Let's consider the partition $\sum_{k=1}^{R}(\beta_k + m_{k-1} + 1)(R - k + 1) - R$.