Axiom 4) of Definition **??** for the endomorphism operad $\mathcal{O}\mathrm{End}(X)$: Applying f to arguments of the form $\tau_{\sigma_i}(g_i)$ yields the same function as first permuting all arguments via $(\sigma_1, \ldots, \sigma_n)^{-1}$ and then applying $f \star \vec{g}$.

$$f \left[\begin{array}{ccccc} & & \uparrow & & \\ & & f & & \\ \leftarrow & g_1 & \rightarrow & \ldots & \leftarrow & g_n & \rightarrow \\ & \downarrow & & & & \downarrow & \end{array} \right] f \star \vec{g}$$

$$\tau_{\sigma_1}(g_1) \left[\begin{array}{ccc} \prec \sigma_1^{-1} \succ & \sigma_n^{-1} & \prec \sigma_n^{-1} \succ \end{array} \right] (\sigma_1, \ldots, \sigma_n)^{-1}$$