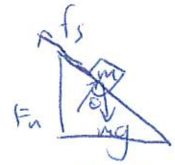
List of Equation for CM.

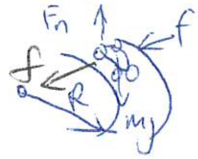
$$f_s \le \mu_s F_n$$

$$f_K = \mu_K F_n$$

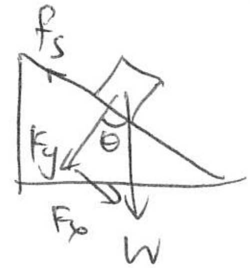
$$\mu_K < \mu_s$$



$\mu_s = \tan\theta_c$



$$f = \mu_s mg = m\frac{V^2}{R}$$



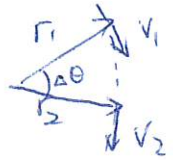
$f_s = \mu_s F_y \quad \frac{F_x}{F_y} = \tan\theta_c.$

$\Rightarrow \boxed{\mu_s = \tan\theta_c}$

Drag & terminal velocity

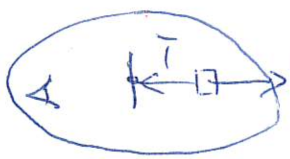
$$F_D = bV^n$$

$$V_t = \left(\frac{mg}{b}\right)^{\frac{1}{n}} \quad \text{free fall}$$

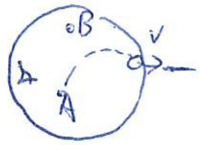


Centripetal acceleration

$$a_c = \frac{V^2}{r}, \quad V = \frac{2\pi r}{T}$$



$F_p = \frac{mv^2}{r}\hat{r}$ centrifugal force.



Coriolis force ⊥ Ball velocity

Work energy.

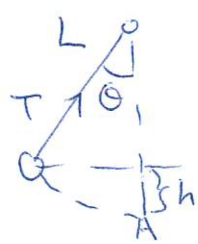
$$dU = -\vec{F}\cdot d\vec{s}$$

$$\Delta U + \Delta K = 0$$

$$\boxed{E = U + K = \text{constant}}$$

Conservation of energy

ex



velocity at A $\quad V = \sqrt{2gL(1-\cos\theta)}$

T at A $\quad T = 2mg(1-\cos\theta)$

General Work-Energy Thm. (w/ non-conservative force)

$F_{net} = F_{nc} + F \qquad F_{nc} = F_{net} - F_c \rightarrow W_{nc} = \Delta K + \Delta U = \Delta E$

$$\begin{cases} W_{nc} = \Delta U + \Delta K = \Delta E & \text{— Work done by } F_{nc} = \text{change in total mech. energy of sys.} \\ E = U + K \\ \Delta E_{sys} = E_{in} - E_{out} \end{cases}$$

Power $\quad P = \vec{F}\cdot\vec{V}$

Center of Mass.

$$M\vec{R}_{cm} = \underbrace{\sum_i m_i \vec{r}_i}_{\text{discrete}} \quad \text{or} \quad \underbrace{\int \vec{r}\, dm}_{\text{cont'}}$$

w/ motion,

Momentum. $M\vec{V}_{cm} = \sum_i m_i \vec{V}_i$

Force $M\vec{A}_{cm} = \sum_i \vec{F}_{i,ext} = \vec{F}_{net,ext}$

if $F_{net,ext} = 0$ then $\vec{V}_{cm} = \text{constant}$

CM Reference Frame

ex

$\vec{u}_i = \vec{V}_i - \vec{V}_{cm}$

$\therefore \sum m_i \vec{u}_i = \sum m_i \vec{V}_i - \sum_i m_i \sum_j \frac{m_j \vec{V}_j}{M}$

$= \sum m_i \vec{V}_i - \sum_j m_j \vec{V}_j$

$= 0$

true always.

- Velocity relative to this frame of center of mass is zero. ⇒ total momentum relatively to this frame is zero.

Conservation of Momentum:

$$\frac{d\vec{P}}{dt} = \vec{F}_{net,ext} = 0 \Rightarrow \vec{P} = \sum \vec{p}_i = \text{const} \Rightarrow \vec{V}_{cm} = \text{const}.$$

Kinetic energy in system of particles.

$K = \frac{1}{2} M V_{cm}^2 + K_{rel}$ — require total momentum relatively to center of mass frame

$K_{rel} = \sum_i \frac{1}{2} m_i \vec{u}_i^{\,2}$ ~~$(\vec{u}_i \; \vec{V}_i \; \vec{V}_{cm})$~~ $(\vec{u}_i = \vec{V}_i - \vec{V}_{cm})$

Pseudowork:

$\int_1^2 \vec{F}_{net} \cdot d\vec{R}_{cm} = \frac{1}{2} M V_{cm,2}^2 - \frac{1}{2} M V_{cm,1}^2$

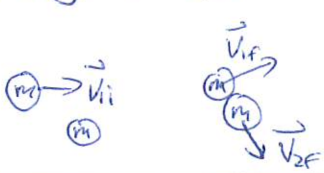
In collision, $\vec{F}_{net,ext} = 0$ ⇒ only K_{rel} can increase or decrease.

Collision. elastic: momentum & kinetic energy conserved

Inelastic: K_{rel} changed

Perfectly inelastic: $K_{rel} \to 0$ objects move together after

ex elastic

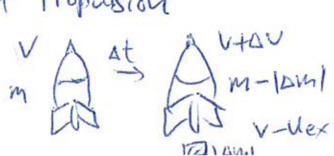


$\vec{v}_{2f} \perp \vec{v}_{1f}$

Impulse & Time Average Force $\vec{F}_{av} = \frac{\vec{I}}{\Delta t}$, $\vec{I} = \int_{t_i}^{t_f} \vec{F} dt = \Delta \vec{P}$

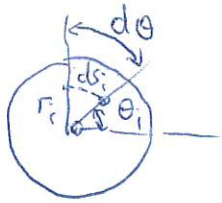
instant change of momentum

Jet Propulsion



$$m\frac{dv}{dt} = \underbrace{u_{ex}\left|\frac{dm}{dt}\right|}_{\text{thurst}} + F_{ext}$$

Rotation.



$\omega = \frac{v_i}{r_i}$ — tangential velocity.

Rolling

$V_{cm} = R\omega$

$A_{cm} = R\alpha$ if accelerating

$a_{it} = r_i \alpha$

$a_{ic} = \frac{v_i^2}{r_i} = r_i \omega^2$

Torque & Moment of Inertia,

Angular momentum

General

$$\boxed{\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F} \\ \vec{\tau} &= \frac{d\vec{L}}{dt} \\ \vec{L} &= \vec{r} \times m\vec{v} \end{aligned}}$$

or $\boxed{\vec{L} = I\vec{\omega}}$ for particles rotates about symmetry axis.

$$\boxed{I = \sum_i m_i r_i^2 \quad \text{or} \quad I = \int r^2 dm}$$

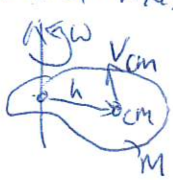
$\tau_{net} = I\alpha$

Conservation of Momentum (Angular) $\tau_{ext,ext} = \frac{d\vec{L}}{dt} = 0$.

Rotational Kinetic Energy. $P = \tau\omega$

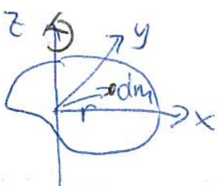
$K = \frac{1}{2} I\omega^2$ (pure rotation). " $K = \frac{1}{2} \sum_i m_i v_i^2$ "

Parallel-Axis Theorem



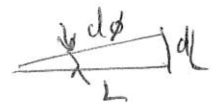
$I = Mh^2 + I_{cm}$

Perpendicular-Axis Theorem (only to plane figure)



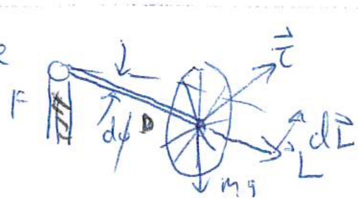
$I_z = I_x + I_y$

$\frac{dL}{dt} = MgD$ — d



$\frac{dL}{dt} = L\omega$

Motion of Gyroscope



$\boxed{\omega_p = \frac{MgD}{L}}$ Angular velocity of precession.

Kepler's Laws

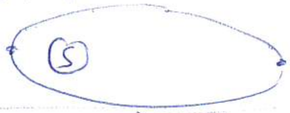
$\Delta t_1 = \Delta t_2$
$\Rightarrow A_1 = A_2$
$\frac{1}{2}|\vec{r}\times\vec{v}dt|$

1) All planets move in elliptical orbits w/ ~~sun~~ sun at one focus.
2) line joining any planet to sun sweeps equal area in equal times

$$dA = \frac{1}{2}|\vec{r}\times\vec{v}dt|$$

3) Square of period of a planet $\propto$ to cub of mean distance from planet to sun

$$T^2 = Cr^3$$



ex Moon's centripetal acceleration.

$$a_m = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} \qquad r = 3.84\times10^8 \text{ (distance between earth \& moon)}$$

$$T = 27.3\,d$$

$$= 2.72\times10^{-3}\,m/s^2$$

Gravitational force.

$$F = \frac{GM_E M}{r^2} \qquad g \simeq \frac{GM_E}{R_E^2}$$

Gravity.

$$ma = \frac{GM_E}{R_E^2} M_G \qquad \underset{\text{gravitational mass}}{M_G} \simeq \underset{\text{inertial mass}}{m} \quad b/c \; a \simeq g \text{ by measurement}$$

Experimental Law

Escape Planet. Integrate gravitational force from R_E to ∞,

$$U(r) = -\frac{GM_E m}{r} \rightarrow \text{Binding Energy } \boxed{U_{max} = -\frac{GM_E m}{R_E} = -mgR_E}$$

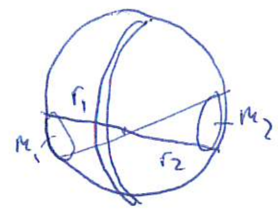
$$\boxed{V_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}}$$

Gravitational field.

spherical shell (uniform)

$$\vec{g} = -\frac{GM}{r^2}\hat{r} \qquad r > R$$

$$\vec{g} = 0 \qquad r < R$$



$$m_2 = m_1\left(\frac{r_2}{r_1}\right)^2$$

Solid sphere

$$g = -\frac{GM}{r^2} \qquad r > R$$

$$g = -\frac{GM}{R^3}r \qquad r < R$$

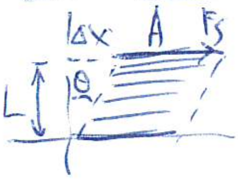
$$M' = \frac{r^3}{R^3}M$$

Fluid & Mechanics

σ Stress $= \frac{F}{A}$

ε Strain $= \frac{\Delta L}{L}$ $\cancel{\frac{\sigma}{L}}$

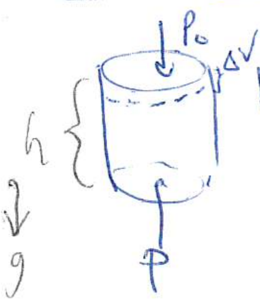
Young's Modulus $= \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L}$ $\frac{FL}{\Delta L A} = \left(\frac{\sigma}{\varepsilon}\right)$



Shear stress $= \frac{F_s}{A}$

Shear strain $= \frac{\Delta x}{L} = \tan\theta$

Shear Modulus $= \frac{\text{shear stress}}{\text{shear strain}} = \frac{F_s/A}{\Delta x/L} = \frac{F_s/A}{\tan\theta}$



$P = P_0 + \rho g h$

Bulk modulus $B = -\frac{P}{\Delta V/V}$

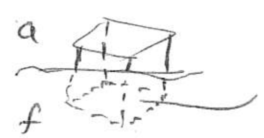
Compressibility $K = \frac{1}{B}$

Capillarity

$\underbrace{\gamma}_{\text{surf. tension}} 2\pi r \cos\theta_c = \rho(\pi r^2 h) g$ $\quad \cancel{\gamma = \frac{\rho g h}{\cos\theta_c}}$ $\quad \gamma = \frac{\rho g r h}{2\cos\theta_c}$

Buoyancy & Archimede's Principle

For floating object



$B = \rho_f g V$ — ~~complete submerged object~~ replaced volume by fluid (either submerged completely or floating)

V (volume of cube)

$\rho_f V' g = \rho V g$

$\Rightarrow \boxed{\frac{V'}{V} = \frac{\rho}{\rho_f}}$

Specify gravity for complete submerged object

$\frac{(W)\ \text{Weight in air}}{(F_B)\ \text{Weight loss in submerge}} = g_s = \frac{\rho}{\rho_f}$

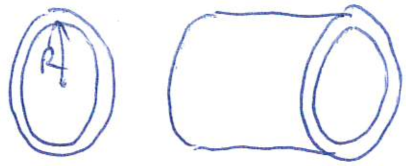
$\cancel{\text{Floating. } g_s = \frac{V\rho}{V'\rho_f}}$

Bernoulli's Equation : $P + \underbrace{\rho g y}_{\text{potential density}} + \underbrace{\frac{1}{2}\rho v^2}_{\text{kinetic density}} = \text{constant}$

Venturi effect : $P + \frac{1}{2}\rho v^2 = \text{const}$ $\quad v\uparrow \quad P\downarrow$

Moment of inertia.

1. hollow hoop/cylinder



MR^2

Solid disc/cylinder

$\frac{1}{2}MR^2$

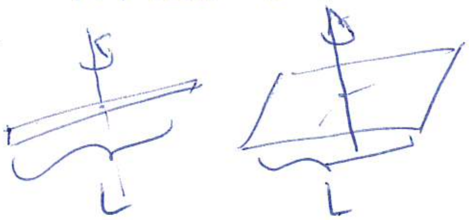
hollow sphere

$\frac{2}{3}MR^2$

Solid Sphere

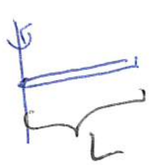
$\frac{2}{5}MR^2$

thin rod/door $\perp$ center



$\frac{1}{12}ML^2$

thin rod $\perp$ end



$\frac{1}{3}ML^2$

$$\begin{aligned} I &= I_{cm} + M\left(\frac{L}{2}\right)^2 \\ &= \frac{1}{12}ML^2 + \frac{ML^2}{4} \\ &= \frac{ML^2}{3} \end{aligned}$$

2. General radius of gyration k

$$I = Mk^2$$

Constraint equations.

holonomic: $f(\vec{r}_1, \vec{r}_2, \ldots, t) = 0$ (else nonholonomic ex $r^2 - a^2 \geq 0$)

Degree of freedom.

Free from constraint. N particles $\Rightarrow$ $3N$ deg. freedom

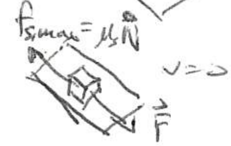
K holonomic constraint equation on N particles $\Rightarrow$ $3N-K$ "

D'Alembert's Principle.

$\sum \vec{f}_i \cdot \delta \vec{r}_i = 0$ $\quad$ $\vec{f}_i$ constraint force, $\delta \vec{r}_i$ virtual dispacement. $\vec{f}_i \perp \delta \vec{r}_i$

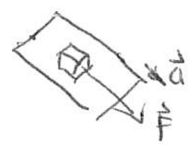
Equilibrium $\vec{F}_i = \vec{F}_i^{(a)} + f_i = 0$

$$\sum \vec{F}_i^{(a)} \cdot \delta \vec{r}_i = 0 \quad \text{(Principle of Virtual Work).}$$



General Motion. $\vec{F}_i - \dot{\vec{P}}_i = 0$

$$\sum (\vec{F}_i^{(a)} - \dot{\vec{P}}_i) \cdot \delta \vec{r}_i = 0 \quad \text{(D'Alembert's Principle)}$$



Motivation to Lagrangés Equation

Virial Theorem.

Statistical expression for N particles system.

$$\boxed{\langle T \rangle = -\frac{1}{2} \langle \sum \vec{F}_i \cdot \vec{r}_i \rangle = -\frac{N}{2} \langle U \rangle}$$

$\langle T \rangle = \frac{N}{2} \langle U \rangle$

where ~~$U = \sum \vec{F}_i d$~~

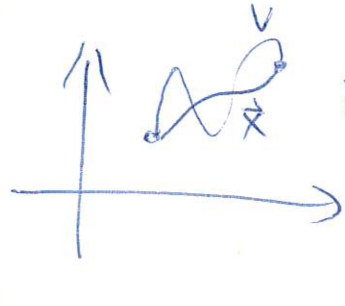
$U = \mp \sum \vec{F}_i \cdot \vec{r}_i$

ex Kinetic Theory ~~of~~ for gases.

$$\boxed{\bar{T}_i = \frac{3}{2} k_B \underset{\text{temp}}{T}}, \quad T = N \bar{T}_i, \quad -\int \vec{P} \hat{n}\, dA = \frac{3VP}{2}$$

$\cancel{\frac{3}{2} N k_B T =}$ $\quad N k_B T = PV$

Principle of Least Action:



$\vec{w} = \vec{x} + \tau \vec{v}$

$$I[\vec{w}(\cdot)] = \int L(\vec{w}(s), \dot{\vec{w}}(s))\, ds$$

$\Rightarrow i[\tau] = I[\vec{x}(\cdot) + \tau \vec{v}(\cdot)]$ set $i'(0) = 0$

$\Rightarrow$ Lagrange Equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$

$$\frac{\partial L}{\partial q} = p \qquad \frac{\partial L}{\partial x} = \dot{p} \qquad \dot{q} = \dot{x}$$

$$L = L(q, x, t)$$

replace q by p

Hamiltonian,

$$\boxed{H = \vec{p} \cdot \dot{\vec{q}} - L} \Rightarrow H = T + V \qquad H = H(p, x, t)$$

~~Apply~~

$$\frac{\partial H}{\partial p} = \dot{q} \qquad \frac{\partial H}{\partial x} = -\dot{p} \qquad \text{ex} \quad H = \frac{p^2}{2m} + mgx$$

$$\frac{dH}{ds} = 0 \Rightarrow H = \text{const} \quad \text{conservation of energy.}$$

Harmonic Oscillator & Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2$$

define $\omega^2 = \frac{k}{m}$

$m\ddot{x} = -kx$

$\omega = \sqrt{\frac{k}{m}}$

$$H = \frac{1}{2m}\left(p^2 + m^2 \omega^2 x^2\right)$$

quantum Harmonic oscillator