Homework 1 ECE 4710J

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

Q1

1. B= $\begin{bmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{bmatrix}$

1. A= $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

1. $AB\vec{v_2} = \vec{x}$

$$\vec{v_2} = \begin{bmatrix} 5.5 \\ 2.20833 \\ 1 \end{bmatrix}$$

In [ ]:
```
B = np.array([[2, 2, 2], [5, 8, 0], [0, 2, 3], [0, 0, 10]])
A = np.array([[2,1,0,0],[1,1,1,1],[0,0,0,10]])
x = np.array([[80], [80], [100]])
AB = np.dot(A,B)
v2 = np.dot(np.linalg.inv(AB), x)
```

Q2

1.
$$\begin{aligned}
\sigma(-x) &= \frac{1}{1+e^x} \\
1-\sigma(x) &= 1-\frac{1}{1+e^{-x}} \\
&= \frac{e^{-x}\cdot e^x}{(1+e^{-x})\cdot e^x} \\
&= \sigma(-x)
\end{aligned}$$

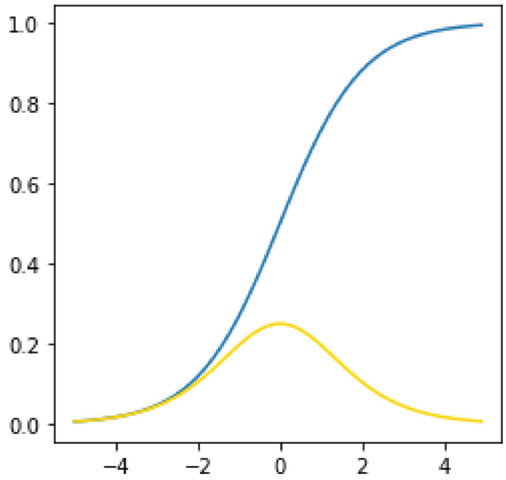
2.
$$\begin{aligned}
\frac{d}{dx}\sigma(x) &= \frac{e^{-x}}{(1+e^{-x})^2} \\
\sigma(x)(1-\sigma(x)) &= \frac{1}{1+e^{-x}}\frac{1}{1+e^x} \\
&= \frac{e^{-x}}{(1+e^x)(1+e^{-x})e^{-x}} \\
&= \frac{e^{-x}}{(1+e^{-x})^2}
\end{aligned}$$

```
In [ ]: def f(x):
            return 1/(1+np.exp(-x))

        def df(x):
            return  (np.exp(-x))/(1+np.exp(-x))**2

        def plot(f, df):
            plt.figure(figsize=(4,4))
            x = np.arange(-5, 5, 0.1)
            plt.plot(x, f(x))
            plt.plot(x, df(x), color='gold')

        plot(f, df)
```



Q3

$$f(c) = \frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$$

$$\frac{d}{dc}f(c) = \frac{1}{n}\sum_{i=1}^{n} -2*(x_i - c)$$

$$= (\frac{1}{n}\sum_{i=1}^{n} -2x_i) + 2c$$

$$\frac{d^2}{dc^2}f(c) = 2 > 0$$

$$\text{Because } f(c) \geq 0, \frac{d^2}{dc^2}f(c) > 0, \frac{d}{dc}f(c) \text{ is always increasing, When } \frac{d}{dc}f(c) = 0, f(c) \text{ will reach the minimum.}$$

$$\text{Let } \frac{d}{dc}f(c) = 0$$

$$c = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Q4

Let B = "the woman has breast cancer" and A = "a positive test".

$$P(B) = 0.01, P(\bar{B}) = 0.99, P(A|B) = 0.8, P(A \cap \bar{B}) = 0.096$$
$$P(A) = P(A \cap B)P(B) + P(A \cap \bar{B})P(\bar{B}) = 0.8 * 0.01 + 0.096 * 0.99 = 0.103$$
$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.8 * 0.01}{0.103} = 0.078$$

Q5

b 6.1

It's a normal distribution and full width at half maximum (FWHM) could be estimated around 15. It relates to the standard deviation σ as FWHM≈2.36 σ for a normal distribution. So The closest answer is b.

Q6

No. Different biases cause this problem. Selection bias is common when analyzing. The selected sample may not cover different populations because of the sampling frame. It also had non-response bias as only 24% people responded.