Partial Wave & Phase Shift

Sph. sym potential $V(r) \neq 0$ then

$$f(\Theta) = \sum_{l=0}^{\infty} (2l+1) f_l(k) P_l(\cos\Theta)$$

substitute G

$$\langle \vec{x} | \psi^+ \rangle = \underbrace{C_l \frac{e^{ikr}}{r}}_{\text{sphr out wave}} - \underbrace{\frac{e^{-i(kr - l\pi)}}{r}}_{\text{sphr. in wave}}$$ in = out conserv. prod.

$C_l = 1 + i2kf_l(k)$ $\quad C_l = 1$ w/o scatter.

$1 + i2kf_l(k) = e^{i2\delta_l}$ $\longrightarrow$ has geo. expression. also w/ scatter $\Rightarrow$ induced phase shift δ_l

$$\sigma_{tot} = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l$$

$$\langle \vec{x} | \psi^+ \rangle = \psi(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \sum i^l (2l+1) A_l(r) P_l(\cos\Theta) \qquad A_l(r) = e^{i\delta_l} [\cos\delta_l \, j_l(kr) - \sin\delta_l \, n_l(kr)]$$

Can rewrite SE in form $\psi'' + (k^2 - \frac{2m}{\hbar^2} V)\psi = 0$

For sphr. sym $V(r)$, we have
& $r < R$ $\qquad u_l'' + (k^2 - \frac{2m}{\hbar^2} V - \frac{l(l+1)}{r^2}) u_l = 0$

w/ $\qquad u_l = r A_l(r)$

Partial Wave analysis good for low energy approx.

High energy $\Rightarrow$ Eikonal

$\psi \sim e^{\frac{iS(x)}{\hbar}}$ $\quad S(x)$ phase w/ $\nabla S \sim \vec{p}$

sub in SE $\Rightarrow$ $\frac{(\nabla S)^2}{2m} + V = \frac{\hbar^2 k^2}{2m}$

$f(\Theta) = f_{refl} + f_{shad}$

$f_{refl} = \frac{1}{2ik} \sum_{l=0}^{kR} (2l+1) e^{i2\delta_l} P_l(\cos\Theta)$

$f_{shad} = \frac{i}{2k} \sum_{l=0}^{kR} (2l+1) P_l(\cos\Theta)$

Ex $\quad$ High energy hard sphere scattering $\qquad kR \gg 1$

$\sigma_{tot} = \frac{4\pi}{k^2} \sum_{l=0}^{kR} (2l+1) \sin^2 \delta_l$, we have $\tan\delta_l = \frac{j_l(kR)}{n_l(kR)} = -\tan(kR - \frac{l\pi}{2})$

$l \sim kb < kR$ $\quad$ so $\quad \delta_l = -(kR - \frac{l\pi}{2})$.

avg. mo

$$\sigma_{tot} = \frac{4\pi}{k^2} \sum_{l=0}^{kR} (2l+1) \cancel{\sin^2(kR - \frac{l\pi}{2})} \; \frac{1}{2} \qquad \text{where} \quad \sum_{l=0}^{kR} (2l+1) = \left(\frac{2kR+1+1}{2}\right)^2 \approx (kR)^2$$

$= 2\pi R^2$ //

Scattering Theory

$$f(\vec{k}',\vec{k}) = -\frac{mL^3}{2\pi\hbar^2}\langle \vec{k}'|V|\psi^+\rangle$$

$$\frac{d\sigma}{d\Omega} = |f(\vec{k}',\vec{k})|^2 \quad \text{or} \quad \frac{d\sigma}{d\Omega} = \left(\frac{mL^3}{2\pi\hbar^2}\right)^2 \langle \vec{k}'|V|\psi^+\rangle$$

$$\langle \vec{k}'|T|\vec{k}\rangle$$

Optical thm

$$\mathrm{Im}\, f(\theta=0) = \frac{k\,\sigma_{tot}}{4\pi}$$

- ~~$T|i\rangle = V|\psi^+\rangle$~~

~~$\langle\vec{k}|$~~ $|\vec{k}'\rangle \to |\vec{k}\rangle$

$\langle\vec{k}| = \langle\psi^+| - \langle\psi^+|VG_0^+$

~~calculate~~ evaluate

$\langle\psi^+|VG_0^+V|\psi^+\rangle$

Caveats

$T|\vec{k}\rangle = V|\psi^+\rangle$ here $|\vec{k}\rangle$ inc wave ket

$T = V + VG_0V + VG_0VG_0V + \cdots$

LS eqn

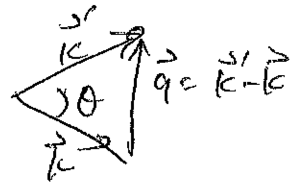
$|\psi^+\rangle = |\vec{k}\rangle + G_0V|\psi^+\rangle$

Green's func

$$G_\pm(\vec{x}',\vec{x}) = \frac{\hbar^2}{2m}\langle\vec{x}'|\frac{1}{E-H_0\pm i\varepsilon}|\vec{x}\rangle$$

Born Approx

- $|\vec{k}'| = |\vec{k}| = k$



~~$q =$~~

$q = 2k\sin\left(\frac{\theta}{2}\right)$

- $V|\psi^+\rangle = T|i\rangle$

$T \simeq V$

$V|\psi^+\rangle = V|i\rangle$

here calculate

$\langle\vec{k}'|V|\psi^+\rangle$

$= \langle\vec{k}'|V|\vec{k}\rangle$