

Coursework 6

YOUR NAME

November 17, 2018

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- **Acknowledgments:** This template takes some materials from course CSE 547/Stat 548 of Washington University:
<https://courses.cs.washington.edu/courses/cse547/17sp/index.html>.
If you refer to other materials in your homework, please list here.
 - **Collaborators:** I finish this template by myself. If you finish your homework all by yourself, make a similar statement. If you get help from others in finishing your homework, state like this:
 - 1.2 (b) was solved with the help from _____.
 - Discussion with _____ helped me finishing 1.3.
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You may use `enumerate` to generate answers for each question:

- 6.1. Type of commonly used notations. Use another `enumerate` to start generate answers for sub-questions:

- (a) Use `$ $` to get an inline equation: $\mathbb{P}(A) = \mathbb{E}[\mathbb{1}_A(\omega)]$.
- (b) Use `equation` to have equation in display math mode:

$$\frac{a+b}{2} \geq \sqrt{ab} \tag{1}$$

- (c) Use `\eqref` to get reference for equations: $(??)$ holds when $a \geq 0, b \geq 0$.
- (d) Now we would introduce some commonly used notations:
 - i. Use `\mathbb{P}`, `\mathbb{R}`, `\mathbb{E}` to type $\mathbb{P}, \mathbb{R}, \mathbb{E}$.
 - ii. Use `\mathcal{A}`, `\mathcal{X}`, `\mathcal{Y}`, `\mathcal{N}` to type $\mathcal{A}, \mathcal{X}, \mathcal{Y}, \mathcal{N}$.
 - iii. Use `\underline{x}`, `\underline{y}` to type vectors $\underline{x}, \underline{y}$.
 - iv. Use `\mathsf{x}`, `\mathsf{y}`, `\mathsf{z}` to type random variables x, y, z . For simplicity, I have defined several macros so you could simply type `\rvx`, `\rvy`, `\rvz`. Don't forget `$ $`!
 - v. Thanks to these macros, we could have $\mathbb{R}, \mathbb{E}[x], \text{Var}(y), \mathbb{P}(A), \perp, \mathbb{1}$ by typing `\reals`, `\E[\rvx]`, `\Var(\rvy)`, `\Prob(A)`, `\independent`, `\1`.
 - vi. Now you can use `\ux`, `\uy`, `\uz` to type vectors $\underline{x}, \underline{y}, \underline{z}$, and use `\urvx`, `\urvy`, `\urvz` to type random vectors $\underline{x}, \underline{y}, \underline{z}$.
 - vii. Remember that $P_{x|y}(x|y) \triangleq \mathbb{P}(x = x|y = y)$.

- α) Writing $\mathbb{P}(x)$ is wrong. \mathbb{P} should only operate on events.
- β) \mathbf{x} is a random variable, while x is a real number.
- (e) You may find <https://en.wikibooks.org/wiki/LaTeX> useful.
- (f) Writing L^AT_EX online may be easier for beginners:
 - i. ShareLaTeX: <https://www.sharelatex.com/>.
 - ii. Overleaf: <https://www.overleaf.com/>.

6.2. You may need aligned equations for your homework, here are several examples:

Total propability rule:

$$\begin{aligned}\mathbb{P}(\mathbf{x} = x) &= \sum_{y \in \mathcal{Y}} \mathbb{P}(\mathbf{x} = x, y = y) \\ &= \sum_{y \in \mathcal{Y}} \mathbb{P}(\mathbf{x} = x | y = y) \mathbb{P}(y = y),\end{aligned}$$

or

$$\begin{aligned}P_{\mathbf{x}}(x) &= \sum_{y \in \mathcal{Y}} P_{\mathbf{x}y}(x, y) \\ &= \sum_{y \in \mathcal{Y}} P_{\mathbf{x}|y}(x|y) P_y(y).\end{aligned}$$

Indicator function:

$$\mathbb{1}_A(\omega) = \begin{cases} 1, & \text{if } \omega \in A, \\ 0, & \text{if } \omega \notin A. \end{cases}$$

6.3. You may need to add figure and source codes in your homework. Figure ?? is an example that compares the empirical distribution (histogram) and probability density function of the Gaussian random variable.

The source code to plot Figure ?? could be found in Appendix ?. Here are the core codes:

```
4 [cnt, x_hist] = hist(data, nbins); % not to plot, only to get
   empirical distribution.

6 cnt = cnt / n / (x_hist(2) - x_hist(1)); % normalization, be
   careful :)

7 bar(x_hist, cnt); % plot the hist using bar()
```

To understand line 6, note that if we have n samples of \mathbf{x} denoted by $x^{(i)}, i = 1, 2, \dots, n$, then the probability density function $p_{\mathbf{x}}$ could be estimated as

$$\begin{aligned}p_{\mathbf{x}}(x_0) &= \left. \frac{d}{dx} \mathbb{P}(\mathbf{x} \leq x) \right|_{x=x_0} \\ &\approx \frac{\mathbb{P}(x_0 - \Delta x < \mathbf{x} \leq x_0)}{\Delta x} \\ &\approx \frac{1}{n\Delta x} \sum_{i=1}^n \mathbb{1}_{x^{(i)} \in (x_0 - \Delta x, x_0]}.\end{aligned}$$

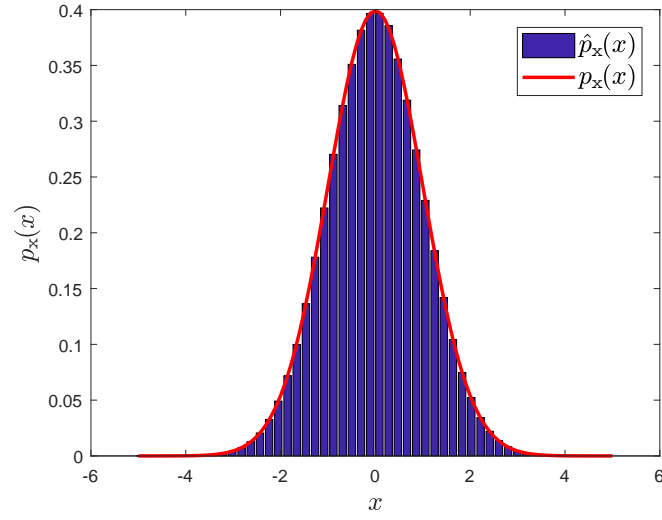


Figure 1: Gaussian PDF and histogram of samples

6.4. An example of hypothesis testing:

$$\log \frac{\mathbb{P}(\mathbf{H} = H_1 | y = y)}{\mathbb{P}(\mathbf{H} = H_0 | y = y)} \underset{\hat{\mathbf{H}}=H_0}{\overset{\hat{\mathbf{H}}=H_1}{\gtrless}} \gamma$$

A Source code

Source code for plotting Figure ?? is shown as follows.

```
1 n = 1e6; % n samples
2 data = randn(1e6, 1); % Generate n Random Gaussian samples.
3 nbins = 50; % bins in your histogram
4 [cnt, x_hist] = hist(data, nbins); % not to plot, only to get
   empirical distribution.
5 figure;
6 cnt = cnt / n / (x_hist(2) - x_hist(1)); % normalization, be
   careful :)
7 bar(x_hist, cnt); % plot the hist using bar()
8 hold on;
9 x = -5 : 0.1 : 5;
10 plot(x, normpdf(x), 'r', 'linewidth', 2);
11 legend({'$\hat{p}_{\sf{x}}(x)$', '$p_{\sf{x}}(x)$'}, 'Interpreter',
   'LaTeX', 'fontsize', 15);
12 xlabel('$x$', 'Interpreter', 'LaTeX', 'fontsize', 15); % You may
   change the size accordingly
13 ylabel('$p_{\sf{x}}(x)$', 'Interpreter', 'LaTeX', 'fontsize', 15);
14 title(<your-title-here>)
```

Listing 1: FigurePlot