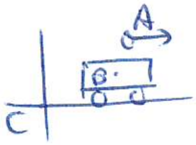
SR

Principle of Relativity

1. Laws of physics apply in all inertial reference sys.
2. Universal speed of light: same in all inertial observers & in all directions.



Galileo's vel addition:

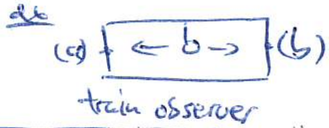
$$V_{ac} = V_{ab} + V_{bc}$$

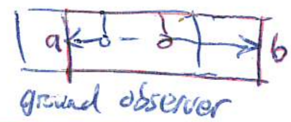
Einstein's vel addition:

$$V_{ac} = \frac{V_{ab} + V_{bc}}{1 + \frac{V_{ab}V_{bc}}{c^2}}$$

Geometry of Relativity

i) Relativity of Simul.: Two events simul. in a inertial sys., are not necessary in another.





event (a) occurs first then (b).

(light speed as time measurement.)

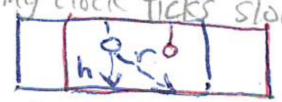
ii) Time dilation: $\gamma \overline{\Delta t} = \underbrace{\Delta t}_{\text{dilate}}$ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ "moving clock ticks slower"

proper time taken in the rest frame of moving clock

$\overline{\Delta t}$ clock on train

Δt clock on ground

time taken on ground measuring duration of event (the moving frame)



longer travel dist. as seen on ground.

Paradox: Observer on train say clock slow on ground.

$\because$ rel. simul. clock syn on train not seen syn on ground.

ex take 1 min to measure light ray reaching the floor on ground while 1s on train so train clock ticks slower.

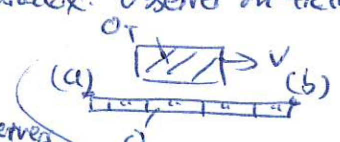
note that ruler moving along w/ $\bar{S}$

iii) Lorentz contraction: $\overline{\Delta x} = \underbrace{\gamma \Delta x}_{\text{contracted}}$ "moving object contracted in direction of motion"

$\Delta x = \frac{\overline{\Delta x}}{\gamma}$ proper length: length of ruler measure on the rest frame of moving $\bar{S}$

length of moving ruler observed from ground

Paradox: Observer on train see length contracted on ground.



$\because$ ~~rel~~ rel. simul.

O_T says O_g measure (b) first then (a).

ex Barn Ladder Paradox.

{daughter runs with ladder into the barn. farther observed from ground}

a. ladders end makes it in door

b. ladders front hit barn wall

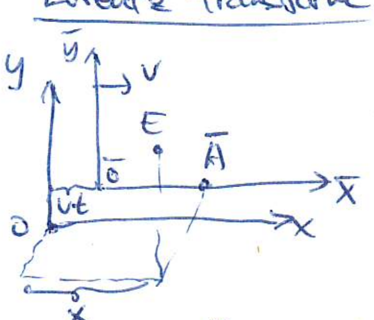
$\Rightarrow$ framer see (a) then (b)

daughter see (b) then (a)

$\because$ rel. simul.

iv) Dimensions $\perp$ to velocity are not contracted!

Lorentz transform.



$\overline{\Delta x}, \bar{x}, \overline{\Delta t}, \bar{t}$ on $\bar{S}$

$\Delta x, x, \Delta t, t$ on S

~~Find event coord. in $\bar{S}$~~

Find event coord. in $\bar{S}$ (i.e. stand in $\bar{S}$)

$$\bar{x} = \gamma(x - vt)$$
$$\bar{y} = y$$
$$\bar{z} = z$$
$$\bar{t} = \gamma\left(t - \frac{v}{c^2}x\right)$$

reverse

$$x = \gamma(\bar{x} + v\bar{t})$$
$$y = \bar{y}$$
$$z = \bar{z}$$
$$t = \gamma\left(\bar{t} + \frac{v}{c^2}\bar{x}\right)$$

or Lorentz transform $x^0 \equiv ct$ $\beta \equiv \frac{v}{c}$

$$\bar{x}^0 = \gamma(x^0 - \beta x^1)$$
$$\bar{x}^1 = \gamma(x^1 - \beta x^0)$$
$$\bar{x}^2 = x^2$$
$$\bar{x}^3 = x^3$$

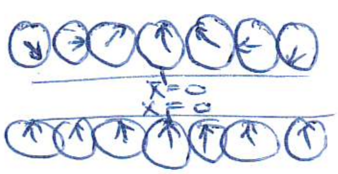
$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Lorentz transf. matrix

$$\bar{x}^\mu = \Lambda^\mu_\nu x^\nu$$

- more exploration on simul. event A, B simul. @ S but not nesscessary on $\bar{S}$: $\bar{t}_A = \gamma(t_A - \frac{v}{c^2}x_A) \neq \bar{t}_B = \gamma(t_B - \frac{v}{c^2}x_B)$
- simul. $\leftrightarrow$ time syn.

@ $t=0, x=0, \bar{t} = -\frac{v}{c^2}\bar{x}$



also explains Time dilation paradox

<u>4 vectors</u>. covariant $a_\mu = (a_0, a_1, a_2, a_3) = (-a^0, a^1, a^2, a^3)$ only $a_0 = -a^0$

contravariant $a^\mu = (a^0, a^1, a^2, a^3)$

<u>4 dimensional scalar product</u> – invariant under Lorentz transform.

$$a_\mu b^\mu = a^\mu b_\mu = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3$$

<u>More 4 vectors</u>: Displacement $x^\mu = \overset{(x^0, \vec{x})}{(ct, \vec{x})}$ Velocity $\eta^\mu = \gamma(c, \vec{v})$

Energy-displacement. $p^\mu = (\frac{E}{c}, \vec{P}) = (\gamma mc, \underbrace{\gamma m\vec{v}}_{\vec{P}\text{ — rel. mom.}})$

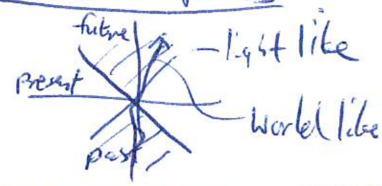
current density $j^\mu = (c\rho, \vec{J})$ wave vector: $k^\mu = (\omega, c\vec{k})$

<u>Interval</u>: $I = \Delta x^\mu \Delta x_\mu = \underline{-c^2t^2 + d^2}$ Invariant under L.T

$I < 0$ Timelike $I > 0$ Spacelike $I = 0$ lightlike

- Lorentz transform change coord. from $(\vec{x}, t)$ to $(\bar{\vec{x}}, \bar{t})$ lies on same hyperbola.

<u>Minkowski diagrams</u>



<u>L.T velocity</u> $\bar{\eta}^\mu = \Lambda^\mu_\nu \eta^\nu$

$\vec{\eta} = \frac{d\vec{l}}{d\tau}$ τ proper time (own time).

<u>ordinary transform</u>. Don't memorized but derive!

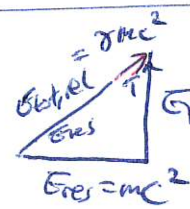
$$\boxed{\bar{u}_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad \bar{u}_{y,z} = \frac{u_{y,z}}{\gamma(1 - \frac{u_x v}{c^2})}}$$

this recovers Einstein's velocity equation

<u>4 momentum</u>: $p^\mu = (\underbrace{\gamma mc}_{\frac{E}{c}}, \underbrace{\gamma m\vec{u}}_{\vec{P}})$ m rest mass

$p^\mu = (p^0, \vec{P})$

$\bar{p}^\mu = \Lambda^\mu_\nu p^\nu$



$E = \gamma mc^2$

$p = \gamma mv$

$m = 0 \Rightarrow E = p \Rightarrow v = c$

<u>ex</u> $\underset{m}{\bigcirc} \rightarrow \frac{3}{5}c$ $\frac{3}{5}c \leftarrow \underset{m}{\bigcirc}$ (bf) $\quad \overset{M = ?}{\bigcirc}$ (af)

<u>Energy conserved</u>

bf. each w/ $E = \gamma mc^2$

af $E = Mc^2$

$\rightarrow$ 3-4-5 rule $M = \frac{5}{2}m$

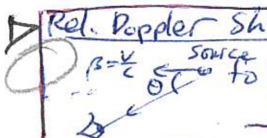
<u>ex</u> pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ Find outgoing muon energy.

$\mu^+ \rightarrow \bar{\nu}_\mu + e^+ + \nu_e$

<u>Bf</u> $E = m_\pi c^2$ $\vec{P} = 0$

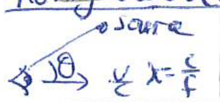
<u>Af</u> $E_\mu + E_\nu$ $\vec{P}_\nu = -\vec{P}_\mu$

<u>Rel. Doppler shift</u>



$$\boxed{f = \frac{\sqrt{1-\beta^2}}{1 - \beta\cos\theta} f_0}$$

<u>~~moving observer~~</u>



$$\boxed{f = f_0\sqrt{\frac{1+\beta}{1-\beta}}\ \ \theta = 0 \text{ toward} \qquad f = f_0\sqrt{\frac{1-\beta}{1+\beta}}\ \ \theta = \pi \text{ away}}$$

<u>Concepts</u>: Invariant: same values in all inertial frame. <u>ex</u> mass (not consv.) <u>ex</u> charge (both)

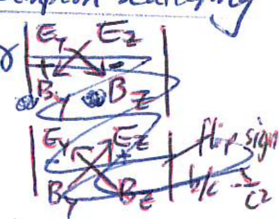
conserved: same values bf and af process. <u>ex</u> energy (not invariant) ex velocity (neither)

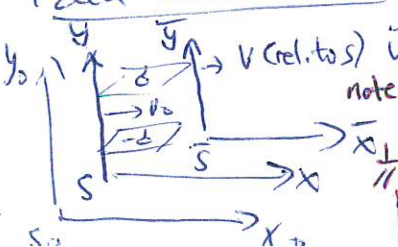
experimental fact: $E_{total,rel} = \gamma mc^2$ & <u>rel. mom.</u> conserved in closed sys.

Elastic collision: T conserved, $\Rightarrow$ E_{res} consv. $\Rightarrow$ mass consv. ex Compton scattering (inelastic)

<u>Field transformation</u>.

$\bar{E}_x = E_x$	$\bar{E}_y = \gamma(E_y - vB_z)$	$\bar{E}_z = \gamma(E_z + vB_y)$
$\bar{B}_x = B_x$	$\bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z)$	$\bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$





$\rightarrow v$ (rel. to S) $\bar{v}$ ($\bar{S}$ rel. to S_0)

note $\vec{E}^\perp = \gamma\vec{E}^\perp$ $\quad E^{\parallel} = E^{\parallel}$

$\perp$ $\vec{E} \perp \vec{v}$

$\parallel$ $\vec{E} \parallel \vec{v}$

$$\boxed{\begin{array}{ll} \text{if } \vec{B} = 0 \text{ in } S & \bar{\vec{B}} = -\frac{1}{c^2}(\vec{v} \times \bar{\vec{E}}) \\ \vec{E} = 0 \text{ in } S & \bar{\vec{E}} = \vec{v} \times \bar{\vec{B}} \end{array}}$$

easily derive

$\bar{B} = B$ — like Lorentz force

$$\boxed{\text{invariant } \quad \vec{E}' \cdot \vec{B}' = \vec{E} \cdot \vec{B} \qquad E'^2 - B'^2 = E^2 - B^2}$$