The induced 
$$P_3$$
 in  $\mathcal{C}_3(P_3)$  generated by  $(\{v_1, v_3\}, C)$ , for  $C = \{c_1, c_2, c_3\}$  as in Eqs. (??).