A IoT-benchmark Data Generation Algorithm

Algorithm 1 describes the data generation strategy. For parameter details, please refer to Table 1.

Table 1: Parameters Description of Data Generator

Notation	Data Features
μ_v	Mean of values
μ_d	Mean of deltas
θ_d	Variance of deltas
γ	Repetition Rate
η	Increase rate
l	Series length

Algorithm 1: Numerical data generator [45]

```
Input: μv, μd, θd, γ, η, length n
Output: TS
DS := empty_list();
while |DS| < n do
    isRepeat := random_index(γ);
    if isRepeat then
    else
        repeat_len := random(8, T);
        DS.append(0, repeat_len);
    end
    isPositive := random_index(η);
    delta := 0;
    if isPositive then
    else
        while delta ≤ 0 do
            delta := random_guass(μd, θd);
        end
    end
    while delta ≥ 0 do
        delta := random_guass(μd, θd);
    end
    DS.append(delta);
end
TS := prefix_sum(DS);
TS.zoom(μv);
return TS;
```

B Proof of Proposition 4.11

The proof of Proposition 4.11 is as follows.

PROOF. Let $Cost\left(I_u\left(c\right)\right)$ denote the cost of the traditional query process of a compressed database, i.e. decompressing first, then restore and query on uncompressed data, where

$$\mathrm{Cost}(I_u\left(c\right)) = Cost\left(\left(Q_u \circ R_u \circ U\right)\left(c\right)\right) = Cost\left(Q_u\left(R_u\left(U\left(c\right)\right)\right)\right)$$

$$= \mathrm{Cost}\left(\left\{u: \begin{array}{l} u_1 \leftarrow op_1\left(u_0\right), u_2 \leftarrow op_2\left(u_1\right), \\ \ldots, u \leftarrow op_n\left(u_{n-1}\right)\end{array} \middle| \begin{array}{c} i \in \{1,\ldots,n\} \\ op_i \in \Pi \end{array}\right\}\right)$$

$$+ \mathrm{Cost}(R_u\left(u\right)) + Cost\left(U\left(c\right)\right)$$

$$= \mathrm{Cost}(U\left(c\right)) + Cost\left(R_u\left(u\right)\right) + Cost\left(\left\{op_1, op_2, \ldots, op_{n-1}\right\}\left(u_1\right)\right).$$

Let $Cost\left(I_c\left(c\right)\right)$ denote the cost of the partial homomorphic query process of a compressed database, where

$$\mathrm{Cost}(I_c\left(c\right)) = Cost\left(\left(I'_u \circ Q_c \circ R_c\right)\left(c\right)\right) = Cost\left(I'_u\left(Q_c\left(R_c\left(c\right)\right)\right)\right)$$

$$= \mathrm{Cost}(I'_u\left(c_j\right)) + Cost\left(Q_c\left(c_0\right)\right) + Cost\left(R_c\left(c\right)\right)$$

$$= \mathrm{Cost}\left(\left\{u: \begin{array}{l} u_j \leftarrow op_j\left(u_{j-1}\right), u_{j+1} \leftarrow op_{j+1}\left(u_j\right), \\ \ldots, u \leftarrow op_n\left(u_{n-1}\right)\end{array} \middle| \begin{array}{c} i \in \{j,\ldots,n\} \\ op_i \in \Pi \end{array}\right\}\right)$$

$$+ \mathrm{Cost}\left(\left\{c_j: \begin{array}{l} c_1 \leftarrow op'_1\left(c_0\right), c_2 \leftarrow op'_2\left(c_1\right), \\ \ldots, c_j \leftarrow op'_j\left(c_{j-1}\right)\end{array} \middle| \begin{array}{c} i \in \{1,\ldots,j\} \\ op'_i \in \Theta \end{array}\right\}\right)$$

$$+ \mathrm{Cost}(U\left(c_j\right)) + Cost\left(R_c\left(c\right)\right)$$

$$= \mathrm{Cost}(R_c\left(c\right)) + Cost\left(\left\{op'_1, op'_2, ..op'_j\right\}\left(c_0\right)\right) + Cost\left(U\left(c_j\right)\right)$$

$$+ \mathrm{Cost}(\left\{op_{j+1}, op_{j+2}, \ldots, op_{n-1}\right\}\left(u_{j-1}\right)),$$

with $\varphi\left(u_i\right) = c_i$, $j \in \{1 ... n-1\}$. Then we have
$\mathrm{Cost}(I_u\left(c\right)) - Cost\left(I_c\left(c\right)\right)$

$$= \mathrm{Cost}(U\left(c\right)) + Cost\left(R_u\left(u\right)\right) + Cost\left(\left\{op_1, op_2, \ldots, op_{n-1}\right\}\left(u_0\right)\right)$$

$$- \left(Cost\left(R_c\left(c\right)\right) + Cost\left(\left\{op'_1, op'_2, ..op'_j\right\}\left(c_0\right)\right) + Cost\left(U\left(c_j\right)\right)\right.$$
$$\left. + \mathrm{Cost}(\left\{op_{j+1}, op_{j+2}, \ldots, op_{n-1}\right\}\left(u_{j-1}\right))\right)$$

$$= \left(Cost\left(U\left(c\right)\right) - Cost\left(U\left(c_j\right)\right)\right) + \left(Cost\left(R_u\left(u\right)\right) - Cost\left(R_c\left(c\right)\right)\right)$$

$$+ \left(Cost\left(\left\{op_1, op_2, \ldots, op_{n-1}\right\}\left(u_0\right)\right) - Cost\left(\left\{op'_1, op'_2, ..op'_j\right\}\left(c_0\right)\right)\right.$$
$$\left. - \mathrm{Cost}(\left\{op_{j+1}, op_{j+2}, \ldots, op_{n-1}\right\}\left(u_{j-1}\right))\right)$$

$$= \left(Cost\left(U\left(c\right)\right) - Cost\left(U\left(c_j\right)\right)\right) + \left(Cost\left(R_u\left(u\right)\right) - Cost\left(R_c\left(c\right)\right)\right)$$

$$+ \left(Cost\left(\left\{op_1, op_2, \ldots, op_j\right\}\left(u_0\right)\right) - Cost\left(\left\{op'_1, op'_2, ..op'_j\right\}\left(c_0\right)\right)\right)$$

Note that when $j = n-1$, query Q is a fully homomorphic query.

By Lemma 4.10, $Size\left(c\right) \geq Size\left(c_j\right)$, thus, $Cost\left(U\left(c\right)\right) \geq Cost\left(U\left(c_j\right)\right)$. And by Definition 4.9, we have

$$Cost\left(\left\{op_1, op_2, \ldots, op_j\right\}\left(u_0\right)\right) \geq Cost\left(\left\{op'_1, op'_2, \ldots, op'_j\right\}\left(c_0\right)\right).$$

And by Definition 4.8, we have

$$Cost\left(R_u\left(u\right)\right) \geq Cost\left(R_c\left(c\right)\right)$$

Thus, we have $Cost\left(I_u\left(c\right)\right) - Cost\left(I_c\left(c\right)\right) \geq 0$, i.e.,

$$Cost\left(I_c\left(c\right)\right) \geq Cost\left(I_c\left(c\right)\right).$$

□