Illustration for Lemma **??**. The boundary Harnack principle cannot be used to estimate increments between y and y' due to the singularity at z. Instead, we demonstrate regularity within the smaller ball by leveraging harmonicity in the larger ball.

Figure 1: Illustration for Lemma **??**: Regularity within the smaller ball $D \cap B(Q, 3\delta_D(y))$ using harmonicity in the larger ball.

The figure above shows the region $D \cap B(Q, 3\delta_D(y))$, which is a smaller ball centered at Q with radius $3\delta_D(y)$, where $\delta_D(y)$ is the distance from y to the boundary ∂D. The larger ball $B(Q, r)$ is used to ensure harmonicity properties, while the singularity at z is handled independently.