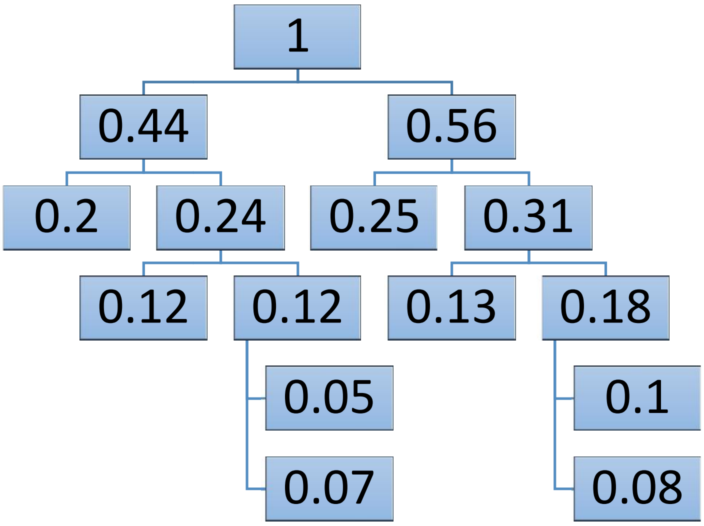
Assignment 5

Chapter 4

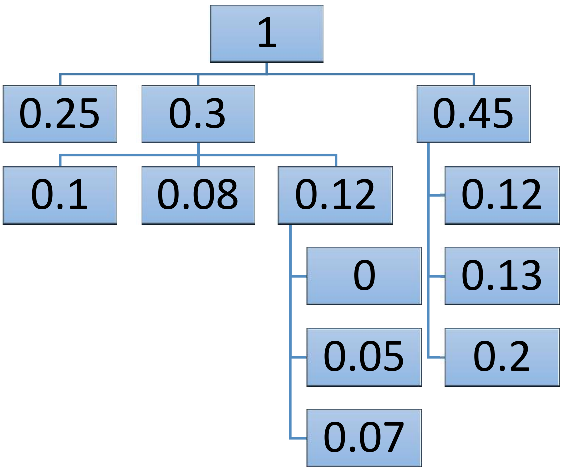
1. Construct a binary Huffman code for the distribution $\{0.25, 0.05, 0.1, 0.13, 0.2, 0.12, 0.08, 0.07\}$.



Left 0, Right 1

2. Construct a ternary Huffman code for the source distribution in Problem 1.

Add one zero probability mass.



3. Show that a Huffman code is an optimal uniquely decodable code for a given source distribution.

4. Construct an optimal binary prefix code for the source distribution in Problem 1 such that all the codewords have even lengths.

We can first construct the 4-nary Huffman code, and convert it into the binary Huffman code with even lengths.

6. Prove that if $p_1 > 0.4$, then the shortest codeword of a binary Huffman code has length equal to 1. Then prove that the redundancy of such a Huffman code is lower bounded by $1 - h_b(p_1)$. (Johnsen [192].)

Chapter 5

1. Show that for any $\epsilon > 0$, $W^n_{[X]\epsilon}$ is nonempty for sufficiently large n.

Proof

For n sufficiently large,

$$(1-\epsilon)2^{n(H(X)-\epsilon)} \leq |W^n_{[X]\epsilon}| \leq 2^{n(H(X)+\epsilon)}. \tag{5.11}$$

For sufficient large n, we can make $(1-\varepsilon)2^{n(H(X)-\varepsilon)} \geq 1$, i.e., nonempty $W^n_{[X]\varepsilon}$.