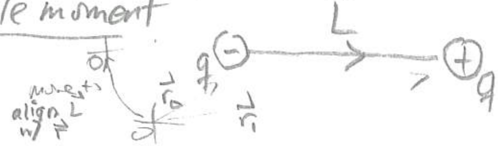
Fields properties
- # line leaving $+q$ and entering $-q \propto q$
- line density at point P $\propto |\vec{E}|$ at P.
- field lines never cross over each other.

Electric dipole moment

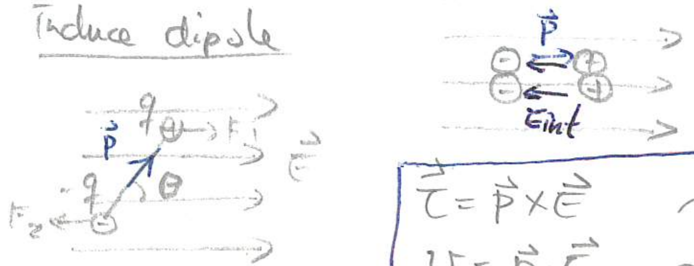


$$\boxed{\vec{p} = q\vec{L} \qquad \vec{p} = \int \vec{x}'\rho(\vec{x}')\,d^3x'}$$

let $\rho(\vec{x}') = \frac{q}{4\pi r^2}(\delta(r-r_1) - \delta(r-r_0))$, $d^3x' = r^2 dr\, d\Omega$, take $\vec{x}' = r\hat{r}$

$$\vec{p} = \hat{r}\frac{q}{4\pi}\int r\frac{(\delta(r-r_1)-\delta(r-r_0))}{r^2} r^2 dr\, d\Omega = \hat{r}q(r_1 - r_0) = q\vec{L}$$

Induce dipole



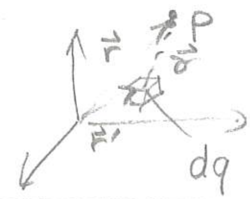
$\vec{p} \parallel \vec{E}_{ext}$ but anti $\vec{E}_{ind}$

$$\boxed{\begin{aligned} \vec{\tau} &= \vec{p}\times\vec{E} \quad \sim \quad \vec{\tau} = \vec{\mu}\times\vec{B} \\ U &= \vec{p}\cdot\vec{E} \quad \sim \quad H = \vec{\mu}\cdot\vec{B} \end{aligned}}$$

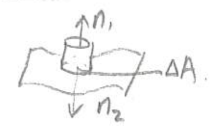
Coulomb's Law.

theory
$$\vec{E} = k\int \frac{\rho(\vec{r}')}{|\vec{r}'-\vec{r}|^2}\hat{\mathscr{r}}\,d^3r'$$

App
$$\boxed{\vec{E} = k\int \frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}},$$

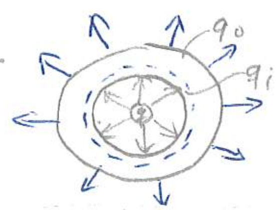


Discontinuity of E_n. All sheet where $\vec{E} \perp \hat{n}$ has jump of $\vec{E}$: $\Delta\vec{E} = \frac{\sigma}{\varepsilon_0}$

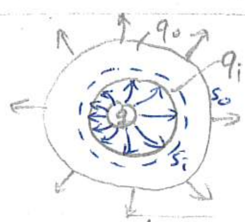


Conductor
- i) Charge free to move s.t net field is zero everywhere inside.
- ii) net charge reside on surface. (Gauss Law)
- iii) $\vec{E}$ outside surface $\perp$ to surface : $E = \frac{\sigma}{\varepsilon_0}$ (Discont of $\vec{E}_\perp$ on sheet).

Cavity



Gauss Law
$q_i = -q$
$q_0 = q$



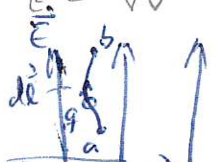
- $\vec{E}$ line density nonuniform on S_i b/c shifted q from center.
- Uniform from S_0 b/c shielded from cavity.

Electric Potential

$$\Delta U = -q\int_a^b \vec{E}\cdot d\vec{\ell}$$

- $\vec{E}$ line points in direction of decreasing electric potential.

$\vec{E} = -\nabla V$



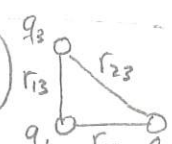
$dU = -\vec{F}\cdot d\vec{\ell}$

Theory
$$\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3} = -\nabla\frac{1}{|\vec{x}-\vec{x}'|} \qquad V = k\int\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x'$$

note: $\nabla^2\frac{1}{|\vec{x}-\vec{x}'|} = -4\pi\delta(\vec{x}-\vec{x}')$

App
$$\boxed{V = k\int\frac{dq}{\mathscr{r}}, \quad \Delta V = -\int_{x_0}^{x}\vec{E}\cdot d\vec{\ell}}$$

Discrete Charge $\vec{E} = \sum_i \vec{E}_i \Rightarrow V = \sum_i \frac{kq_i}{r_{i0}}$; (Work need to bring charge from ∞ to final position r_i)



$W = \frac{k}{2}\sum_{\substack{i,j=1 \\ i\neq j}}^{3} q_i \ldots$

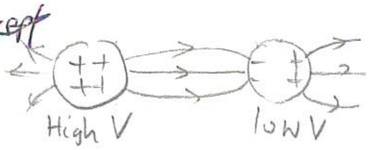
Concept

$\int \vec{E}\cdot d\vec{\ell}$ { $\vec{\ell} \not\parallel$ surface yield potential difference; $\vec{\ell} \parallel$ surface (since $\vec{E}\perp\vec{\ell}$) yield equipotential.

Equipotential surface
- Vol. occupied by conductor is equipotential ($\because \vec{E}=0 \Rightarrow \nabla^2 V = 0$) $\Rightarrow$ surface equipotential
- or $\int_a^b \vec{E}\cdot d\vec{\ell} = 0 \Leftrightarrow V_a = V_b$ (along equipotential surface) $\Rightarrow$ field lines $\perp$ equipotential s... con... give...

* $dV = -\vec{E}\cdot d\vec{r}$ E large, dV fixed, $\Rightarrow dr$ small. Potential line closely packed.

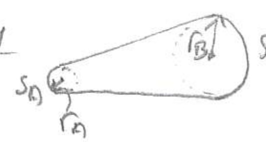
Concept



- $\vec{E}$ points to lower V $\Rightarrow$ conductors seperated in space don't have same potential
- ΔV between conductors depends on geometry, seperation, not charge.

dielectric breakdown: When air become Ionized. Dielectric strength of air $E_{max} = 3\times10^6\ V/m$

P.X Charge density and geometry



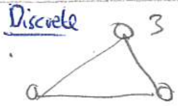
$\sigma_{SA} > \sigma_{SB}$ b/c
$E_{SA} > E_{SB}$

sphere potential

$V = \frac{kq}{r} \propto r\sigma$ & $V_A = V_B$

$= \frac{\sigma}{\varepsilon_0} r$

Energy of Point Charge: Discrete



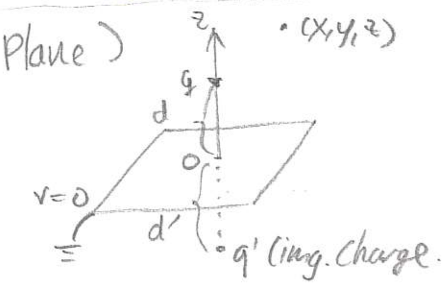
$$W = \frac{1}{2}\sum_{j=1}^{n} q_j V(\vec{r}_j)$$?

Continuous $W = \frac{1}{2}\int \rho V\, d\tau$ volume

recall $W = \frac{1}{2}\sum_i q_i \sum_j k\frac{q_j}{r_{ij}} = \frac{1}{2}\sum_i q_i V(\vec{r}_i)$

if continuous, $W = \frac{1}{2}\int dq\, V(\vec{r})$ $\Rightarrow$ energy density: $\boxed{\eta = \frac{\varepsilon_0}{2}E^2}$

MOI: (Plane)



$$V(x,y,z) = \frac{kq}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{kq'}{\sqrt{x^2+y^2+(z-d')^2}}$$

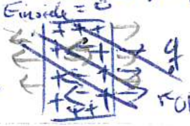
B.C $V(0,0,0)=0 \Rightarrow \boxed{q' = -q}$ check!
$V(x,y,0)=0 \Rightarrow \boxed{d' = -d}$

Note

Induced surface charge: $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \Rightarrow \sigma = -\varepsilon_0\frac{\partial V}{\partial n}$ @ $z=0$ $\quad \sigma(x,y) = \frac{-qd}{2\pi(x^2+y^2+d^2)^{3/2}}$

should it be $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$? No

all sheet has jump $\frac{\sigma}{\varepsilon_0}$

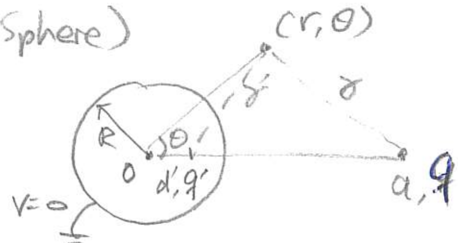


Force: $\vec{F} = -k\frac{q^2}{(2d)^2}\hat{z}$

Energy: $W = -\frac{k}{2}\frac{q^2}{2d}$ due to single charge & conducting plane w/ symmetry, energy store in field of upper plane.

MOI (Sphere)

induced charge from outside.



$\boxed{q' = -\frac{R}{a}q \\ d' = \frac{R^2}{a}}$

outside charge
a distance of out charge.

$V(\vec{r}) = k\left(\frac{q}{\gamma} + \frac{q'}{\gamma'}\right)$

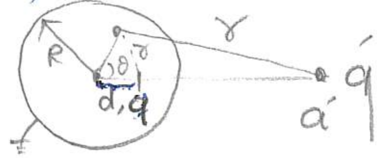
$\gamma = \sqrt{r^2+a^2-2ra\cos\theta}$

$\gamma' = \sqrt{r^2+d'^2-2rd'\cos\theta}$

$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{(a-d')^2}$

What if q inside q' outside?

(Shell)



by sym.

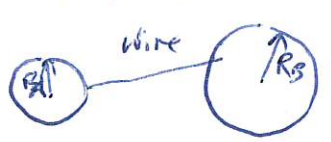
$d = \frac{R^2}{a'} \longmapsto \boxed{a' = \frac{R^2}{d}}$

$q = -\frac{R}{a'}q' \longmapsto \boxed{q' = -\frac{R}{d}q}$

dimensional analysis

Charge inside.
d distance of inside charge

reconstruct P.X as:



nected by wire to
es equipotential

generally a sphere.

$E = \frac{kQ}{r^2} \Rightarrow V = \frac{kQ}{r}$

at R, $V = \frac{\sigma R}{\varepsilon_0}$

since equipotential, we have

$\sigma_A R_A = \sigma_B R_B$

$R_B > R_A \Rightarrow \sigma_B < \sigma_A$

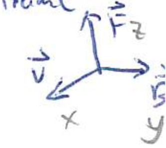
and $E = \frac{\sigma}{\varepsilon_0} \Rightarrow E_B < E_A$!

B fields — Magnetic force on current carrying wire $d\vec{F} = I d\vec{l} \times \vec{B}$

q charge, n density, v_d, A, l, B

Force $\vec{F} = qnAl\vec{v}_d \times \vec{B}$

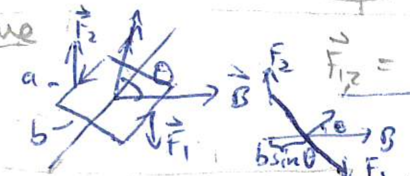
treat $\vec{F} \sim \vec{v} \times \vec{B}$ as coord. frame



Magnetic dipole moment $\mu = AI$ (area)

$\vec{\mu} = \frac{1}{2}\int \vec{x}' \wedge \vec{J}(x')\, d^3x'$

Torque



$F_{12} = IBa$, $\tau = \underbrace{NBAI\sin\theta}_{\vec{\mu} \wedge \vec{B}}$ if N loops

Analog	
E field	B field
$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{\mu} \times \vec{B}$
$\vec{p} = q\vec{L}$	$\vec{\mu} = I\vec{A}$
$H = \vec{p} \cdot \vec{E}$	$H = \vec{\mu} \cdot \vec{B}$

If Bar magnetic — Define pole strength q_m

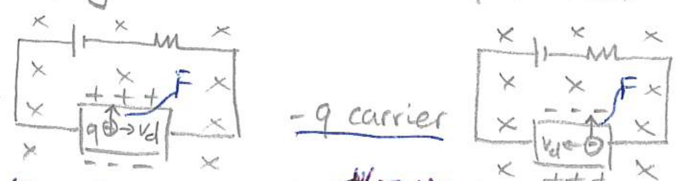
$\vec{F} = q_m \vec{B}$, $\vec{\mu} = q_m \vec{L}$ — vector from South to North pole

Hall effect: Charge seperation in current carrying wire due to external B field.

App • Find sign and density of charge carrier
• measure $\vec{B}$

$R_H = \frac{1}{qn}$ when saturated.

+q carrier | -q carrier



note: Carries in metallic conductor are negative

Hall voltage V_H

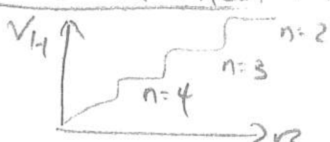
$qE = qV_dB$ (force balance)

$V_H = Ew$ (parallel plate)

just like motion emf $V = Blv$ w/ $v = v_{drift}$

$V_H = v_d B w$ — small b/c v_d; use $I = qnAV_d$ & $A = wt$ to compute n (density), V_H

Quantum Hall Effect: At very low temp. & high B fields, Hall voltage is quantized.



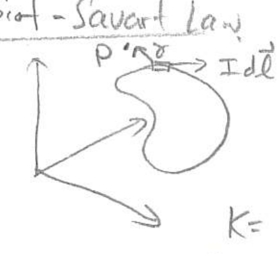
$R_H = \frac{V_H}{I} = \frac{R_K}{n}$ (Hall Resistance; R_K: von Klitzing const.) $n = 1, 2, 3$ $R_K = \frac{h}{e^2} = 25813\Omega$

if n fractional #, fractional Hall effect

Biot-Savart Law

App $d\vec{B} = k\frac{I d\vec{l} \times \hat{r}}{r^2}$ "B field induced by current circulation"

Theory $\vec{B} = k\int d^3x' \frac{J(\vec{x}') \wedge (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}$



$k = \frac{\mu_0}{4\pi}$

$dI = I d\vec{l}$ (discrete) or $= dq\vec{v}$

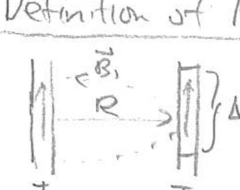
Magnetic force

$\vec{F}_{12} = q_2\vec{v}_2 \times k\frac{q_1\vec{v}_1 \times \hat{r}_{21}}{r_{21}^2}$ $\vec{F}_{21} = q_1\vec{v}_1 \times k\frac{q_2\vec{v}_2 \times \hat{r}_{12}}{r_{12}^2}$

$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

$\vec{F}_{12} = \vec{F}_{21}$ but not along line joining charges. Thus doesn't obey N.3

Definition of Ampere



for now $B = \frac{\mu_0 I}{2\pi R}$

$\vec{F}_{12} = \Delta l_2 I_2 B_1$

$\frac{\vec{F}_{12}}{\Delta l_2} = k\frac{2I_1I_2}{R}$ or $\frac{\mu_0}{4\pi}\frac{2I_1I_2}{R}$

$I_1I_2 = 1A^2$ is st for $R = 1$ $\left|\frac{\vec{F}_{12}}{\Delta l_2}\right| = 2\times10^{-7}\ N/m$

$\mu_0 = 4\pi\times10^{-7}\ N/A^2$

Ampère's Law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c$ (induced B field by current; I_c: enclosing current)

ex Straight wire has $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$

$I_c = \int \vec{J} \cdot d\vec{a}$ (cross section of wire; da area, current)

Steady state $\nabla \times \vec{B} = \mu_0 \vec{J}$

cont' $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$, $\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}$ displacement current

Maxwell

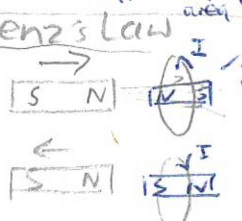
$\nabla \cdot D = \rho$ $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

$\nabla \cdot B = 0$ $\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}$

• Ampère Law not apply in discontinuous current in space.

• Apply for symmetric system for easy computation.

Lenz's Law



change flux induced emf

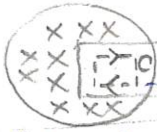
$\mathcal{E} = -\frac{d\Phi_m}{dt}$ $\mathcal{E} = \oint \vec{E} \cdot d\vec{l}$

Faraday's Law $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

direction induced emf oppose to change

note $\Phi_m = \int \vec{B} \cdot d\vec{A}$

Eddy Current.

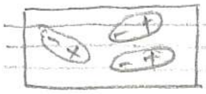


slap — $\vec{v}$ — Eddy current

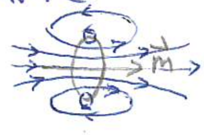
Pull slap $\Rightarrow$ induced $\mathcal{E}$ w/ I flow in C.

selected closed path C

Dipole moment.



$\vec{E}$

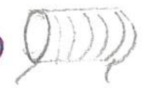


$\vec{B}$ — align magnetic moment due to intrinsic atomic motion

$\vec{m}$ — I_{net} — $I=0$ — l

Solenoid + material

Magnetization $\vec{M} = \frac{d\vec{m}}{dV} = \frac{dI}{dl}$



$\vec{B}_m = \mu_0 \vec{M}$ — due to magnetization.

$\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$

$\vec{B} = \vec{B}_0 (1+\chi_m)$

$\vec{M} = \chi_m \frac{\vec{B}_0}{\mu_0}$ — magnetized susceptibility.

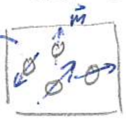
para $0<\chi_m \ll 1$ Temp. depend.

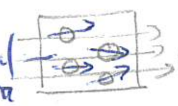
diama $-1<\chi_m<0$ Temp. independ.

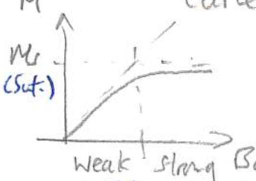
Magnetic Materials

Para w/o Bext — w Bext

weakly interact — $0<\chi_m \ll 1$ — thermal motion



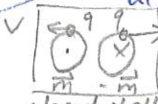




Curie's Law $M = M_S \left(\frac{mB}{3kT}\right)$ — energy due to magnetization / due to thermal

$\vec{B} = \vec{B}_{app} + \mu_0 \vec{M}$ — all materials are diama at suff. high temp.

Diama — net $\vec{m}$ point in — closed shell



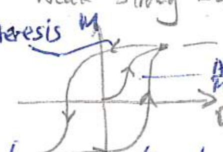
F, P, D — mag. hierarchy

Ferro — pure iron, cobalt, nickel — magnetic domain

$T >$ Curie temp. ferro $\to$ para.

below — at which material loss permanent magnetization when $T>T_C$

hysteresis



— reaches M_s
— Area $\Leftrightarrow$ heat dissipation
— Magnetization on previous history

ex. bismuth

supercond. "perfect diama" $\chi_m = -1$

Energy Momentum — what's the energy density of EM field at free space?

$\eta_e = \frac{1}{2}\varepsilon_0 E^2 \quad \eta_m = \frac{1}{2\mu_0}B^2$ free space $\boxed{cB = E}^{(3)}$ $\because \vec{B} \perp \vec{E} \Rightarrow \eta = \eta_e + \eta_m = \frac{EB}{\mu_0 c}$

Instantaneous intensity, (4) $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ Poynting vector if $\vec{E} \perp \vec{B}$ then $\eta = \frac{|S|}{c}$ (5)

Maxwell's equation

$$\boxed{\begin{array}{ll} \nabla\cdot\vec{D} = \rho & \nabla\times\vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} \to (1) \\ \nabla\cdot\vec{B} = 0 & \nabla\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \to (2) \end{array}} \qquad \boxed{\begin{array}{l} \vec{D} = \varepsilon\vec{E} \\ \vec{H} = \frac{\vec{B}}{\mu} \end{array}}$$

$\varepsilon = \varepsilon_0$ reduce to vaccum

Dielectric / Permeability

$\hat{n}\cdot(\vec{D}_2 - \vec{D}_1) = \Sigma$

$\hat{n}\times(\vec{E}_2 - \vec{E}_1) = 0$

$\hat{n}\cdot(\vec{B}_2 - \vec{B}_1) = 0$

$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{K}$

B.C. Conductor

$E^{\parallel}_{out} = E^{\parallel}_{in} \quad E^{\perp}_{out} - E^{\perp}_{in} = \frac{\sigma}{\varepsilon_0}$ (cont' if no surf. charge)

$B^{\perp}_{out} = B^{\perp}_{in} \quad \vec{B}^{\parallel}_{out} - \vec{B}^{\parallel}_{in} = \mu_0 \vec{K}\times\hat{n}$ (cont' if no surf. current)

Wave Equation $\boxed{\text{free space}}$ $\rho, \vec{J} = 0$ from maxwell's eqn $\Rightarrow^{(1)} \vec{E}_{tt} - c^2\Delta\vec{E} = 0$, $\vec{B}_{tt} - c^2\Delta\vec{B} = 0$

$\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}$

plane wave: $e^{i(kx-\omega t)}$, $c = \frac{\omega}{k}$ $k = \omega\sqrt{\mu_0\varepsilon_0}$ $\left(n = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}} > 1\right)$, general $k = \omega\sqrt{\mu\varepsilon}$, $v = \frac{\omega}{k}$

$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

In general $c\vec{B} = \hat{n}\times\vec{E}^{(2)}$

$\hat{n} = \frac{\vec{k}}{k}$ — index of reflection $c = nv$

$\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$ (Fourier transform) propagation direction

make plane wave $\vec{E} \sim e^{i(kx-\omega t)}\hat{x}$, $\vec{B} \sim e^{i(kx-\omega t)}\hat{y}$

Dipole. Multipole Expansion:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int r'^n P_n(\cos\theta')\rho(\vec{r}')d^3\vec{r}'$$

n=1, monopole $V_0(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ n=2 dipole $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}\cdot\hat{r}}{r^2}$

or from ex. $E_x = \frac{2kP}{x^3}$ $B_x = \frac{2km}{x^3}$

Radiation

Accelerating charge at long distance for $v \ll c$ (non-relativistic) radiate total power:

Oscillating dipole $\vec{P}(t) = P_0\cos(\omega t)\hat{z}$.

Larmor formula $\boxed{P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}}$ — acceleration

$P \sim q^2 a^2$

intensity — No radiation along dipole axis / max @ $\theta = 90°$

$\langle S\rangle = \left(\frac{\mu_0}{32\pi^2 c}\right)\frac{P_0^2\omega^4}{r^2}\sin^2\Theta \quad \langle P\rangle_E = \left(\frac{\mu_0}{12\pi c}\right)P_0^2\omega^4 \sim \langle P\rangle_m = \left(\frac{\mu_0}{12\pi c^3}\right)m_0^2\omega^4$

Circuit I. Capacitance

$\circ\ C \equiv \frac{Q}{V}$ unit $1F = \frac{1C}{V}$ how many charge given by a volt.

$\circ\ E = \frac{\sigma}{\varepsilon_0}x$ $\quad V = \frac{\sigma d}{\varepsilon_0}$

$\Downarrow\ V = \frac{\sigma}{\varepsilon_0}x$ $\quad \circ\ V = Ed$

important fact $\rightarrow$ $C = \frac{\varepsilon_0 A}{d}$ $\quad C = \frac{Q}{Ed}$

* easier route: $E = \frac{Q}{\varepsilon_0}$ $V = \frac{\sigma d}{\varepsilon_0}$, $V = \frac{Qd}{A\varepsilon_0}$, $C = \frac{\varepsilon_0 A}{d}$.

Dielectrics

- non conducting materials that weakened $\vec{E}$ field btw capacitor.
- increase capacitance by K given Q.

— Given V yields Q then remove V, inserted dielectric then C' increase. Replug V then Q increase.

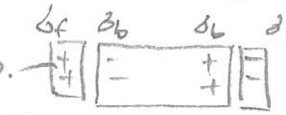
polarized dipole

induced E_{int}

$$\boxed{K \equiv \frac{\varepsilon'}{\varepsilon_0} \ (K > 1) \quad E' = \frac{E_0}{K} \quad C' = KC_0 \quad \varepsilon' = K\varepsilon_0 \quad C' = \frac{\varepsilon' A}{d}}$$

$E = \frac{E}{K}$ $\left(\int \vec{E} \cdot d\vec{l}\right) \Rightarrow V' = \frac{V}{K}$ $\left(V = \frac{Q}{C}\right) \Rightarrow C' = C$

σ_b bound charge density.

Cap. 

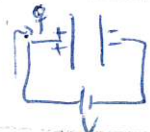
$E_b = \frac{\sigma_b}{\varepsilon_0}$ $\quad E_0 = \frac{\sigma_f}{\varepsilon_0}$ $\quad E_0 - E_b = \frac{E_0}{K} \Rightarrow$ define $\boxed{\sigma_b = \frac{K-1}{K}\sigma_f}$

Energy Storage

Work need to create field in conductor and stored in E field.

Capacitor: $dU = V\,dq \rightarrow U = \frac{1}{2}\frac{Q^2}{C}$ or $\frac{1}{2}CV^2$

energy density $\eta = \frac{U}{Ad}$, $\eta = \frac{1}{2}\varepsilon E^2$



$C = \frac{q}{V}$

Current

$I = \frac{dQ}{dt}$ $\quad I = qnAV_d$

cross-section area (A), drift velocity (V_d), # electrons (n).

Analogous: electric — thermal

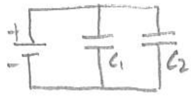
$V = IR$ $\quad \Delta T = IR$

resistivity $\rho = \frac{1}{\sigma}$ conductivity

$\boxed{R = \frac{L}{\sigma A}}$ $\quad R = \frac{\Delta x}{KA}$

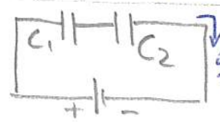
* Superconductor has ~~$\sigma = 0$~~ $\rho = 0$ below T_c.

Circuit.



$Q = Q_1 + Q_2$

$C_{eq} = C_1 + C_2$



$V = V_1 + V_2$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

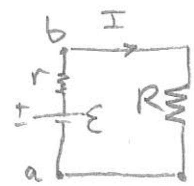
Emf

Converts chemical, mechanical into electrical

ideal battery

$P = \mathcal{E} I$

real battery



$\mathcal{E} = (r+R)I$

$P_{net} = RI^2 - rI^2$

Electric conduction (Classical)

$\vec{J} = \sigma\vec{E}$ — $I = enAV_d = \frac{1}{\rho}EA$, $m_e\frac{V_d}{\tau} = eE \Rightarrow \rho = \frac{m_e}{ne^2\tau}$, $\lambda = V_{av}\tau \Rightarrow \rho = \frac{m_e V_{av}}{ne^2\lambda}$ $\circ$

Ohm's Law

Classically, ρ depends on ions size & density but indept. E. Not all true in quantum.

conductivity

- overestimate ρ at high temp.
- incorrect temp dependence $V_{av} \propto \sqrt{T}$ (Maxwell-Boltzmann distribution)
- electron obeys Fermi-Dirac distribution where V_{av} indept. of temp.

mean-free-path:

$$\lambda = \frac{1}{N}\frac{V}{\pi r^2} = \frac{1}{n\pi r^2} = \frac{1}{n\sigma}$$

ions in V $\quad$ ion density $\quad$ crossection

- $R = \rho\frac{L}{A}$ $\quad \rho = \frac{1}{\sigma}$
- $\vec{J} = \sigma\vec{E}$, $J = \frac{I}{A}$
-  $\vec{E}$

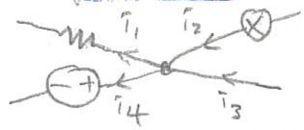
  $qE = m\frac{V_d}{\tau}$
- ~~$\tau = \frac{\bar{V}}{}$~~ $\tau = \frac{\lambda}{\bar{V}}$

at thermal equilibrium $\frac{1}{2}m_e V_{rms}^2 = \frac{3}{2}KT$

$$\rho = \frac{m_e\bar{V}}{nq^2\lambda}$$

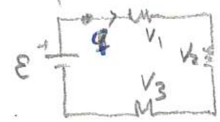
Kirchhoff's rules

junction rule: charge conservation



$i_2 + i_3 = i_1 + i_4$

Loop rule

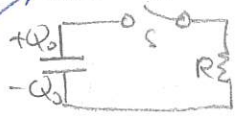


- energy gain = energy lost
- q at P has energy qV

$\mathcal{E} = V_1 + V_2 + V_3$

RC circuit → discharge

switch closed

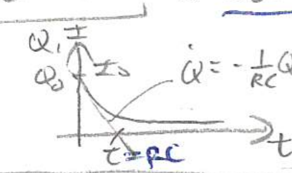


I.C: $I_0 = \frac{Q_0}{RC}$

$I = -\frac{dQ}{dt}$

$IR = \frac{Q}{C}$ (kirchhoff)

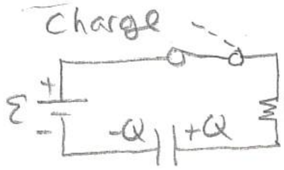
$\boxed{\dot{Q} + \frac{1}{RC}Q = 0}$ $\quad Q = Q_0 e^{-t/\tau}$ $\quad \tau = RC$



$\dot{Q} = -\frac{1}{RC}Q_0$

$\tau = RC$

Charge



I.C: $Q(0) = 0$ $\quad \boxed{I(0) = \frac{\mathcal{E}}{R}}$

$I = \frac{dQ}{dt}$

$\mathcal{E} - V_R - V_C = 0$ (kirchhoff)

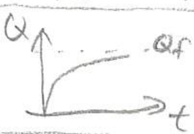
$\boxed{\dot{Q} + \frac{1}{RC}Q - \frac{\mathcal{E}}{R} = 0}$ $\quad Q = Q_f(1 - e^{-t/\tau})$

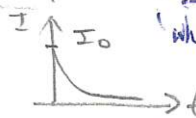
final charge

what's the form? at $t \to \infty$ $I \to 0$ left

$Q_f = \frac{\mathcal{E}C}{\cancel{\mathcal{E}}}$

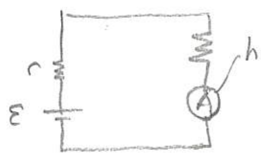
Finding this it's counter intuitive





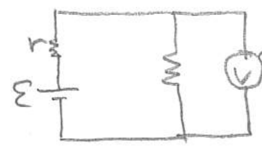
Ammeter & Voltmeter

amp.



has small R

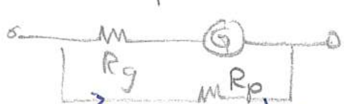
vol.



has large R

Principal component: galvanometer, has $\underset{\text{shunt}}{R_g}$ resistance thru by I_g passing current.

amp



note R_p small

Set $R_g \gg 1$

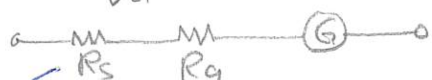
s.t $R_{eq} \approx R_p$

overall amp has small R

$R_p \ll R_g \quad R_{eq} \simeq R_p \ll R_g$

shunt

vol



shunt

$R_s \gg R_g \quad R_{eq} \gg R_g$

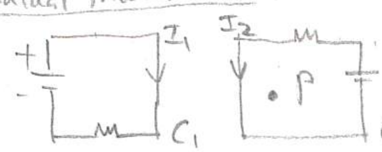
Magnetic Inductance

Self Inductance.

$\boxed{\phi_m = LI, \quad \mathcal{E} = -L\frac{dI}{dt}}$

$\mathcal{E} = -\frac{d\Phi}{dt}$

Mutual Inductance



$\phi_{m2} = L_2 I_2 + M_{12} I_1$

$\phi_{m1} = L_1 I_1 + M_{21} I_2$

M_{ij} mutual inductance and $M_{12} = M_{21}$ generally

LR Circuit

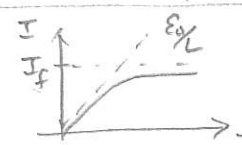
just like charge & discharge in RC

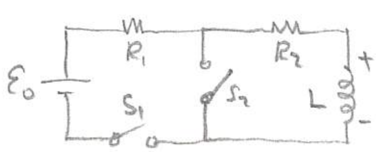
$\boxed{\text{I.C} \quad I(0) = 0 \quad LI'(0) = \mathcal{E}_0}$

$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$

$\mathcal{E}_0/R \quad \tau = \frac{L}{R}$

$I = I_f(1 - e^{-t/\tau})$



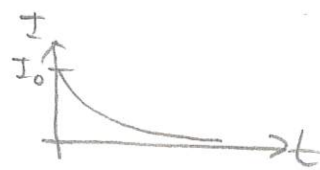


S_1 close till steady then S_2 close S_1 open. $I_f = I_0$

I.C $\quad I_0 = \frac{\mathcal{E}_0}{R_1 + R_2}$

$IR_2 + L\frac{dI}{dt} = 0$

$I = I_0 e^{-t/\tau}$



Magnetic Energy

$$\underbrace{\mathcal{E}_0 I}_{\text{battery output}} = \underbrace{I^2R}_{\text{dissipation}} + \underbrace{LI\frac{dI}{dt}}_{\text{input inductor}}$$

$\frac{dU_m}{dt} = LI\frac{dI}{dt} \Rightarrow$

Magnetic: $U_m = \frac{1}{2}LI_f^2$

Electric: $U_e = \frac{1}{2}CV^2$

Now $B = \mu_0 nI$, $L = An^2\mu_0 l$ for $U = \frac{1}{2}LI^2$ $\Rightarrow$ $\eta_m = \frac{1}{2}\frac{B^2}{\mu_0}$

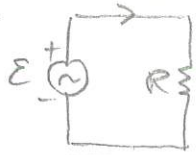
$\eta_e = \frac{1}{2}\varepsilon_0 E^2$

energy density stored in the field

D.C: RC, LC $\qquad$ A.C: R, L, C; LC, LRC

Circuit II (AC)

- Resistor



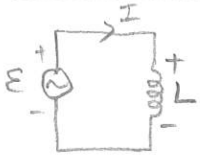
$\varepsilon = \varepsilon_{max}\cos\omega t$, $I = \overset{\varepsilon_{max}/R}{I_{max}}\cos\omega t$, $I_{rms} = \frac{I_{max}}{\sqrt{2}}$; $I_{rms} = \sqrt{\langle I^2\rangle}$

$\langle P\rangle = \langle I^2\rangle R$

Energy dissipated $P_{av} = I_{rms}^2 R = \frac{1}{2} I_{max}^2 R$

delievered $P_{av} = \varepsilon_{rms} I_{rms} = \frac{1}{2}\varepsilon_{max} I_{max}$

- Inductor

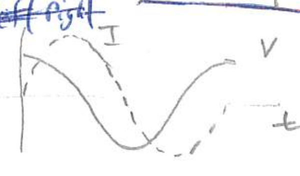


$\varepsilon = L\frac{dI}{dt}$, $I = I_{max}\cos(\omega t - \frac{\pi}{2})$ ~~travel to left right~~

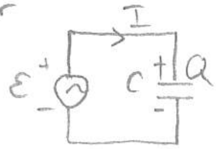
Voltage lead current by 90°

$I_{max} = \frac{\varepsilon_{max}}{X_L}$ $X_L = \omega L$ inductive reactance

$I_{rms} = \frac{\varepsilon_{rms}}{X_L}$



- Capacitor

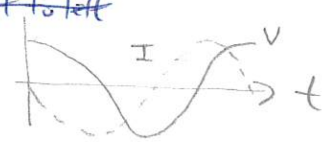


$\varepsilon = \frac{Q}{C}$, $Q = \overset{\varepsilon_{max} C}{Q_{max}}\cos\omega t$, $I = I_{max}\cos(\omega t + \frac{\pi}{2})$ Voltage lags current by 90°

~~shift to left~~

$I_{max} = \frac{\varepsilon_{max}}{X_C}$ $X_C = \frac{1}{\omega C}$ capacitive reactance.

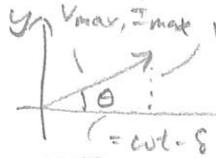
$I_{rms} = \frac{\varepsilon_{rms}}{X_C}$



$X_R = R \quad X_L = \omega L \quad X_C = \frac{1}{\omega C}$ $\quad I_{rms} = \frac{\varepsilon_{rms}}{X}$ $\quad I_{max} = \frac{\varepsilon_{max}}{X}$

Phasors

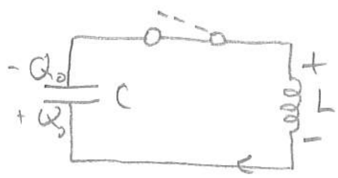
Treat I, V as vectors. $V = V_{max}\cos(\omega t - \delta)$ $\quad I = I_{max}\cos(\omega t - \delta)$



series circuit Add V's $\quad V_{max} = I_{max} R$

parallel Add I's

LC

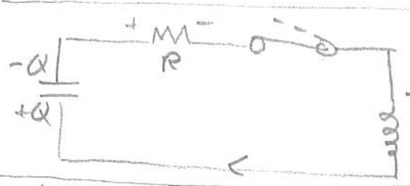


I.C $Q(0) = Q_0$ Before switch closed

$L\ddot{Q} + \frac{1}{C}Q = 0$ $\quad L \sim m$ $\quad \frac{1}{C} \sim k$ $\quad \omega_0 = \sqrt{\frac{k}{m}} \sim \omega_0 = \frac{1}{\sqrt{LC}}$

$U_{tot} = \frac{1}{2}\frac{Q_0^2}{C}$

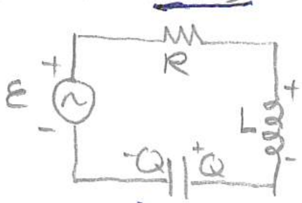
LCR



$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = 0$ $\quad R \sim b$ damping factor

energy form $\frac{d}{dt}\left(\frac{1}{2}LI^2\right) + \underbrace{I^2 R}_{\text{heat diss.}} + I\frac{Q}{C} = 0$

LCR generator (series)

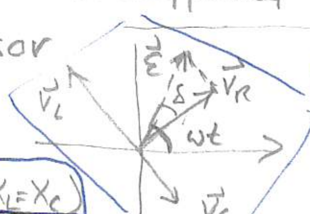


$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = \varepsilon_{max}\cos\omega t \sim m\ddot{x} + b\dot{x} + kx = F_0\cos\omega t$

$I = \frac{\varepsilon_{max}}{Z}\cos(\omega t - \delta)$ $\quad x = \frac{F_0\cos(\omega t - \delta)}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}}$

$\tan\delta = \frac{X_L - X_C}{R}$ $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$ $\quad \tan\delta = \frac{b\omega}{m(\omega_0^2 - \omega^2)}$

* Phasor



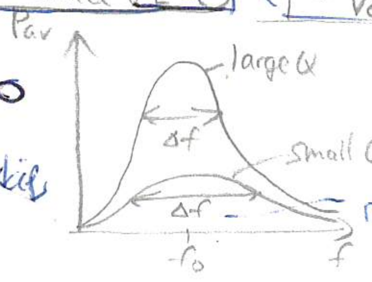
$\vec{\varepsilon} = \vec{V}_R + \vec{V}_L + \vec{V}_R$

Resonance $\omega \to \omega_0$ (As $X_L = X_C$)

~~and~~ $\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \sim \sqrt{\frac{k}{m}}$

$\varepsilon_{max} = \sqrt{V_{R,max}^2 + (V_{L,max} - V_{C,max})^2} = I_{max} Z$

Resonance ($X_L = X_C$)



diff

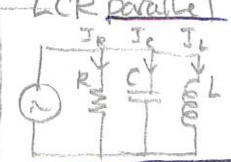
Quality factor

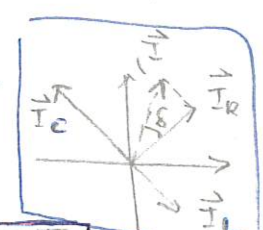
LRC	spring mass
$Q = \frac{\omega_0 L}{R}$	$Q = \frac{\omega_0 m}{b}$

for $Q > 2, 3$

$\boxed{Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}} \longrightarrow \Delta\omega, \Delta f$

LCR parallel



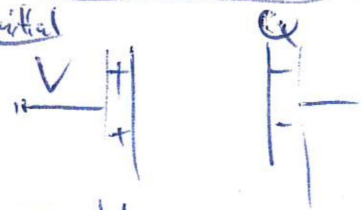


$I = \sqrt{I_R^2 + (I_C - I_L)^2} = \frac{V}{Z}$

$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$

Dielectric revisite

initial

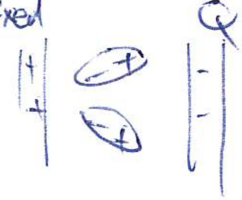


$$C = \frac{Q}{Ed} = \frac{\varepsilon_0 A}{d}$$

definition for K is natural not convenient however!

removed b to keep Q fixed

& Add dielectric materials.



$$E' = \frac{E_0}{K} \quad K > 1$$

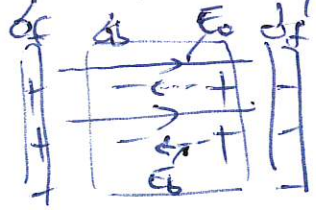
$$V' = E'd$$

$$C' = \frac{Q}{E'd} = K C_0 \quad \text{or} \quad C' = \frac{\varepsilon A}{d}$$

from $E' = \frac{E_0}{K}$

w/ capacitor surface charge σ: $\frac{\sigma}{\varepsilon} = \frac{\sigma}{K\varepsilon_0}$ $\Rightarrow$ $\boxed{K = \frac{\varepsilon}{\varepsilon_0}}$ — permittivity of dielectric. — use this as definition for K.

bound charged density in dielectric (σ_b)



$$E_b = \frac{\sigma_b}{\varepsilon_0}$$

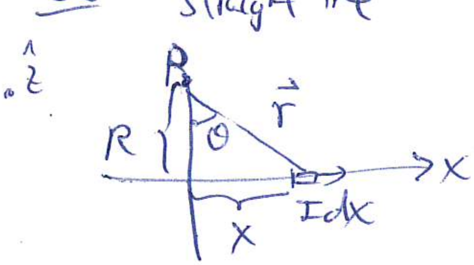
$$E_0 = \frac{\sigma_f}{\varepsilon_0}$$

$$E_0 - E_b = \frac{E_0}{K}$$

why?, $\boxed{\sigma_b = \frac{K-1}{K}\sigma_f}$

---

ex straight line



$$d\vec{B} = K\frac{I d\vec{l} \wedge \vec{r}}{r^3} \quad ; \quad I d\vec{l} \wedge \vec{r} = rI dx \cos\theta$$

$$\vec{B} = K\int \frac{I dx \cos\theta}{r^2} \quad \hat{z}$$

w/ $x = R\tan\theta$

$R = r\cos\theta$

$$B = \frac{KI}{R}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$B = \frac{KI}{R}(\sin\theta_2 + \sin\theta_1)$$

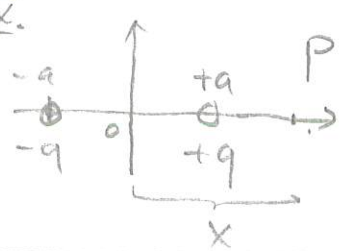
As wire $\to \infty$

$$B = \frac{\mu_0 I}{2\pi R}$$

Magnetic flux

$$\phi_m = \int \vec{B} \cdot \hat{n} \, dA$$

ex.

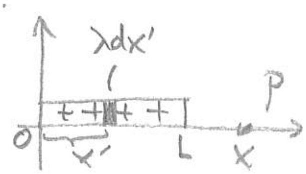


Find $\vec{E}$ on x-axis at P.

For $x \gg a$ $\quad \vec{E} \simeq \frac{4kqa}{r^3}\hat{i} \quad \vec{E} = \frac{2kp}{x^3}\hat{i}$

Coulomb's Law (Field)

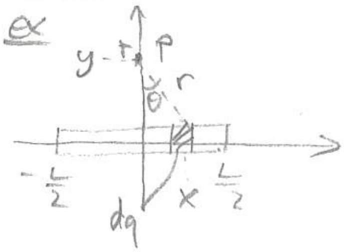
ex



$\vec{E} = \hat{x}\, k \int_0^L \frac{\lambda dx'}{|x-x'|^2} \qquad E_x = \frac{kQ}{x(x-L)}$, $\quad$ as $L \ll x$, $E_x = \frac{kQ}{x^2}$ (as expect)

$E_x = \frac{kQ}{x\, r_{end}}$

ex



$dE = \frac{k\,dq}{r^2}$

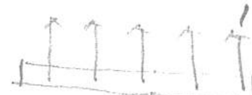
symmetry

$dE_y = \frac{2k\lambda dx}{r^2}\cos\theta \quad \rightarrow \quad E_y = \frac{2k\lambda \frac{L}{2}}{y\sqrt{\frac{L^2}{4}+y^2}}$

$E_y = \frac{kQ}{y\, r_{end}}$

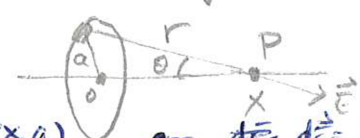
if $y \gg L$, point charge like

if $L \to \infty$, $E_y = \frac{2k\lambda}{y}$



$\boxed{E \propto \frac{1}{y}}$

ex Uniform Ring Charge $\quad \vec{E}$ on axial.



Sym. $dE_x = \frac{k\,dq}{r^2}\cos\theta \qquad \boxed{E_x = \frac{kxQ}{(a^2+x^2)^{3/2}}}$

for $\vec{r} = (x, a)$ or $d\vec{E} = \frac{k\,dq}{r^3}\vec{r} \quad dE_x = \frac{k\,dq\,x}{(a^2+x^2)^{3/2}}$ yield immediately

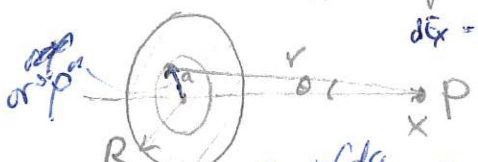
$E_x = \frac{kQ}{r^2}\hat{x}$, $\hat{x} = \frac{x}{r}$

$x \gg a$

$E_x \simeq \frac{kQ}{x^2}$.

$\boxed{E \propto \frac{1}{x^2}}$

ex Uniform Disk Charge



$dE_x = \frac{k\,dq\,x}{(a^2+x^2)^{3/2}}$

Sym. $dE_x = \frac{k\,dq}{r^2}\cos\theta$

$dq = \sigma\, a\, da\, d\phi$

$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$

$E_x = k\int \frac{dq}{r^2}\cos\theta$, $dq = \sigma\rho\, d\rho\, d\phi$ use $E_x = k\frac{dq}{r^3}\vec{r}$ more natural

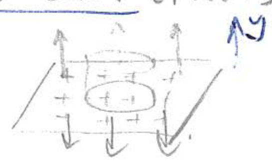
If $x \gg R$, point like charge

If $\boxed{R \gg x \quad E_x = \begin{cases} 2\pi k\sigma & x>0 \\ -2\pi k\sigma & x<0 \end{cases}}$

$\vec{E}$ jump by "$4\pi k\sigma$" $\frac{\sigma}{\varepsilon_0}$

Gauss Law (Field)

ex.

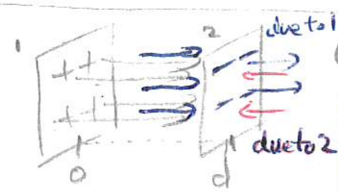


$\underbrace{\int \vec{E}\cdot\hat{n}\, ds}_{2\pi r^2 E} = \frac{\pi r^2 \sigma}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}} \quad y>0$

$-\frac{\sigma}{2\varepsilon_0} \quad y<0$

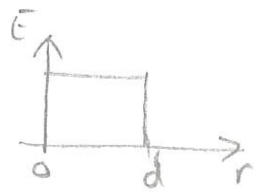
jump $\frac{\sigma}{\varepsilon_0}$ agree

ex

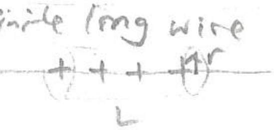


$E_1 = \frac{\sigma}{2\varepsilon_0} \quad E_2 = \frac{\sigma}{2\varepsilon_0} \quad E = E_1 + E_2$

$\boxed{E = \begin{cases} \frac{\sigma}{\varepsilon_0} & 0<x<d \\ 0 & x>|d| \end{cases}}$

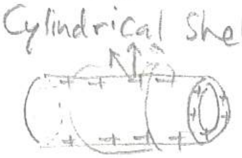


ex Infinite long wire



$\underbrace{\int \vec{E}\cdot\hat{n}\, ds}_{2\pi r L E} = \frac{\lambda L}{\varepsilon_0} \Rightarrow E = \frac{2k\lambda}{r} \quad \boxed{E \propto \frac{1}{r}}$

ex Cylindrical Shell

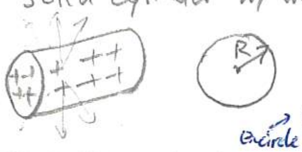


$Q = 2\pi R L \sigma \qquad E\, 2\pi r L = \frac{Q}{\varepsilon_0}$

$\lambda = \frac{Q}{L}$

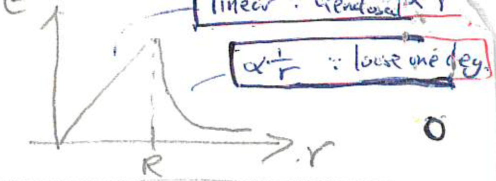
$E = \begin{cases} \frac{2k\lambda}{r} & r>R \\ 0 & r<R \end{cases}$

Solid cylinder w/ uniform charge



$\cancel{\lambda = \sigma \pi R^2}$

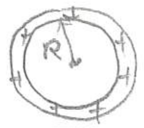
$$E = \begin{cases} \frac{2k\lambda}{r} & r>R \\ \frac{2k\lambda}{R^2} r & r<R \end{cases}$$



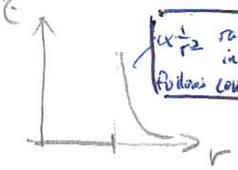
0

encircle by r $\vec{Q}' = \frac{Q r^2}{R^2}$, $\lambda = \frac{Q}{L}$

Spherical Shell (U.C)

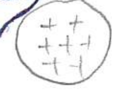


$$E = \begin{cases} \frac{kQ}{r^2} & r>R \\ 0 & r<R \end{cases}$$

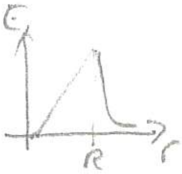


$\propto \frac{1}{r^2}$ radial in all direc. follows coulomb's

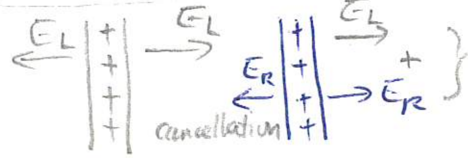
Sphere (U.C)



$$E = \begin{cases} \frac{kQ}{r^2} & r>R \\ \frac{kQ}{R^3} r & r<R \end{cases}$$

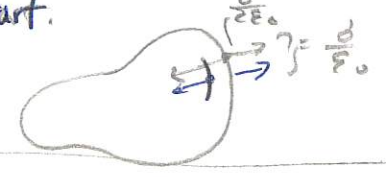


ex



$\} = \frac{\sigma}{\varepsilon_0}$

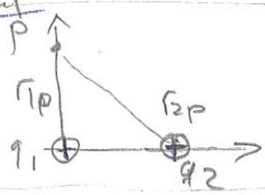
for arbitrary surf.



$\} = \frac{\sigma}{\varepsilon_0}$

potential

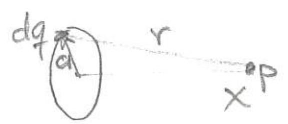
ex



$V(\infty) = 0$ $\quad V = \frac{kq_1}{r_{1p}} + \frac{kq_2}{r_{2p}}$

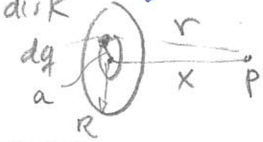
$V = k\int \frac{dq}{r}$ $\quad dq = \rho\,d\rho\,d\phi$

ex ring



$V = k\int\frac{dq}{r}$ $\quad \boxed{V = \frac{kQ}{r}}$

disk

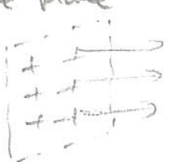


$dq = \sigma 2\pi a\,da$ $\quad \sigma = \frac{Q}{A}$

$V = 2\pi k\sigma(\sqrt{x^2+R^2} - x)$

ex Infinite plane

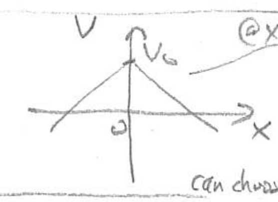
note $x<0$ $\vec{E} = -\frac{\sigma}{2\varepsilon_0}\hat{x}$



$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{x}$

$\Delta V = -\int \vec{E}\cdot d\vec{l} = \frac{\sigma}{2\varepsilon_0} x$

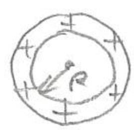
$$V = \begin{cases} V_0 + \frac{\sigma}{2\varepsilon_0}x & x<0 \\ V_0 - \frac{\sigma}{2\varepsilon_0}x & x>0 \end{cases}$$



@ x=0 E has jump
V con't everywhere

can choose V=0 at any point

ex Spherical Shell

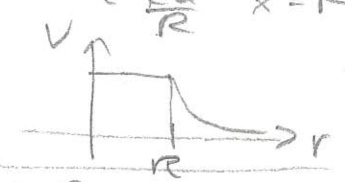


$\vec{E} = \frac{kQ}{r^2}\hat{r}$ $\quad V = -\int_\infty^r \vec{E}\cdot d\vec{r} = +\frac{kQ}{r}$ $\quad V(\infty) = 0 \Rightarrow$

$$V = \begin{cases} \frac{kQ}{r} & x \ge R \\ \frac{kQ}{R} & x \le R \end{cases}$$

$\because (\vec{E}=0)_{inside}$, $\nabla\cdot\vec{E} = -\nabla^2 V = 0 \Rightarrow V = \text{const}$

* Vol. occupies by conductor is equipotential.



ex Infinite line charge

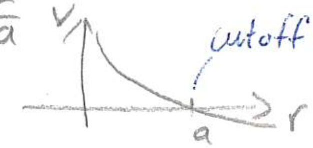
$\vec{E} = \frac{2k\lambda}{r}\hat{r}$

$\int_{r_0}^{r} dV = -\int \vec{E}\cdot d\vec{r} = -2k\lambda \int_{r_{cut}=r_0}^{r} \frac{dr}{r}$

$V = V_0 - 2k\lambda \ln r \Rightarrow V = -2k\lambda \ln\frac{r}{a}$

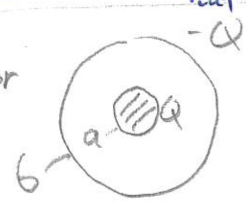
choose $r=a$ $V=0$

for $a \gg 1$, $V \sim V(\infty) = 0$



capacitance

ex cylindrical capacitor



$E_r = \frac{Q}{2\pi\varepsilon_0 L}\frac{1}{r}$; $V = -\int_b^a \vec{E}\,d\vec{r}$ $\begin{matrix} a \text{ (close)} \\ b \text{ (afar)} \end{matrix}$

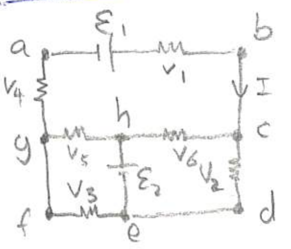
$V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{b}{a}$ $\quad C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$

agree w/ intuition

$\boxed{C \propto L}$

$\Rightarrow \boxed{C \propto \frac{1}{\ln\frac{r_0}{r_1}}}$ b, a

Kichhoff



loop rule

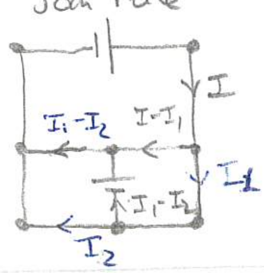
abcga

efghe

cdehc

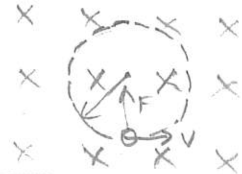
(ex $\varepsilon_2 = V_2 + V_6$)

join rule



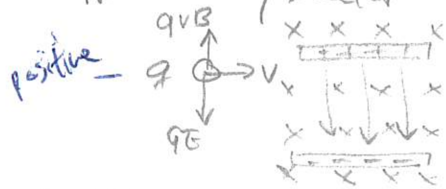
Magnetic Force

ex



$ma_r = qvB$, $r = \frac{mv}{qB}$ $\omega = \frac{qB}{m}$ $T = \frac{2\pi m}{qB}$ $f = \frac{qB}{2\pi m}$

ex App. Velocity selector

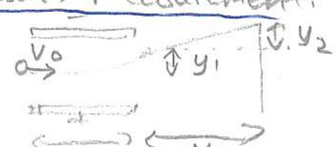


Force balance $v_b = \frac{E}{B}$

if $v > v_b$ q deflects in $\vec{F}_B$ direction

if $v < v_b$ q deflects in $\vec{F}_E$ direction

ex Thomson's Measurement of $\frac{q}{m}$ ratio for electrons.



$\Delta y = \Delta y_1 + \Delta y_2$ $\Delta y_1 = \frac{1}{2} a t_1^2$ $a = \frac{qE}{m}$

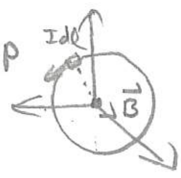
$\Delta y = \frac{1}{2}\frac{qE}{m}\left(\frac{x_1}{v_0}\right)^2 + \frac{qE}{m}\frac{x_1 x_2}{v_0^2}$ $\Delta y_2 = v_y t_2$ $v_y = a t_1$

ex Mass Spectrometer: find $\frac{m}{q}$ ratio, measuring mass of isotopes

$\frac{1}{2}mv^2 = q\mathcal{E}$. $v = \frac{rqB}{m}$ $\Rightarrow$ $\frac{m}{q} = \frac{B^2 r^2}{2\mathcal{E}}$ — B, $\mathcal{E}$ known, r measured.

Biot-Savart Law

ex Center of Loop

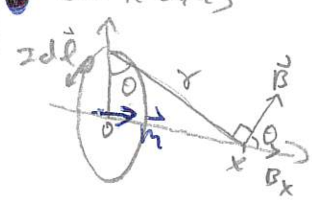


$dB = k\frac{I d\vec{l} \times \hat{r}}{R^2}$

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

$k\frac{2\pi R^2 I}{R^3}$

• on x-axis



$dB = k\frac{I d\vec{l} \times \hat{r}}{r^2}$ — $k\frac{2\pi R^2 I}{r^3}$

$B_x = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(x^2+R^2)^{3/2}}$ $m = IA$

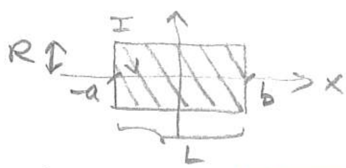
$B_x = \frac{2km}{x^3}$ $x \gg R$

as dipole from afar.

analog:

$E_x = \frac{2kp}{x^3}$

* ex Solenoid . make uniform field like cap.

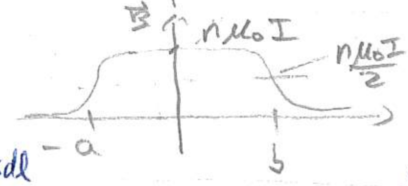


loop density (inside) : $n = \frac{N}{L}$ # turns

$dB_x = k\frac{2\pi R^2 n I dx}{(x^2+R^2)^{3/2}}$

or $B = k\int \frac{d\vec{I} \wedge \vec{r}}{r^3}$ $d\vec{I} = I n dx dl$

If $R \ll a, b$ $\boxed{B = n\mu_0 I}$

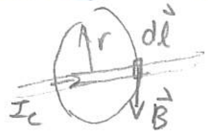


can be understand using Ampere law

$\int \frac{dx}{(x^2+R^2)^{3/2}} = \frac{1}{R^2}\frac{x}{\sqrt{x^2+R^2}}$

Ampére's Law

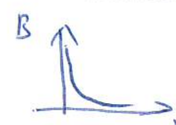
ex Straight line



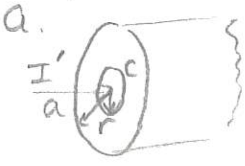
sym.

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_c$

$$\boxed{B = \frac{\mu_0 I_c}{2\pi r}}$$



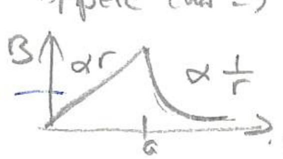
• ex Long wire of radius a.



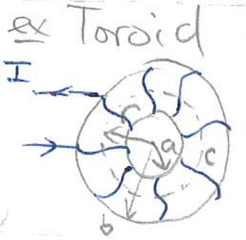
$I' = \frac{r^2}{a^2} I$ ∵ current ∝ area.

Ampère law ⇒ $B = \frac{\mu_0 I}{2\pi a}\left(\frac{r}{a}\right)$ $I < a$

or $\frac{\mu_0 I}{2\pi a^2} r$



ex Toroid



Ampère law ⇒ $B = \frac{\mu_0 N I}{2\pi r}$ $a < r < b$

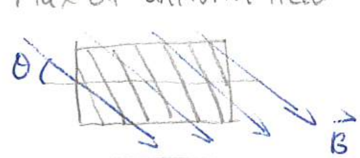
If define $n_t = \frac{N}{2\pi r}$

then $B = n_t \mu_0 I$ like long Solenoid. $\frac{\mu_0 N I}{L}$ for L↑

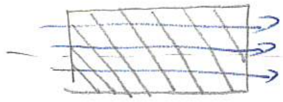
Magnetic flux

$\phi_m \equiv \int \vec{B}\cdot\hat{n}\,dA$ magnetic flux

ex Flux of uniform field through solenoid.

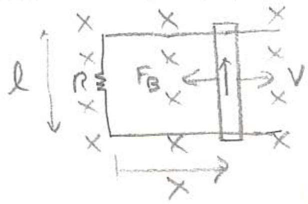


$\phi_m = NBA\cos\theta$



$\phi_m = NBA$

ex Motion Emf



$\phi_m = Blx \quad |\mathcal{E}| = \frac{d\phi_m}{dt} = \underline{Blv}$

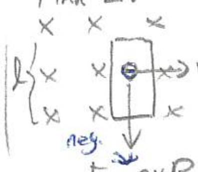
Induced current c.c.w

$\because$ N S S N

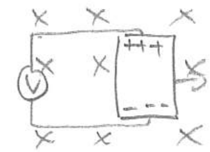
Need Emf induced B_i to keep B_{ic} increases

$\mathcal{E}$ by relative motion of $\vec{B}$ and current path.

Find ΔV



neg. $F = qvB$



Force balance

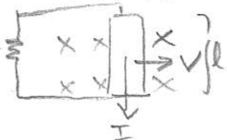
$E = vB$

$\Delta V = El$

$\underline{\Delta V = Blv}$

(kind remind the Hall effect.

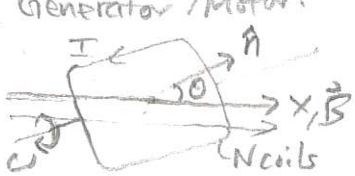
ex Find power input due to induced emf. What's the Joule heating?



Force on rod: $F = IlB$ (Lorentz force) $\quad P = IlBv$

Joule heating: $IlBv = I^2R$

ex Generator/Motor



$\phi_m = NBA\cos(\omega t + \delta)$

$\mathcal{E} = \underbrace{-\omega NBA}_{\mathcal{E}_{max}}\sin(\omega t + \delta)$

ex Atomic Magnetic Moments

para, ferro

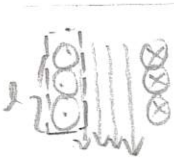
$L = m_q v r \ , \ \mu = I\pi r^2 = \frac{1}{2}qvr \quad$ since $I = \frac{q}{T}$

$\vec{\mu} = \frac{q}{2m_q}\vec{L}$ due to angular momentum.

Inductance

ex Find self inductance

$\phi_m \equiv LI$



$B = n\mu_0 I$

$\phi_m = NBA$

$\Rightarrow \phi_m = \underbrace{n^2 A\mu_0 l}_{L}\, I \qquad [L] = H = \frac{T\cdot m^2}{A}$

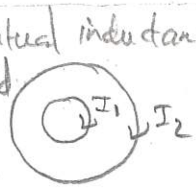
$* \ \mathcal{E} = -L\frac{dI}{dt}$

ex Find mutual inductance.

tightly co-solenoid

length l

r_1, r_2, N_1, N_2

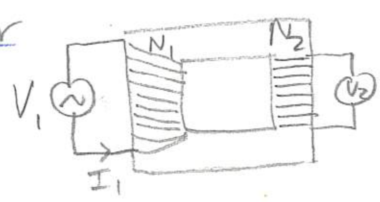


$\phi_{m_{12}} = N_2 B_1 A_1 \Rightarrow M_{12} = \mu_0 n_2 n_1 l\pi r_1^2$

$\phi_{m_{21}} = N_1 B_2 A_1 \Rightarrow M_{21} = \mu_0 n_1 n_2 l\pi r_1^2$

$M_{12} = M_{21}$ in general.

ex Transformer



Given V_1, I_1 find V_2, I_2

• same flux through both coils ϕ_m. • No power loss.

$V_1 = N_1\frac{d\phi_m}{dt} \ , \ V_2 = N_2\frac{d\phi_m}{dt}$

$\frac{V_1}{N_1} = \frac{V_2}{N_2}$ since $V_1 I_1 = I_2 V_2 \Rightarrow \underline{I_1 N_1 = I_2 N_2}$

Energy momentum

ex Plane Wave $E = E_0\sin(kx - \omega t)$

$B = B_0\sin(kx - \omega t)$

$\eta = \frac{EB}{\mu_0 c}$

$= \frac{E_0 B_0 \sin^2(kx - \omega t)}{\mu_0 c}$

$\langle S \rangle = \frac{1}{2}\frac{E_0 B_0}{\mu_0}$

ex Plane Wave given $E_y = E_{y0}\sin(kx - \omega t)$ find B_z.

① $\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$

$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} = -kE_{y0}\cos(kx - \omega t)$

$\therefore B_z = \frac{k}{\omega}E_{y0}\sin(kx - \omega t)$

② or use

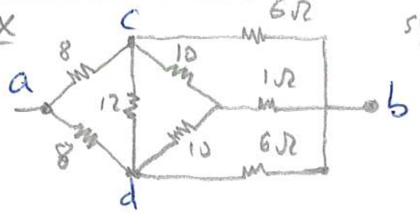
$c\vec{B} = \hat{n}\times\vec{E}$

$B_z = \frac{1}{c}E_y$

$B_z = \frac{k}{\omega}E_y.$

Circuit by symmetry

ex



what's R_{eq}? 6Ω

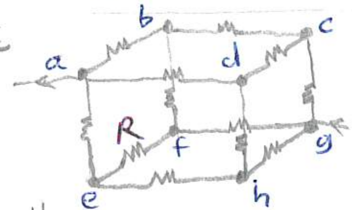
sym. not current through 12Ω

Given

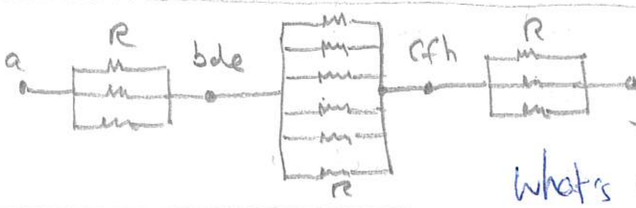
use $R_{eq} \to V_{ab} \to$ solve I_{ab}.

$R_{eq} = 6\Omega$

ex



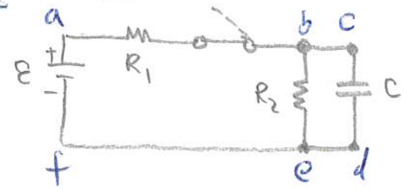
sym. from ag
glue bde
and cfh



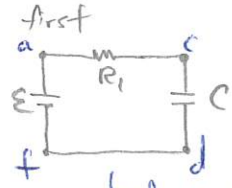
What's R_{eq}? $\frac{5R}{6}$

RC circuit

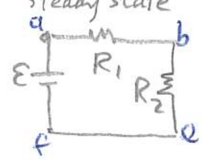
ex



first



Steady state



$Q_f = \varepsilon C$

Work done by battery: $W = \varepsilon^2 C$

energy stored: $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}W$

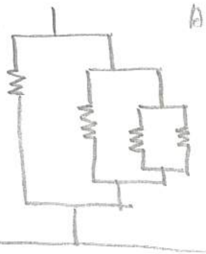
energy lost: $\frac{dW_R}{dt} = I^2 R$, $W_R = \frac{1}{2}\varepsilon C^2$

Joule heating

whats I here?

(half lost in R, half store in C)

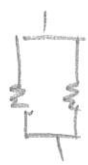
ex



All R Find R_{eq}.

$$\frac{1}{R_{eq}} = \frac{4}{R}$$

Note.



$$\frac{1}{R_{eq}} = \frac{2}{R}$$

$P = \varepsilon I$

$W = \varepsilon Q$

$W_{max} = \varepsilon Q_f$

$= \varepsilon^2 C$

also $\frac{Q_f^2}{C}$ total

& $U = \frac{1}{2}\frac{Q_f^2}{C}$ stored in cap.

half stored in Capacitor
half loss in Resistor as heat,

- E-field
- Scaler potential
- B-field
- vector potential
- Circuit
- Maxwell's equations