Attack-Resilient Supervisory Control with Intermittently Secure Communication

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Abstract— **In this work, we study the supervisory control of discrete event systems under network-based attacks on information delivered to and from the supervisors to the plant's sensors and actuators. The attackers are modeled by FSTs having the ability to nondeterministically rewrite a word to any word of a regular language. A desired language is called *controllable* if there exists a supervisor such that the restricted language executed by the plant for all possible attacking behavior is the desired one; such supervisors are referred to as *attack-resilient*. First, we solve the problem of computing the maximal controllable sub-language (MCSL) of a desired language and propose the design algorithm for an attack-resilient supervisor, in scenarios where no security guarantees exists for communication between the plant and the supervisor. Then, we consider the case that the supervisor has active but intermittent access to a size-limited secure channel that ensures integrity and availability of the data transmitted over it. Specifically, we propose the notion of accessibility as a measure of distance between a language and its sub-language, and show that a desired language is controllable with intermittently secure communication if and only if its difference from its MCSL without secure channel is bounded by the accessibility measure.**

I. Introduction

Control resiliency is a key issue in the deployment of cyber-physical systems (CPS) in a wide range of safety-critical but attack-subjective environments, such as smart grids [1], medical devices [2], and distributed control systems [3]. In general, in these applications, adversarial attacks can happen from both the cyber and physical domains; attacks targeting on the sensors or actuators of the plant, or the communication between the controller and the plant, or even the physical environment of the plant, have raised the urgency to develop methods to analyze systems under attacks as well as design attack-resilient systems (e.g., [4], [5], [6]). Such attack-resilient systems provide strong Quality-of-Control guarantees even under intelligent and coordinated attacks [7], [6] that introduce malicious behaviors more complicated than merely generating random failures (which can be handled with fault tolerant control).

In this work, we focus on supervisory control of discrete event system in the presence of network-based attacks on communication between the supervisor and the plant's sensors/actuators, as illustrated in Figure 1. We focus on network-based attacks, such as *Man-in-the-Middle* or *Denial-of-Service* attacks, since network connectivity, which is prevalent in CPS, allows for a *remote* attacker to affect control performance by tampering with the transmitted sensor measurements and actuator commands. As proposed in our companion paper [8], we model such attacks using finite state transducers (FST); in [8], we show that FSTs enable capturing of most forms of malicious activity in CPS, which may occur as a result of network-based attacks that corrupt the data communicated between the plant and controller, as well as non-invasive attacks that affect the environment of the plant (e.g. such as GPS spoofing attacks on autonomous vehicles [9]). These attacks generalize the ones studied in the recent growing literature in the attack-resilient discrete event systems, considering possible attacks between the communication between the supervisor and the plant [10], [11], [12], [13]. We also use FSTs as the supervisor for the plants modeled by deterministic finite automata, giving it the power to revise symbols instead of simple insertion or deletion, as it did in previous works [10], [11], [12], [13].

Note that FSTs have found applications in a wide-range of application domains, such as applications in speech recognition [14], [15]. In this work, nondeterministic FSTs are employed to model all possible malicious behaviors of the network-based attacks. Additional advantage of using FSTs to model attacks is that the class of FSTs is closed under inversion and composition, which are easily computable. Consequently, the problem of designing attack-resilient supervisory control for complex system configurations and coordinated attacks from multiple attack-points, where each attack is modeled by an FST, can be simplified to a few basic configurations.

We are mainly concerned with two problems in the attack-resilient supervisory control of discrete-event system plants commonly modeled as deterministic finite state automata (e.g.,as in [19]): **(i)** control of plants in the presence of FST-modeled (i.e., regularly-rewriting) attacks on the communication between the actuators and sensors to the supervisor, and **(ii)** the same problem with the supervisor having active and intermittent access to a secure size-limited secure channel, as illustrated in Figure 1. The active and intermittent access to a size-limited secure channel captures the common practice of providing intermittent security guarantees for communication over potentially compromised networks. For example, in CAN-based automotive systems, the continuous use of cryptographic security primitives to protect communication, may not be possible due to resource constraints in the bandwidth for real-time transmission and the processing speed of the

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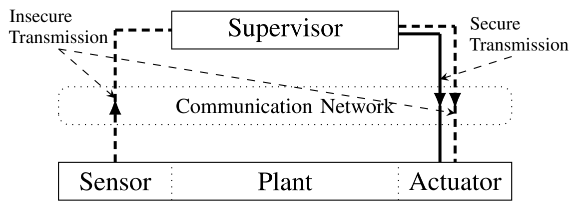


Fig. 1: Supervisory control in the presence of attacks on the communication between the plant's sensors and actuators and the supervisor; in this work, we use finite-state transducers to capture a very general class such network-based attacks.

computers [16], [17], [18]. Note that the network as well as the processors used for control computation are commonly shared between more than one control loop. Thus, increasing communication/computation requirements in every control cycle may not be possible if done for all control loops; however, if additional resources are only intermittently used for each control loop, the resource overhead can be 'spread' around for all control loops, making the system schedulable on deployed platforms.

The major contribution of this work is twofold: **(i)** We solve the problem of computing the maximal controllable sublanguage (MCSL) of a desired language and the corresponding attack-resilient supervisor design. These result can be viewed as an supplement and continuation of the attack-resilient supervisory control problems proposed in [8]. **(ii)** We study the benefit of the supervisor having active and intermittent access to a size-limited secure channel in attack-resilient supervisory control. Specifically, we propose a graph-theoretic metric for the difference between an automaton and its subautomaton, and use the metric to derive a controllability condition for a desired language, under the presence of such a secure channel; we show that the desired language is controllable and the system can be made resilient to attacks, if and only if the language's difference from its MCSL measured by accessibility is within the secure channel's repairing ability.

The rest of the article is organized as follows. The preliminaries on discrete events systems and FSTs are provided in Section II. In Section III, the problem formulation is presented. In Section IV, we provide a procedure to compute the MCSL of a desired language K without any secure transmission, and design the corresponding attack-resilient supervisor. Based on this, we present the controllability theorem and the supervisor design for a desired language with the resilient supervisor having active but intermittent access to a size-limited secure channel via a graph-theoretic approach in Section V. Finally, we conclude the work in Section VI.

II. Preliminaries

The empty string is denoted by ε. A finite-length sequence taken from a given finite set of symbols is called a *word*. A set of words is called a *language* of those symbols. The cardinality of set A is denoted by $|A|$. For two sets A and B, let $A \backslash B = \{x \in A \mid x \notin B\}$. For $n \in \mathbb{N}$, let $[n] = \{0, 1, \dots, n-1\}$. For a word $\mathsf{i}_1\mathsf{i}_2\dots\mathsf{i}_n$, we call $\mathsf{i}_1\mathsf{i}_2\dots\mathsf{i}_k$, with $k \leq n$, a prefix of I. For a language L, its prefix-closure is defined by $\overline{L} = \{I \mid I \text{ is a prefix of } J, J \in L\}$. The language L is *prefix-closed* if $L = \overline{L}$. The length of a word is denoted by $|\cdot|$. We adopt the following convention on generating regular expressions: for languages L_1 and L_2, L_1^* stands for repeating L_1 finitely many times, L_1, L_2 for the disjunction of L_1 and L_2, and $\overline{L_1}$ for the prefix closure of L_1.

A relation $\mathcal{R}$ between two sets A, B is a set $\mathcal{R} \subseteq A \times B$. For $a \in A$, let $\mathcal{R}(a) = \{b \in B \mid (a, b) \in \mathcal{R}\}$. The relation $\mathcal{R}$ is a partial function if $\mathcal{R}(a)$ is empty or a singleton for any $a \in A$. More generally, for $A' \subseteq A$, while slightly abusing the notation, let $\mathcal{R}(A') = \{b \in B \mid (a', b) \in \mathcal{R}, a' \in A'\}$. Clearly, $\mathcal{R}(\cdot)$ defines a function $2^A \to 2^B$. For relation $\mathcal{R} \subseteq A \times B$, its inversion is defined by $\mathcal{R}^{-1} = \{(b, a) \in B \times A \mid (a, b) \in \mathcal{R}\}$. For two relations $\mathcal{R} \subseteq A \times B$ and $\mathcal{R}' \subseteq B' \times C$, their (serial) composition is defined by $\mathcal{R} \circ \mathcal{R}' = \{(a, c) \in A \times C \mid \exists b \in B \cap B' : (a, b) \in \mathcal{R} \wedge (b, c) \in \mathcal{R}'\}$.

A. Discrete Event Systems

Discrete event systems (DES) are deterministic finite state automata (FSA) [19], [20].

Definition 1 (Finite State Automata)**.** *An FSA $\mathcal{P}$ is a tuple $\mathcal{P} = (\mathsf{S}, \mathsf{s}_{\text{init}}, \mathbf{I}, \mathsf{Trans}, \mathsf{S}_{\text{final}})$ where*

- S *is a finite set of* states*;*
- $\mathsf{s}_{\text{init}} \in \mathsf{S}$ *is the* initial state*;*
- $\mathbf{I} \cup \{\varepsilon\}$ *is a finite set of* inputs*;*
- $\mathsf{Trans} : \mathsf{S} \times \mathbf{I} \to \mathsf{S}$ *is a* transition *relation;*
- $\mathsf{S}_{\text{final}} \subseteq \mathsf{S}$ *is a finite set of* final states.

The FSA $\mathcal{P}$ is deterministic if Trans *is a partial function. The sequence* $(\mathsf{s}_{\text{init}}, \mathsf{i}_0, \mathsf{s}_0)(\mathsf{s}_0, \mathsf{i}_1, \mathsf{s}_1)\dots(\mathsf{s}_{n-2}, \mathsf{i}_{n-1}, \mathsf{s}_{n-1})$ *is called an* execution *of $\mathcal{P}$, if $\mathsf{s}_i \subseteq \mathsf{Trans}(\mathsf{s}_{i-1}, \mathsf{i}_i)$ for $i \in [n]$ with $\mathsf{s}_{-1} = \mathsf{s}_{\text{init}}$. The state s_{n-1} is called reachable by the input word $I = \mathsf{i}_0\mathsf{i}_1\dots\mathsf{i}_n$. The input word I is* accepted *by $\mathcal{P}$, if there exists an execution ending at $\mathsf{S}_{\text{final}}$. The set of* words *accepted by $\mathcal{P}$ is called the* language *accepted by $\mathcal{P}$, denoted by $L(\mathcal{P})$. On the other hand, $\mathcal{P}$ is called a* realization *or* model *of $L(\mathcal{P})$.*

B. Finite State Transducers

Finite State Transducers (FSTs) extend discrete event systems by generating a sequence of outputs nondeterministically during execution, by augmenting each transition with a regular output language.

Definition 2 (Finite State Transducer)**.** *An FST is a tuple $\mathcal{A} = (\mathsf{S}, \mathsf{s}_{\text{init}}, \mathbf{I}, \mathbf{O}, \mathsf{Trans}, \mathsf{S}_{\text{final}})$ where*

- S *is a finite set of* states*;*
- $\mathsf{s}_{\text{init}} \in \mathsf{S}$ *is the* initial state*;*
- $\mathbf{I}$ *is a finite set of* inputs*;*
- $\mathbf{O}$ *is a finite set of* outputs*;*
- $\mathsf{Trans} : \mathsf{S} \times \mathrm{Re}(\mathbf{I}^*) \times \mathrm{Re}(\mathbf{O}^*) \to \mathsf{S}$ *is a partial* transition *function;*
- $\mathsf{S}_{\text{final}} \subseteq \mathsf{S}$ *is a finite set of* final states.

Specially, the FST is normal *if the input and output labels on the transitions are $\mathbf{I} \cup \{\varepsilon\}$ and $\mathbf{O} \cup \{\varepsilon\}$, respectively.*

The definitions of *input/output words*, *executions*, *acceptance*, and *realization* are similar to discrete event systems (Definition 1). The automaton $\mathcal{A}_{\text{in}}$ derived by removing the output symbols of the FST $\mathcal{A}$ is called *the input automaton* of $\mathcal{A}$; its acceptable language is called the *input languages* accepted by $\mathcal{A}$, denoted by $L_{\text{in}}(\mathcal{A})$; similarly, the automaton $\mathcal{A}_{\text{out}}$ derived by removing the input symbols of the FST $\mathcal{A}$ is called the output automaton of $\mathcal{A}$; its acceptable language is called *output languages* generated by $\mathcal{A}$, denoted by $L_{\text{out}}(\mathcal{A})$.

C. Regular Relations

An FST $\mathcal{A}$ defines a relation $\mathcal{R}_{\mathcal{A}}$ between the inputs $\mathbf{I}^*$ and the outputs $\mathbf{O}^*$ as follows: $(\mathsf{i}, \mathsf{o}) \in \mathcal{R}_{\mathcal{A}}$ if and only if there exists an execution $(\mathsf{s}_{\text{init}}, \mathsf{i}_0, \mathsf{o}_0, \mathsf{s}_0)(\mathsf{s}_0, \mathsf{i}_1, \mathsf{o}_1, \mathsf{s}_1)\ldots(\mathsf{s}_{n-2}, \mathsf{i}_{n-1}, \mathsf{o}_{n-1}, \mathsf{s}_{n-1})$ such that $\mathsf{i} = \mathsf{i}_0\mathsf{i}_1\ldots\mathsf{i}_{n-1}$ and $\mathsf{o} = \mathsf{o}_0\mathsf{o}_1\ldots\mathsf{o}_{n-1}$. This relation is called a *regular* relation between $\mathbf{I}^*$ and $\mathbf{O}^*$. On the other hand, a relation $\mathcal{R} \subseteq \mathbf{I}^* \times \mathbf{O}^*$ is *regular*, only if it is realized by a finite state transducer. Clearly, $L_{\text{out}}(\mathcal{A}) = \mathcal{R}(\mathbf{I}^*)$ and $L_{\text{in}}(\mathcal{A}) = \mathcal{R}^{-1}(\mathbf{O}^*)$. Specially, a discrete event system $\mathcal{M}$ defines a regular relation $\mathcal{R}_{\mathcal{M}} = \{(I, I) \mid I \in L(\mathcal{M})\}$ on $\mathrm{Re}(\mathbf{I}^*) \times \mathrm{Re}(\mathbf{I}^*)$.

Despite the addition of an output, the computational complexity of solving problems on FSTs does not increase significantly from discrete event systems. A discrete event system can be lifted to a special FST with identical inputs and outputs. On the other hand, a normalized FST can be viewed as a discrete event system with labels in the set $(\mathbf{I} \cup \{\varepsilon\}) \times (\mathbf{O} \cup \{\varepsilon\})$.

III. System Model and Problem Description

In this work, we consider the setups from Figure 2. Here, the supervisor $\mathcal{S}$, modeled by an FST, controls the behavior of the plant $\mathcal{P}$ by observing the symbols that the plant generates and then sending the possible control symbols back to the plant. In a companion paper [8], we show how such attacks can be modeled as nondeterministic FSTs; the attack FSTs can regularly rewrite an word, i.e., replace a symbol nondeterministically with an arbitrary word taken from some predefined regular language, including, for example, injection, replacement and deletion. In addition, we show that such FSTs can be used to capture a very wide class of attacks including all previously considered attacks on discrete-event systems (e.g., false data-injection, nondeterministic denial of service, rewriting), as well as additional attacks reported in recent security incidents (e.g., replay attack). In such models, the nondeterminism captures all possible actions of the attacker for a specific set of compromised resources (e.g., sensors, actuators), as well as all potential limitations imposed on the attacker's actions by the system design (e.g., the use of cryptographic primitives on some communication messages to prevent false-data inserting attacks over the network).

Consequently, in this work we model network-based attacks on the information delivered to and from the supervisor by two FSTs $\mathcal{A}_a$ and $\mathcal{A}_s$ affecting the input and output of the plant, respectively (Figure 2). The control objective is to restrict the symbols passing to the plant $\mathcal{P}$ within some desired language $\mathcal{K} \subseteq L(\mathcal{P})$. We first consider the case that the communication

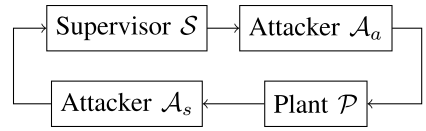


Fig. 2: Supervisor control without a secure channel.

is always subject to attack, as shown in Figure 2, and study the problem of computing the maximal controllable sublanguage (MCSL) $\underline{\mathcal{K}} \subseteq \mathcal{K}$ of the desired language, namely, the largest subset of the desired language that is achievable by some supervisor under the attacks. In addition, we study the synthesis problem for such attack-resilient supervisor.

In the presence of network-based attacks captured by $\mathcal{A}_a$ and $\mathcal{A}_s$, the MCSL $\underline{\mathcal{K}}$ can be significantly smaller than the desired language $\mathcal{K}$. Thus, we also study the controllability problem with an intermittently accessible secure channel that works asynchronously with the insecure channel subject to the attacker $\mathcal{A}_a$, as shown in Figure 3. Suppose that the supervisor wants to ensure that any controls in $\mathcal{K}$ is sent to the plant safely – i.e., the plant receives words in $\mathcal{K}$ under all possible attacks. Clearly, if $\mathcal{K} \not\subseteq \underline{\mathcal{K}}$, then sending a control in $I \in \mathcal{K} \backslash \underline{\mathcal{K}}$ entirely through the insecure channel is undesirable. One solution is to send any word $I \in K$ through a secure channel, where security is usually enforced by encryption. However, this can impose excessive communication and computation overheads, limiting its use in resource-constrained systems.

A more cost-efficient way is to find another control $J \in \underline{\mathcal{K}}$ that only mismatches with I in short fragments; namely, there exist compositions $I = I_1 I_2 I_3 I_4 I_5 ...$ and $J = I_1 J_2 I_3 J_4 I_5 ...$, where $I_i, J_i \in \mathbf{I}^*$, such that for all $n \in \mathbb{N}$, $|I_{2n}| \leq l_1$ and $|J_{2n}| \leq l_2$ for some $l_1, l_2 \in \mathbb{N}$. The idea is that in this case, the supervisor can send via insecure channel fragment-by-fragment the control $I'_1 J'_2 I'_3 J'_4 I'_5 \ldots$ that is modified from the original controls $I = I_1 I_2 I_3 I_4 I_5 ...$, and always stays within $\underline{\mathcal{K}}$ under the attacks (in Section V, we show how to compute these fragments from I and J). On the other hand, the pairs $\{(I_{2n}, J_{2n})\}_{n \in \mathbb{N}}$ indicating the difference between I and J are sent via the secure channel, where J_{2n} serves as anchors for the restoration. Accordingly, the *authenticator* only receives a control within $\underline{\mathcal{K}}$ via the insecure channel; in certain cases, due to attacks it will receive J and can restore the original control I, which may be $\in \mathcal{K} \backslash \underline{\mathcal{K}}$, using the secure transmissions.

As the restoration of J to I is only performed on the small fragments J_{2n}, the secure channel merely needs the capacity of intermittently transmitting a pair of words of length less than l_1 and l_2. Note that in this scenario, the transmission of the anchors J_{2n} is the overhead cost (i.e., additional communication packets). Therefore, if $l_2 \leq l_1$, the savings of not having to secure every transmitted symbol (i.e., in the order of $l_1 + l_2$) will be large due to the very high cost of protecting communication packets (e.g., with the use of standard cryptographic primitives). For example, let's assume that the transmission in the secure and insecure channels happens asynchronously and take the time to send each of fragments $\{I'_n J'_n\}_{n \in \mathbb{N}}$, as well as to send each of

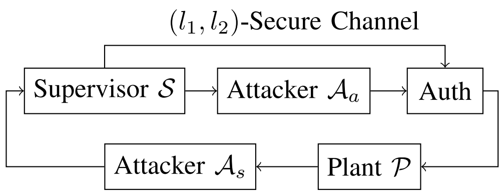


Fig. 3: Supervisor control with a secure channel.

the pairs $\{(I_{2n}, J_{2n})\}_{n\in\mathbb{N}}$ as 1. Then, the transmission of the secure channel is only needed every other time. For other time measures, the maximal frequency constraints on the secure channel changes accordingly.

Assumptions: We assume that $L_{\text{out}}(\mathcal{A}_a) = L_{\text{in}}(\mathcal{A}_s) = L(\mathcal{P})$, namely, the input attacker cannot generate words unacceptable to the plant and the output attacker does not receive those words. The latter is achievable by trimming the attacker $\mathcal{A}_s$; the former is achievable by restricting the plant — the results derived in the rest of paper holds for $L_{\text{out}}(\mathcal{A}_a)$ instead of $L(\mathcal{P})$ when $L_{\text{out}}(\mathcal{A}_a) \neq L(\mathcal{P})$. In addition, we assume that for simplicity that for the plant $\mathcal{P}$, the supervisor $\mathcal{S}$ and the input and output attackers $\mathcal{A}_a, \mathcal{A}_s$, all states are final $\mathsf{S}_{\text{final}} = \mathsf{S}$, i.e., both the sets of their inputs and outputs are prefix-closed. In addition, the desired language is also prefix-closed $\mathcal{K} = \bar{\mathcal{K}}$ and regular. The regularity of $\mathcal{K}$ is to ensure that it is controllable by supervisors modeled by FSTs. The prefix-closeness requires that the supervision can be implemented step-by-step.

IV. Maximal Controllable Sub-Language

In this section, we study the problem of computing the MCSL of a desired language $\mathcal{K} \subseteq L(\mathcal{P})$, under the presence of input and output attacks and without the use of secure channel, as shown in Figure 2.

Definition 3. *Let $\mathcal{K}$ be the desired language of the plant $\mathcal{P}$ with $\mathcal{K} \subseteq L(\mathcal{P})$, and $\mathcal{A}_a$ and $\mathcal{A}_s$ be the FSTs modeling attacks on the plant's input and output. The desired language $\mathcal{K}$ is controllable if there exists a supervisor $\mathcal{S}$ such that*

$$L(\mathcal{P}|\mathcal{S}, \mathcal{A}_a, \mathcal{A}_s) = \mathcal{K},$$

where $L(\mathcal{P}|\mathcal{S}, \mathcal{A}_a, \mathcal{A}_s) = \mathcal{K}$ is the language executed by the plant $\mathcal{P}$ in the presence of supervisor $\mathcal{S}$, and attackers $\mathcal{A}_a$ and $\mathcal{A}_s$. The controllable language $\underline{\mathcal{K}}$ is the maximal controllable sub-language (MCSL) of $\mathcal{K}$, denoted by $\underline{\mathcal{K}} \subseteq_{\max} K$, if every controllable sub-language of $\mathcal{K}$ is contained in $\tilde{K}$. Clearly, the desired language $\mathcal{K}$ is controllable if and only if $\mathcal{K} \subseteq \underline{\mathcal{K}}$.

The computation of MCSL $\underline{\mathcal{K}}$ of the desired language $\mathcal{K}$ depends on the operator $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$. For any words $k, k' \in L_{\text{out}}(\mathcal{A}_a)$, it holds that $k' \in \mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(k)$ if and only if $\mathcal{A}_a^{-1}(k') = \mathcal{A}_a^{-1}(k)$ — namely, k and k' may result from the same supervisory control. Therefore, the following lemma holds.

Lemma 1. *For any $k, k' \in L_{\text{out}}(\mathcal{A}_a)$, it holds that*

(i) $k \in \mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(k)$, and

(ii) $$k' \in \mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(k) \Leftrightarrow \mathcal{A}_a^{-1}(k) = \mathcal{A}_a^{-1}(k') \Leftrightarrow k \in \mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(k').$$

For any language $\mathcal{K}$, let $\mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})$ be the least fixed point (LFP) of $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$ containing $\mathcal{K}$ – i.e., the smallest superset of $\mathcal{K}$ such that

$$\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}\big(\mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})\big) = \mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}).$$

As $L_{\text{out}}(\mathcal{A}_a)$ is a fixed point of $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$, such an LFP exists. The LFP is characterized by the following lemma.

Lemma 2 (Least Fixed Point)**.** *For any $\mathcal{K} \subseteq L_{\text{out}}(\mathcal{A}_a)$, the LFP of $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$ is given by*

$$\mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}) = \bigcup_{i=0}^{\infty} \mathcal{R}^{i}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}),$$

where $\mathcal{R}^{i}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$ is the n-fold composition of $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$. Clearly, for any $i \in \mathbb{N}$, $\mathcal{R}^{i}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}) \subseteq \mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})$.

When $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$ is idempotent on $\mathcal{K}$, namely, there exists n such that $\mathcal{R}^{n+1}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}) = \mathcal{R}^{n}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})$, then the LFP can be computed in finite time by $\mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}) = \mathcal{R}^{n}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})$. Otherwise, computing $\mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})$ me be challenging.

In [8], we show that given the MCSL $\underline{\mathcal{K}}$ satisfying $\underline{\mathcal{K}} = \mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\underline{\mathcal{K}})$, the supervisor designed by $\mathcal{S} = \mathcal{A}_s^{-1} \circ \mathcal{M}_{\underline{\mathcal{K}}} \circ \mathcal{A}_a^{-1}$ restricts the language received by the plant to $\underline{\mathcal{K}}$. Similar to [8] and differing from other previous works [13], [19], [10], the MCSL only depends on the input attacker $\mathcal{A}_a$, because the supervisor, modeled by an FST instead of automata, has more power in generating control commands, and can totally counter the effect of the output attacker by composing with $\mathcal{A}_s^{-1}$. Given $\mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K})$, the computation of the MCSL of $\mathcal{K}$ and the supervisor can be done as captured by Theorem 1 and Algorithm 1.

Theorem 1 (Maximal Controllable Sub-Language)**.** *The maximal controllable sub-language (MCSL) of the desired regular language $\mathcal{K} \subseteq L_{\text{in}}(\mathcal{P})$ under the input and output attacks modeled by FSTs $\mathcal{A}_a$ and $\mathcal{A}_s$ is*

$$\underline{\mathcal{K}} = \mathcal{K} \backslash \mathcal{R}^{\infty}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}\big(\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(\mathcal{K}) \backslash \mathcal{K}\big), \tag{1}$$

under the supervisor

$$\mathcal{S} = \mathcal{A}_s^{-1} \circ \mathcal{M}_{\underline{\mathcal{K}}} \circ \mathcal{A}_a^{-1}. \tag{2}$$

Proof. For simplicity, let $\mathcal{R} = \mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}$ in this proof. First, we show that the supervisor given by (2) controls the plant to the MCSL given by (1), namely, $\mathcal{R}(\underline{\mathcal{K}}) \subseteq \underline{\mathcal{K}}$. By (1), for any $k \in \underline{\mathcal{K}}$, we have $k \notin \mathcal{R}^{\infty}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big)$, namely, $k \notin \mathcal{R}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big)$. Thus, we have $\mathcal{R}(k) \notin \mathcal{R}(\mathcal{K})\backslash\mathcal{K}$ by Lemma 1, namely, $\mathcal{R}(\mathcal{K}) \in K$. Now, assume that $\mathcal{R}(k) \cap \mathcal{R}^{\infty}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big) \neq \emptyset$, then by Lemma 1 it holds that $k \in \mathcal{R}\big(\mathcal{R}^{\infty}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big)\big) = \mathcal{R}^{\infty}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big)$. This is in contradiction with (1). Thus, $R(k) \subseteq \underline{\mathcal{K}}$ for any $k \in \underline{\mathcal{K}}$.

Next, we show that $\underline{\mathcal{K}}$ given in (1) is the MCSL of $\mathcal{K}$. Let $\mathcal{K}'$ be a controllable sub-language of $\mathcal{K}$. Since $\mathcal{R}(\mathcal{K}') = \mathcal{K}' \subseteq \mathcal{K}$, i.e., $\mathcal{R}(\mathcal{K}') \cap \big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big) = \emptyset$, we have that $\mathcal{K}' \cap \mathcal{R}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big) = \emptyset$ by Lemma 1. Again by $\mathcal{R}(\mathcal{K}') = \mathcal{K}'$, we have that $\mathcal{R}(\mathcal{K}') \cap \mathcal{R}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big) = \emptyset$, leading to $\mathcal{K}' \cap \mathcal{R}^2\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big) = \emptyset$ by Lemma 1. By repeating the above procedure and using Lemma 2, it follows that $\mathcal{K}' \subseteq \mathcal{K}\backslash\mathcal{R}^{\infty}\big(\mathcal{R}(\mathcal{K})\backslash\mathcal{K}\big) = \underline{\mathcal{K}}$. $\square$

Algorithm 1 Design of a Supervisor for Maximal Controllable Sub-Language under Input and Output Attacks

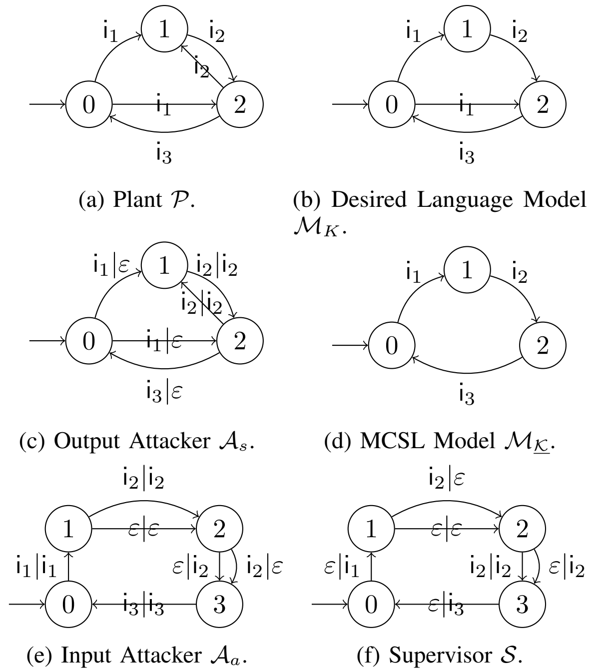
Require: Plant $\mathcal{P}$, input attacker $\mathcal{A}_a$, output attacker $\mathcal{A}_s$, Desire language $\mathcal{K}$.

Compute MCSL by (1) and find a model $\mathcal{M}_{\underline{\mathcal{K}}}$.

Compute supervisor $\mathcal{S}$ by (2).

return Supervisor $\mathcal{S}$.

Example 1 (Supervisor design for intermittent secure transmissions)**.** *Consider the supervisory control of a plant $\mathcal{P}$ shown in Figure 4a with the set of symbols $\mathbf{I} = \{\mathsf{i}_1, \mathsf{i}_2, \mathsf{i}_3\}$. The (prefix-closed) desired language is $\mathcal{K} = \overline{(\mathsf{i}_1(\mathsf{i}_2, \varepsilon)\mathsf{i}_3)^*}$, modeled by an automaton shown in Figure 4b. The input and output attacks $\mathcal{A}_a$ and $\mathcal{A}_s$ of the plant are modeled by FSTs shown in Figures 4c and 4e, respectively. By Theorem 1, the MCSL is $\underline{\mathcal{K}} = \overline{(\mathsf{i}_1\mathsf{i}_2\mathsf{i}_3)^*}$, which is strictly contained in $\mathcal{K}$, as shown in Figure 4d. It is easy to check the maximality of $\underline{\mathcal{K}}$. The supervisor computed by Algorithm 1 is shown in Figure 4f after minor simplification.* $\triangleleft$



(a) Plant $\mathcal{P}$. (b) Desired Language Model $\mathcal{M}_K$. (c) Output Attacker $\mathcal{A}_s$. (d) MCSL Model $\mathcal{M}_{\underline{\mathcal{K}}}$. (e) Input Attacker $\mathcal{A}_a$. (f) Supervisor $\mathcal{S}$.

Fig. 4: Example supervisor for input and output attacks.

Remark 1. *By Theorem 1, attacks on the output do not affect controllability, as opposed to previous works [10], [11], [13]. This is because the plant is deterministic, so its state is known from the perspective of the supervisor, and the supervisor as an FST can generate controls for the next step by itself without using the sensing information of the plant. These sensing information will become useful to learn the state of the plant when it is nondeterministic, which will be studied in the future.*

Remark 2. *Note that the supervisor derived in Example 1 is nondeterministic — there are transition with ε input symbol that can be triggered spontaneously. The controllability theorem guarantees that in the presence of attacks, the union of all possible words received by the plant under all these allowable controls is exactly the desired language $\mathcal{K}$. In implementation, the nondeterminism can be resolved by choosing one of the allowable controls. Accordingly, the possible words received by the plant is contained in $\mathcal{K}$.*

V. Controllability with Active Access to Secure Channel

In this section, we extend the results in Section IV to the case that the supervisor has access to a secure channel. Recalling Section III, the supervisor can decompose the desired control $I = I_1 I_2 I_3 I_4 I_5 ... \in \mathcal{K}\backslash\underline{\mathcal{K}}$ into fragments and find a segmentally mismatching but attack-resilient control $J = I_1 J_2 I_3 J_4 I_5 ....$ To counter the attacks, the supervisor can send the modified control fragments $I_1' J_2' I_3' J_4' I_5' \ldots$ with $I_1' = \mathcal{R}_{\mathcal{A}_a^{-1}}(I_1)$, $I_1' J_1' = \mathcal{R}_{\mathcal{A}_a^{-1}}(I_1 J_1)$, and so on, via the insecure channel, and the pairs $\{(I_{2n}, J_{2n})\}_{n\in\mathbb{N}}$ to indicate the difference between I and J via the secure channel. Accordingly, the authenticator will receive from the insecure channel a word in $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(J) \subseteq \underline{\mathcal{K}}$. Upon receiving exactly J, it can use the pairs $\{(I_{2n}, J_{2n})\}_{n\in\mathbb{N}}$ to restore J back to I, where J_{2n} serves as anchors for the restoration. In other words, the control received by the plant is always in $\mathcal{K}$ and is exactly the original one I when the attacker $\mathcal{A}_a$ takes certain attacks. When $\mathcal{R}_{\mathcal{A}_a^{-1}\circ\mathcal{A}_a}(J) \subseteq \underline{\mathcal{K}} = \{J\}$ – i.e., J is revertible from all possible attacks of $\mathcal{A}_a$, the recovery to the original control I is guaranteed.

The above solution depends critically on whether for any word $\mathcal{K}$, we can find a word in $\underline{\mathcal{K}}$ with bounded fragmentary mismatches. This naturally incurs a difference measure between $\mathcal{K}$ and $\underline{\mathcal{K}}$. To solve this problem, we propose to build automata models $\mathcal{M}_{\mathcal{K}}$ and $\mathcal{M}_{\underline{\mathcal{K}}}$ realizing $\mathcal{K}$ and $\underline{\mathcal{K}}$, respectively, with the model $\mathcal{M}_{\underline{\mathcal{K}}}$ contained in the model $\mathcal{M}_{\mathcal{K}}$. Then the measurement of the difference between $\mathcal{K}$ and $\underline{\mathcal{K}}$ is converted to the the measurement of the difference of two graph models $\mathcal{M}_{\underline{\mathcal{K}}}$ and $\mathcal{M}_{\mathcal{K}}$, using graph-theoretic tools.

A. Accessibility

Definition 4 (Sub-Automata)**.** *Let $\mathcal{P} = (\mathsf{S}, \mathsf{s}_{\text{init}}, \mathbf{I}, \mathsf{Trans}, \mathsf{S}_{\text{final}})$ be an FSA with $\mathsf{S}_{\text{final}} = \mathsf{S}$. We call $\mathcal{P}' = (\mathsf{S}', \mathsf{s}'_{\text{init}}, \mathbf{I}', \mathsf{Trans}')$ with $\mathsf{S}'_{\text{final}} = \mathsf{S}'$ a sub-automata of $\mathcal{P}$, denoted by $\mathcal{P}' \subseteq \mathcal{P}$, if $\mathsf{S}' \subseteq \mathsf{S}$, $\mathsf{s}'_{\text{init}} = \mathsf{s}_{\text{init}}$, $\mathbf{I}' \subseteq \mathbf{I}$, and $\mathsf{Trans}'(\mathsf{s}', \mathsf{i}') = \mathsf{Trans}(\mathsf{s}', \mathsf{i}')$ for any $\mathsf{s}' \in \mathsf{S}'$ and $\mathsf{i}' \in \mathbf{I}'$. For any prefix-closed $\mathcal{K} \subseteq L(\mathcal{P})$, there exists a maximal sub-automaton $\mathcal{P}' \subseteq \mathcal{P}$ such that $\mathcal{K} = L(\mathcal{P}')$, and all such sub-automata are sub-automata of $\mathcal{P}'$.*

To quantify the difference between two automata $\mathcal{P}' \subseteq \mathcal{P}$, we introduce the definition of (l_1, l_2)-step accessibility.

Definition 5 ((l_1, l_2)-Step Accessibility)**.** *For two automata $\mathcal{P}'$ and $\mathcal{P}$, such that $\mathcal{P}' \subseteq \mathcal{P}$, we say that $\mathcal{P}$ is (l_1, l_2)-accessible from $\mathcal{P}'$ if there is no execution $(\mathsf{s}_0, \mathsf{i}_0, \mathsf{s}_1) \ldots (\mathsf{s}_{n-1}, \mathsf{i}_{n-1}, \mathsf{s}_n) \subseteq \mathsf{Trans}\backslash\mathsf{Trans}'$ with $n > l_2$. In addition, for any such execution with $n \leq l_2$, there exists an execution $(\sigma_0, \mathsf{i}_0, \sigma_1) \ldots (\sigma_{m-1}, \mathsf{i}_{m-1}, \sigma_m) \subseteq \mathsf{Trans}$ with*

Algorithm 2 Constructing $\mathcal{M}_{\underline{\mathcal{K}}} \subseteq \mathcal{M}_{\mathcal{K}}$ for $\mathcal{K}$ and $\underline{\mathcal{K}}$.

Require: Languages $\underline{\mathcal{K}} \subseteq \mathcal{K}$.

Find minimal realization $\mathcal{M}_{\mathcal{K}}$ and $\mathcal{M}_{\underline{\mathcal{K}}}$ of $\mathcal{K}$ and $\underline{\mathcal{K}}$.
Convert $\mathcal{M}_{\mathcal{K}}$ and $\mathcal{M}_{\underline{\mathcal{K}}}$ to FSTs by adding ε as output and input symbol for each transition, respectively.
$\mathcal{M} = \mathcal{M}_{\mathcal{K}} \circ \mathcal{M}_{\underline{\mathcal{K}}}$.
Trim off transitions with input symbol ε in $\mathcal{M}$.
return $\mathcal{M}_{\text{in}}$ and $\mathcal{M}_{\text{out}}$.

Algorithm 3 Design supervisor and authenticator with active access to secure channel

Require: Plant $\mathcal{P}$, input attacker $\mathcal{A}_a$, output attacker $\mathcal{A}_s$, Desire language $\mathcal{K}$.

Compute MCSL by (1) and find FSA models $\mathcal{M}_{\underline{\mathcal{K}}} \subseteq \mathcal{M}_{K}$.
Compute $\mathcal{N}_K$ by (3) and supervisor $\mathcal{S}$ by (4).
return Supervisor $\mathcal{S}$.

$m \leq l_1$, $\sigma_0 = \mathsf{s}_0$ *and* $\sigma_m = \mathsf{s}_n$.

For an FSA $\mathcal{P}$, let the induced the directed graph be $\mathrm{Graph}_{\mathcal{P}} = (\mathsf{S}, \{(\mathsf{s},\mathsf{s}') \in \mathsf{S}^2 \mid (\mathsf{s},\mathsf{i},\mathsf{s}') \in \mathsf{Trans}\})$. Accessibility can be checked by the following graph-theoretic condition.

Theorem 2 ((l_1, l_2)-Step Accessibility)**.** *For two automata $\mathcal{P}'$ and $\mathcal{P}$, such that $\mathcal{P}' \subseteq \mathcal{P}$, $\mathcal{P}$ is (l_1, l_2)-accessible from $\mathcal{P}'$ if and only if (i) the subtracted graph $\mathrm{Graph}_{\mathcal{P}} \backslash \mathrm{Graph}_{\mathcal{P}'}$ is a tree with paths no longer than l_1; and (ii) for any such path, there is a path no longer than l_2 with same start and end in $\mathrm{Graph}_{\mathcal{P}'}$.*

Proof. It suffices to show necessity. First, (ii) follows directly from Definition 5. For (i), let us assume $\mathrm{Graph}_{\mathcal{P}} \backslash \mathrm{Graph}_{\mathcal{P}'}$ has a loop with labels I. Now, consider a path of $\mathrm{Graph}_{\mathcal{P}} \backslash \mathrm{Graph}_{\mathcal{P}'}$ containing the loop and with labels J_1, I, J_2. Then any word of the form $J_1 I^* J_2$ is word accepted by $\mathcal{P}$ but not $\mathcal{P}'$. This contradicts Definition 5. ☐

These two graph-theoretic conditions in Theorem 2 can be examined with well-studied algorithms (e.g., from [21]).

B. Supervisor Design for Attack-Resilience

To measure the distance between $\mathcal{K}$ and $\underline{\mathcal{K}}$, we build two automata $\mathcal{M}_{\underline{\mathcal{K}}} \subseteq \mathcal{M}_{\mathcal{K}}$, where $\underline{\mathcal{K}}$ is the MCSL of $\mathcal{K}$, as given in Theorem 1. This is done as follows. Let $\mathcal{M}_{\underline{\mathcal{K}}}$ and $\mathcal{M}_{\mathcal{K}}$ be the minimal realization of $\mathcal{K}$ and $\underline{\mathcal{K}}$. Then we can convert them into FSTs by adding ε as input and output symbol for each transition, respectively. Let $\mathcal{M} = \mathcal{M}_{\mathcal{K}} \circ \mathcal{M}_{\underline{\mathcal{K}}}$ and trim the transitions of $\mathcal{M}$ with labels ε input symbol. Clearly, we have $\mathcal{M}_{\text{out}} \subseteq \mathcal{M}_{\text{in}}$ $L(\mathcal{M}_{\text{in}}) = \mathcal{K}$ and $L(\mathcal{M}_{\text{out}}) = \underline{\mathcal{K}}$. This is summarized by Algorithm 2. In the rest of this section, let $\mathcal{M}_{\mathcal{K}} = (\mathsf{S}, \mathsf{s}_{\text{init}}, \mathbf{I}, \mathsf{Trans}, \mathsf{S}_{\text{final}})$ be $\mathcal{M}_{\text{in}}$ and $\mathcal{M}_{\underline{\mathcal{K}}} = (\underline{\mathsf{S}}, \underline{\mathsf{s}_{\text{init}}}, \underline{\mathbf{I}}, \underline{\mathsf{Trans}}, \underline{\mathsf{S}_{\text{final}}})$ be $\mathcal{M}_{\text{out}}$.

The transitions in $\mathcal{M}_{\underline{\mathcal{K}}}$ are resilient to attacks, thus do not require transmissions from the secure channel, while other transitions in $\mathcal{M}_{\mathcal{K}}$ are non-resilient and need secure transmission. By Definition 5, if $\mathcal{M}_{\mathcal{K}}$ is (l_1, l_2)-accessible from $\mathcal{M}_{\underline{\mathcal{K}}}$, then for any execution of $\mathcal{M}_{\mathcal{K}}$, there is at most l_1 consecutive non-resilient transitions and it is replaceable by an l_2 secure transitions. This pair of words, transmitted through the (l_1, l_2)-secure channel, are used to repair the corrupted word from the insecure channel. This is summarized by Theorem 3.

Theorem 3 (Controllability with Intermittent Active Access to Secure Channel)**.** *The desired regular language $\mathcal{K} \subseteq L(\mathcal{P})$ is controllable with active (l_1, l_2)-secure channel access if $\mathcal{M}_{\mathcal{K}}$ is (l_1, l_2)-accessible from $\mathcal{M}_{\underline{\mathcal{K}}}$, where $\underline{\mathcal{K}}$ is the MCSL of $\mathcal{K}$ as given by Theorem 1 and $\mathcal{M}_{\mathcal{K}}$ and $\mathcal{M}_{\underline{\mathcal{K}}}$ are given by Algorithm 2.*

Proof. Let $T \subseteq \mathsf{Trans}$ be the execution of $k \in L(\mathcal{M}_{\mathcal{K}})$. Then it can be uniquely decomposed into $T = R_0 T_0 \dots R_{n-1} T_{n-1} R_n$ where $R_i \subseteq \underline{\mathsf{Trans}}$, $T_i \subseteq \mathsf{Trans} \backslash \underline{\mathsf{Trans}}$, $|R_i| > 0$ and $|T_i| > 0$ for $i \in [n]$. Since $\mathcal{M}_{\mathcal{K}}$ is (l_1, l_2)-accessible from $\mathcal{M}_{\underline{\mathcal{K}}}$, we have for each $i \in [n]$, $|T_i| \leq l_1$ and there exists $T_i' \subseteq \underline{\mathsf{Trans}}$ such that $|T_i'| \leq l_2$ and $T = R_0 T_0' \dots R_{n-1} T_{n-1}' R_n$ is the execution of some $k' \in L(\mathcal{M}_{\underline{\mathcal{K}}})$. Therefore, the word k can be transmitted by sending the pairs $(T_i, T_i')_{i \in [n]}$ through the secure channel and k' through the insecure channel. The fact that $|R_i| > 0$ for $i \in [n]$ ensures that the secure channel is not activated consecutively. The word received by the authenticator from the insecure channel is in $\mathcal{R}_{\mathcal{A}_a^{-1} \circ \mathcal{A}_a}(k') \ni k'$: on receiving k', the authenticator revises it to k using the pairs from the secure channel. ☐

Now, the supervisor resilient to such attacks can be realized by an FST with two outputs for the secure and insecure channel, respectively. Specifically, let $\mathcal{M}_{\mathcal{K}} = (\mathsf{S}, \mathsf{s}_{\text{init}}, \mathbf{I}, \mathsf{Trans}, \mathsf{S}_{\text{final}})$ and $\mathcal{M}_{\underline{\mathcal{K}}} = (\underline{\mathsf{S}}, \underline{\mathsf{s}_{\text{init}}}, \underline{\mathbf{I}}, \underline{\mathsf{Trans}}, \underline{\mathsf{S}_{\text{final}}})$ be deterministic FSA models for $\underline{\mathcal{K}} \subseteq \mathcal{K}$, with $\mathcal{M}_{\underline{\mathcal{K}}} \subseteq \mathcal{M}_{\mathcal{K}}$. We construct an FST $\mathcal{N}_{\mathcal{K}} = (\mathsf{S} \times \underline{\mathsf{S}}, (\mathsf{s}_{\text{init}}, \underline{\mathsf{s}_{\text{init}}}), \mathbf{I}, \mathbf{I} \times \{+, -\}, \mathsf{Trans}', \mathsf{S}_{\text{final}} \times \underline{\mathsf{S}_{\text{final}}})$ with

$$
\begin{aligned}
\mathsf{Trans}' = &\left\{\left((\underline{\mathsf{s}}, \underline{\mathsf{s}}), \mathsf{i}, (\diamond, \mathsf{i}), (\underline{\mathsf{s}}', \underline{\mathsf{s}}')\right) \mid (\underline{\mathsf{s}}, \mathsf{i}, \underline{\mathsf{s}}') \in \underline{\mathsf{Trans}}\right\} \\
&\left\{\left((\mathsf{s}, \underline{\mathsf{s}}), \mathsf{i}, (\mathsf{i}, \diamond_1), (\mathsf{s}', \underline{\mathsf{s}})\right) \mid (\mathsf{s}, \mathsf{i}, \mathsf{s}') \in \mathsf{Trans} \backslash \underline{\mathsf{Trans}}, \underline{\mathsf{s}} \in \underline{\mathsf{S}}\right\} \\
&\left\{\left((\underline{\mathsf{s}}, \underline{\mathsf{s}}'), \mathsf{i}, (\mathsf{i}, \diamond_2), (\underline{\mathsf{s}}, \underline{\mathsf{s}}'')\right) \mid (\underline{\mathsf{s}}', \mathsf{i}, \underline{\mathsf{s}}'') \in \underline{\mathsf{Trans}}, \underline{\mathsf{s}} \in \underline{\mathsf{S}}\right\}
\end{aligned} \tag{3}
$$

The output $(\diamond, \mathsf{i})$ stands for sending i to the insecure channel while closing the secure channel; $(\mathsf{i}, \diamond_1)$ and $(\mathsf{i}, \diamond_2)$ denote sending i to the first and second component of the pair of words transmission via the secure channel while closing the insecure channel. Then the supervisor $\mathcal{S}$ is constructed by the composition

$$
\mathcal{S} = \mathcal{N}_K \circ_2 \mathcal{A}_a^{-1} \tag{4}
$$

similar to [8]. Here, $\circ_2$ stands for the composition between the input of $\mathcal{A}_a$ with the second component of the output of $\mathcal{N}_K$, while keeping the first component.

Example 2 (Supervisor design for maximal controllable sub-languages)**.** *Following Example 1, the desired language $\mathcal{K} = \overline{(\mathsf{i}_1(\mathsf{i}_2, \varepsilon)\mathsf{i}_3)^*}$ is achievable with intermittent access to*

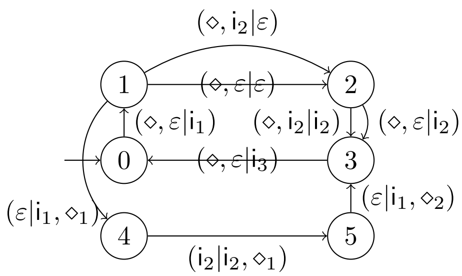


Fig. 5: Supervisor $\mathcal{S}$ with active access to secure channel.

$(2,1)$-secure channel. The supervisor designed by Algorithm 3 is shown in Figure 5, in which the upper half of the FST is identical to Figure 4f handling words in MCSL $\underline{\mathcal{K}}$ and the lower half handles the word in $\mathcal{K} \backslash \underline{\mathcal{K}}$. Upon i_1 at the beginning, the supervisor may actively activate the secure channel to send $(\mathsf{i}_1\mathsf{i}_2, \mathsf{i}_1)$, so that if the plant receives $\mathsf{i}_1\mathsf{i}_2\mathsf{i}_3$, the authenticator repairs it back to $\mathsf{i}_1\mathsf{i}_3$. $\triangleleft$

VI. Conclusions

In this work, we have studied the supervisory control of discrete event systems under regularly-rewriting attacks on their actuators and sensors with intermittent authentication. The attackers are modeled by FSTs having the ability to nondeterministically rewrite a word to any word of a regular language. A desired language is called controllable if there exists a supervisor such that the restricted language executed by the plant for all possible attacking behavior is the desired one – for such supervisor, we refer to the system as attack-resilient. First, we have solved the problem of computing the maximal controllable sub-language (MCSL) of a desired language and proposed the design algorithm for the attack-resilient supervisor, when there is no protection for communication between the plant and the supervisor. Then, we have considered the case that the supervisor has active but intermittent access to a size-limited secure channel that ensures the plant-supervisor communication. Specifically, we have proposed the notion of accessibility as a measure of distance between a language and its sub-language, and shown that a desired language is controllable with intermittent security guarantees if and only if its difference from its MCSL without secure channel is bounded by the accessibility measure.

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