$$S_{10} = 1$$
 $S_{8} = 2 \cdots S_{10} = 1 \cdots 1$ $S_{9} = 1$ $S_{6} = S_{7} = 2 \cdots S_{9} = 1 \cdots 2$ $S_{8} = 2 S_{5} = S_{6} = 2 \cdots S_{8} = 1 \cdots 3$ $S_{4} = 2 \cdots S_{8} = 1 \cdots 4$ $S_{3} = S_{4} = 2 \cdots S_{6} = 1 \cdots 5$ $S_{2} = S_{3} = 2 \cdots S_{6} = 1 \cdot S_{5} = 3$ $S_{2} = 2 \cdots S_{4} = 1 \cdots 7$ $S_{4} = 3$ $S_{1} = 2 \cdots S_{8} = 1 \cdots 8$ $S_{2} = 1 \cdots 9$ $S_{2} = 2 \cdots S_{1} = 1 \cdots 9$ $S_{1} = 1 \cdots 9$ $S_{1} = 1 \cdots 9$