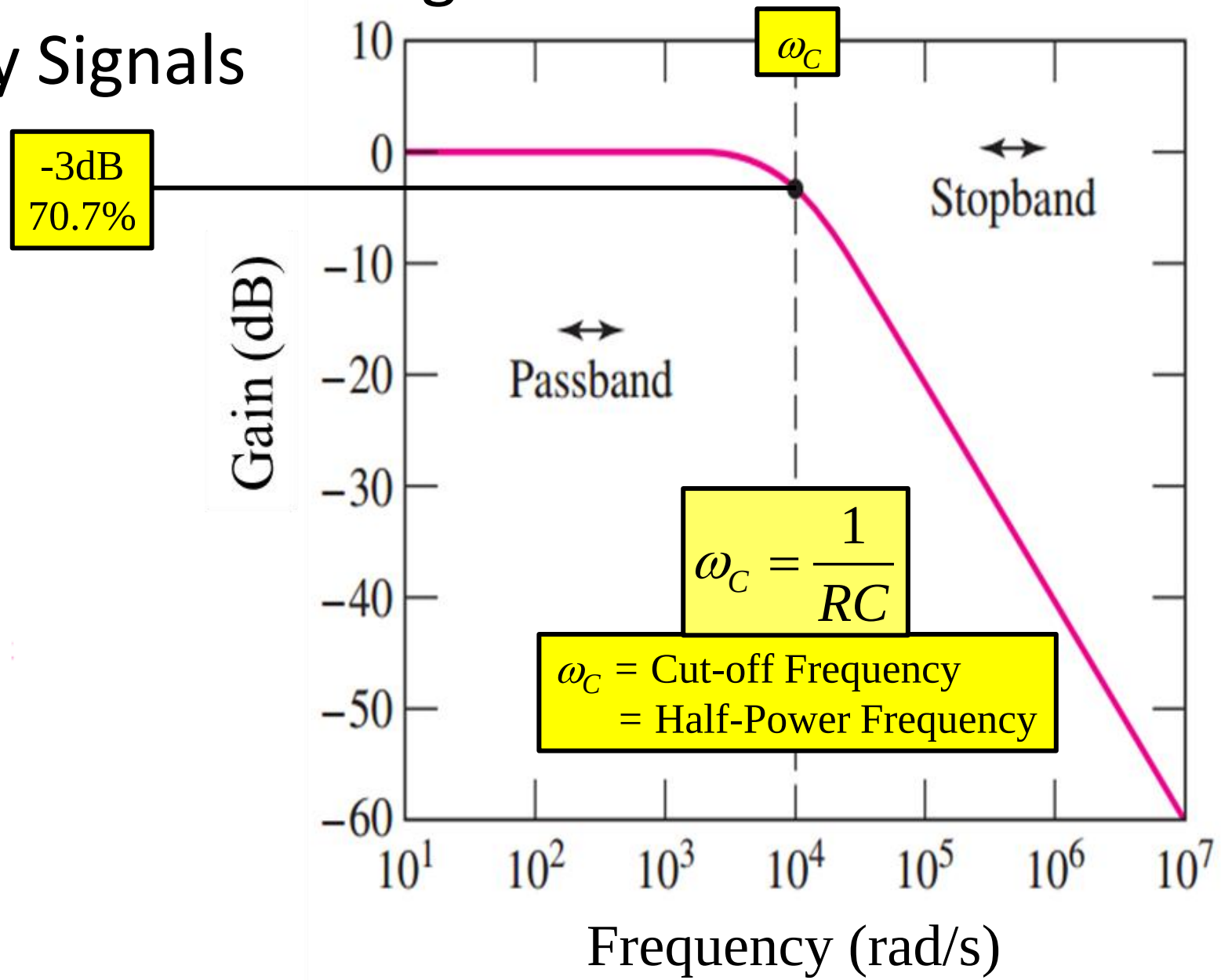


COMING ATTRACTIONS: Anti-Aliasing Filter:
Reduce High Frequency Signals

$$Gain(\text{dB}) = 20\log_{10}\frac{|V_{out}|}{|V_{in}|}$$



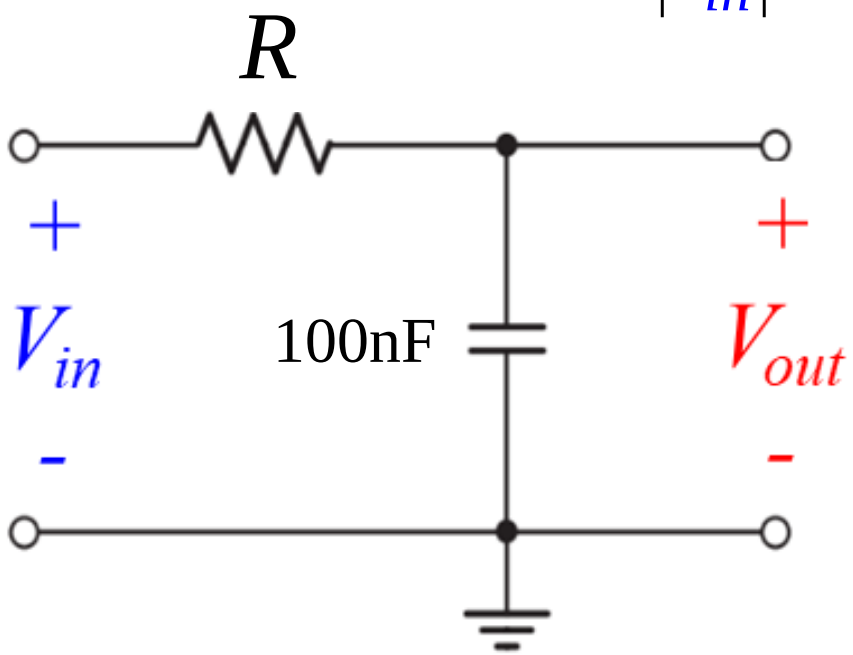


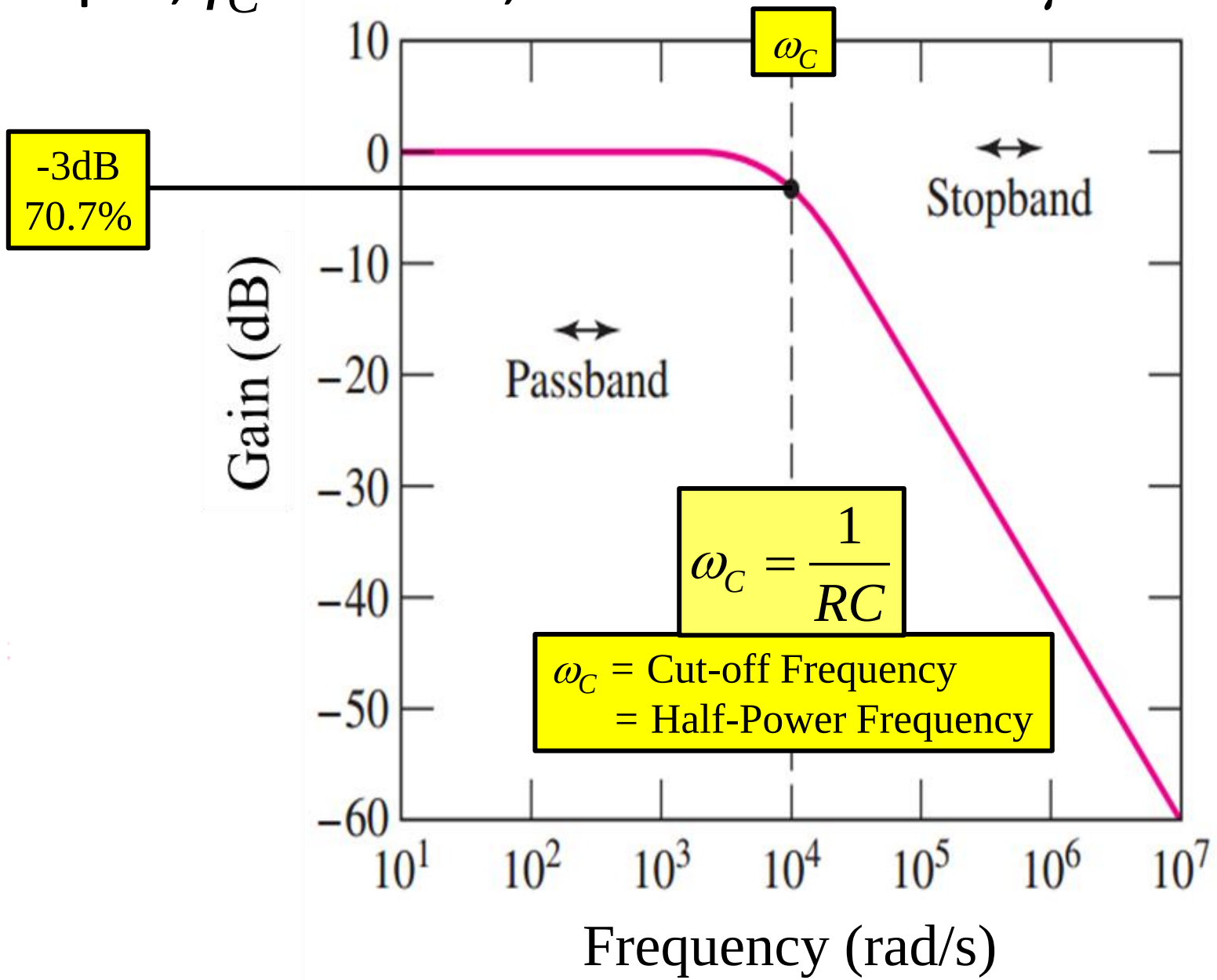
Electrical & Computer ENGINEERING

Anti-Aliasing Filter: Example, $f_C = 30\text{Hz}$, $C = 100\text{nF} = 0.1\mu\text{F}$

Find R

$$Gain(\text{dB}) = 20\log_{10}\frac{|V_{out}|}{|V_{in}|}$$





Do I Really Need to Add a Signal Conditioning Circuit in Front of an ADC Input?

- Short answer: Yes

- Long answer: Yes, but sometimes it's already done for you.
- Many ADCs have a pretty good analog filter built in
  - Integrated filters typical have f_C near 50% of their maximum sampling rate (Nyquist-Shannon)
  - Works well if you are sampling at maximum rate (max power, max busy….)
  - Does not work well if you are sampling below the maximum rate
- An additional filter will only further reduce higher frequency components

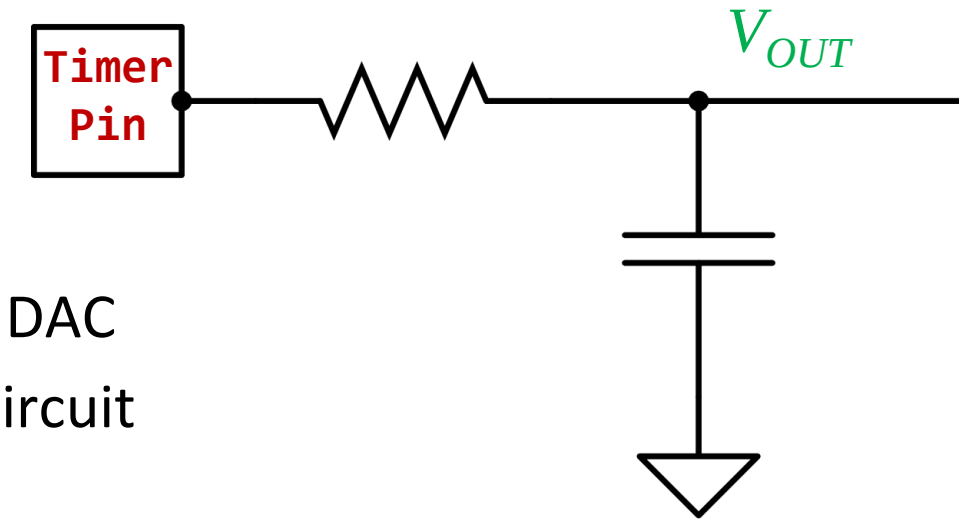
Agenda

- Analog and Digital Worlds – The Problem with Pizza
- Analog to Digital Converters (ADCs)
- Digital to Analog Converters (DACs)

Do It Yourself DAC

- DACs are expensive circuits
- Vary rare for a MCU to have a DAC
- Usually, they are an external circuit you have to purchase

Do It Yourself DAC



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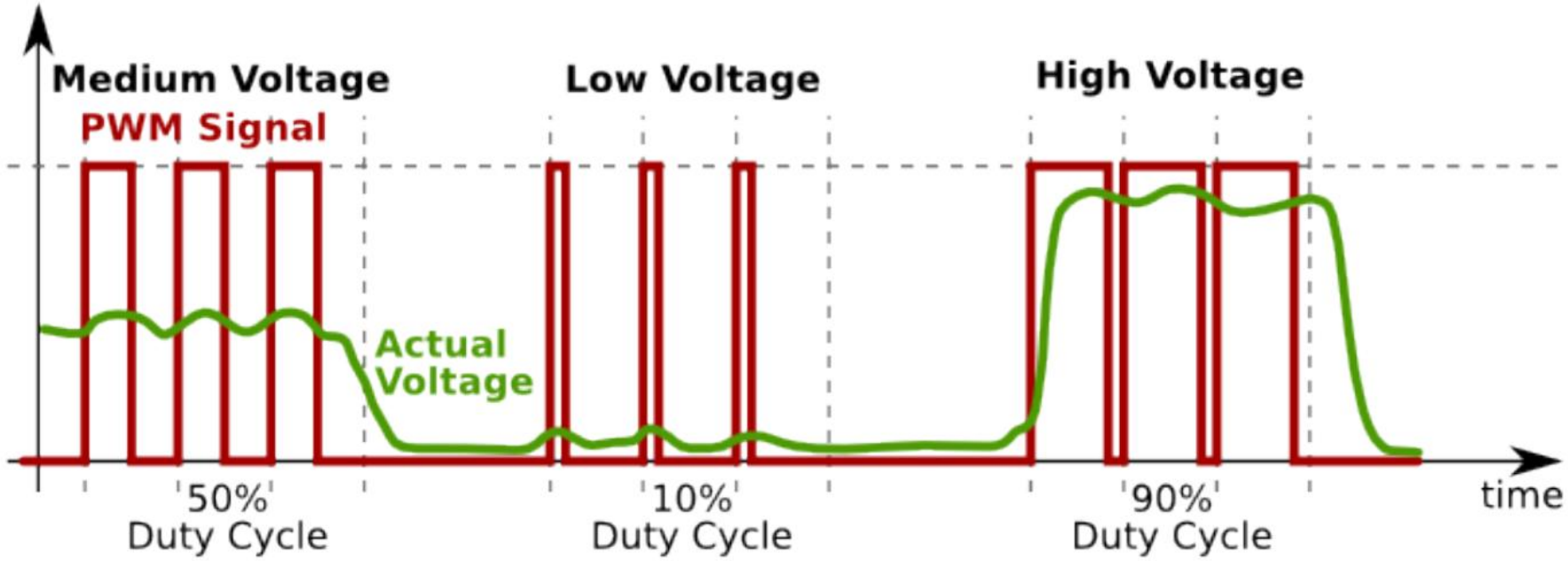
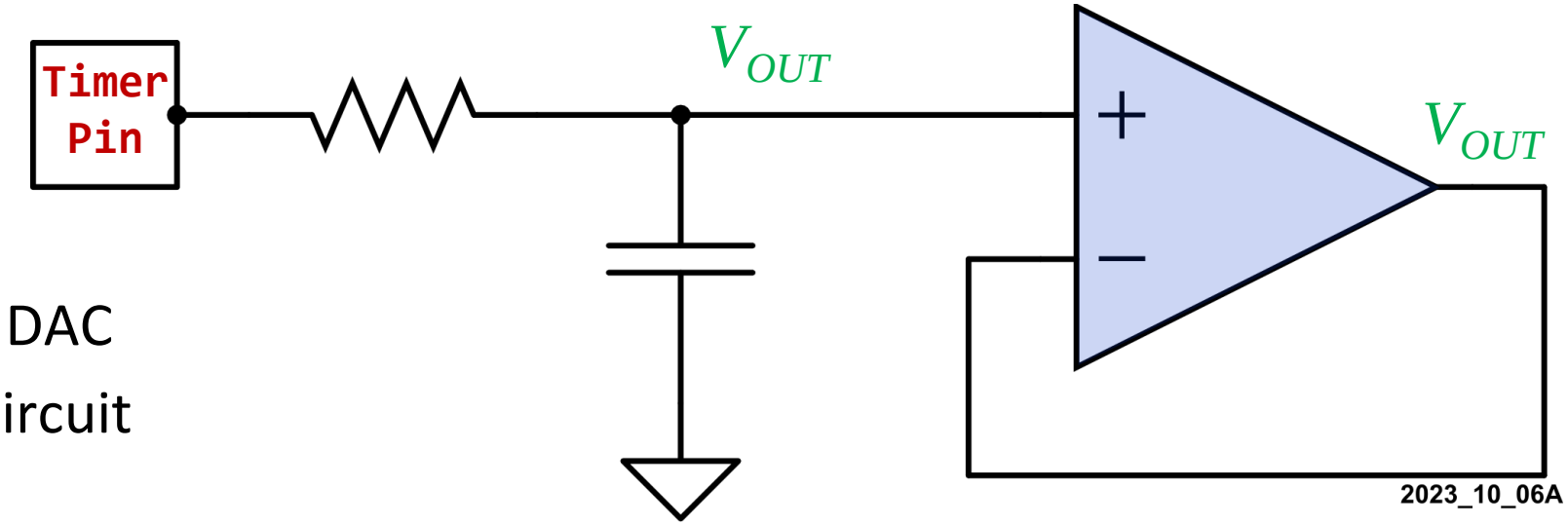
- You can make a low-quality DAC with just a PWM timer and an RC circuit



Do It Yourself DAC



- DACs are expensive circuits
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- Usually, they are an external circuit you have to purchase
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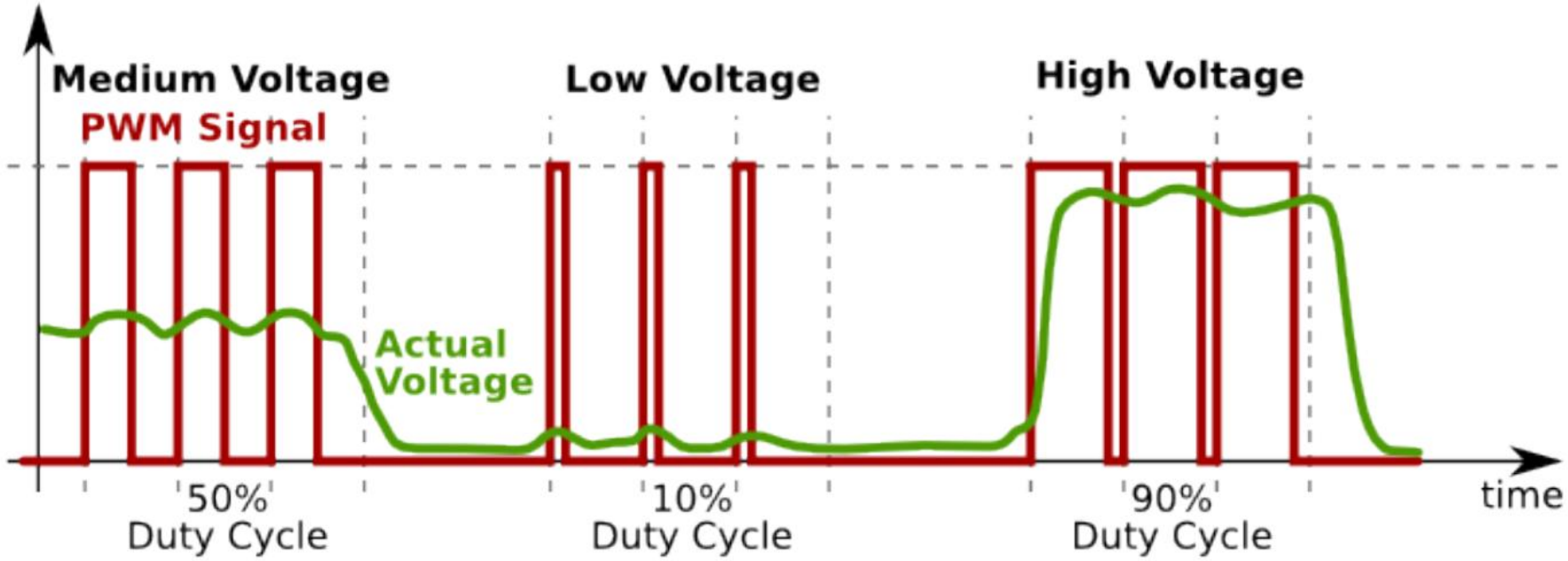
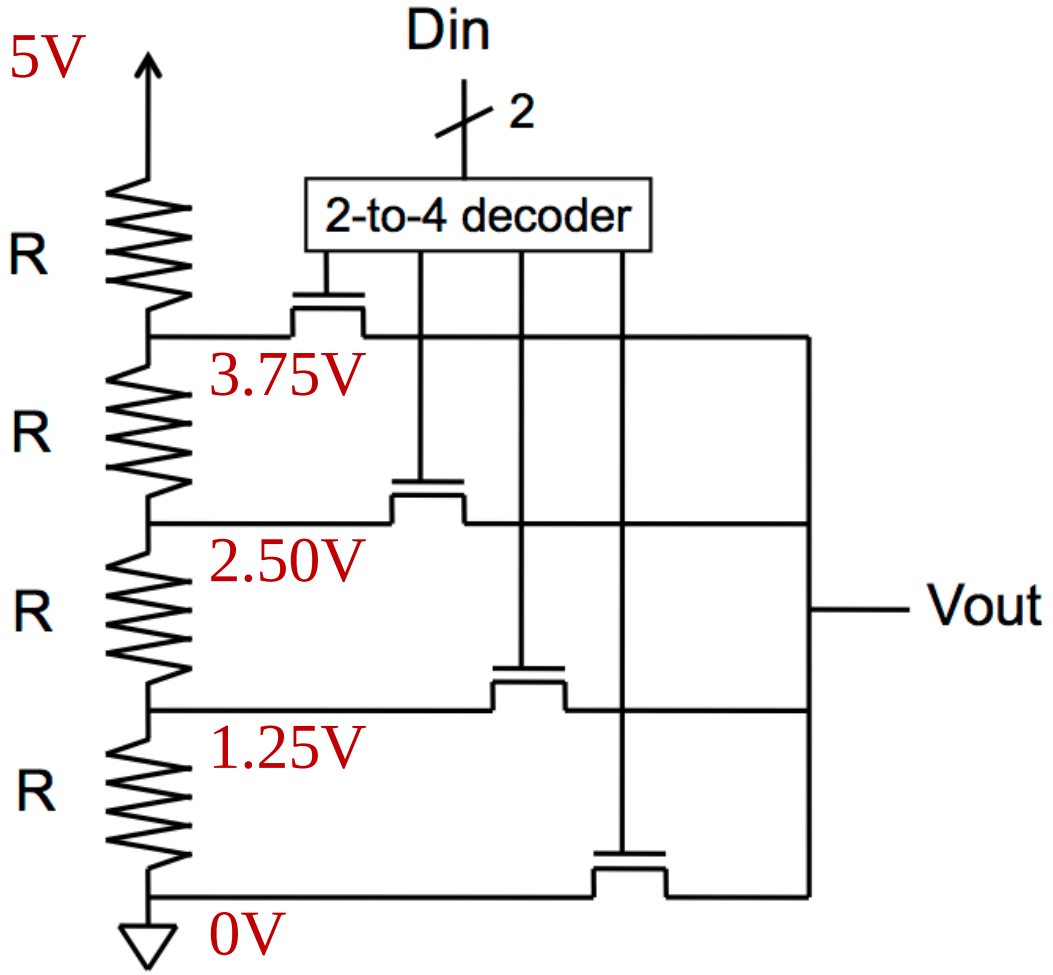
- **Use an op amp to increase output current**



Resistance Ladder DAC

- Advantages
  - Extremely Fast!
  - Monotonic: Output voltage always increases with DAC step count



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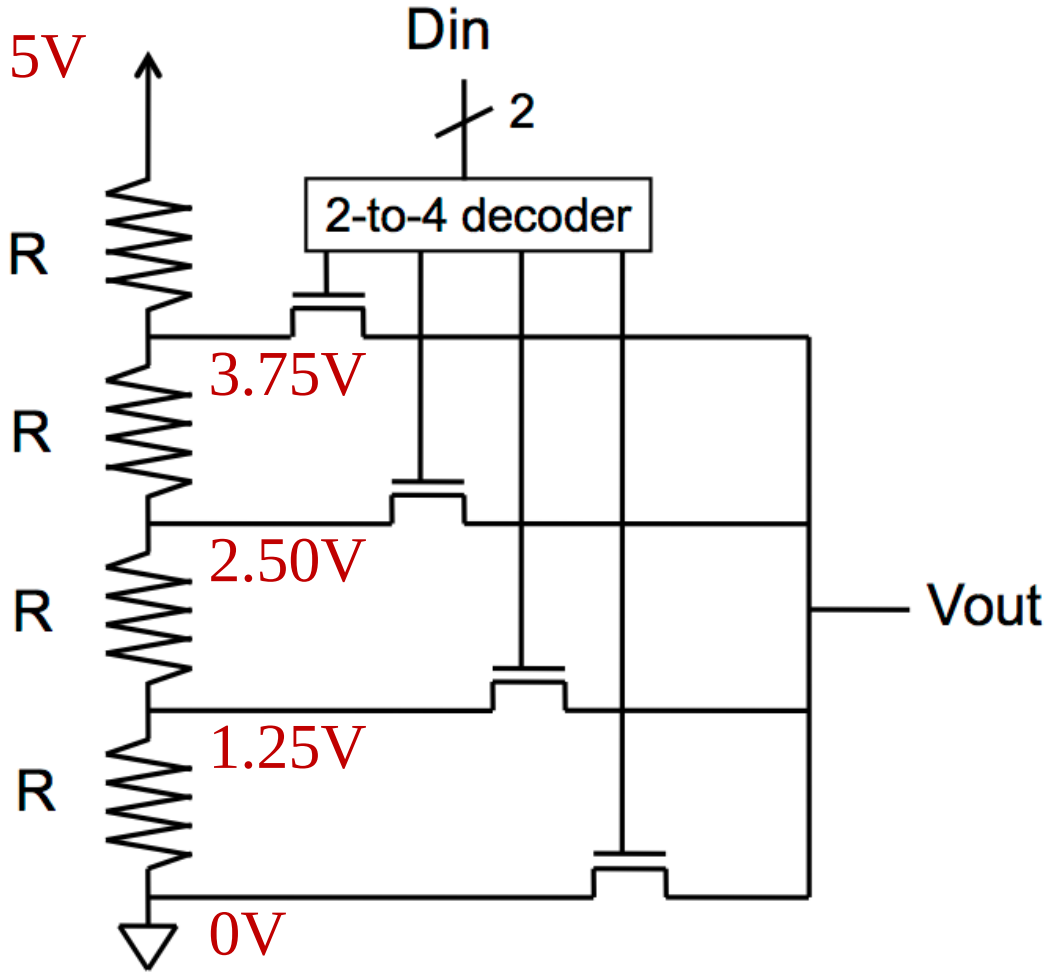
- Disadvantages
  - One resistor and one transistor required per each output voltage level (N-bit DAC requires 2^N resistors and transistors)
  - Large decoder needed $\log_2$N-to-N
  - Each extra bit doubles the size



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  - Accuracy requires matched resistors
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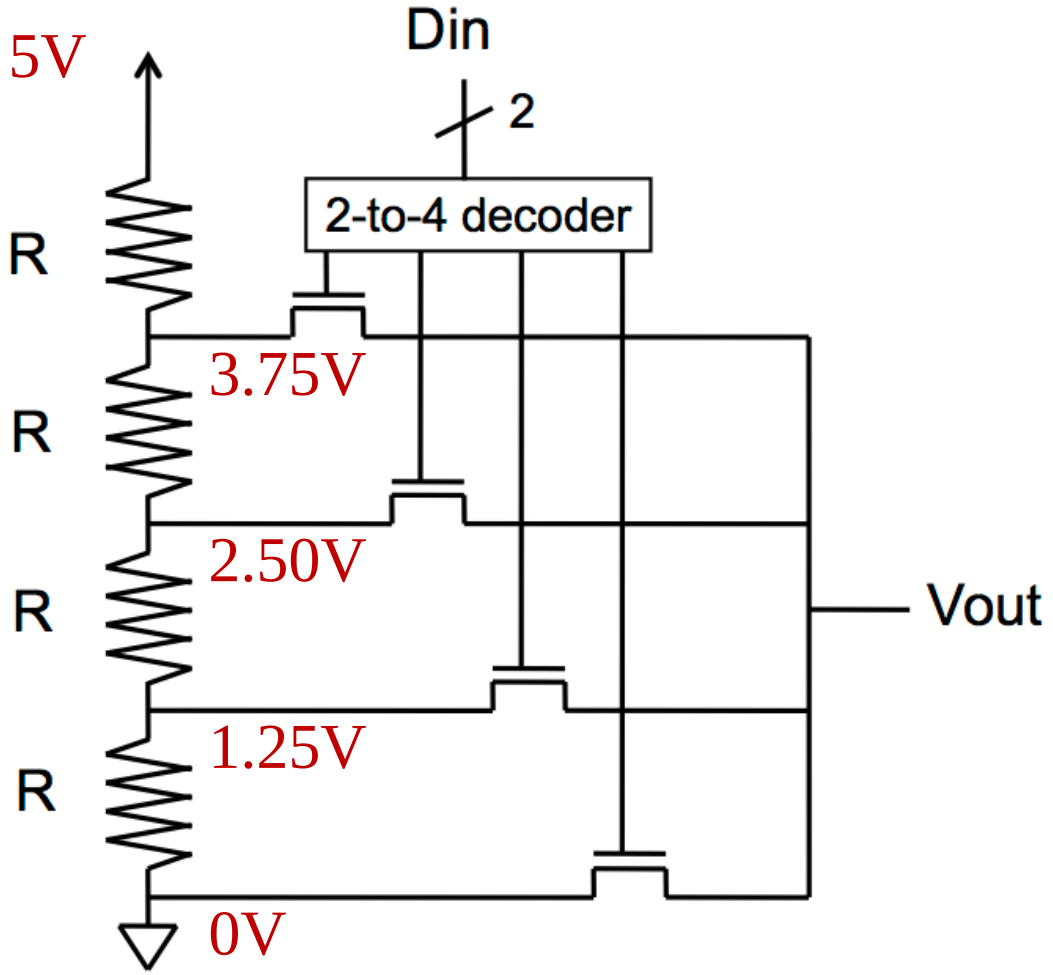
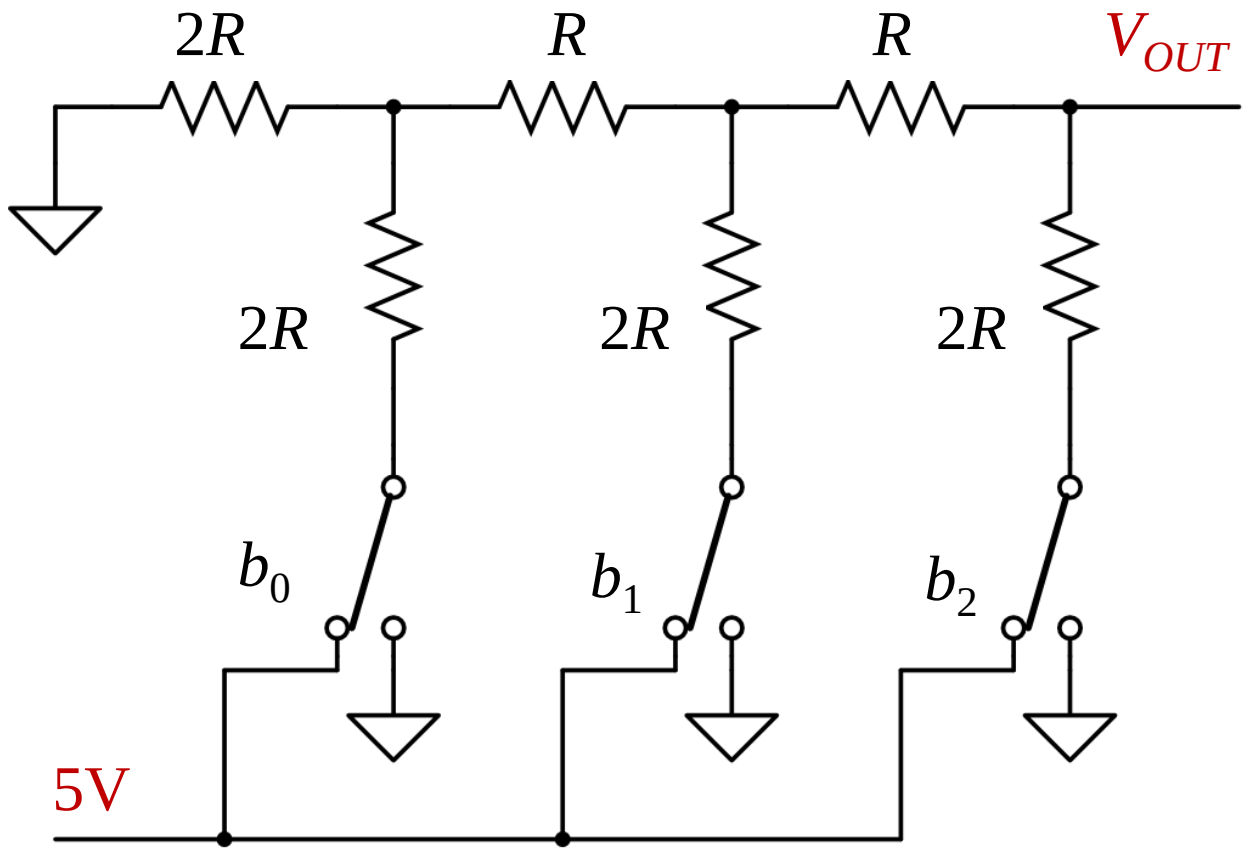


R-$2R$ Ladder DAC

- Resistive ladder constructed from R and $2R$ resistor values
- Switch positions controlled by the DAC's N-bits of inputs (3-bit DAC shown)

- b_X = **1** Switch to +5V
- b_X = **0** Switch to 0V

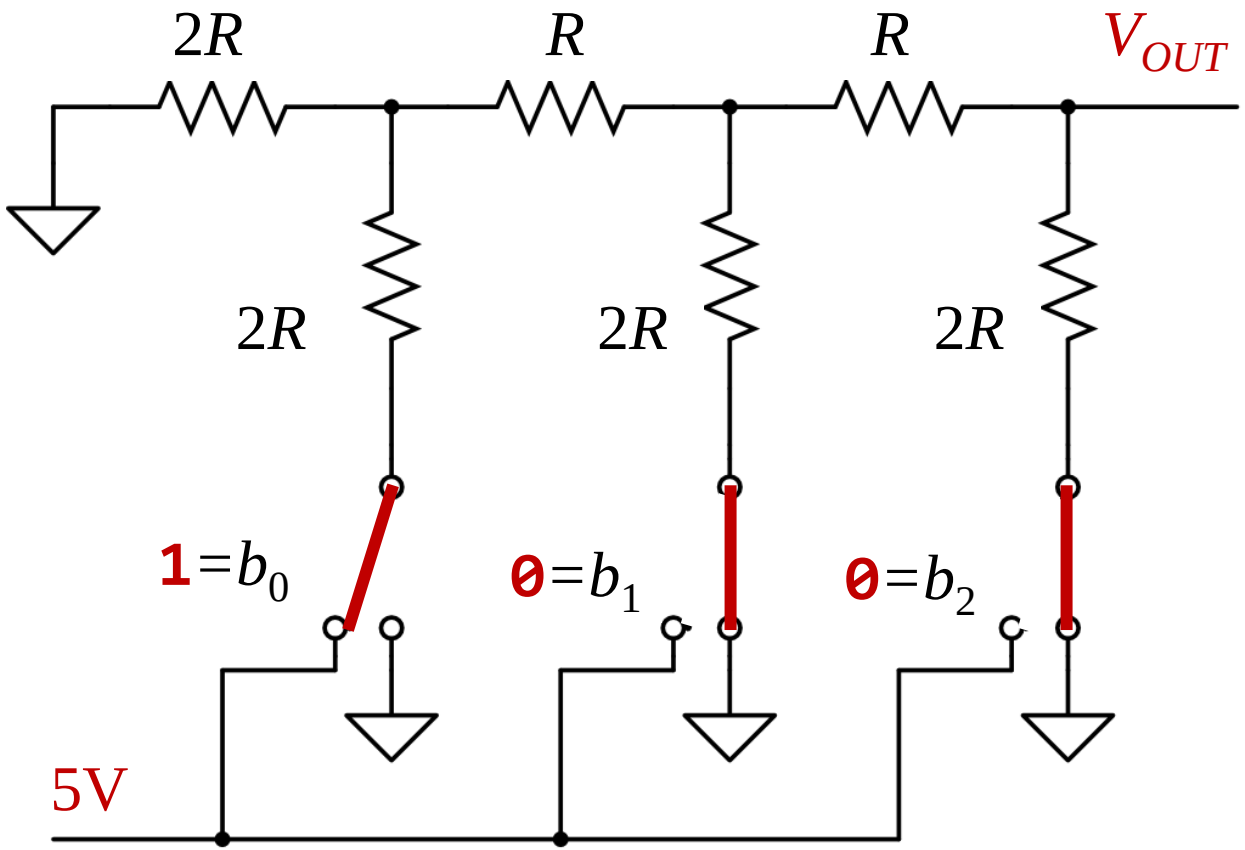
- 3-bit DAC shown has input **111**



2023_10_06B

R-$2R$ Ladder DAC

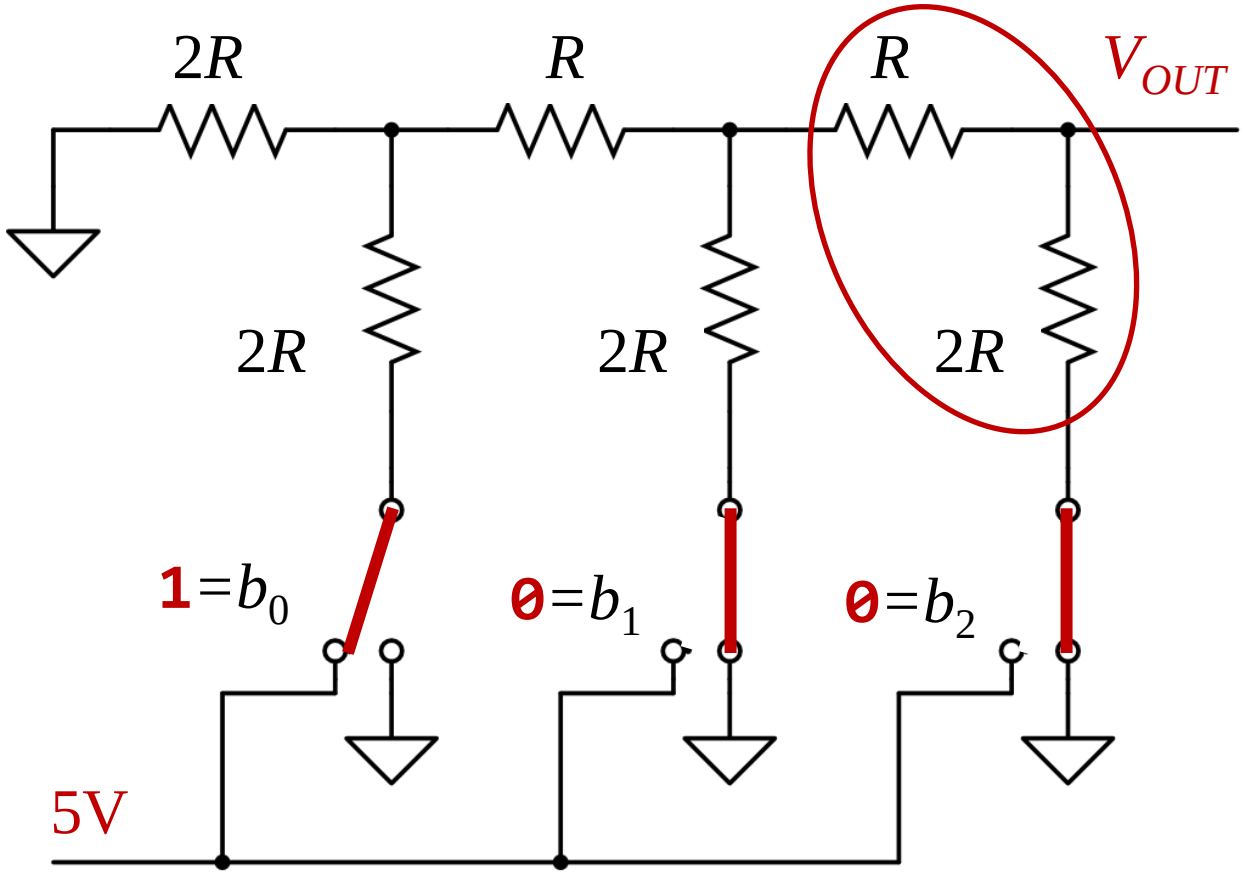
- Consider input **001**



2023_10_06B

R-$2R$ Ladder DAC

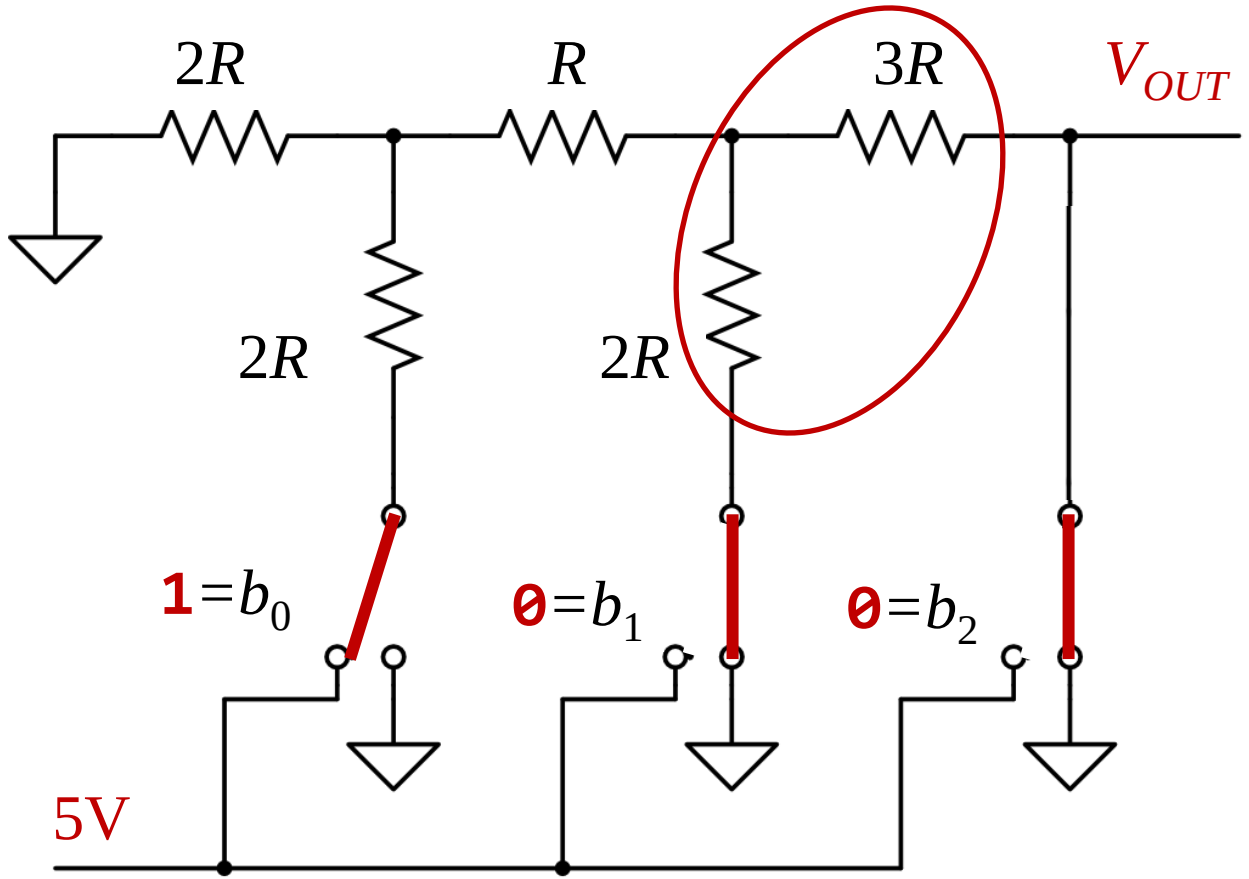
- Consider input **001**



2023_10_06B

R-$2R$ Ladder DAC

- Consider input **001**



2023_10_06B

R-$2R$ Ladder DAC

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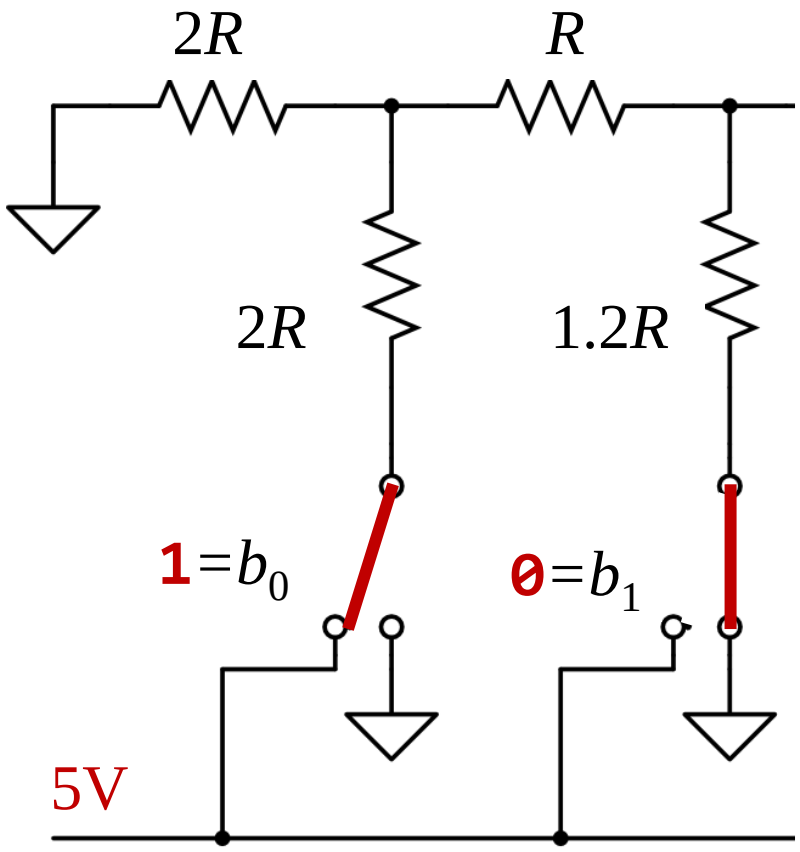

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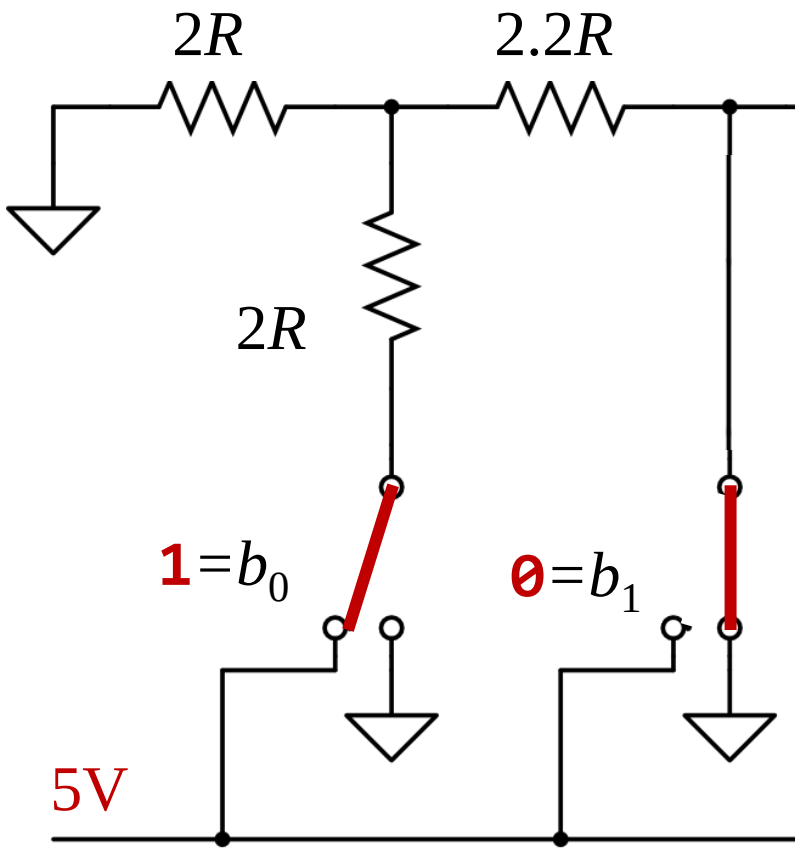

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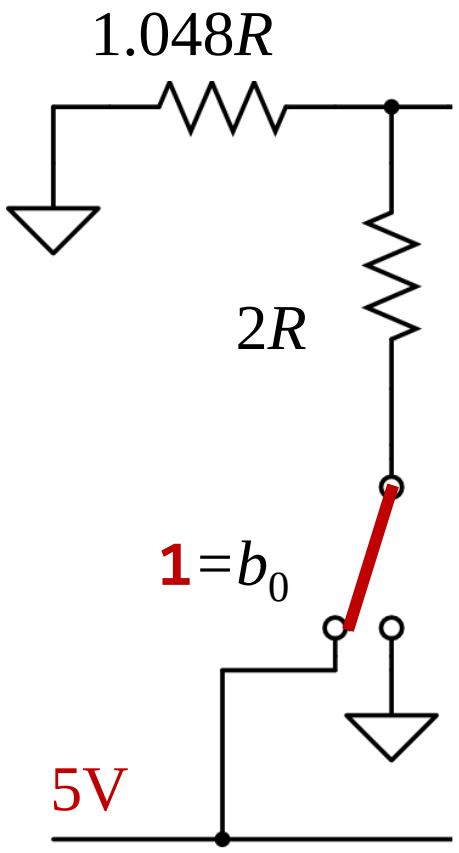

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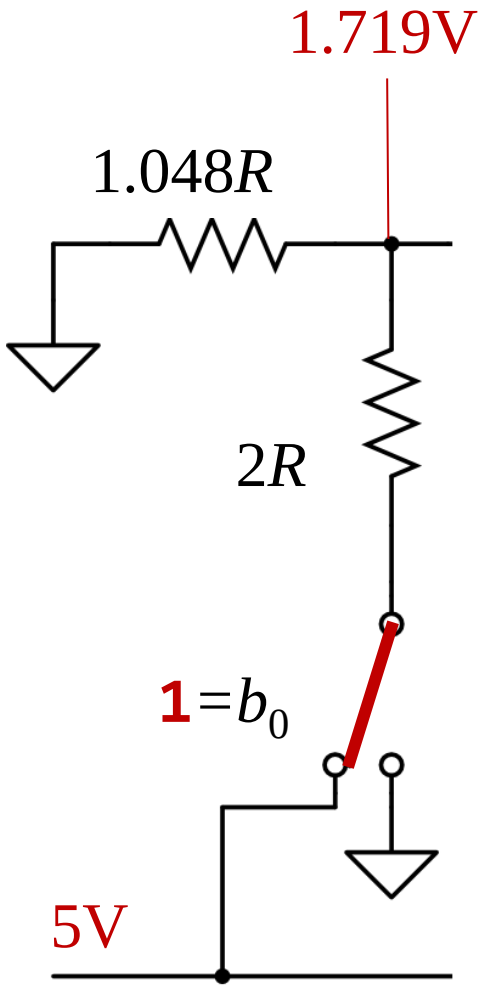

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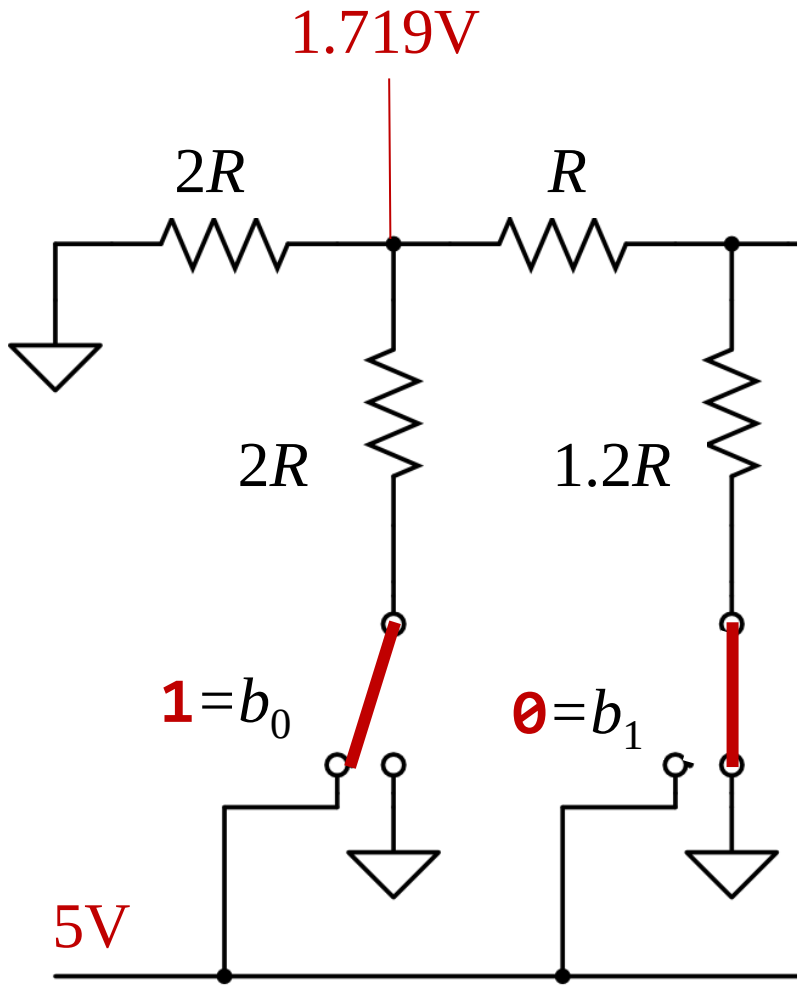

R-$2R$ Ladder DAC

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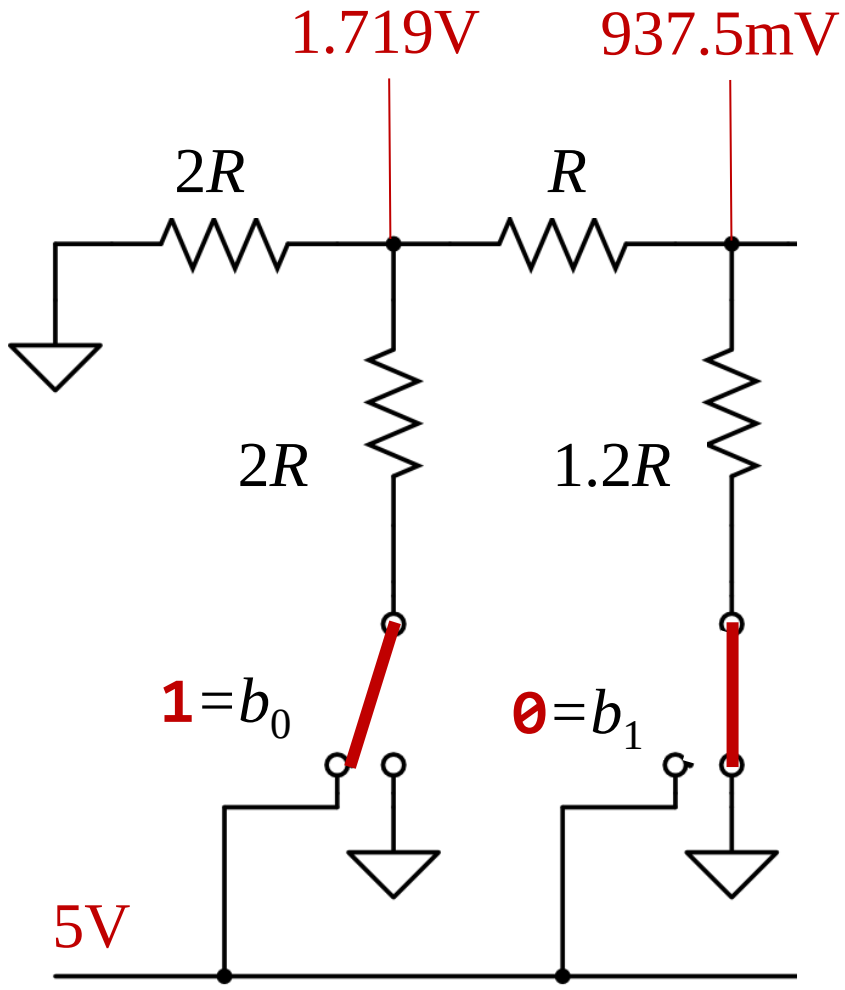

R-$2R$ Ladder DAC

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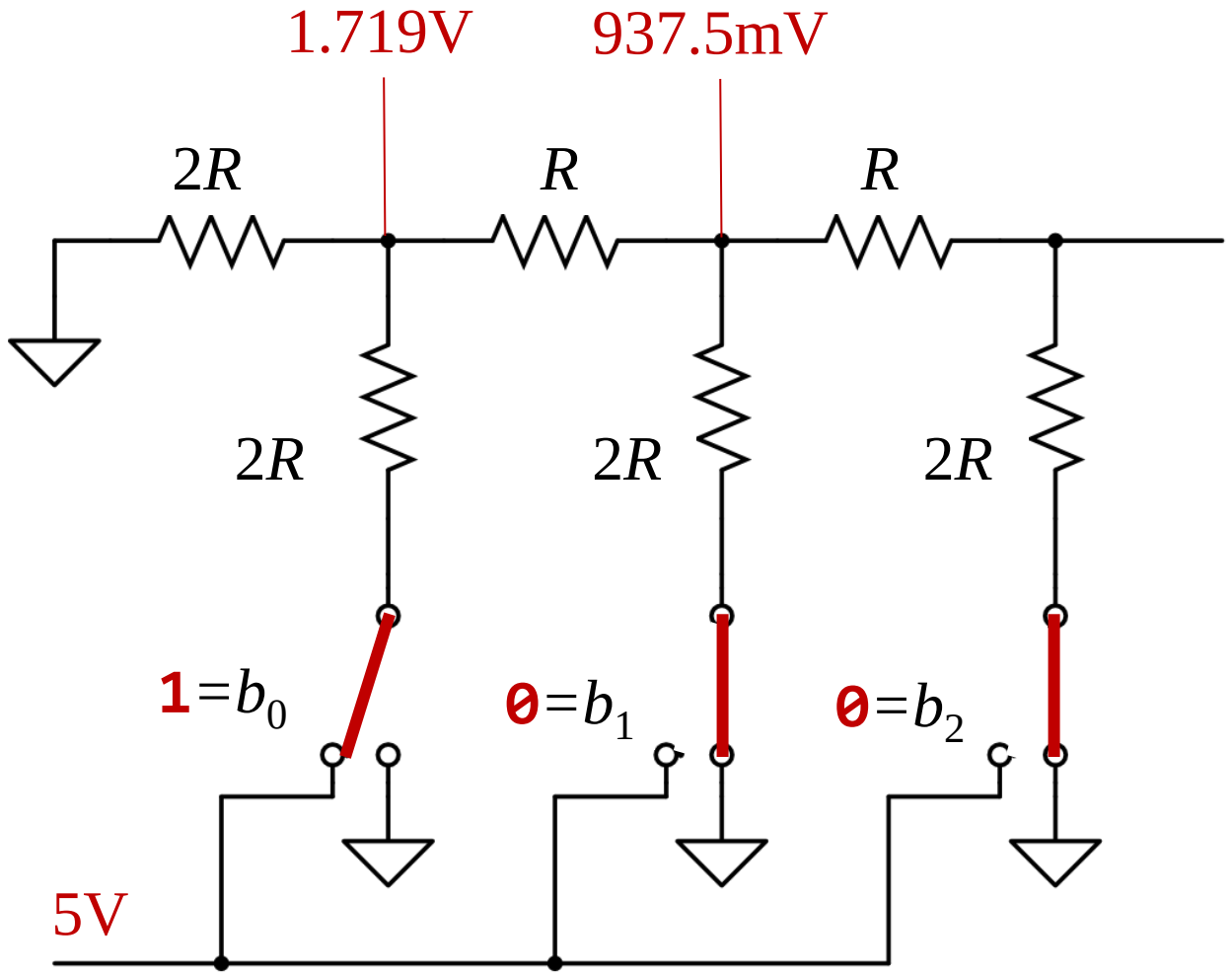

2023_10_06B

R-$2R$ Ladder DAC

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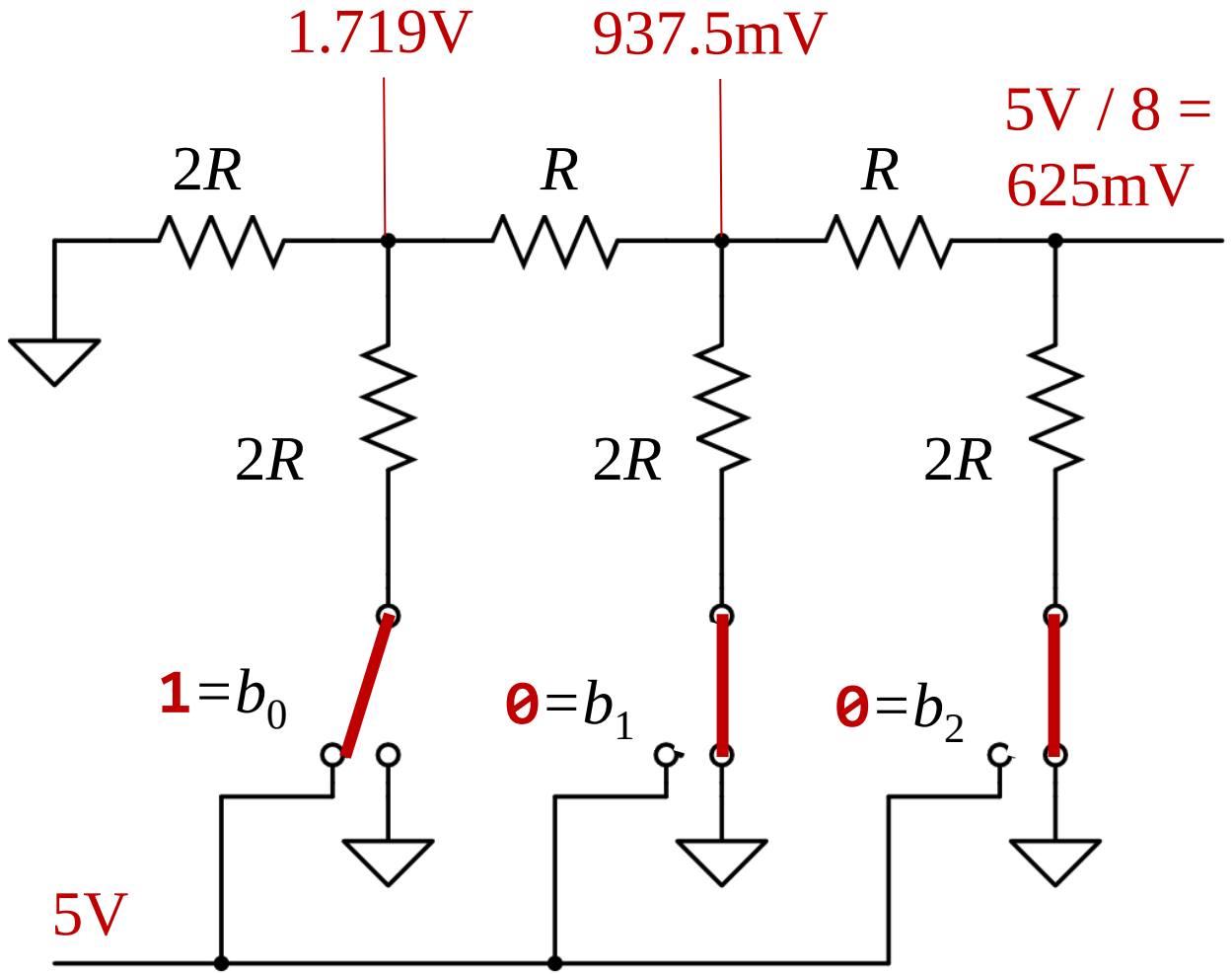
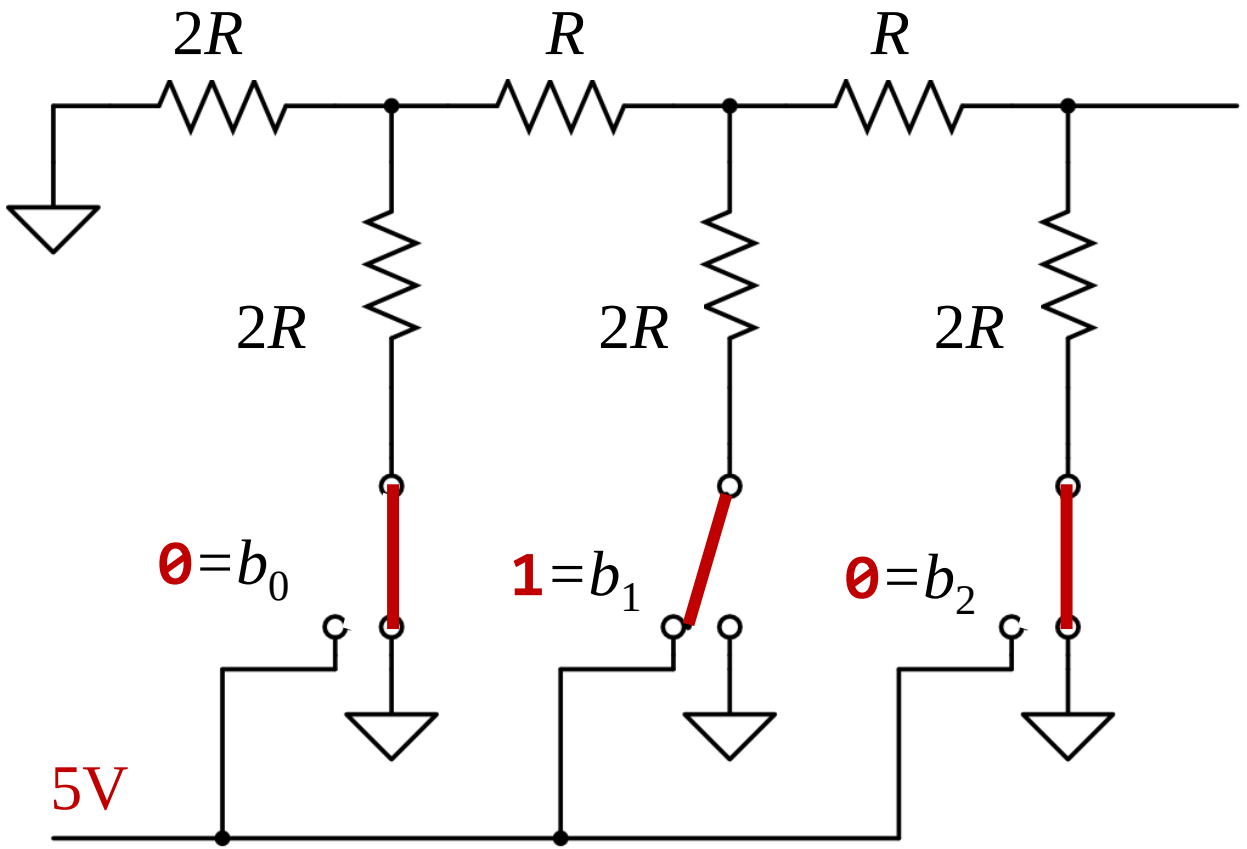


2023_10_06B

R-$2R$ Ladder DAC

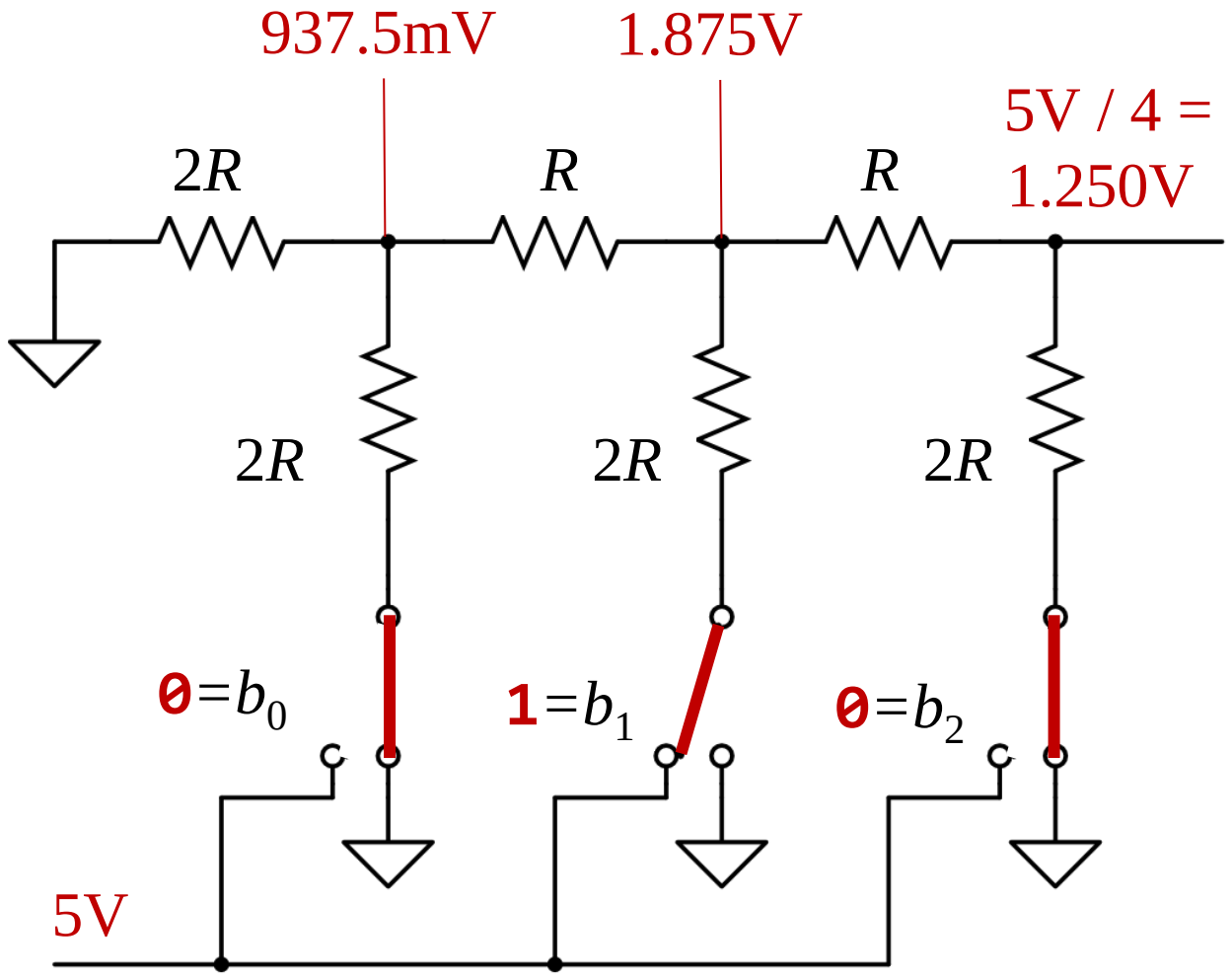
- Consider input **010**



2023_10_06B

R-$2R$ Ladder DAC

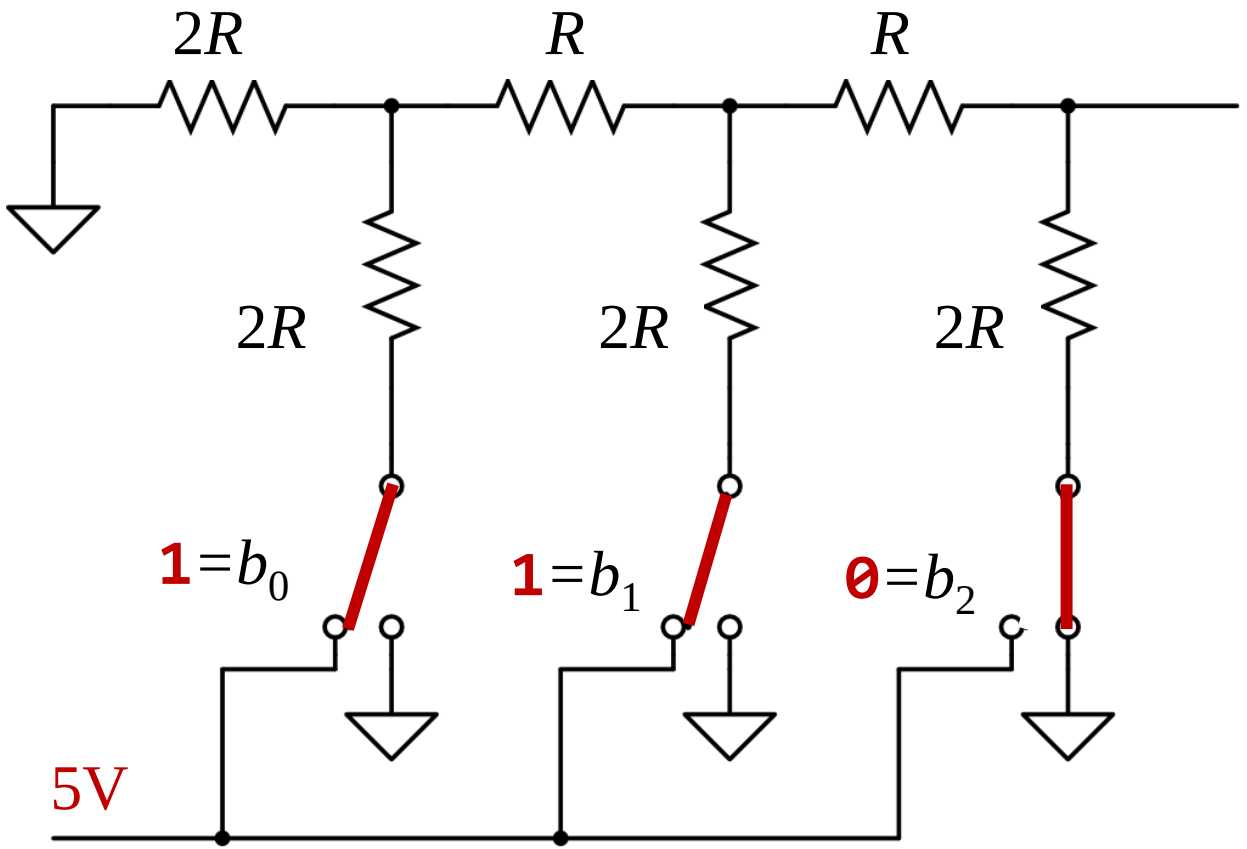
- Consider input **010**



2023_10_06B

R-$2R$ Ladder DAC

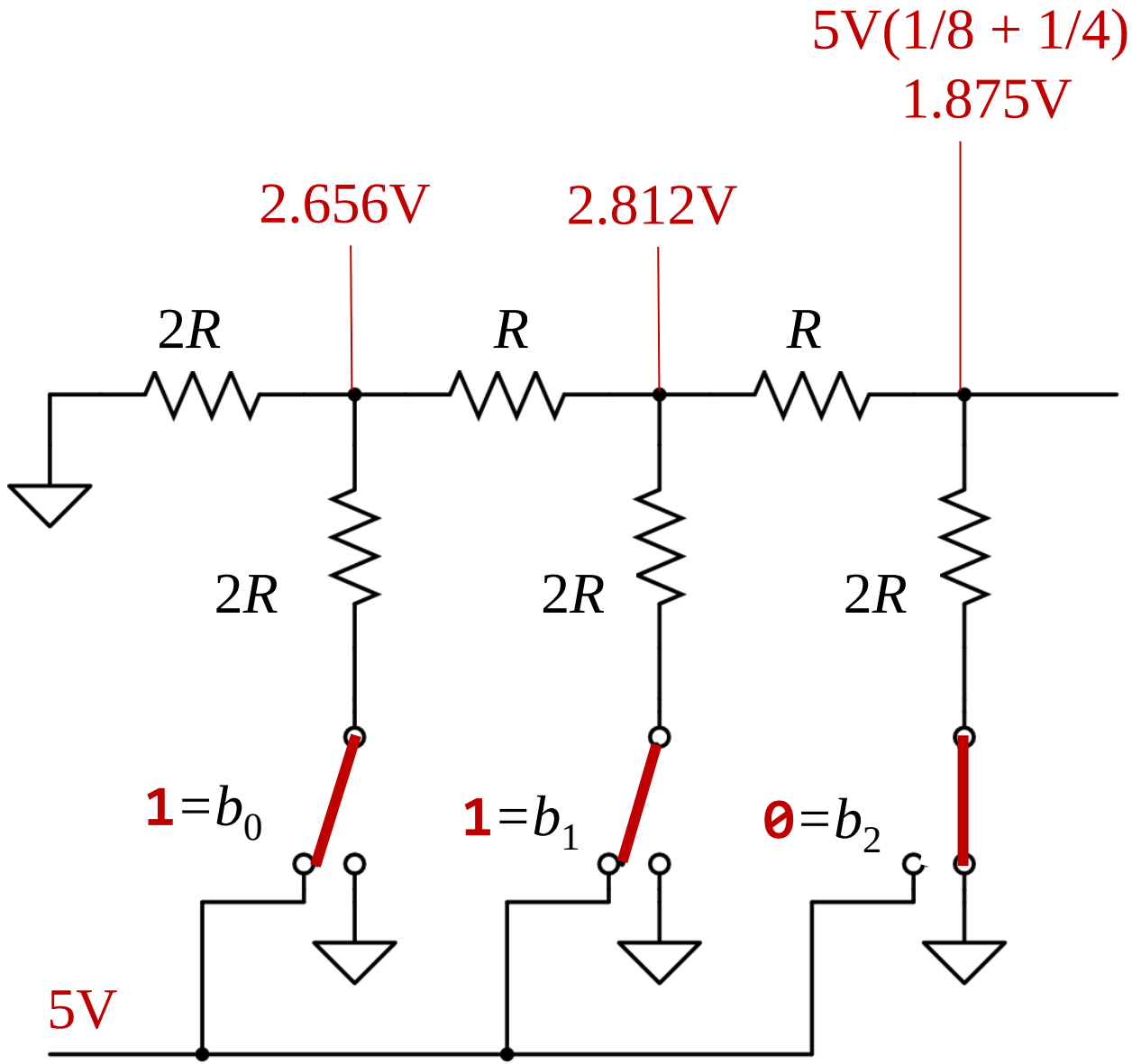
- Consider input **011**



2023_10_06B

R-$2R$ Ladder DAC

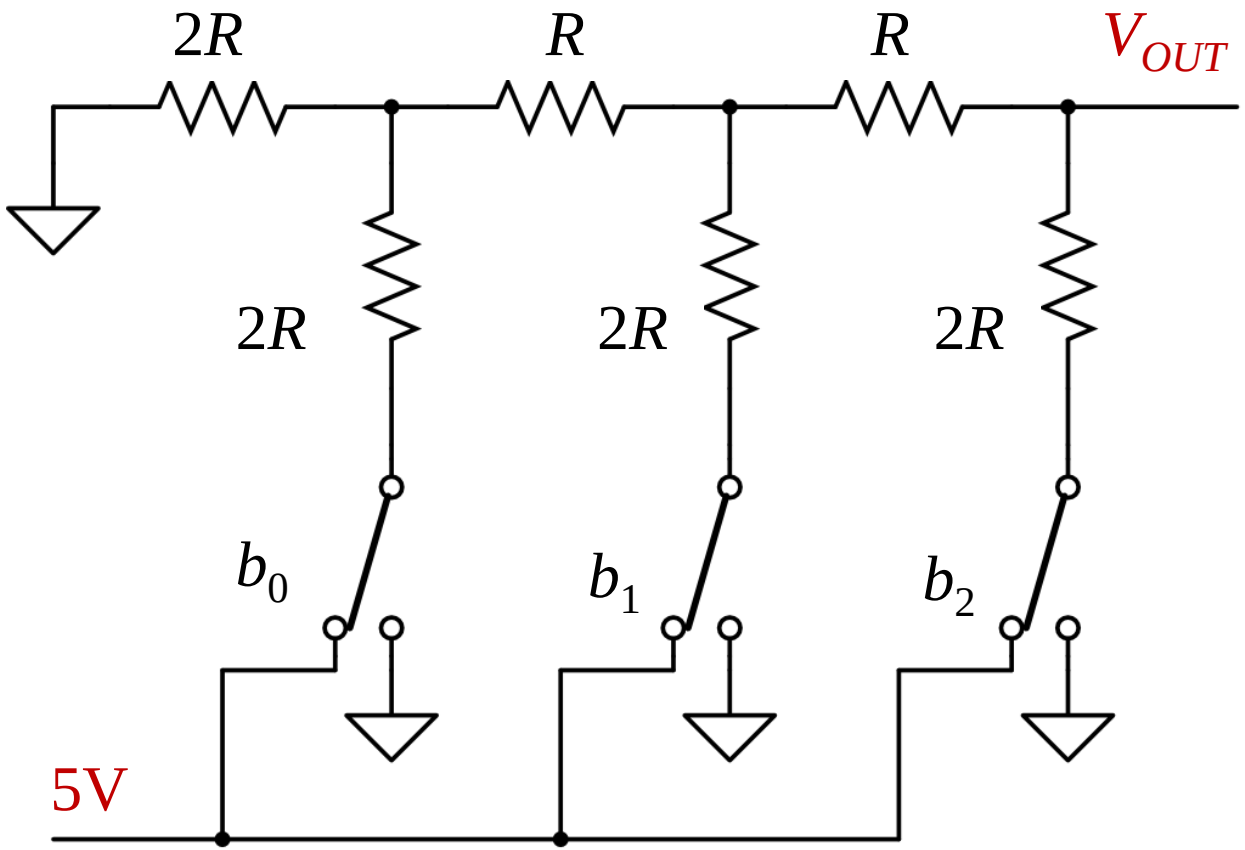
- Consider input **011**



2023_10_06B

R-$2R$ Ladder DAC

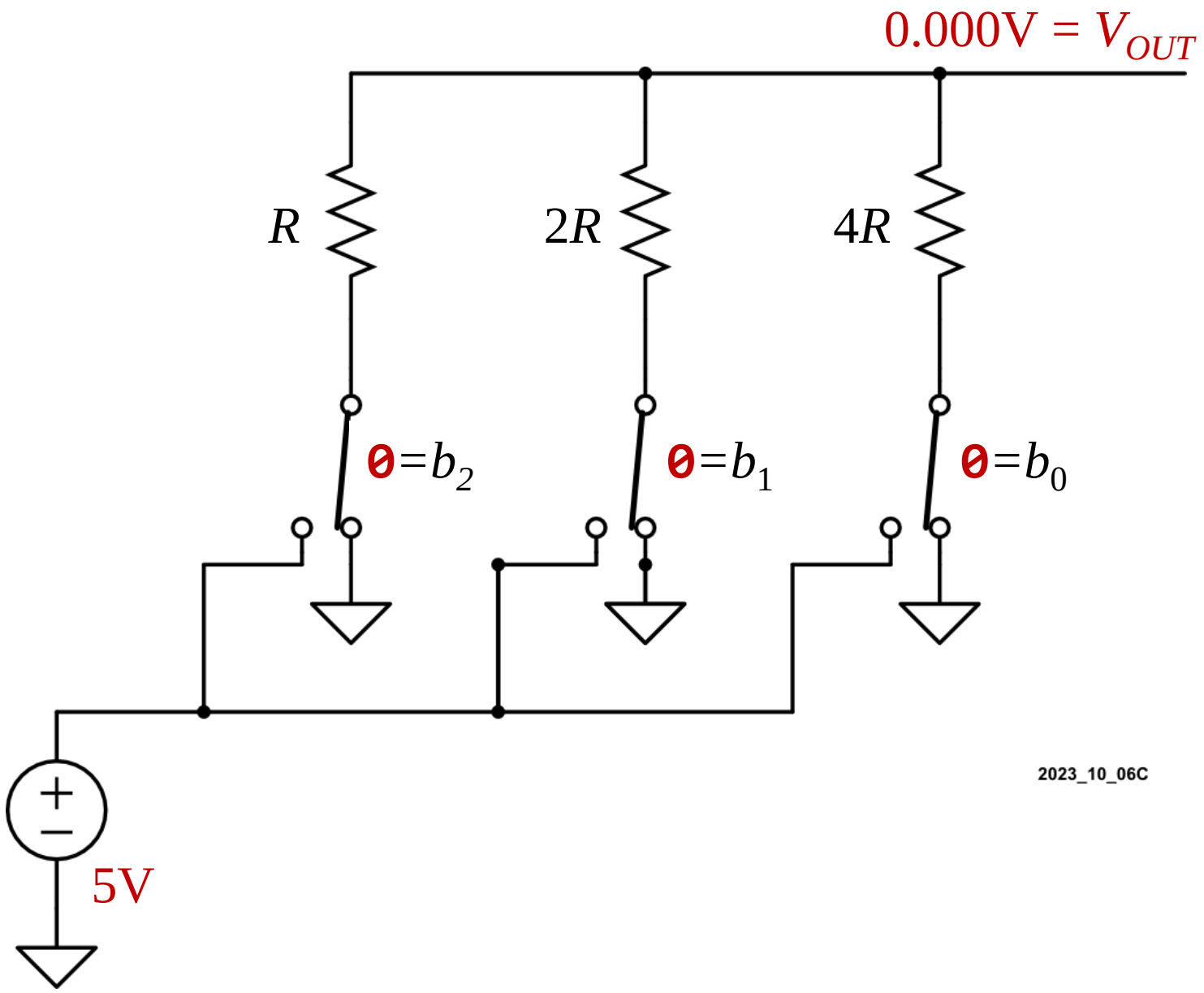
- Advantages
  - Fast!
  - Easy to manufacture for low-bit counts
- Disadvantages
  - Monotonicity problems
  - If resistors aren't completely accurate, increasing the input across 2^X boundaries can decrease the output voltage (01111111 → 10000000)
  - Use an op amp buffer to increase current drive



2023_10_06B

Binary-Scaled DAC

- Similar to a R-2R DAC, but resistors are weighted



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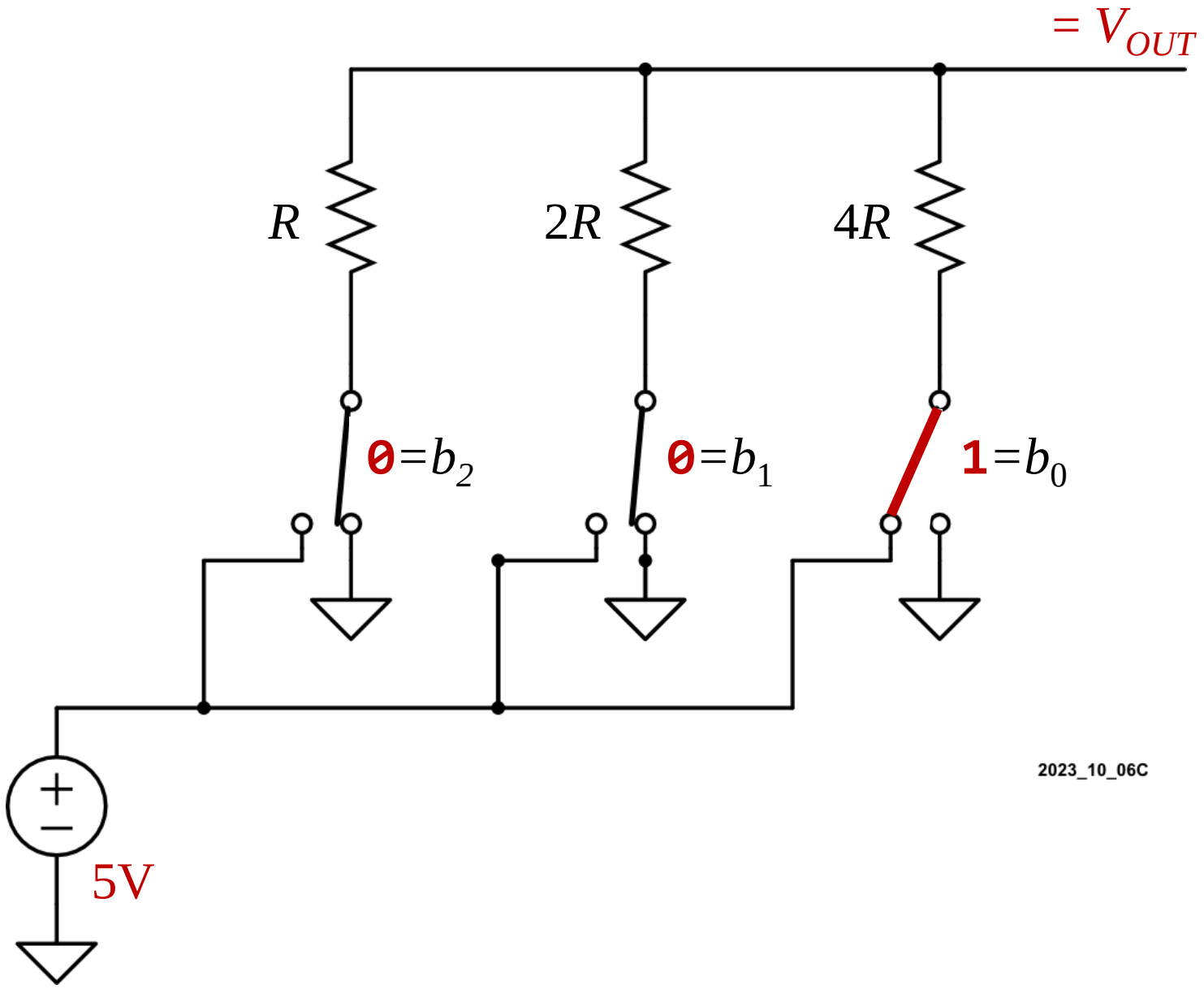

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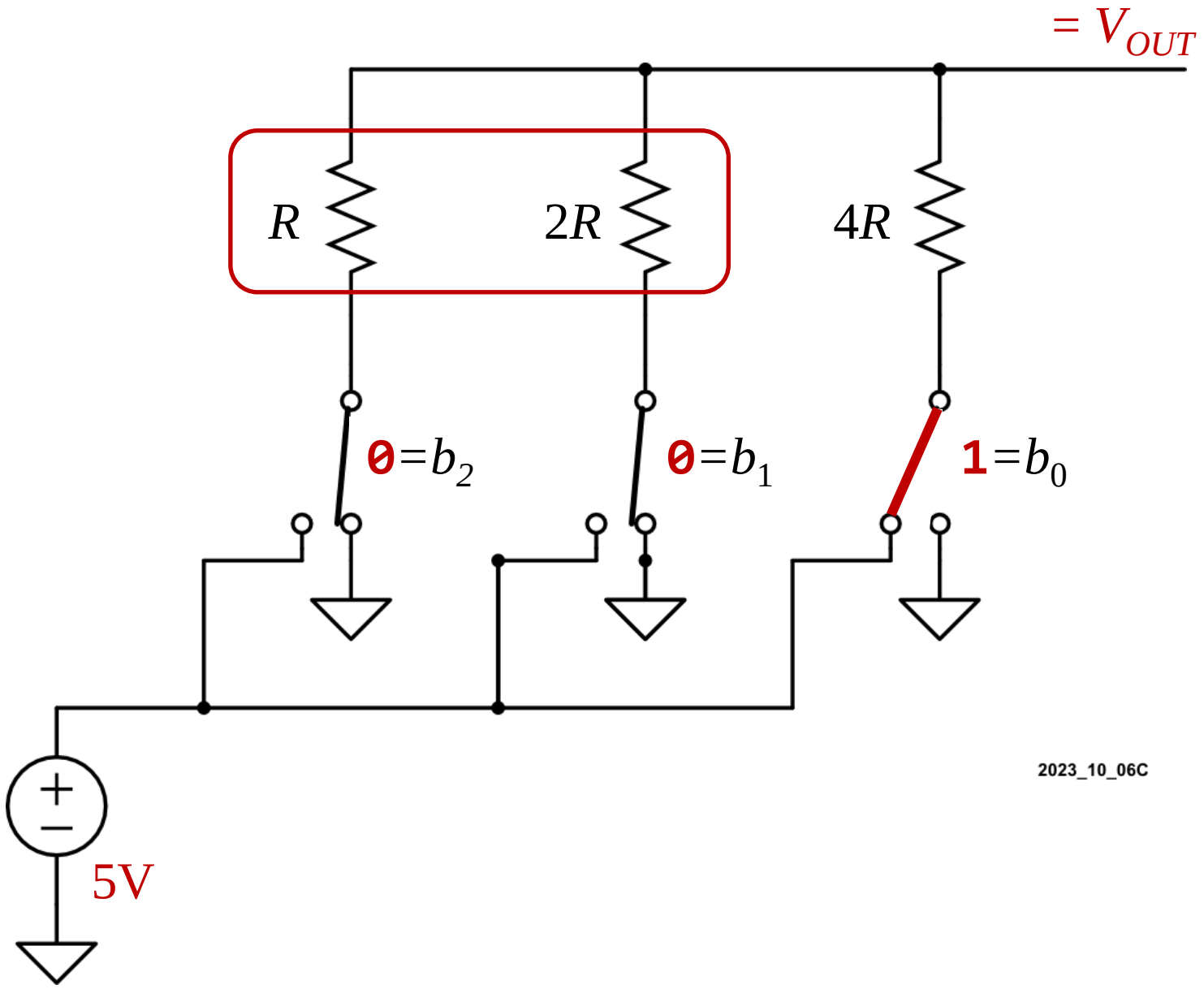

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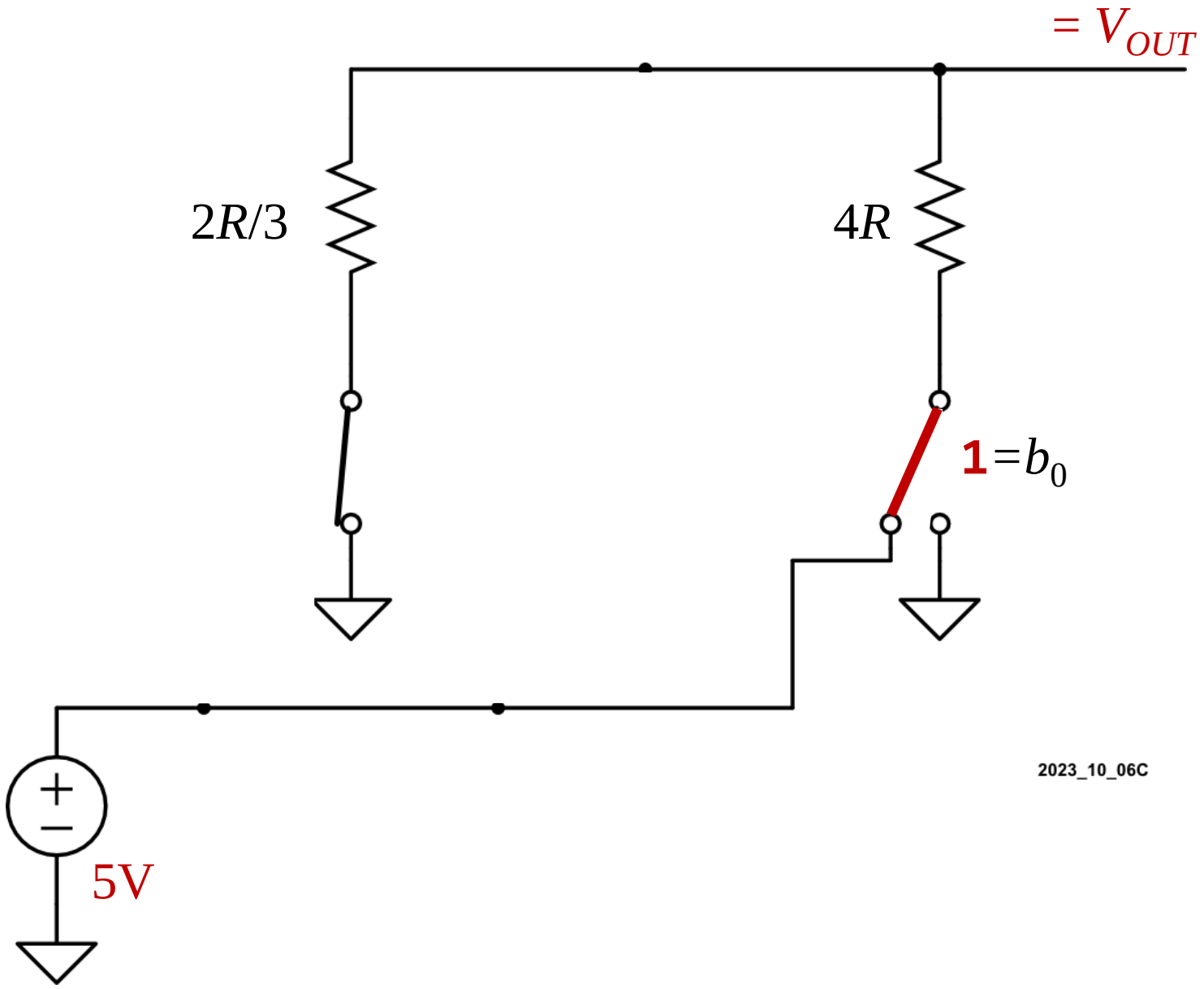

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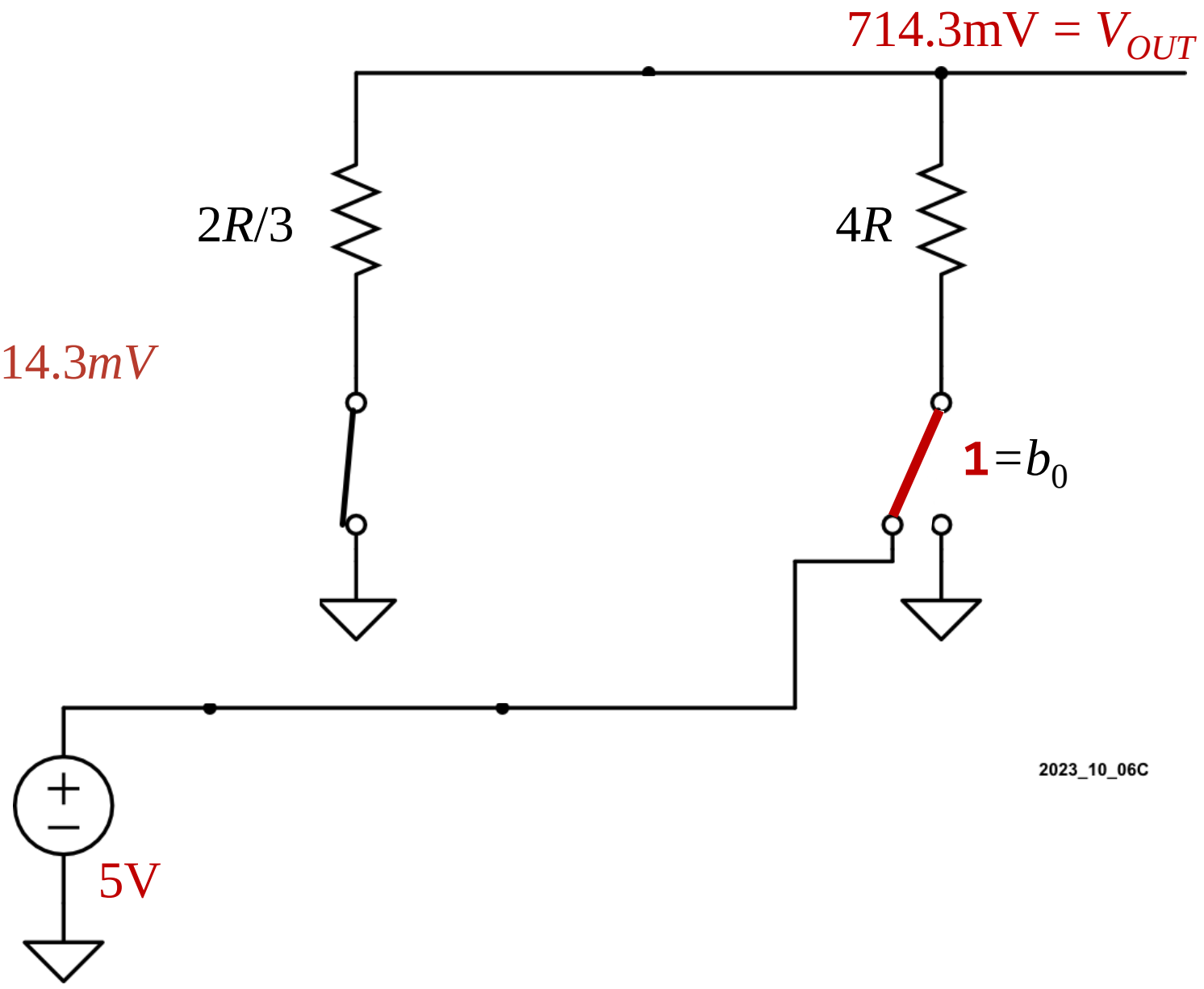
- Similar to a R-2R DAC, but resistors are weighted

$$V_{OUT} = 5V\left(\frac{2R/3}{4R + 2R/3}\right) = 5V\left(\frac{1}{7}\right) = 714.3mV$$



Binary-Scaled DAC

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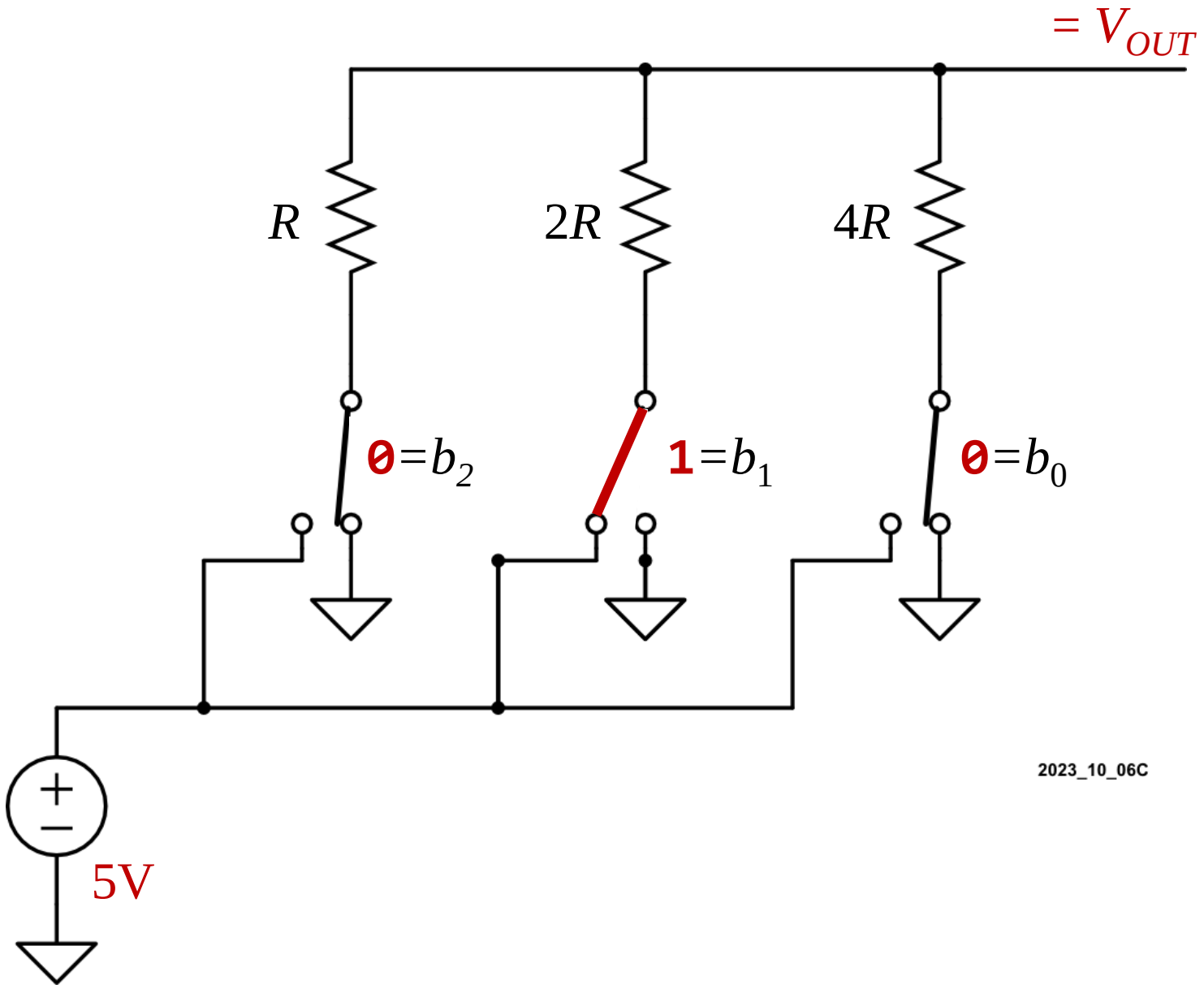

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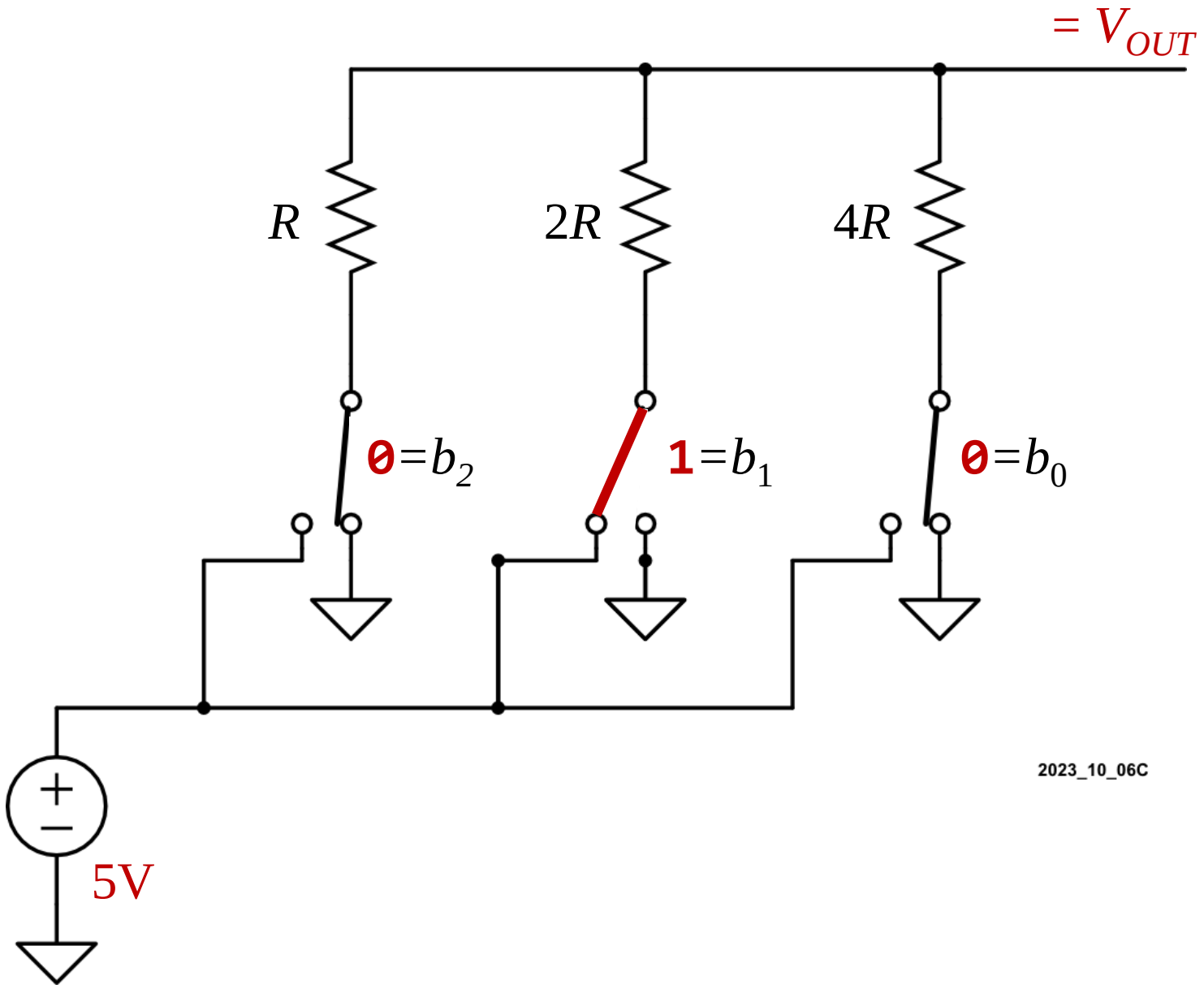

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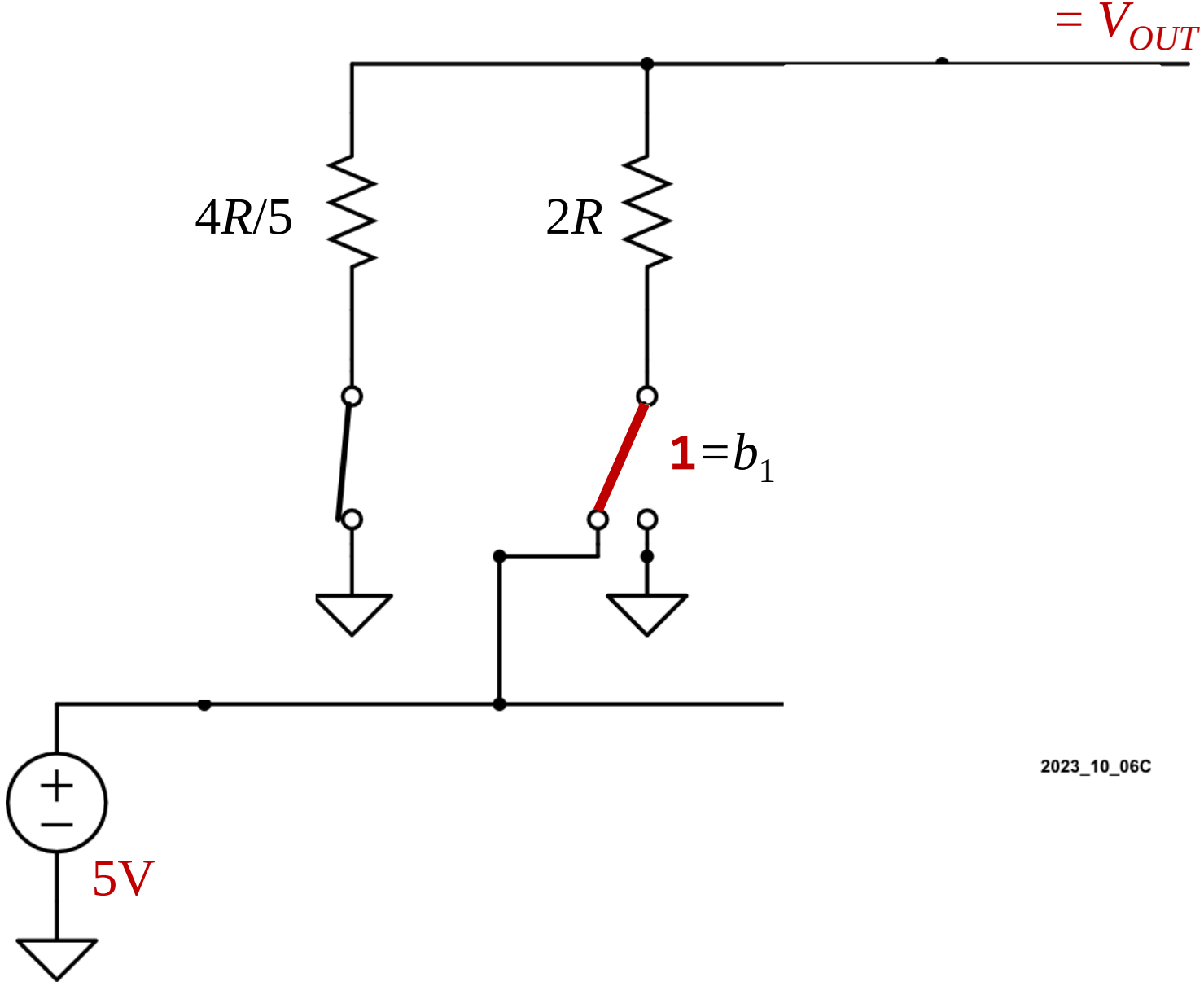

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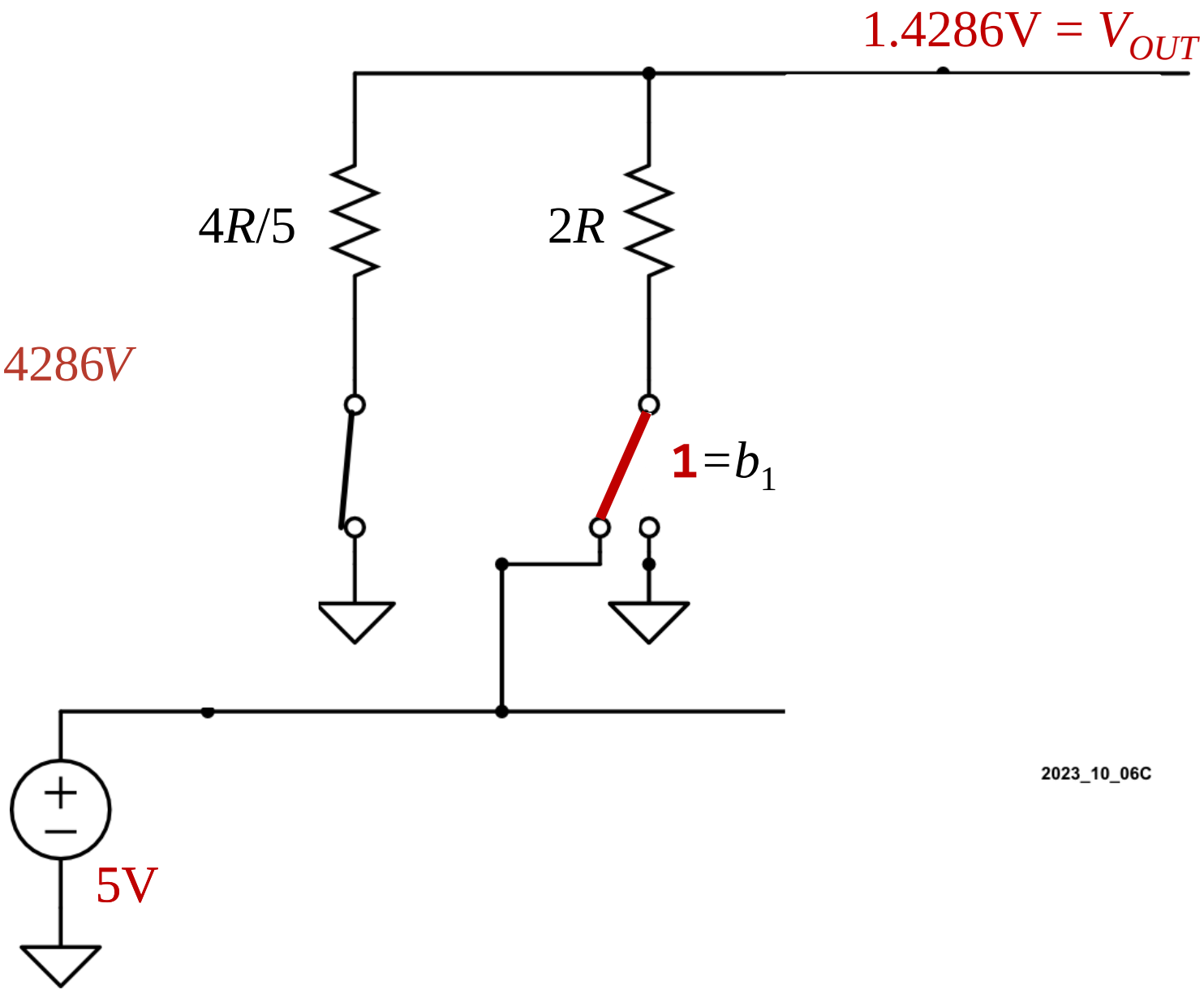
$$V_{OUT} = 5V\left(\frac{4R/5}{2R + 4R/5}\right) = 5V\left(\frac{2}{7}\right) = 1.4286V$$



2023_10_06C

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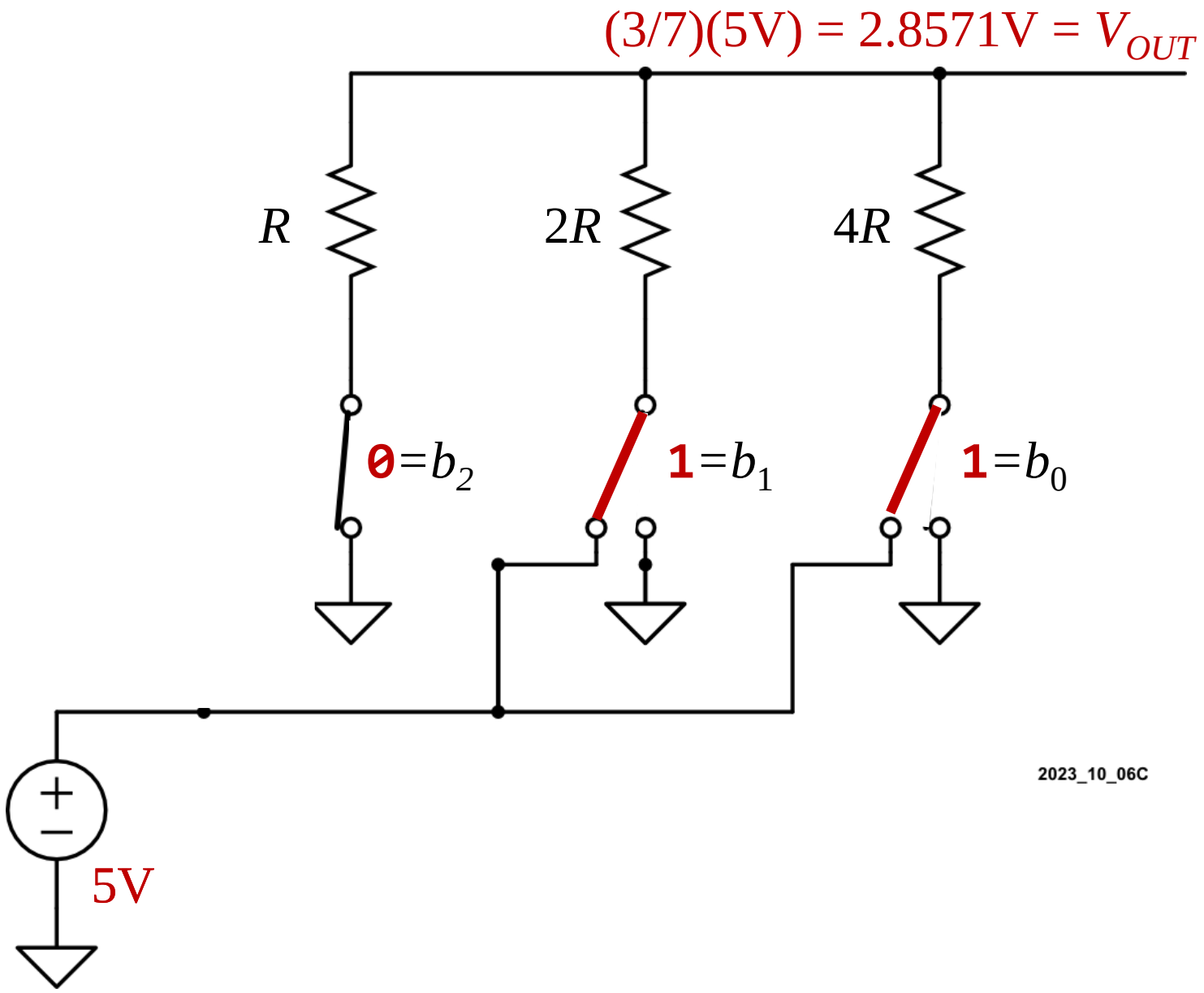
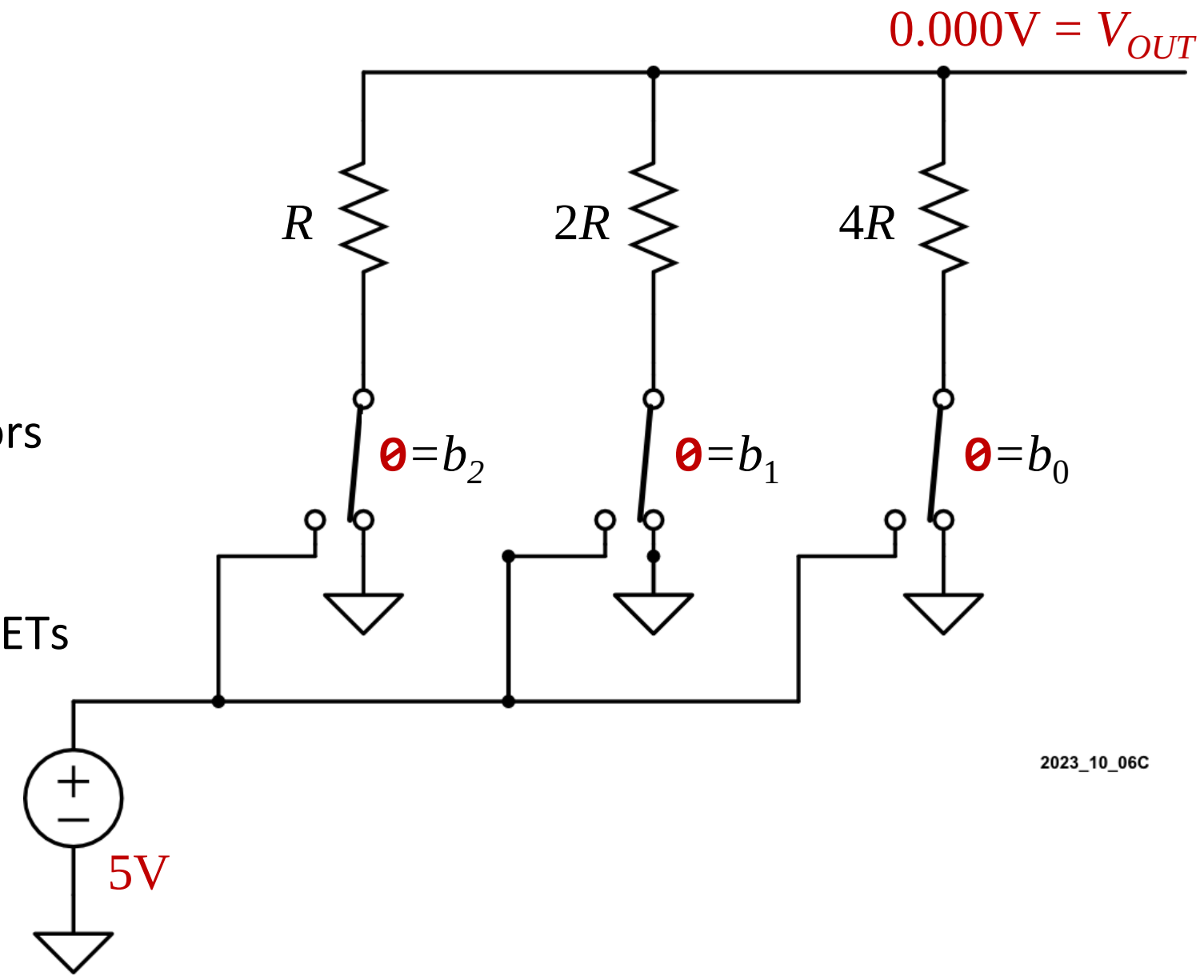


Binary-Scaled DAC

- Advantages:
  - It is simple in construction
  - It provides fast conversion
- Disadvantages:
  - Requires large range of resistors
  - Requires high precision for low value resistors.
  - Requires low resistance MOSFETs
  - Can be expensive
  - Use an op amp buffer to increase current drive



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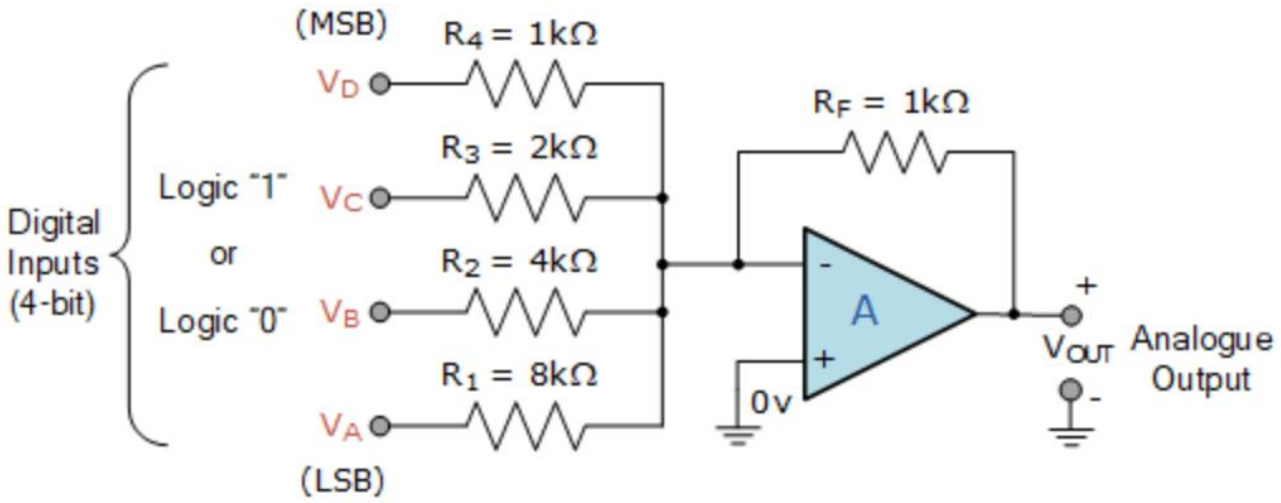

$$V_{OUT} = -\left[\frac{R_F}{R_4}V_D + \frac{R_F}{R_3}V_C + \frac{R_F}{R_2}V_B + \frac{R_F}{R_1}V_A\right]$$

$$V_{OUT} = -\left[\frac{1k\Omega}{1k\Omega}V_D + \frac{1k\Omega}{2k\Omega}V_C + \frac{1k\Omega}{4k\Omega}V_B + \frac{1k\Omega}{8k\Omega}V_A\right]$$

$$V_{OUT} = -\left[1V_D + \frac{1}{2}V_C + \frac{1}{4}V_B + \frac{1}{8}V_A\right]$$

https://www.electronics-tutorials.ws/combination/digital-to-analogue-converter.html

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    011 → (3/8)(5V) = 1.875V

    111 → (7/8)(5V) = 4.375V

    110 → (6/8)(5V) = 3.750V

    100 → (4/8)(5V) = 2.500V