Consider the set of distributions $\{P_{Y|X}(\bullet|x) : x \in \mathbb{X}\}$ lying in a transformation model $\mathcal{P}_{\mathbb{G}} = \{g_* P_0 : g \in \mathbb{G}\} \subseteq \mathcal{P}(\mathbb{Y})$. For each $x \in \mathbb{X}$, there exists an element g_x of the group $\mathbb{G}$ such that $P_{Y|X}(\bullet|\phi x) = g_x * P_0$. When transforming x with ϕ, there is also an element $g_{\phi x}$ such that $P_{Y|X}(\bullet|x) = g_{\phi x} * P_0$. This enables direct transition from $P_{Y|X}(\bullet|x)$ to $P_{Y|X}(\bullet|\phi x)$ via the coboundary $c(\phi, x) : (\phi, x) \mapsto g_{\phi x} g_x^{-1} \in \mathbb{G}$. The coboundary $c(\phi, x)$ does not depend on P_0 and thus applies to (infinitely) many other conditional distributions. Modelling the coboundary instead of $P_{Y|X}$ helps improve robustness against model mis-specification.

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