

**Fig. 1.** The instances created by Alice and Bob in the proof of Theorem ?? for an instance of  $\operatorname{Index}_{L-2}$  with X[J]=1. The dashed intervals on the upper part correspond to the zero elements of the bitvector X. The red intervals  $I_1$ ,  $I_2$  correspond to expired intervals.  $I_J$  is the only non-expired interval disjoint with the special interval  $I_{L-1}$ .

of size 1.

Since X[J] = 1, the optimal solution is of size 2. If X[J] was equal to 0 then the interval  $I_J$  would not be disjoint with  $I_{L-1}$ , and, thus, an optimal solution would be