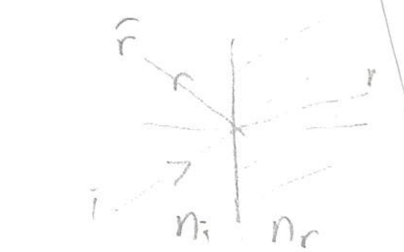
Optics

* reflection $\boxed{\theta_i = \theta_r}$ intensity $I_f = \left(\frac{n_i - n_r}{n_i + n_r}\right)^2 I_o$



$R = \frac{I_f}{I_0}$ $T = \frac{I_r}{I_0}$

$1 = R + T$

* refraction

Snell's Law $\boxed{\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_r}{n_i}}$ Intensity $I_0 = I_f + I_r$

plane wave $e^{i(kx - \omega t)}$
$n = \frac{c}{v}$ $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $v = \frac{1}{\sqrt{\mu\varepsilon}}$
$\frac{\omega}{k} = v$ $k = \sqrt{\mu\varepsilon}\,\omega$

total internal reflection
$\theta_r = 90°$ $(n_i > n_r)$ $\boxed{\sin\theta_c = \frac{n_r}{n_i}}$

from $n = \frac{c}{v}$ → dispersion under same frequency $\boxed{\lambda_r = \frac{\lambda}{n_r}}$

$n=1$ | n_r w/ snell's law $n_r \sin\theta_r = \frac{\lambda}{\lambda_r} \sin\theta_r = \text{const}$

~~$\lambda_r \nearrow n_r \searrow \theta_r$~~

λ_r → red → violet (analogue)

Quantum (Scattering)

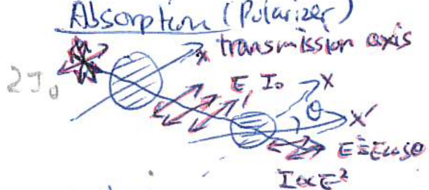
$R = \left(\frac{k-k'}{k+k'}\right)^2$

step potential, refraction

$n_r = \frac{\lambda}{\lambda_r}$ dependence of refrac index on λ_r
so λ fixed $\lambda_r \searrow \sin\theta_r \searrow \theta_r \searrow$
$\lambda_r \nearrow \sin\theta_r \nearrow \theta_r \nearrow$
blue ⟹ red
380nm 780nm

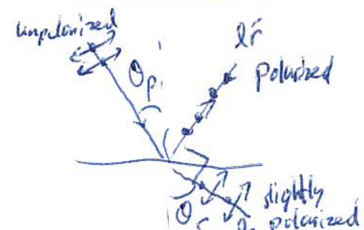
* Polarization.

Absorption (Polarizer)



Malus's Law
$\boxed{I = I_0 \cos^2\theta}$

Reflection.



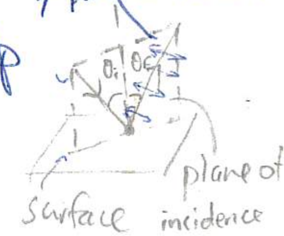
Brewster's Law: At θ_p reflected ray polarized and $\theta_p + \theta_r = 90°$

$\boxed{\tan\theta_p = \frac{n_r}{n_i}}$ Brewster's angle

• Polarized incident ray has no reflected ray at θ_p

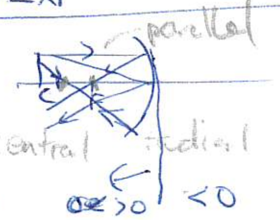
$l_f \perp l_r$

reflected light linearly polarized: $\vec{E}$ vector
$\perp$ plane of incident
$\parallel$ plane of surface



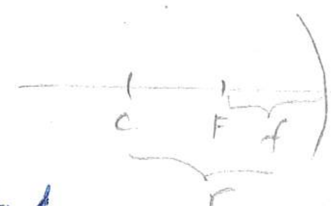
Spherical mirror

• ray diagram
• sign convention
r, f + concave − convex



$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad f = \frac{r}{2}}$

$m = \frac{y'}{y} = -\frac{s'}{s}$



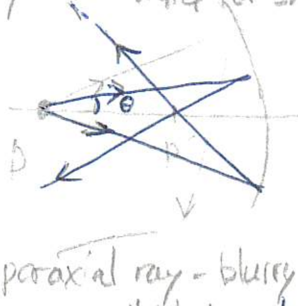
~~Img tracing~~ Image tracing (concave mirror)

behind F Img real inverted
S behind C Img between CF small
S between CF Img behind C large
S at F $y' \sim \infty$
S between FV, Img erected large virtual

(convex)

Img erected & virtual between VF
y' at V, $y'_{max} \simeq y$

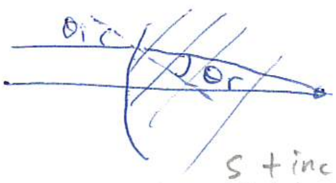
paraxial ray:
• same focus point
• parallel rays are paraxial
• valid for small θ



nonparaxial ray - blurry image "spherical aberation"

worth remember

Image by refraction



s + inc
s' + trans.
r, f − inc
+ trans.

$$\boxed{\frac{n_i}{s} + \frac{n_r}{s'} = \frac{n_r - n_i}{r}}$$

$$\boxed{m = \frac{y'}{y} = -\frac{n_i}{n_r}\frac{s'}{s}}$$

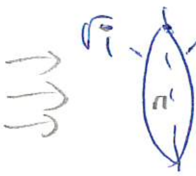
- sign convention carries to lens

$$\frac{n_i}{s} + \frac{n_r}{s'} = \frac{n_r - n_i}{r}$$

Thin lens

lens maker eqn.

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{r_i} - \frac{1}{r_t}\right)}$$

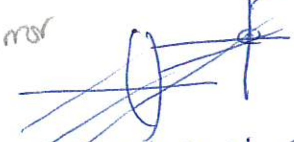


transmission.
same f on both side

thin lens eqn.

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

same as mirror



focal plane.

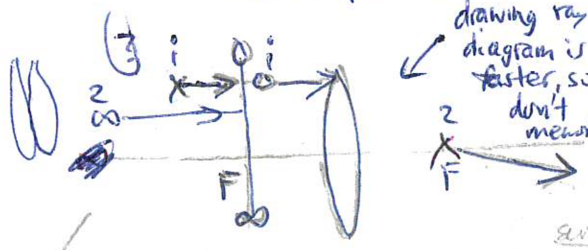
sign conversion
r, f + convex ∵ use r, f on tran. side
r, f − concave ∵ use r, f on inc. side

$$\boxed{m = \frac{y'}{y} = -\frac{s'}{s}}$$

parallel light falls on focal plane

thin lens in contact

$$\boxed{\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}}$$



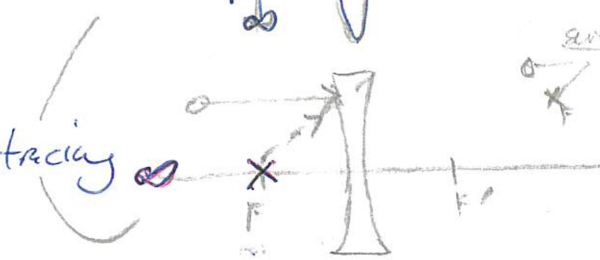
drawing ray diagram is faster, so don't memorize

lens Power

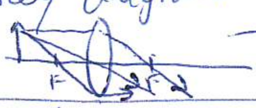
$$\boxed{P = \frac{1}{f}}$$

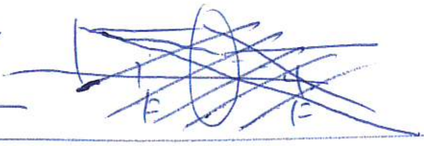
~~diopter~~ diopters [m^{-1}]

img tracing



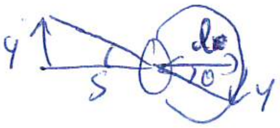
ray diagram





Instruments

eye



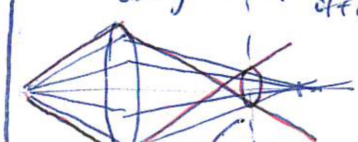
$\theta = \frac{y}{s}$ (conventional measure for retaining size)

$y' = \frac{y}{s} l_r$

l_r length of retina. $l_r \approx l$.

Lens Aberration

astigmatism - object off axis



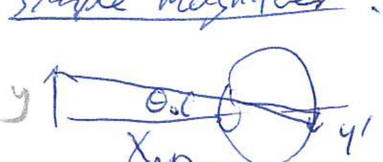
circle of confusion

type of aberration

- coma
- astigmatism
- chromatic

not from lens defect

Simple Magnifier



see object as from afar

$$M = \frac{\theta}{\theta_0} = \boxed{\frac{x_{np}}{f}}$$

Camera

define size of aperture

- f-num. $= \frac{f}{D}$ (f: lens focal length; D: aperture diameter)

(f-num 22 = write f/22) ⇒ $D = \frac{f}{\text{fnum}} = \frac{f}{22}$

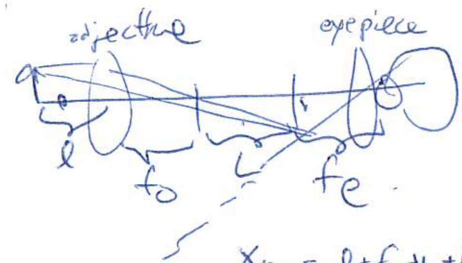
D large exposure time ~~small~~ short $D^2 \propto \frac{1}{t}$

or $t \propto \frac{1}{D^2}$

instrument

Compound microscope

see close object
magnify by eye-piece
as seeing it afar

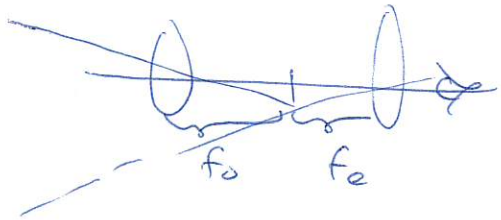


$X_{np} = l + f_o + L + f_e$

$$\boxed{M = m_e m_o = -\underbrace{\frac{X_{np}}{f_e}}_{m_e}\underbrace{\frac{L}{f_o}}_{m_o}}$$

Telescope

see distant objective
magnify by eye piece



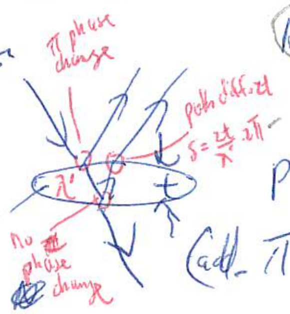
$$\boxed{M = -\frac{f_o}{f_e}}$$

Δx path diff.
δ phase diff.
based mainly on
$\delta = \frac{\Delta x}{\lambda} 2\pi$
&
amplitude
$2A_o\cos\left(\frac{\delta}{2}\right)$
of course w/ bracket eqns for double slits

Interference

thin film

Amplitude $2A_o\cos\left(\frac{\delta}{2}\right)$ harmonic wave.



$\lambda_r = \frac{\lambda}{n_r}$

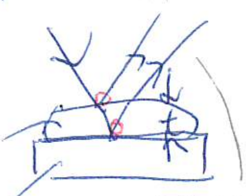
$\delta_{tot} = \delta + \pi$

path diff $2t$

(add. π phase change)

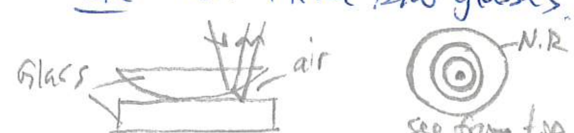
$$\left[\begin{array}{ll} \frac{2t}{\lambda'} = m & \text{destr.} \\ \frac{2t}{\lambda'} = m + \frac{1}{2} & \text{constr.} \end{array}\right]$$

(no add π) $\delta_{tot} = \delta = \frac{2t}{\lambda'}$



$$\left[\begin{array}{ll} \frac{2t}{\lambda'} = m & \text{constr.} \\ \frac{2t}{\lambda'} = m + \frac{1}{2} & \text{destr.} \end{array}\right]$$

ex air film btw glasses



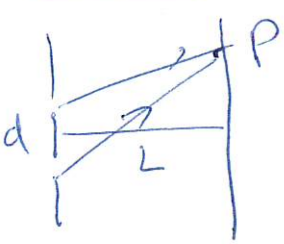
$\delta_{tot} = 2\pi \frac{2t}{\lambda'} + \pi$

near contact path diff. $= 0$
$\Rightarrow$ destructive central region.

Double slits

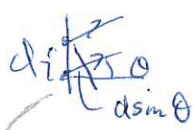
(No π) path diff $= d\sin\theta$

phase change
$\delta = \frac{2\pi}{\lambda} d\sin\theta$



$$\left[\begin{array}{ll} d\sin\theta = m\lambda & \text{constr.} \\ d\sin\theta = (m+\frac{1}{2})\lambda & \text{destr.} \end{array}\right]$$

or $\frac{d\sin\theta}{\lambda} = \begin{cases} m & (C) \\ m+\frac{1}{2} & (D) \end{cases}$



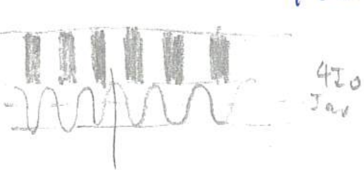
rays near parallel
assume L far enough.

fringe spacing Δy when $\Delta m = 1$

$$\boxed{y_m = m\lambda\frac{L}{d}}$$

Intensities

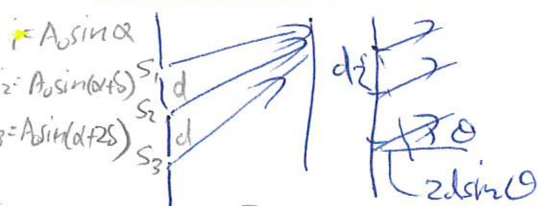
$$\boxed{I = 4I_o\cos^2\left(\frac{\delta}{2}\right)}$$



$I_{av} = 2I_o$

from EM wave
$E_1 = A_o\sin\omega t$
$E_2 = A_o\sin(\omega t + \delta)$
$E = 2A_o\cos(\frac{\delta}{2})\sin(\omega t + \frac{\delta}{2})$
$I \propto E^2$

Multiple slits



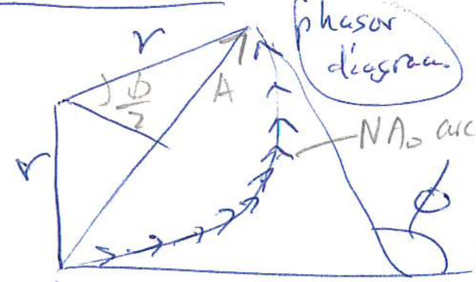
@ $\delta = 0$ Amplitude $3A_0 \Rightarrow I = 9I_0$

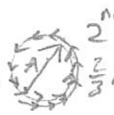
$$\boxed{\frac{d\sin\theta}{\lambda} = m \quad m \in \mathbb{N}}\ \text{max} \quad \text{as usual}$$

for N # slits

intensity $\boxed{I = N^2 I_0 \qquad N-2 \quad 2^{nd} \text{ max}}$

Diffraction





$A = \frac{2}{3\pi} A_{max}$

$$\boxed{I = I_0 \left(\frac{\sin\left(\frac{\phi}{2}\right)}{\frac{\phi}{2}} \right)^2}$$

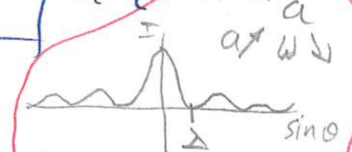
as usual

$$\boxed{\frac{a\sin\theta}{\lambda} = m \quad m \in \mathbb{N}}\ \text{min}$$

$\phi = \frac{2\pi}{\lambda} a \sin\theta$ phase change

$w \propto \frac{1}{a}$

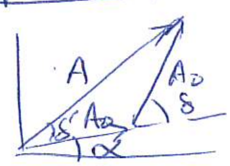
$a \nearrow w \searrow$



$\delta = \frac{2\pi}{\lambda} d\sin\theta$

$\underbrace{\delta_{tot}}_{\phi} = \frac{2\pi}{\lambda} a \sin\theta$

phasor

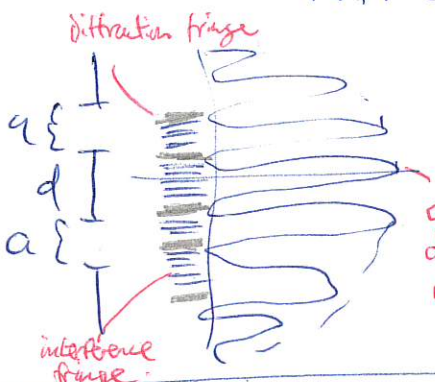


$\vec{E}_1 = A_0 \sin(\alpha)$ $\quad \vec{E}_2 = A_0 \sin(\alpha + \delta)$

$\Rightarrow \delta' = \frac{\delta}{2}$

$E = E_1 + E_2 = 2A_0 \cos\left(\frac{\delta}{2}\right) \sin\left(\alpha + \frac{\delta}{2}\right)$

Double slits Interference. Diffraction



$$I = 4I_0 \cos^2\left(\frac{\delta}{2}\right)\left(\frac{\sin\frac{\phi}{2}}{\frac{\phi}{2}}\right)^2.$$

$a\sin\theta = m\lambda$, min diff.

$d\sin\theta = m\lambda$ max inter.

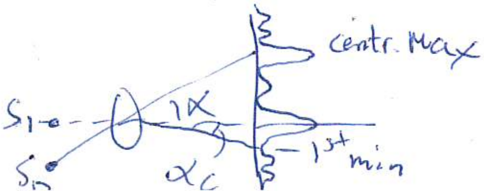
Fraunhofer diffrac. pattern

Fresnel

closer to screen.

Fresnel

Diffraction & Resolution



Rayleight's criterion for resolution

$\alpha_c = 1.22 \frac{\lambda}{D}$ — for microscope $\lambda \searrow$

— for telescope $D \nearrow$

grating Diffrac.

$R = \frac{\lambda}{\Delta\lambda} = mN$

Power for spectroscope.

N # of slits

Interference :

Superposition of harmonic wave.

Based on path difference Δx
& phase difference δ

$\delta = 2\pi \frac{\Delta x}{\lambda}$ ~~$\delta = k\Delta x$~~

$\delta = k\Delta x$; $k = \frac{2\pi}{\lambda}$

* Amplitude :

$\boxed{2A_0 \cos\left(\frac{\delta}{2}\right)}$ for ~~rational~~ ~~int~~ val. $\frac{\Delta x}{\lambda} = \begin{cases} m \in \mathbb{Z} & (C) \\ m + \frac{1}{2} & (D) \end{cases}$ $\in \mathbb{Q}$ $\in \mathbb{Q}$

some details

$y_1 = A\sin(kx_1 - \omega t)$

$y_2 = A\sin(kx_2 - \omega t + \delta)$

same k, ω $y = 2A\cos\left(\frac{\delta}{2}\right)\sin\left(kx - \omega t + \frac{\delta}{2}\right)$

in phase $\delta = 0$ $A \to 2A$ (C)

out phase $\delta = \pi$ $A \to 0$ (D)

coherent source : 2 sources having const. phase . diff

incoherent : phase diff. varies randomly.

example. thin films

π phase change : 1st ray hits $n' > 1$

assume the reflected refracted rays ⊥ surface $\boxed{\lambda' = \frac{\lambda}{n'}}$

air, n', t, air

$\delta_{tot} = 2\pi \frac{2t}{\lambda'} + \pi$

$= k'(2t) + \pi$

no π phase change splits ray goes from $n' > 1$ to $n \sim 1$

$\frac{2t}{\lambda'} = \begin{cases} m & (D) \\ m + \frac{1}{2} & (C) \end{cases}$ $m = 0, 1, 2, \ldots$

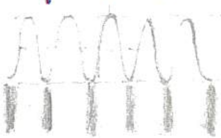
or

n: 1, 1.3, 1.5

$\delta_{tot} = 2\pi \frac{2t}{\lambda'}$ $\frac{2t}{\lambda'} = \begin{cases} m & (C) \\ m + \frac{1}{2} & (D) \end{cases}$

double slits.

Assume L long enough rays appear //



$\delta = 2\pi \frac{d\sin\theta}{\lambda}$

$\frac{d\sin\theta}{\lambda} = \begin{cases} m & (C) \\ m + \frac{1}{2} & (D) \end{cases}$

d, θ, L, y_m

fringe position $\boxed{y_m = m\frac{\lambda L}{d}}$

Intensity

$\boxed{I = 4I_0\cos^2\left(\frac{\delta}{2}\right)}$

$m = 0, 1, 2 \ldots$

multiple slits

$\frac{d\sin\theta}{\lambda} = \begin{cases} m & (C) \\ m + \frac{1}{2} & (D) \end{cases}$

max $I = N^2 I_0$ # slits

# of 2nd max $N-2$ 2nd max

ex 4 slits 1st min
$\frac{\pi}{2} = \delta \Rightarrow \frac{\lambda}{4d} = \sin\theta$
solve θ then $\tan\theta = \frac{y}{L}$
can find y. if θ small
$\sin\theta \sim \tan\theta$; $y = \frac{\lambda L}{4d}$

see ex for technique on phasor diagram.

Diffraction

$\frac{A}{2}$, $\frac{\phi}{2}$, A_{max}, $A_{max} = r\phi$, $\frac{A}{2} = r\sin\frac{\phi}{2}$ ~~$I = A_{max}$~~

$I \sim A^2$

$\delta = \frac{2\pi}{\lambda}d\sin\theta$

single slits

$\delta_{tot} = \frac{2\pi}{\lambda}a\sin\theta$
ϕ

$\frac{a\sin\theta}{\lambda} = \begin{cases} m & \text{min (d)} \\ m + \frac{1}{2} & \text{max (c)} \end{cases}$

$I = I_0\left(\frac{\sin\left(\frac{\phi}{2}\right)}{\frac{\phi}{2}}\right)^2$

$m = 1, 2, 3 \ldots$

phasor diagram ; 2nd max

overall length of the curve

$C = \frac{1}{3}A_{max} = \frac{\pi A}{2}$

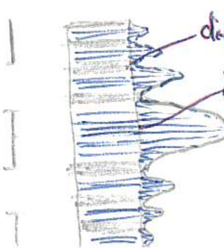
$\Rightarrow A = \frac{2}{3\pi}A_{max}$ $I \sim A^2 \Rightarrow I = \frac{4}{9\pi^2}I_0$

$\boxed{\omega \propto \frac{1}{a}}$

$a \ll \lambda$ $\omega \to \infty$ point source no zero I

$a \gg 1$ $\omega \to 0$

Double slits Interference Diffraction.



$\frac{a\sin\theta}{\lambda} = m_d$ min

$\frac{d\sin\theta}{\lambda} = m_i$ max

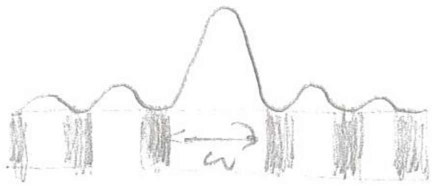
$d > a$. So higher freq. for interference & diffraction ⇒ envelope

$$I = 4I_0\underbrace{\cos^2\left(\frac{\delta}{2}\right)}_{\text{int.}}\underbrace{\left(\frac{\sin\frac{\phi}{2}}{\frac{\phi}{2}}\right)^2}_{\text{diff}}$$

$m = 0, 1, 2, \ldots$

Classification of Diffraction

- Fraunhofer pattern



assumptions.

(1) plane wave incident normally on slit

(2) $L >> a$

note $a >> \lambda$ (ie 1000λ) no fraunhofer pattern. b/c very small minium (why?)

~~b/c $\left(\frac{a\sin\theta}{\lambda} = m\right)_{min}$ θ and w small, rays appears parallel always.~~

Fresnel pattern

as L near a,

App. of Fraunhofer diffraction

"Resolution"

large angles to seperate light source

when 1^{st} min of S_1 falls on central max of S_2 we have first diffraction minimum substended by angle

$$\frac{D\sin\theta_c}{\lambda} = 1.22 \qquad \theta_c \text{ small}$$

define. $\boxed{\theta_c = 1.22\frac{\lambda}{D}}$ Rayleigh's criterion.

$\theta > \theta_c$  image not overlap.

$\theta \sim \theta_c$  unresolved resolution

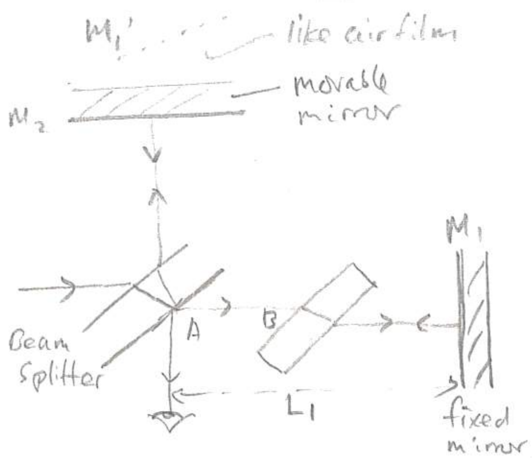
For clear resolution,

$\lambda \searrow$ for microscope

$D \nearrow$ for telescope.

~~b/c for large angle.~~ Want θ_c small to fit in our vision.

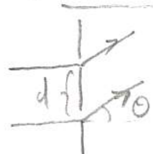
Michelso Interferometer (act like air film)



App

- redefine standard meter (measured Krypton 86 emitted light)
- measure n_{air} $\lambda' = \frac{\lambda}{n}$ (↓ air)
- measure light speed difference.

Diffraction Gratings



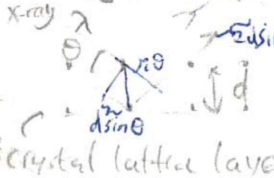
$d \sim 10^{-4}$ cm

interference due to reflection on ridges.

- maxima; $\frac{d\sin\theta}{\lambda} = m$ — order number
- more source shaper maxima.
- resolution power of spectroscope for nearly equal λ_1, λ_2 assume $\lambda_1 \sim \lambda_2 \sim \lambda$

- $R = \frac{\lambda}{|\Delta\lambda|} = mN$ — # slits, m — order #

Bragg's Diffraction



$$\frac{2d\sin\theta}{\lambda} = m \text{ max}$$

m — order

$m = 1, 2, 3, \ldots$

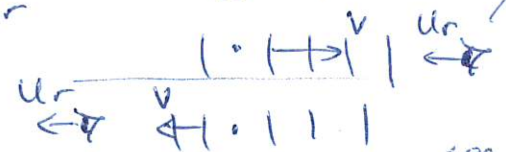
App. Hologram.

Wave and Oscillation

Doppler effect. (moving observer) approach

~~wave velocity~~ propertie as stationary: v, f_0, λ_0

$v > u_r$ (assume)



away

see λ_0 fixed
see more # wave
$f \nearrow$

see less # wave
$f \swarrow$

f_+
$f_{ap} = \dfrac{v + u_r}{\lambda_0}$

f_-
$f_{aw} = \dfrac{v - u_r}{\lambda_0}$

$v = \lambda_0 f_0$ $\quad \cancel{\lambda_0 = \dfrac{v}{f_0}}$

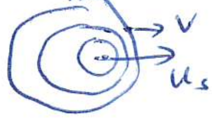
Simplified

$$\boxed{f_{\pm} = f_0\left(\frac{v \pm u_r}{v \mp u_s}\right)}$$

upper sign approach
lower sign away

(moving source) approach

$v > u_s$ (assume)

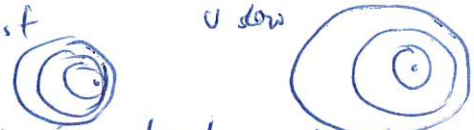


fixed.
see packed wave
diff. λ than λ_0
wave see in Δt

λ_+
$\lambda_{ap} = \dfrac{v - u_s}{f_0}$

λ_-
$\lambda_{aw} = \dfrac{v + u_s}{f_0}$

v fast v slow



closely packed loosely packed.
f_0 fixed by source

actual freq. experience by observer.

$f_{ap,act} = \dfrac{v}{\lambda_{ap,act}}$
$f_{\pm} = \dfrac{v}{\lambda_{\pm}}$ $\quad \lambda_+ < \lambda_+ \Rightarrow f_+ > f_-$

Rel. Doppler Shift

freq. source's rest frame.

~~moving source~~ general $\quad f = \dfrac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} f_0$

$\theta = 0$ at
$\theta = \pi$ away.

~~moving observer~~ $\quad f = f_0 \sqrt{\dfrac{1 \pm \beta}{1 \mp \beta}}$ same sign convention

Wave. Transport eqn. $a u_x + b u_y = 0$

$\nabla u \cdot \vec{v} = 0$

$\Rightarrow u$ constant along $ay - bx = $ const.

$\Rightarrow u(x,y) = f(ay - bx)$

Geometry

Wave eqn.

$u_{tt} - c^2 u_{xx} = 0 \qquad -\infty < x < \infty$

soln. $u(x,t) = f(x+ct) + g(x-ct)$.

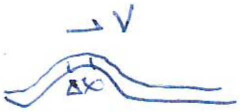
IVP If Given $u(x,0) = \phi(x)$
$u_t(x,0) = \psi(x)$

the D'Alembert soln:

$$u(x,t) = \frac{1}{2}[\phi(x+ct) + \phi(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(s)\,ds.$$

Energy transmitted by string. total $E = \frac{1}{2}KA^2 \quad K = m\omega^2 \quad \cancel{m = v\Delta t\rho}$

$\to v$



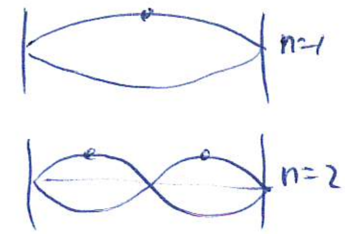
$\Delta x = vt$
$m = \rho\Delta x$

$\cancel{P = \frac{1}{2}\omega^2 A^2 v \mu}$ $\quad P = \frac{1}{2}\rho v \omega^2 A^2$.

wave properties material properties.

Standing Wave

(fixed ends)

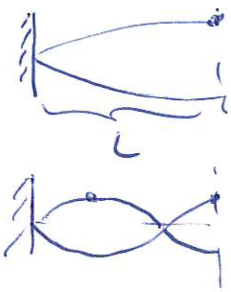


$n=1 \Rightarrow \lambda_n = \frac{2L}{n}$ (n is node the peak of slope/hill)

$f_n = \frac{v}{\lambda_n}$ $v = \sqrt{\frac{T}{\rho}}$

T tension
ρ density

(fixed one end)

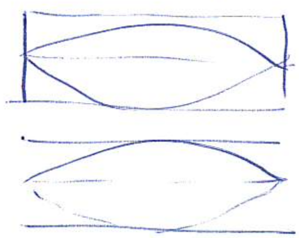


$\lambda_n = \frac{4L}{n}$ $n = 1, 3, 5 \ldots$

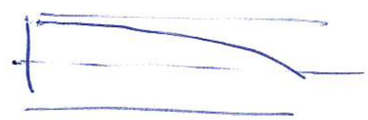
So even harmonic is missing.

Standing sound wave.

close pipe / Open pipe



half open pipe



Beats

Inteference of two waves of different but nearly eq. freq. result in alternate loud and soft tone intensity.

beat frequency

$2\pi\Delta f = \Delta\omega$

$2\pi f_{av} = \frac{\omega_1+\omega_2}{2}$

$P_1 = P_0 \sin \omega_1 t$

$P_2 = P_0 \sin \omega_2 t$

$\omega_1 \sim \omega_2$

$p = 2p_0 \cos\left(\frac{2\pi\Delta f}{2} t\right) \sin(2\pi f_{av} t)$

↑ envelop

Decibels (dB)

human hearing, sound intensities ~ logarithmic.

$\boxed{\beta = 10 \log \frac{I}{I_0}}$

I_0 hearing threshold ~~10^{-12} W/m² ~ 1 W/m~~

$(10^{-12} \sim 1)\ W/m^2$

Rayleigh Scattering

size of particle a, given wavelength λ

3 regim: $\underbrace{\lambda \ll a}_{\text{geometric optics}}$ $\underbrace{\lambda \sim a}_{\text{Mie scattering theory}}$ $\underbrace{\lambda \gg a}_{\text{Rayleigh Scattering}}$

$I \propto I_0 \lambda^{-4} a^6$

$\lambda^{-4} \Rightarrow$ short wavelength scatter more strongly
So we see blue

- Sunset, we receiving light that not scattered away, we see red