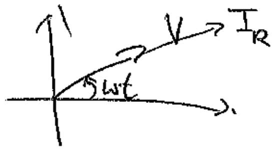
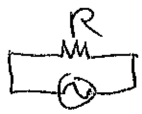
AC circuit

$V(t) = V_s \sin(\omega t)$



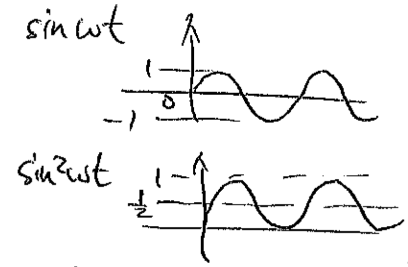
← phasor diagram



$IR = V_s \sin(\omega t)$

b/c oscillating, we take time average $\langle\,\rangle$. But $\langle \sin\omega t\rangle = \langle\cos\omega t\rangle = 0$

so take I^2 instead.



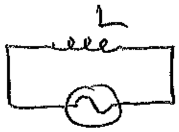
$I^2(t) = \frac{V_s^2}{R^2}\sin^2(\omega t)$

$\langle I^2\rangle = \frac{V_s^2}{2R^2}$

$I_{rms} \equiv \sqrt{\langle I^2\rangle} = \frac{V_s}{\sqrt{2}R} = \frac{V_{rms}}{R}$ w/ $V_{rms} = \frac{V_s}{\sqrt{2}}$ here $Z = R$ Z - ~~impedance~~ impedance

is circuit resistance in AC.

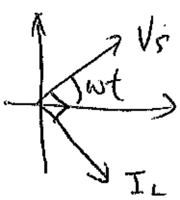
Consider



$L\frac{dI}{dt} = V_s \sin(\omega t)$

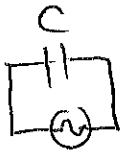
$I = \frac{-V_s}{\omega L}\cos(\omega t)$

$= \frac{V_s}{\omega L}\sin(\omega t - \frac{\pi}{2})$



$X_L \equiv \omega L$

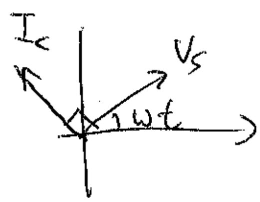
$I_{rms} = \frac{V_{rms}}{Z}$ w/ $Z = X_L$



$\frac{Q}{C} = V_s \sin(\omega t)$

$I = \omega C V_s \cos(\omega t)$

$I = \frac{V_s}{X_C}\sin(\omega t + \frac{\pi}{2})$



$I_{rms} = \frac{V_{rms}}{X_C}$ $X_C = \frac{1}{\omega C}$

$Z = X_C$

LR

$L\frac{dI}{dt} + IR = V_s \sin(\omega t)$

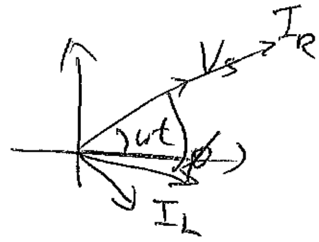
$L\dot{I} + IR = V_s e^{i\omega t}$ (using complex)

Ansatz

$I = I_0 e^{i\omega t + i\phi} = I_0 e^{i\omega t} e^{i\phi}$

$(i\omega L + R) I_0 e^{i\omega t} e^{i\phi} = V_s e^{i\omega t}$

$I_0 = \frac{V_s}{\sqrt{R^2 + X_L^2}}$

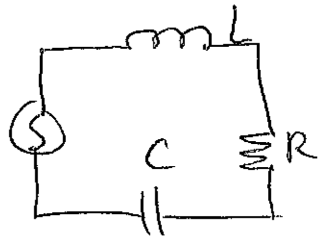


so $I = \frac{V_s}{\sqrt{R^2 + X_L^2}}\sin(\omega t - \phi)$

$I_{rms} = \frac{V_{rms}}{Z}$ w/ $Z = \sqrt{R^2 + X_L^2}$

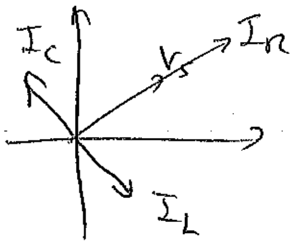
From phasor diagram, $\tan\phi = \frac{X_L}{R}$ (i.e. $\tan\phi = \frac{I_L}{I_R} = \frac{X_L}{R} = \frac{\omega L}{R}$)

Now LRC



$L\dot{I} + IR + \frac{Q}{C} = V_s \sin(\omega t)$

using combination from previous phasor diagram



we have $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$I_{rms} = \frac{V_{rms}}{Z}$$

$$\phi: \tan\phi = \frac{\cancel{R}\; X_C - X_L}{R}$$

or from the math:

Ansatz $\quad Q = Q_0 e^{i\omega t + i\phi}$

$$-\omega^2 L Q + i\omega R Q + \frac{1}{C} Q = V_s e^{i\omega t}$$

$$-\left(iR + (X_C - X_L)\right) \underbrace{i\omega Q_0 e^{i\phi}}_{I_0} = V_s$$

$$I_0 = \frac{V_s}{\sqrt{R^2 + (X_C - X_L)^2}} \quad \text{or} \quad \frac{V_s}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\cancel{Z} \qquad I_{rms} = \frac{V_{rms}}{Z} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Using the phasor diagram, the impedance of AC RC circuit is trivail.