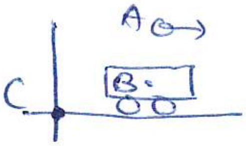
SR

Principle of Relativity

1. Laws of physics apply in all inertial reference system.
2. Universal speed of light:
   - same light speed for all inertial observers regardless source's motion
   - same light speed in all direction



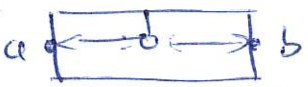
Galileo's vel. addition : $V_{ac} = V_{ab} + V_{bc}$

Einstein's vel. addition : $V_{ac} = \dfrac{V_{ab}+V_{bc}}{1+\frac{V_{ab}V_{bc}}{c^2}}$

Geometry of Relativity

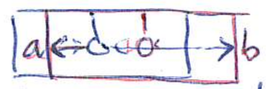
i) Relativity of simultaneity : Two events simultaneous in one inertial sys. are not, in general, simultaneous in another.

ex



observer on train

light reaches a, b simultaneously



observer on ground

light travel long dist to b than a, takes longer time to b than a.

$\boxed{\text{"event (a) occurs first than (b)"}}$

└ use light speed as time measurement

ii) Time dilation: $\boxed{\gamma \bar{\Delta t} = \underbrace{\Delta t}_{\text{dilate}}}$ $\quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\bar{\Delta t}$ clock on train

Δt clock on ground

$\bar{\Delta t} = \frac{h}{c}$ what it seen on ground take longer dist., time dilate $\Delta t > \bar{\Delta t}$.

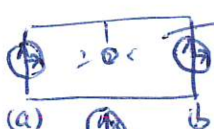
$\Delta t = \frac{r}{c}$

note 3-4-5-rule

0.6 $v = \frac{3}{5}c$ $\quad \gamma = \frac{5}{4}$ 1.25

0.8 $v = \frac{4}{5}c$ $\quad \gamma = \frac{5}{3}$ 1.67

Paradox: observer on train say clock is slower on ground!

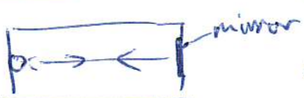


clock syn on train

not syn on ground (being seen)

(a) occurs first than (b) — rel. simultaneity.

iii) Lorentz contraction: $\boxed{\underbrace{\bar{\Delta x}}_{\text{contracted}} = \gamma \Delta x}$



c dist. travel $2\bar{\Delta x}$

time $\bar{\Delta t} = \frac{2\bar{\Delta x}}{c}$

$\bar{\Delta x}$ dist. measure on train

Δx dist. measure on ground

dist. from back to mirror take Δt_1

dist. from mirror to back take Δt_2

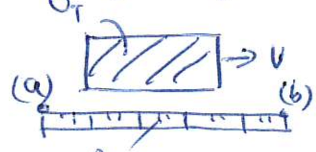
$\Delta x + v\Delta t_1 \Rightarrow \Delta t_1 = \frac{\Delta x + v\Delta t_1}{c}$

$\Delta x - v\Delta t_2 \Rightarrow \Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$

thus $\gamma \bar{\Delta t} = \Delta t = \Delta t_1 + \Delta t_2$

Gives. $\bar{\Delta x} = \gamma \Delta x$

Paradox: Observer on train see (observer) length contracted on ground!



O_T says O_g measure (b) first then a moment later (a)!

└ rel. simul.

ex Barn Ladder paradox

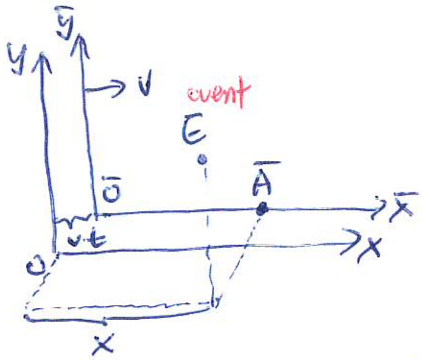
a. ladder's end makes it in the door

b. ladder's front hit barn end wall.

$\Rightarrow$ farmer see (a) first then (b)
daughter see (b) first then (a)
rel. simul.

iv) Dimensions $\perp$ to velocity are not contracted!

Lorentz transformation.



$\bar{\Delta x}, \bar{x}, \bar{t}, \bar{\Delta t}$ measured at $\bar{S}$

$\Delta x, x, t, \Delta t$ measured at S

$S = \{(x,y,z)$ $\quad \bar{S} = (\bar{x}, \bar{y}, \bar{z})$

Find event coord. in $\bar{S}$

$\bar{x} = \gamma\underbrace{(x - v \cdot t)}_{\text{like moving ruler.}}$ — x, vt measure in S exp. contraction need correct factor γ

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = \gamma(t - \frac{v}{c^2}x)$ — In $\bar{S}$, then $x = \gamma(\bar{x} + v\bar{t})$
w/ $\bar{x} = \gamma(x - v t)$
solve $\bar{t}$ in t.

Event A, B simult. @ S

$\bar{t}_A = \gamma(t - \frac{v}{c^2}x_A)$ $\quad \bar{t}_B = \gamma(t - \frac{v}{c^2}x_B)$

not simul. @ $\bar{S}$ ie $\bar{t}_A \neq \bar{t}_B$
necessary.

reverse

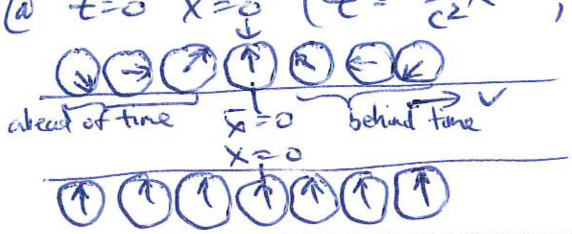
$x = \gamma(\bar{x} + v\bar{t})$ $\quad t = \gamma(\bar{t} + \frac{v}{c^2}\bar{x})$

$y = \bar{y}$

$z = \bar{z}$

stand at $\bar{S}$
S travels to the left of $\bar{S}$ thus $-v$

Also
Simul., time syn., $\quad$ @ $t=0$ $x=0$ $\left(\bar{t} = -\frac{v}{c^2}\bar{x} \right.$, $\left. t=0 \ \forall x \right)$



Spacetime structure.

$x^0 \equiv ct$ $\quad \beta \equiv \frac{v}{c}$

$\bar{x}^0 = \gamma(x^0 - \beta x^1)$

$\bar{x}^1 = \gamma(x^1 - \beta x^0)$

$\bar{x}^2 = x^2$

$\bar{x}^3 = x^3$

in matrix form

$$\begin{pmatrix} \bar{x}^0 \\ \bar{x}^1 \\ \bar{x}^2 \\ \bar{x}^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$\bar{x}^\mu = \Lambda^\mu_\nu x^\nu$

Λ^μ_ν — Lorentz tranformation matrix.

4 vectors

when raising index, it causes a minus sign

covariant $a_\mu = (a_0, a_1, a_2, a_3) = (-a^0, a^1, a^2, a^3)$ $\quad$ only 0th component $a_0 = -a^0$

Contravariant $a^\mu = (a^0, a^1, a^2, a^3)$

Four-dimensional scalar product — invariant under Lorentz transform.

$$a^\mu b_\mu = a_\mu b^\mu = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3$$

more 4 vectors

Displacement $x^\mu = (ct, \vec{x})$ $\quad$ Velocity $\eta^\mu = \gamma(c, \vec{v})$

Energy-momentum $p^\mu = (\frac{E}{c}, \vec{P}) = (\gamma m c, \underbrace{\gamma m \vec{v}}_{\vec{p} \text{ — rel. mom.}})$; $E = \gamma m c^2$ $\quad \vec{p} = \gamma m \vec{v}$

Current density $j^\mu = (c\rho, \vec{J})$

Wave vector $k^\mu = (\omega, c\vec{k})$

Interval: $\Delta x^{\mu} = x^{\mu}_A - x^{\mu}_B$, $I = \Delta x^{\mu} \Delta x_{\mu} = -c^2 t + d^2$; t, d change I invariant under Lorentz transform.

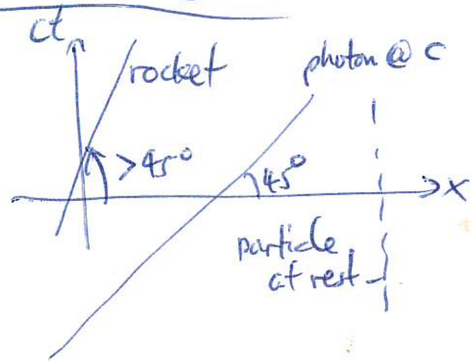
Consider $I = -c^2 t^2 + d^2$ should be $I = -c^2 \Delta t^2 + \Delta x^2$

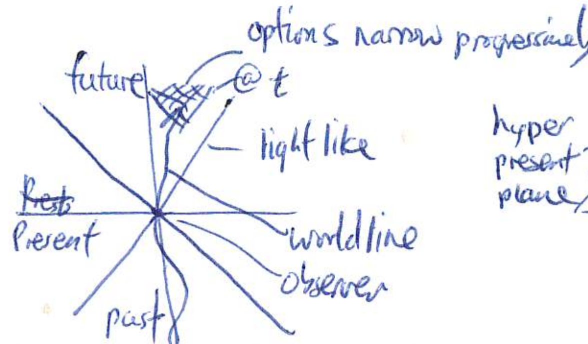
Timelike (seperated by time) ($I<0$) i.e two events occur at same location.

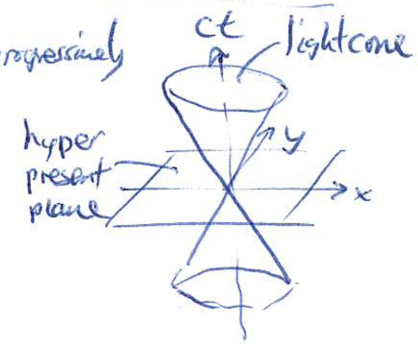
Spacelike (seperated by space) ($I>0$) i.e two events occur at same time.

Lightlike ($I=0$) $I=0 \Rightarrow$ $c(t_2 - t_1) = x_2 - x_1$, ~~$t_2 - t_1 = \frac{x_2}{c} - \frac{x_1}{c}$~~, $\frac{\Delta x}{\Delta t} = c$ i.e two events connected by signal traveling at light speed.

Minkowski diagrams





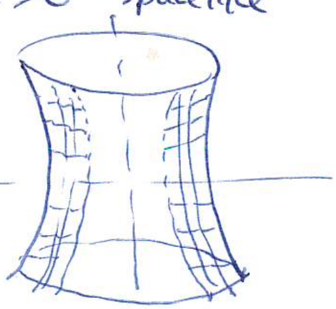


Moving along worldline my watch τ runs slow as clock t on wall ticks.

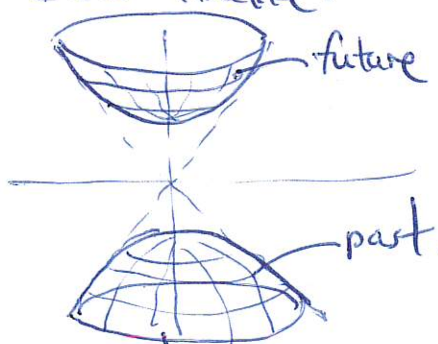
$\gamma d\tau = dt$ (τ: proper time "own")

Consider $I = x^2 + y^2 - c^2 t^2$

$I>0$ spacelike



$I<0$ timelike.



Lorentz transform.
coordinate change from (x,t) to $(\bar{x}, \bar{t})$
new $(\bar{x}, \bar{t})$ on same hyperbola.

Invariant interval between causally related event is always timelike.
Their temporal ordering is same for all inertial observers.

Ordinary velocity: $\vec{u} = \frac{d\vec{l}}{dt}$ proper velocity: $\vec{\eta} = \frac{d\vec{l}}{d\tau}$ is 4velocity $\eta^{\mu} = \frac{dx^{\mu}}{d\tau}$

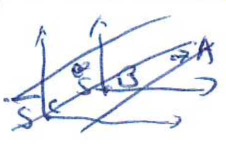
Lorentz transform.

velocity observed at $\bar{S}$ perspective $\rightarrow \bar{\eta}^{\mu} = \Lambda^{\mu}_{\nu} \eta^{\nu}$

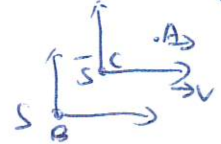
ordinary transform.

$$\bar{u}_x = \frac{d\bar{x}}{d\bar{t}} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$\bar{u}_x$ recovers Einstein velocity eqn. b/c:



$\bar{u} = V_{AC}$
$u = V_{AB}$
$v = -V_{BC}$



$$\bar{u}_y = \frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})} \qquad \bar{u}_z = \frac{u_z}{\gamma(1 - \frac{u_x v}{c^2})}$$

4 momentum: $p^\mu = (\gamma mc, \gamma m\vec{u})$ $\quad$ m (rest mass).

total rel. energy $\rightarrow \frac{E_{tot,rel}}{c}$ $\quad$ $\vec{p}$ — rel. mom.

- Experimental fact that $E_{tot,rel}$ & $\vec{p}$ are conserved in closed sys.

Concepts:

Invariant - same value in all inertial sys.

conserved - same value bf and af process.

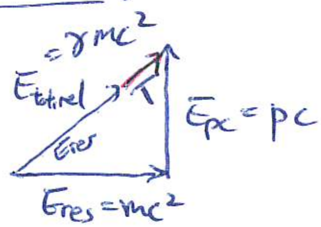
ex mass invariant ($\because p^\mu p_\mu = -m^2c^4$) but not conserved ($\because E = mc^2$)

energy conserved but not invariant ($\because K=0$ in one frame, $K \neq 0$ after boost)

charge both conserved & invariant.

velocity is neither.

Geometry of 4momentum:



Simplify $E_{tot,rel}$

- $E^2 = p^2c^2 + m^2c^4$
- $T = E - mc^2 = (\gamma - 1)mc^2$
- $\left.\begin{array}{l} E = \gamma mc^2 \\ p = \gamma mv \end{array}\right\}$ when $m=0 \Rightarrow E = pc \Rightarrow v = c$.

Lorentz transform.

$p^\mu = (p^0, \vec{p})$ $\quad$ $\bar{p}^\mu = \Lambda^\mu_\nu p^\nu$

$$\bar{p}^0 = \gamma(p^0 - \beta p^1)$$
$$\bar{p}^1 = \gamma(p^1 - \beta p^0)$$
$$\bar{p}^2 = p^2$$
$$\bar{p}^3 = p^3$$

Kinematic: App of rel. mom., rel. energy, (conserved property)

ex Two clumps of clay each w/ mass m collide head on at $\frac{3}{5}c$, then stick together, what is mass M of the composite lump.

$\underset{m}{\bigcirc} \rightarrow \frac{3}{5}c \quad \leftarrow \underset{m}{\bigcirc}$ $\qquad$ $\bigcirc$

bf $\qquad$ af

(i) momentum zero (trivial)

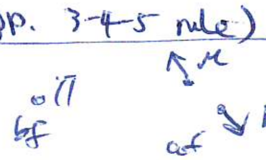
(ii) energy conserved: bf each w/ $E = \gamma mc^2 = \frac{5}{4}mc^2$

af $E = Mc^2 \Rightarrow M = \frac{5}{2}m$

(app. 3-4-5 rule)

ex pion decay. $\pi^+ \rightarrow \mu^+ + \bar{\nu}_\mu$ or $\pi^- \rightarrow \mu^- + \nu_\mu$

Find energy of ongoing muon.



Bf $E = m_\pi c^2$ $\quad$ $\vec{p} = 0$

Af $E_\mu + E_\nu$ $\quad$ $\vec{P}_\nu = -\vec{P}_\mu$

$E_\nu^2 = P_\nu^2c^2 + \underset{0}{(m_\nu c^2)^2} = P_\mu^2 c^2$

conserved energy

$E_\mu + E_\nu = E_\pi$

$E_\mu^2 = P_\nu^2c^2 + m_\mu^2c^4$

sub. $E_\mu + \sqrt{E_\mu^2 - m_\mu^2c^4} = m_\pi c^2$

$$E_\mu = \frac{(m_\pi^2 + m_\mu^2)c^2}{2m_\pi}$$

Conservation of mom., energy in classic vs relativistic.

Classical: mom. & energy conserved always; kinetic ~~doesn~~ isn't.

Relativistic: total energy, rel. mom conserved always; mass and kinetic ~~isn~~ aren't.

Elastic collision:

(Kinetic energy always conserved) $\Rightarrow$ rest energy conserved $\Rightarrow$ mass also.

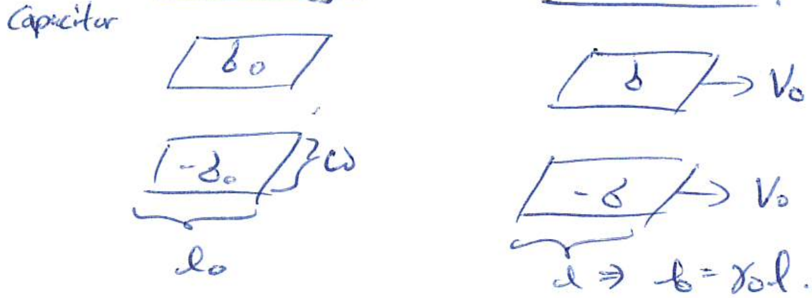
physical meaning: particle comes out as it went in

ex Compton scattering.

Field transformation.

Stationary. In motion.

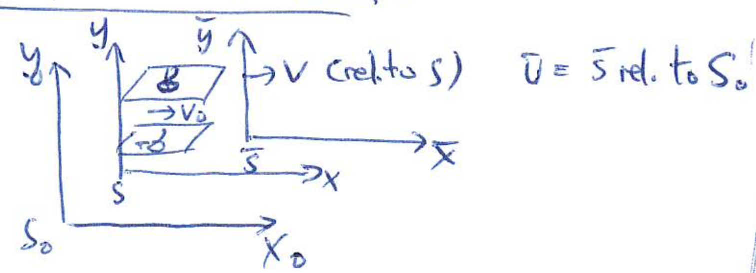
Capacitor



$$E_0^{\perp} = \frac{\sigma_0}{\varepsilon_0} \Rightarrow E^{\perp} = \gamma_0 E_0^{\perp}$$
$$E^{\parallel} = E_0^{\parallel}$$

$$\gamma \begin{vmatrix} E_y & E_z \\ -vB_y & -vB_z \end{vmatrix}$$

Now w/ Both EB-field.



$\bar{v} = \bar{S}$ rel. to S_0

then $E_y = \frac{\sigma}{\varepsilon_0}$, $B_z = -\mu_0 \sigma V_0$

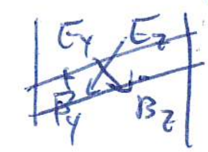
in $\bar{S}$ $\bar{E}_y = \frac{\bar{\sigma}}{\varepsilon_0}$ $\bar{B}_z = -\mu_0 \bar{\sigma} \bar{v}$

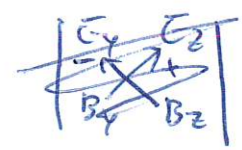
$$\bar{v} = \frac{V + V_0}{1 + \frac{VV_0}{c^2}} \quad \bar{\gamma} = \frac{1}{\sqrt{1 - \frac{\bar{v}^2}{c^2}}} \quad \gamma = \frac{1}{\sqrt{1 - V^2/c^2}}$$

After some algebra:

$$\bar{E}_x = E_x, \quad \bar{E}_y = \gamma(E_y - vB_z), \quad \bar{E}_z = \gamma(E_z + vB_y)$$

$$\bar{B}_x = B_x, \quad \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right), \quad \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$





$$\gamma \begin{vmatrix} B_y & B_z \\ \frac{v}{c^2}E_y & \frac{v}{c^2}E_z \end{vmatrix}$$

If $\vec{B} = 0$ in S

$$\bar{B} = \frac{-1}{c^2}(\vec{v} \times \bar{E})$$

If $\vec{E} = 0$ in S

$$\bar{E} = \vec{v} \times \bar{B}$$

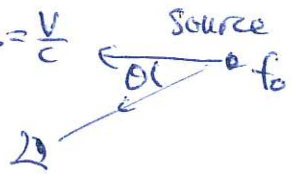
Some invariant dot products

$$\vec{E}' \cdot \vec{B}' = \vec{E} \cdot \vec{B}$$

$$E'^2 - B'^2 = E^2 - B^2$$

Relativistic Doppler shift

$\beta = \frac{v}{c}$



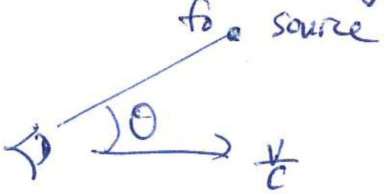
$$f = \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} f_0$$

@ $\theta = 0$

$$f = f_0\sqrt{\frac{(1-\beta)(1+\beta)}{(1-\beta)^2}}$$

special case,

If observer is moving



$f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$ $\theta = 0$ toward

$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$ $\theta = \pi$ away

$$\lambda = \frac{c}{f}$$