Let's consider the partition
$$\sum_{k=1}^R (\beta_k + m_{k-1} + 1)(R - k + 1) - R$$
.
$$\beta_2 + m_1 + 1$$

$$R - 1$$

$$\beta_R + m_{R-1} + 1$$

$$1$$

$$1$$

$$1$$

$$1$$