$m\ddot{x} + \lambda\dot{x} + kx = F(t)$ $\quad \gamma \equiv \frac{\lambda}{2m}$ $\quad \omega_0^2 = \frac{k}{m}$ $\quad \gamma^2 + \omega^2 \equiv \omega_0^2$ Q-factor $Q = \frac{\omega_0}{2\gamma}$

$x(t) = x_0 + \int_0^t dt'\, G(t-t')F(t')$ Green function $G(t) = \begin{cases} 0 & t<0 \\ \frac{1}{m\omega}e^{-\gamma t}\sin\omega t & t>0 \end{cases}$

Spherical coordinate

$$\nabla \times \vec{F} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(F_\varphi \sin\theta) - \frac{\partial F_\theta}{\partial\varphi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\varphi} - \frac{\partial}{\partial r}(rF_\varphi)\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right)\hat{\varphi}$$

$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\hat{\varphi}$; Area by simple closed curve $A = \frac{1}{2}\int_{s_0}^{s_1}(x\dot{y} - y\dot{x})\,ds$

Kelper's 3rd Law $T = \frac{2\pi R}{V} \rightarrow T^2 = \frac{4\pi^2 R^3}{GM_e}$; Solar mass $\tilde{M}_a = \frac{M_a}{M_s}$

Kelper's 2nd Law $dA = \frac{1}{2}r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = \frac{J}{2m}$

Elliptic orbits $(E<0,\ 0\le e\le 1)$ $\frac{(x+ae)^2}{a^2} + \frac{y^2}{b^2} = 1$, $a = \frac{l}{1-e^2} = \frac{|k|}{2|E|}$ $\quad b^2 = al = \frac{J^2}{2m|E|}$

Viral thm $\langle V\rangle_{av} = -2\langle T\rangle_{av}$

Effective potential $U_{eff}(\vec{r}) = \frac{J^2}{2mr^2} + U(\vec{r})$

Scattering Xsection hard sphere $b = R\cos\frac{\theta}{2}$, $d\sigma = \frac{1}{4}R^2\sin\theta\, d\theta\, d\varphi$

generally, $d\sigma = b|db|d\varphi$, $\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$

mean free path $\lambda = \frac{x}{n\sigma x}$ (n density), particle flux (f) $\frac{df}{dx} = -f(x)n\sigma$, $f = f_0 e^{-\frac{x}{\lambda}}$

Rutherford Scattering $b = a\cot\frac{\theta}{2}$, $a = \frac{qq'}{4\pi\varepsilon_0}\left(\frac{1}{mv^2}\right)$; $\frac{d\sigma}{d\Omega} = \frac{a^2}{4\sin^4\frac{\theta}{2}}$

Rate particles enter detector $d\omega = Nf\frac{d\sigma}{d\Omega}\frac{dA}{L^2}$ $\quad$ L dist. detector from target

N # atom contains by target.

Effective xsection

$= \pi b^2$

Con. E : $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 - \frac{GMm}{R}$; $mbV = mRV_1$

Rotating frame $\frac{d\vec{r}}{dt} = \vec{\omega}\wedge\vec{r}$, $\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} + \vec{\omega}\wedge\vec{r}$, $\frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} + 2\vec{\omega}\wedge\dot{\vec{r}} + \vec{\omega}\wedge(\vec{\omega}\wedge\vec{r})$

point mass

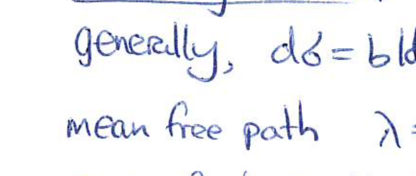
$m\ddot{\vec{r}} = m\vec{g} + \vec{F} \underbrace{-2m\vec{\omega}\wedge\dot{\vec{r}}}_{(*)} \underbrace{-m\vec{\omega}\wedge(\vec{\omega}\wedge\vec{r})}_{(*)}$

acceleration w.r.t rotating frame

acceleration w.r.t inertial frame

(*) are fictitious effect only seen by observer on rotating frame

Foucault Pendulum



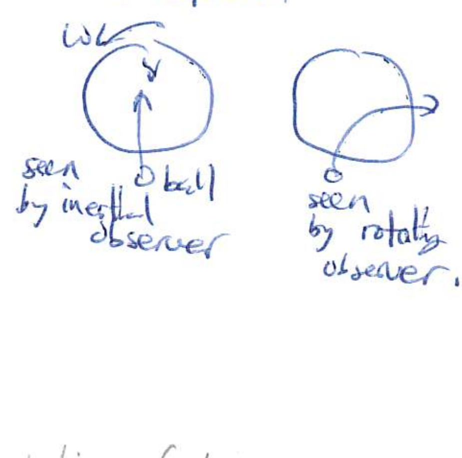
$\vec{\omega} = (0, \omega\sin\theta, \omega\cos\theta)$

z small $\quad -2m\vec{\omega}\times\dot{\vec{r}}$ then

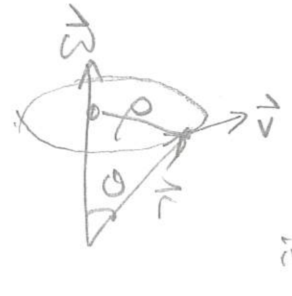
$m\ddot{x} = -\frac{mg}{l}x + 2m\omega_0\dot{y}\cos\theta$

$m\ddot{y} = -\frac{mg}{l}y - 2m\omega_0\dot{x}\cos\theta$

thus explains.



Key Rotating Frame

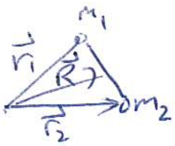


$|\vec{\omega}\wedge\vec{r}| = \omega \underbrace{r\sin\theta}_{\rho} = V$

$\Rightarrow \frac{d\vec{r}}{dt} = \vec{\omega}\wedge\vec{r}$

in general $\frac{d\vec{a}}{dt} = \vec{\omega}\wedge\underbrace{\vec{a}}_{\text{any vector fixed in rotating body.}}$

2-body



$\vec{r}_{1,2} = R \pm \frac{m_{2,1}}{M}\vec{r}$, CM: $\vec{R}=0$, denote $\vec{r}_{1,2}^{\,*} = \pm\frac{M_{2,1}}{M}\vec{r}$

momenta in CM: $m_1\dot{\vec{r}}_1^{\,*} = -m_2\dot{\vec{r}}_2^{\,*} = \mu\dot{\vec{r}} = p^*$.

relate to other frame, $\dot{\vec{r}}_{1,2} = \dot{\vec{R}} + \dot{\vec{r}}_{1,2}^{\,*}$

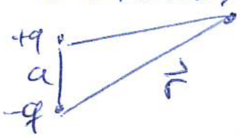
many body — Rocket $-\frac{dv}{V} = \frac{dM}{M}$ w/ thrust $M\ddot{z} = -mg + \alpha u$

Electric Potential — spherical shell $\Phi(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}$ sphere $\Phi(\vec{r}) = \begin{cases} \frac{q}{4\pi\varepsilon_0 r} & r>a \\ \frac{q}{4\pi\varepsilon_0}\left(\frac{3}{2a} - \frac{r^2}{2a^3}\right) & r<a \end{cases}$

energy stored: $U = \frac{1}{2}\int \rho\Phi\, dV$; magnetic moment $\mu = \frac{1}{2}\int \rho(\vec{r})(\vec{r}\wedge\vec{v})\,dv$, $\vec{\tau} = \vec{\mu}\times\vec{B}$

Classical ang. Momentum $J_{shell,s} = \frac{2}{3}mR^2\omega\,\hat{k}$

Oblateness, dipole, Quadrupole



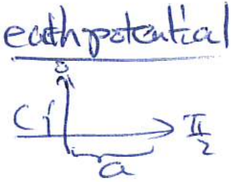
$\phi(\vec{r}) = \frac{q}{4\pi\varepsilon_0|\vec{r}-\vec{a}|} - \frac{q}{4\pi\varepsilon_0|\vec{r}|}$

dipole — dipole moment $\vec{d} = q\vec{a}$: $\phi(\vec{r}) = \frac{\vec{d}\cdot\vec{r}}{4\pi\varepsilon_0 r}$

Quadrupole: $\phi(\vec{r}) = \frac{Q}{16\pi\varepsilon_0 r^3}(3\cos^2\theta - 1)$; $Q = 4da$

generally $Q = \int \rho(\vec{r})(2z^2 - x^2 - y^2)\,d^3\vec{r}$; oblateness $\varepsilon = \frac{a-c}{a}$

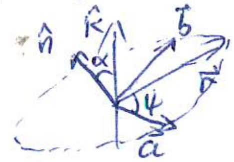
earth potential



$\Phi(\vec{r}) = -\frac{GM}{r} + \frac{GMa^2J_2}{2r^3}(3\cos^2\theta - 1) - \frac{1}{2}\omega^2r^3\sin\theta$, $J_2 = \frac{2}{3}\varepsilon$ (ρ uniform)

moment of earth: $m\vec{r}\times\vec{g}$, $\vec{r} = (r,0,0)$

$\vec{g} = (g_r, g_\theta, 0)$ $\vec{g} = -\nabla\Phi$



length of equivalent simple pendulum

$$L = \frac{I}{MR}$$

Rigid bodies

Euler angle

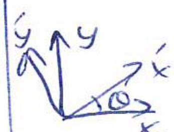
$T = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$; $\dot{\phi} = \Omega$

$\omega_1 = \dot{\phi}\sin\alpha\sin\psi + \dot{\alpha}\cos\psi$

$\omega_2 = \dot{\phi}\sin\alpha\cos\psi - \dot{\alpha}\sin\psi$

$\omega_3 = \dot{\phi}\cos\alpha + \dot{\psi}$

rotational matrix



$\hat{x} = x\cos\theta - y\sin\theta$

$\hat{y} = x\sin\theta + y\cos\theta$

$\hat{z} = z$

Lagrangian of sym top $L = \frac{1}{2}I_1(\dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta) + \frac{1}{2}I_3(\dot{\psi} + \dot{\varphi}\cos\theta)^2 - MgR\cos\theta$

Also $\frac{d\vec{J}}{dt} = \dot{\vec{J}} + \vec{\omega}\wedge\vec{J} = \vec{G} = \sum \vec{r}\wedge\vec{F}$

in component form
$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = G_1$
$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 = G_2$
$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = G_3$

phasor's method $A_1\cos(\omega_1 t) + A_2\cos(\omega_1 t + \delta) = A'\cos(\omega_1 t + \delta')$

w/ $A' = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sin\delta}$ where $\delta = \delta(t)$

Effective potential and central potential

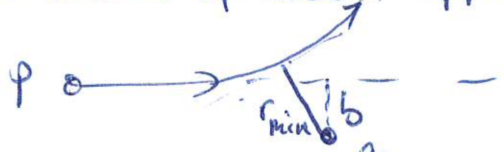
ex polar coordinate.

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - \underbrace{V(r)}_{\text{central potential.}}$$

$$J = mr^2\dot{\theta}$$

$$E = \frac{1}{2}m\dot{r}^2 + \underbrace{\frac{J^2}{2mr^2} + V(r)}_{\text{effective potential.}}$$

ex Distance of closest approach.



p approaches q at distant b from afar w/ velocity v, find r_{min}.

$$\frac{1}{2}mv^2 = \frac{1}{2}m\dot{r}^2 + \frac{J}{2mr^2} + \frac{k}{r}$$

for the closest distance $\dot{r} = 0$,

$$\frac{1}{2}mv^2 = \frac{J}{2mr^2} + \frac{k}{r}. \Rightarrow r^2 - \frac{2k}{mv^2}r - \frac{J^2}{m^2v^2} = 0$$

let $a = \frac{qq'}{4\pi\varepsilon_0 mv^2}$ $b = \frac{J}{mv}$; $r_{min} = a + \sqrt{a^2+b^2}$ //