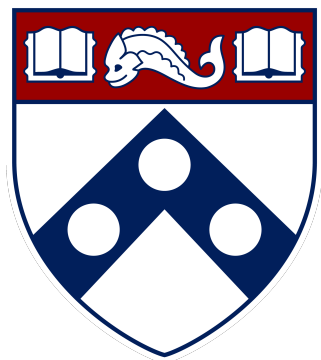
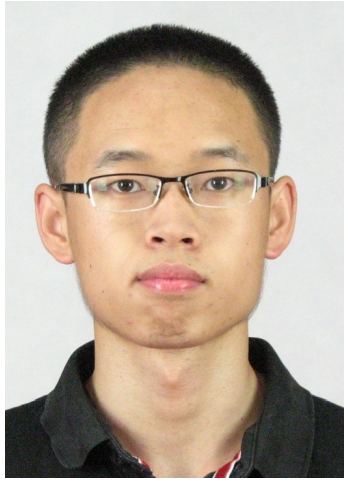
Minimax-optimal reward-agnostic exploration in reinforcement learning

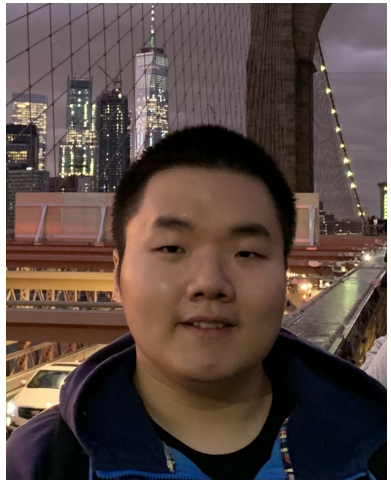


Yuxin Chen

Statistics & ESE, UPenn

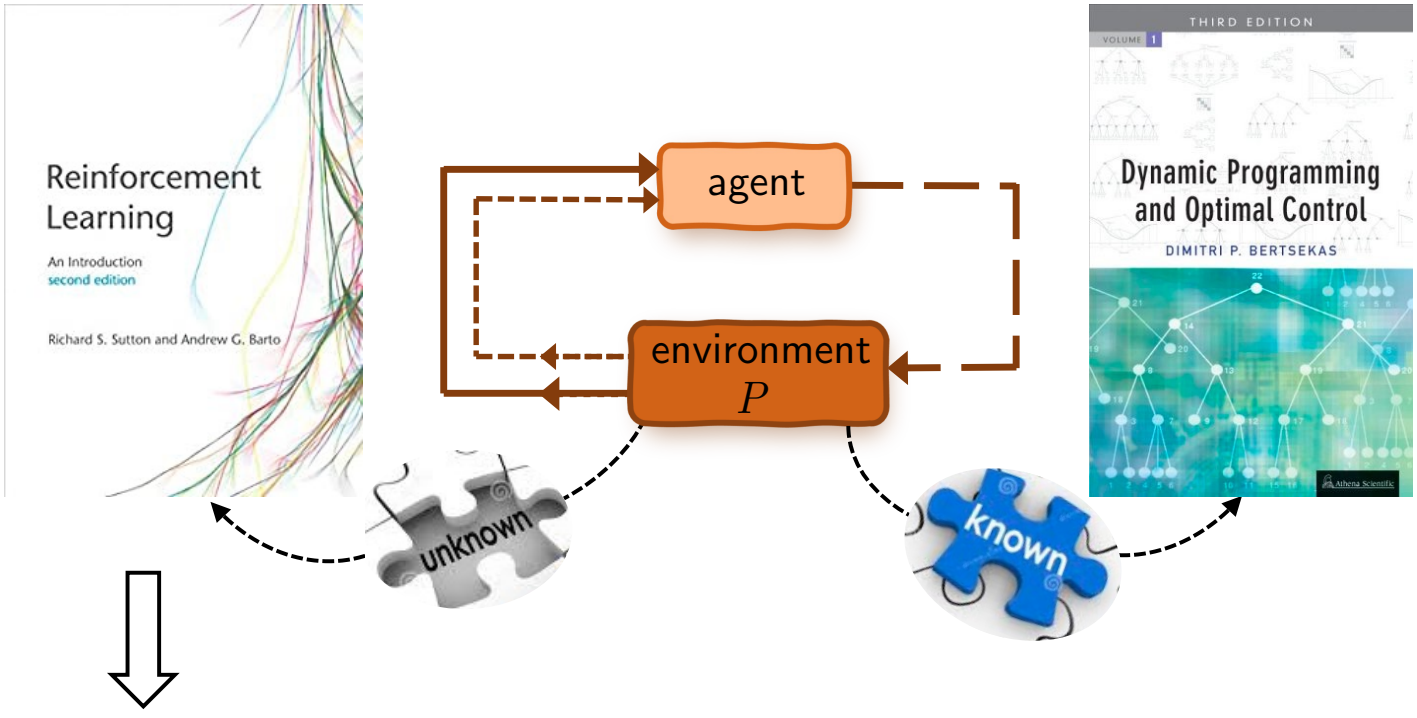


Gen Li
CUHK

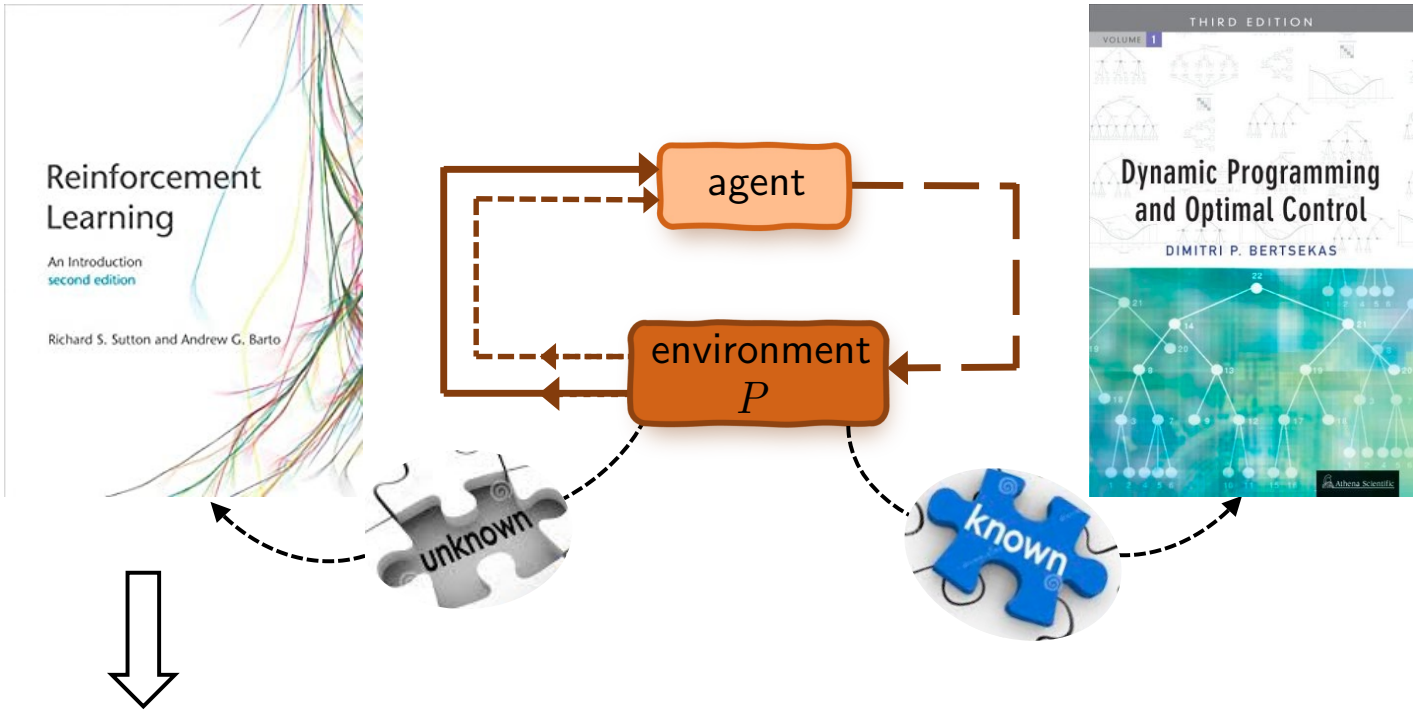


Yuling Yan
MIT

Jianqing Fan
Princeton

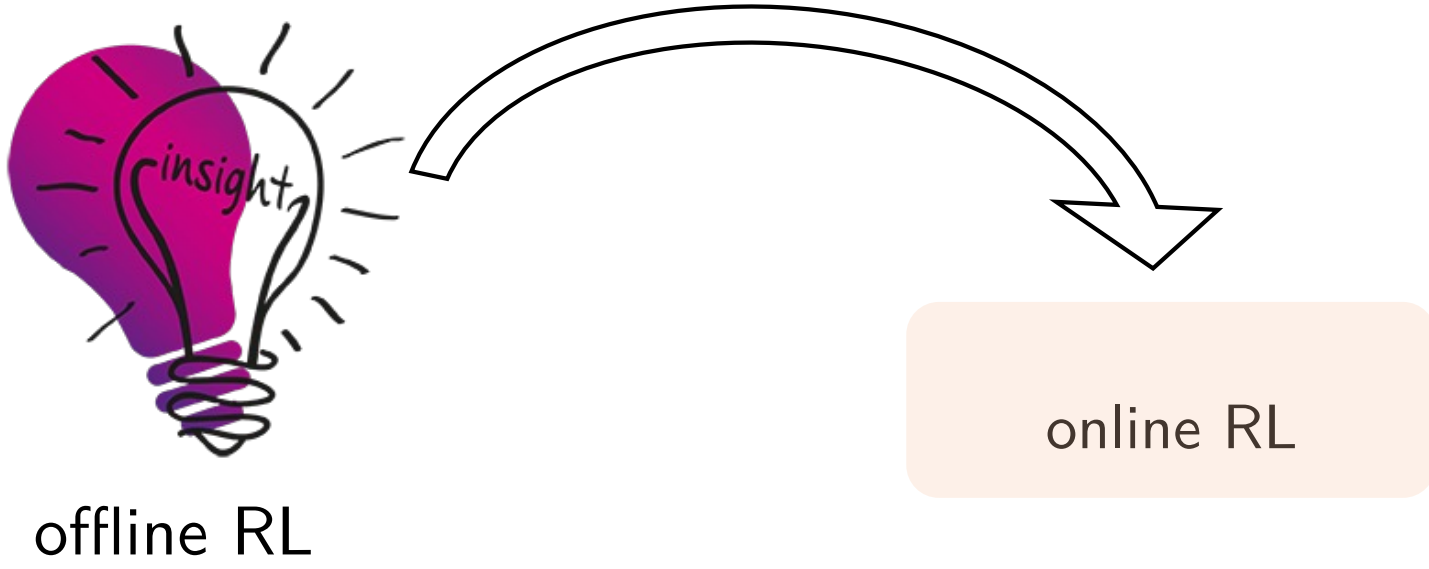


In RL, we need to collect data to learn unknown environments

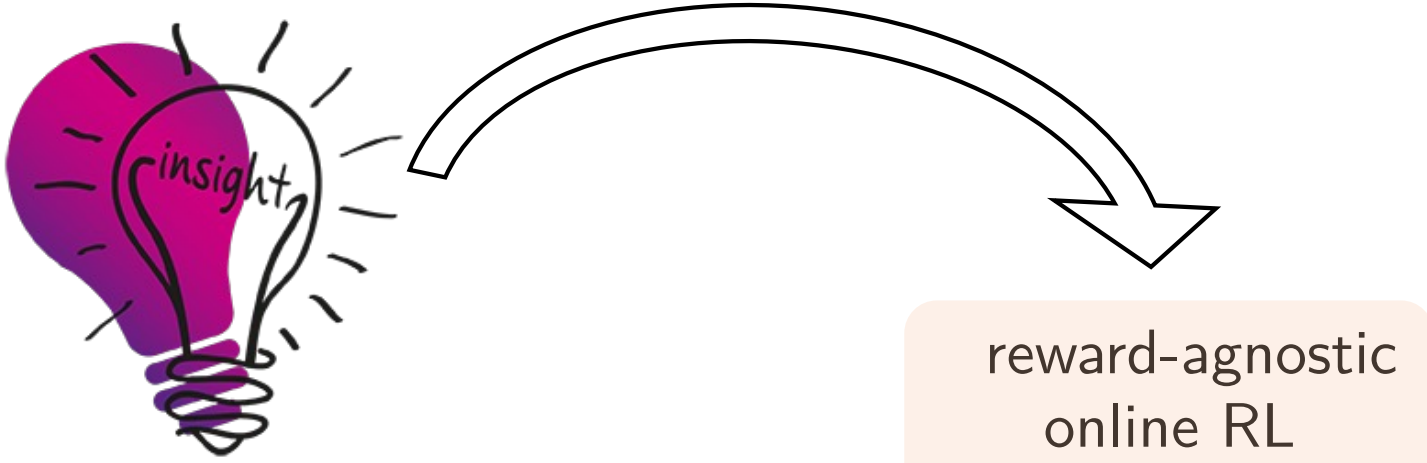


In RL, we need to collect data to learn unknown environments

1. simulator (Li, Wei, Chi, Chen '24, Operations Research)
2. **online RL** **(Zhang, Chen, Lee, Du '24, COLT)**
3. **offline RL** **(Li, Shi, Chen, Chi, Wei '24, Annals. Stats)**



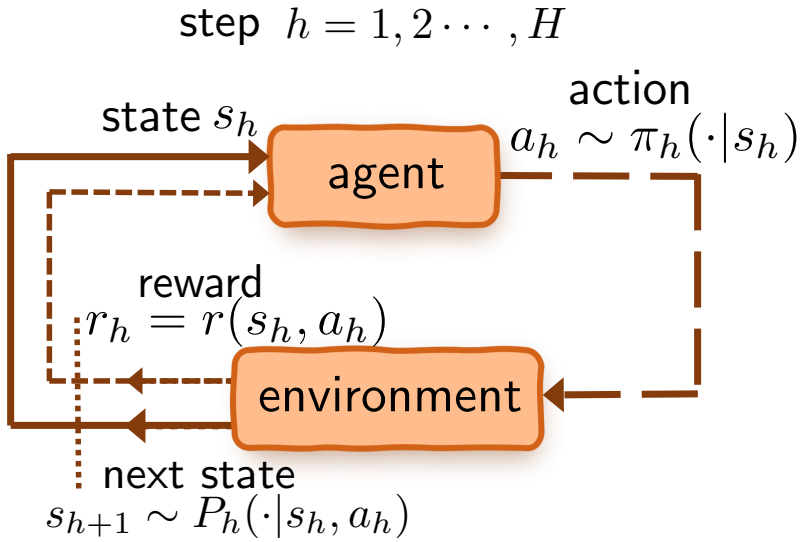
Key takeaway of this talk: insights from offline RL can inspire online RL algorithms



Key takeaway of this talk: insights from offline RL can inspire (reward-agnostic) online RL algorithms

Preliminaries

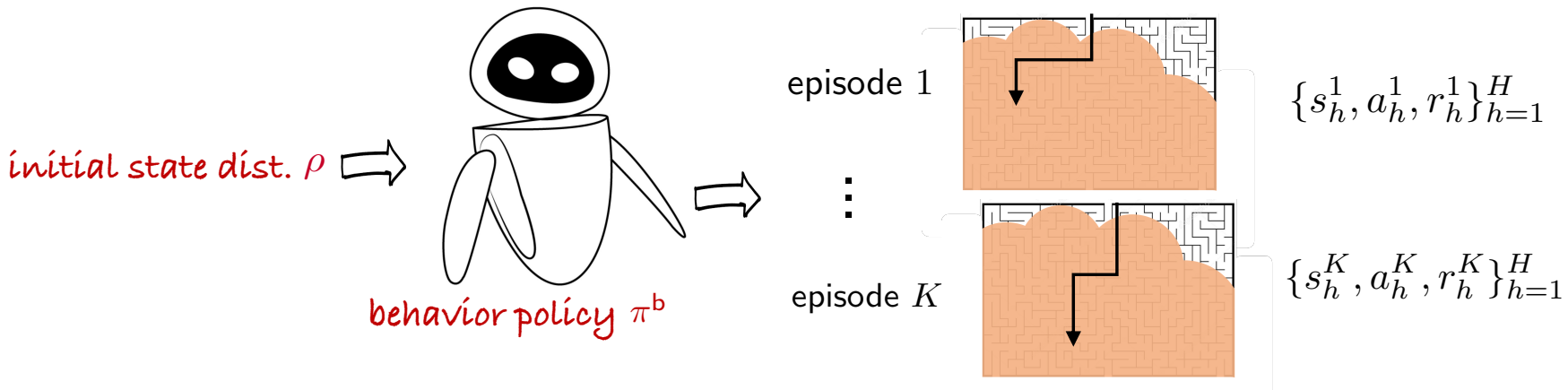
Finite-horizon Markov decision process (MDP)



- H: horizon length (large)
- $\mathcal{S} = \{1, \ldots, S\}$: state space (large)
- $\mathcal{A} = \{1, \ldots, A\}$: action space (large)

A mathematical model for offline RL

A historical dataset $\mathcal{D}$ containing K episodes generated by π^{b}:



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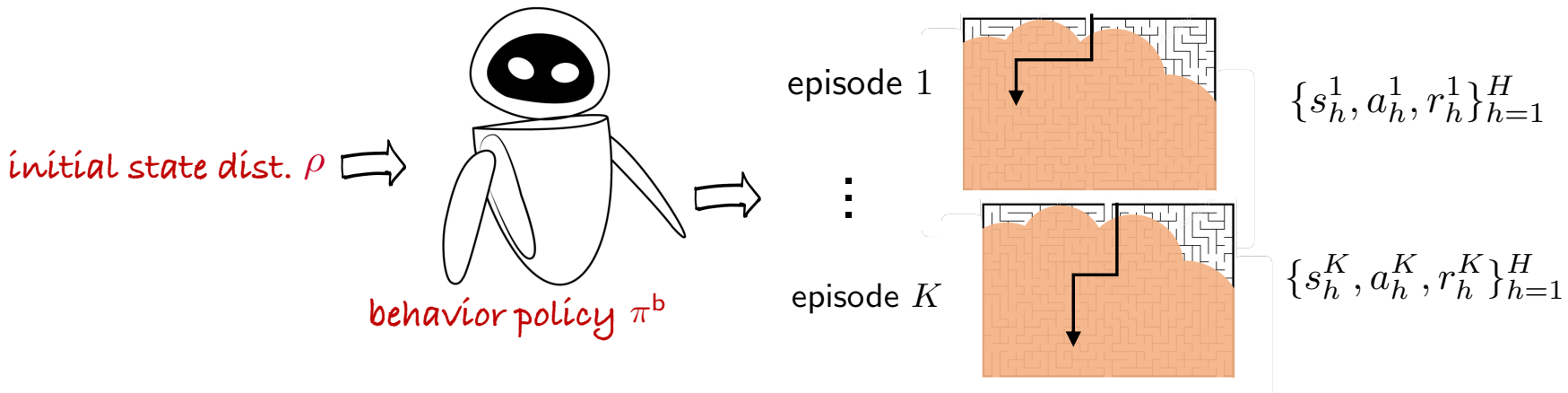
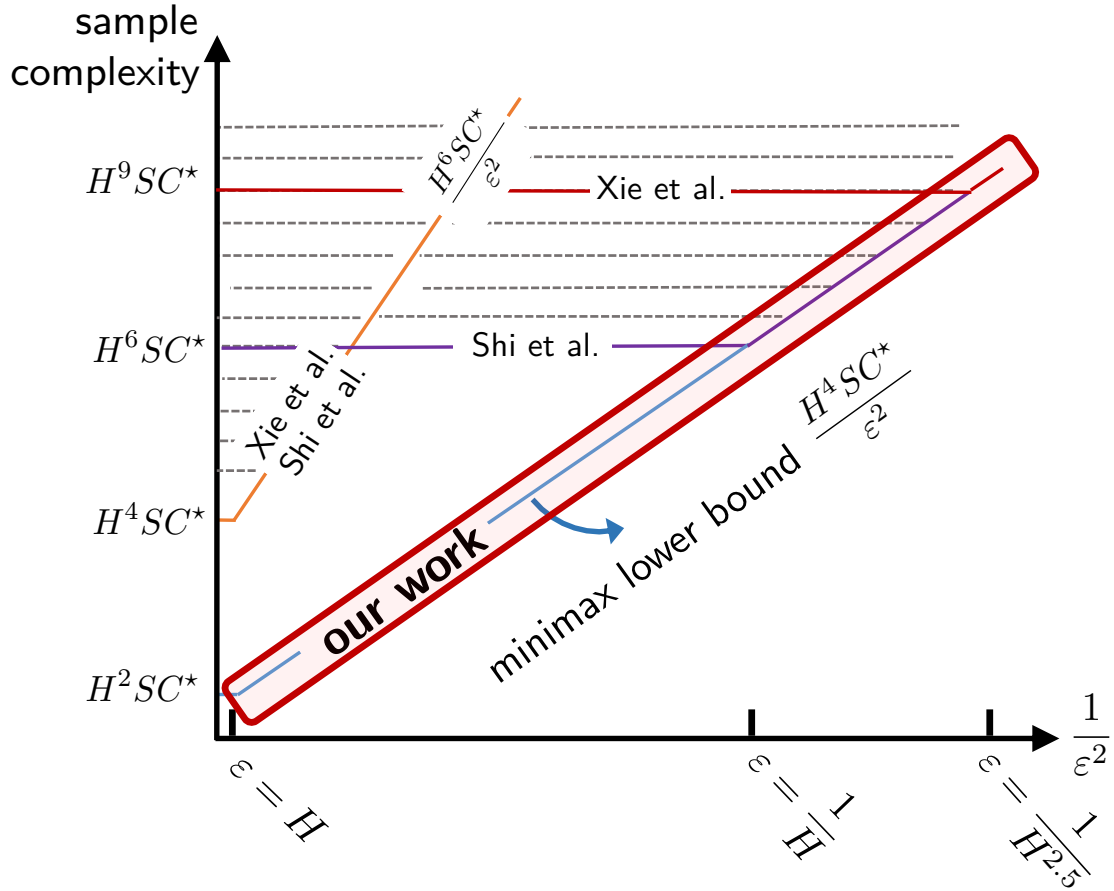


- single-policy concentrability coefficient: $C^\star := \left\| \frac{d^{\pi^\star}}{d^{\pi^{\mathsf{b}}}} \right\|_\infty$



Theorem 1 (Li, Shi, Chen, Chi, Wei '24)

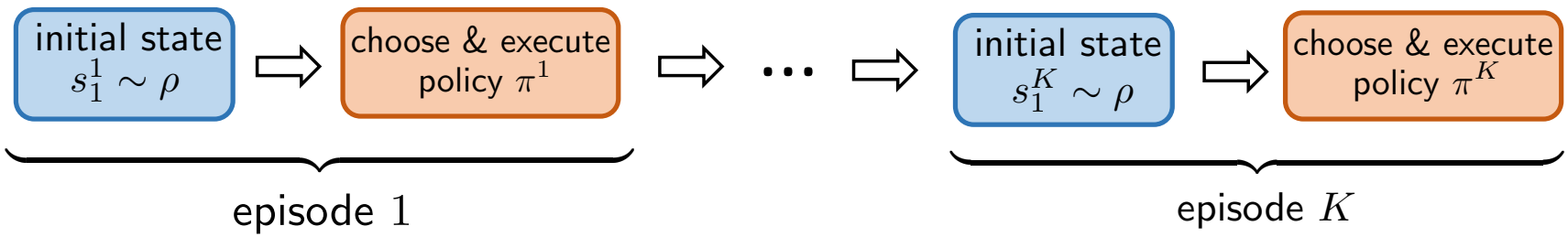
For any $0 < \varepsilon \le H$, we can design a pessimistic model-based algorithm that achieves $V_1^\star(\rho) - V_1^{\widehat{\pi}}(\rho) \le \varepsilon$ with

$$\widetilde{O}\left(\frac{H^3SC^\star}{\varepsilon^2}\right) \text{ episodes} \qquad \text{or} \qquad \widetilde{O}\left(\frac{H^4SC^\star}{\varepsilon^2}\right) \text{ samples}$$

This talk: reward-agnostic exploration

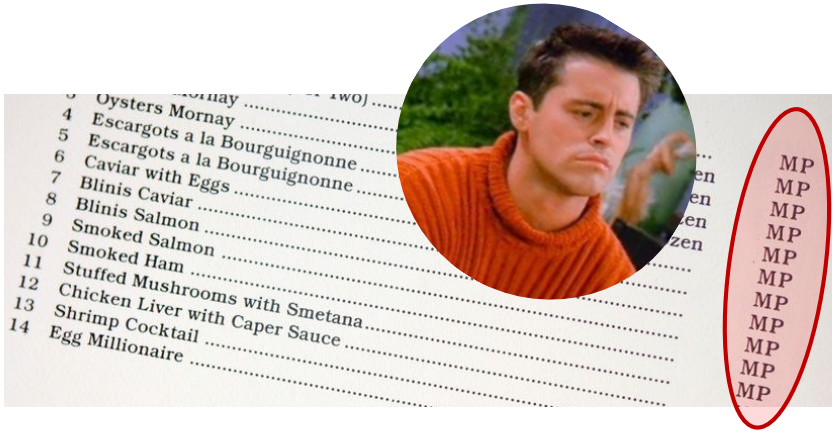
Online RL: interacting with real environments

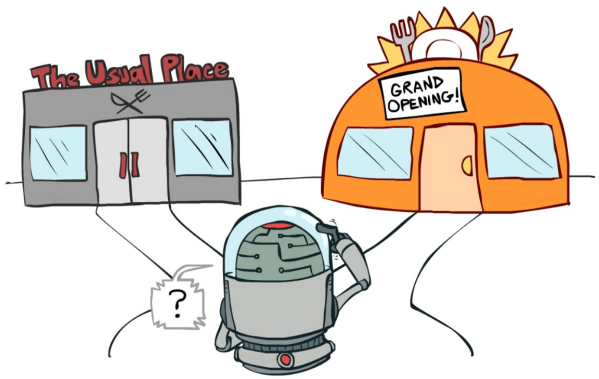
Sequentially execute MDP for K episodes, each containing H steps



Reward-agnostic exploration?

The learner is unaware of the rewards during exploration ...





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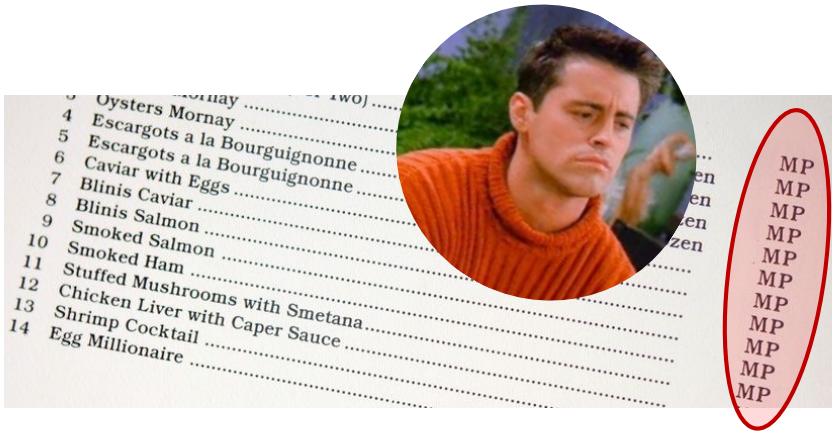

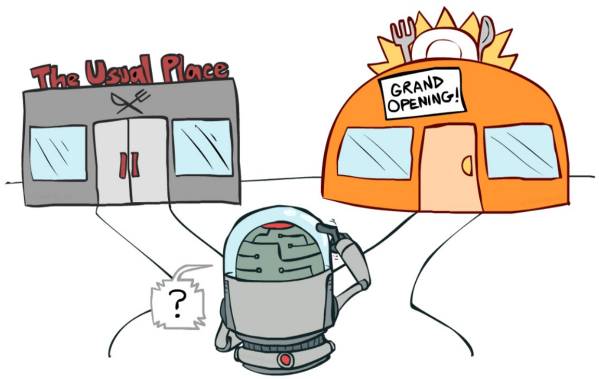

Motivation

- (significantly) delayed feedback
- reward functions keep changing
- many reward functions of interest

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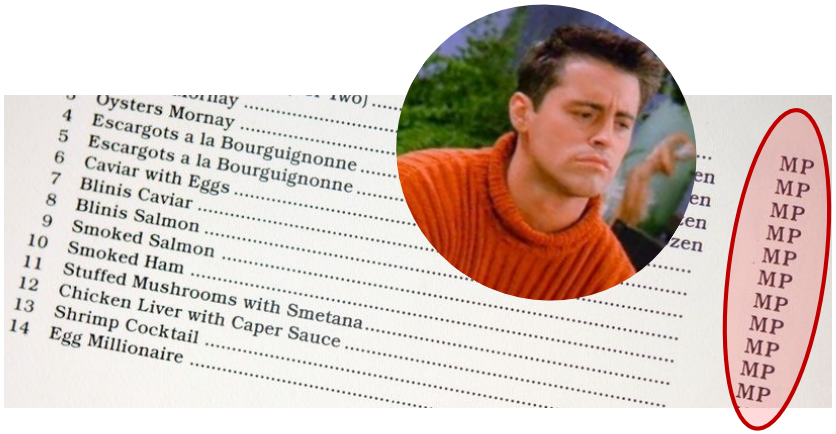

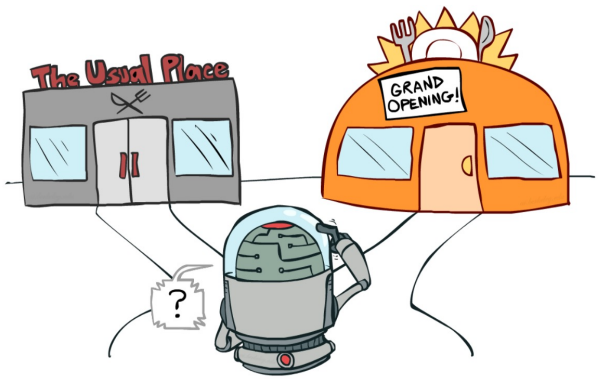

Motivation

- (significantly) delayed feedback
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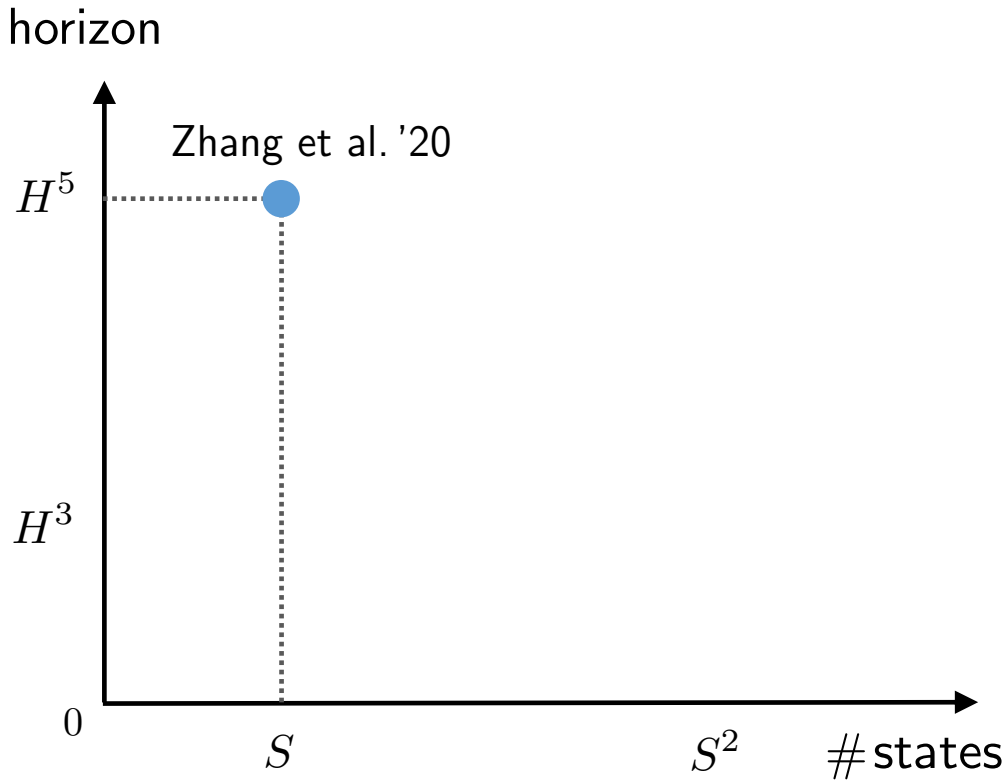
- many reward functions of interest

Question: can we perform pure exploration just once but achieve efficiency for many unseen reward functions at once?

Prior art: sample complexity upper bounds

Suppose there is a fixed (but unseen) reward function of interest ...



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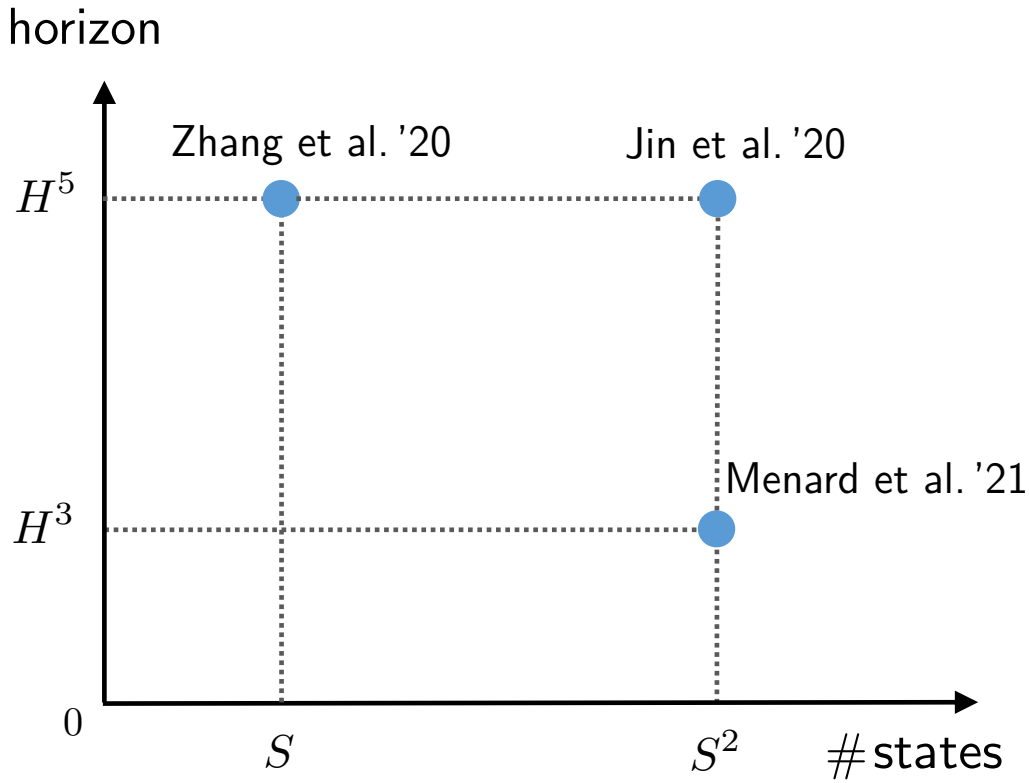

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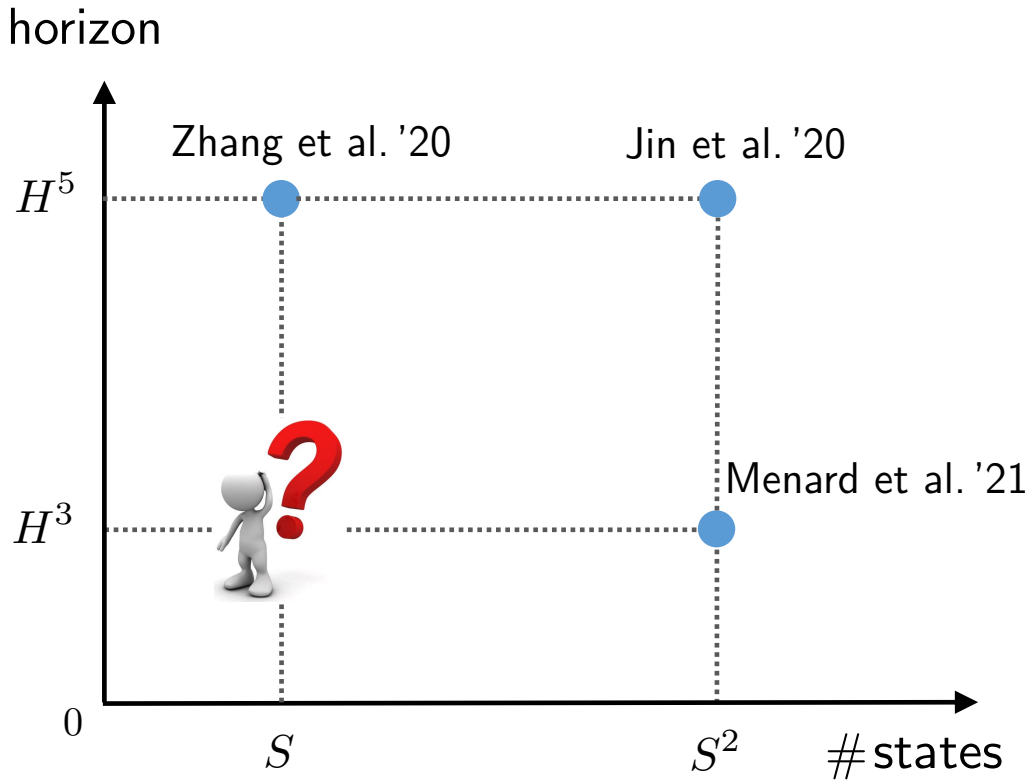

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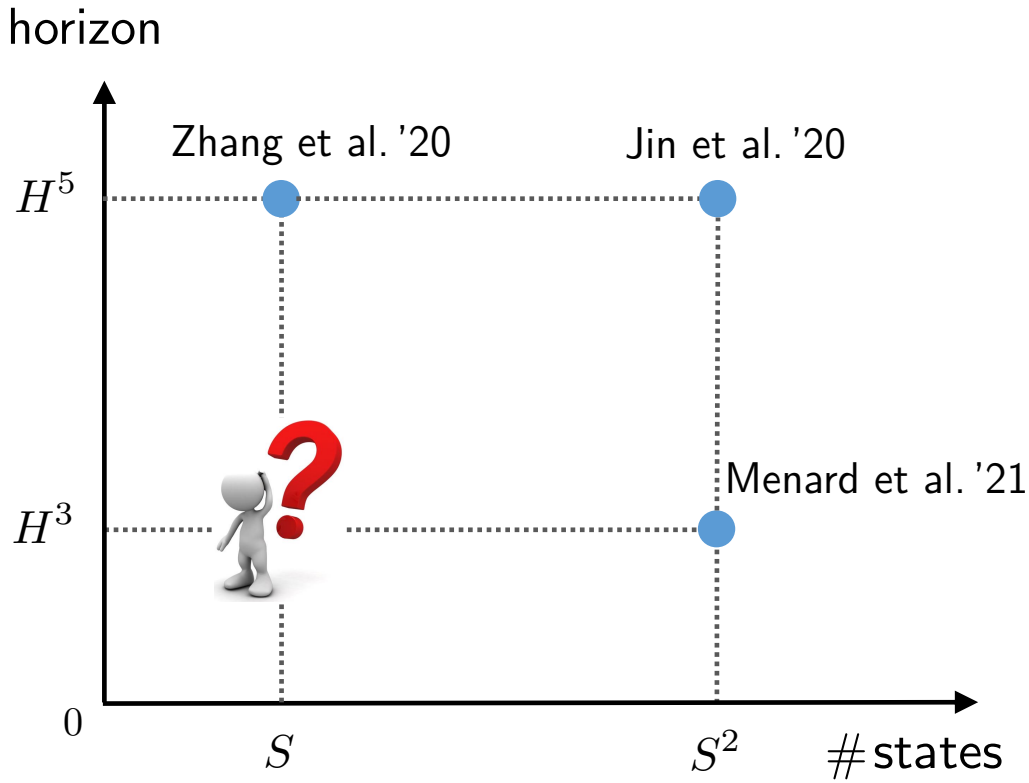
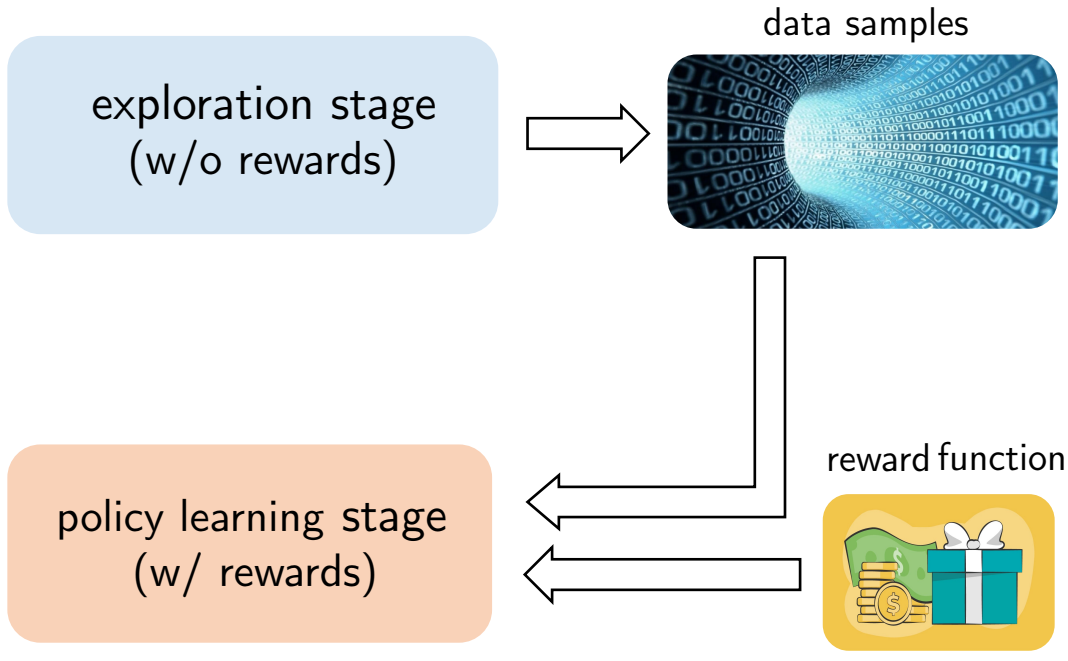
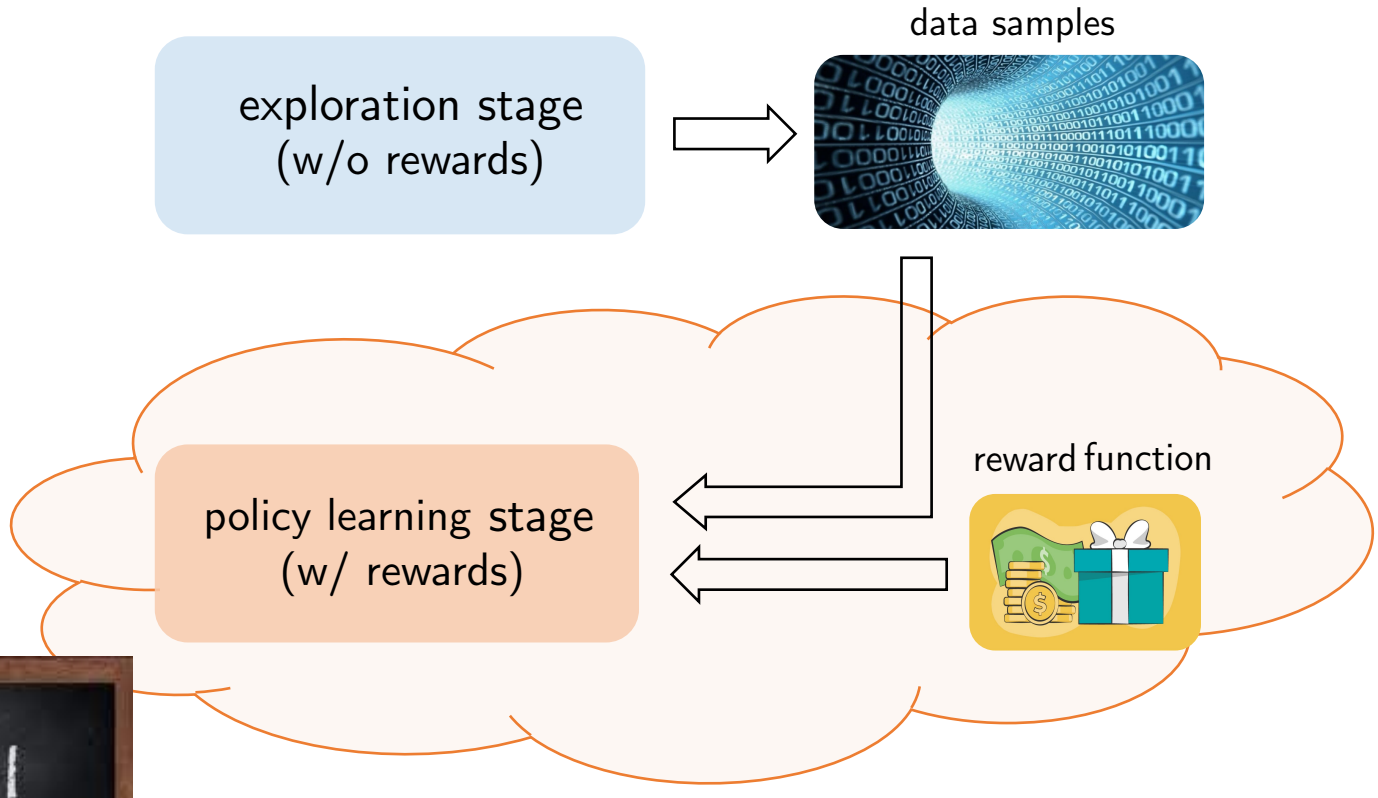


Question: can we simultaneously optimize dependency on S & H?

exploration stage
(w/o rewards)


exploration stage
(w/o rewards)
data samples
policy learning stage
(w/ rewards)
reward function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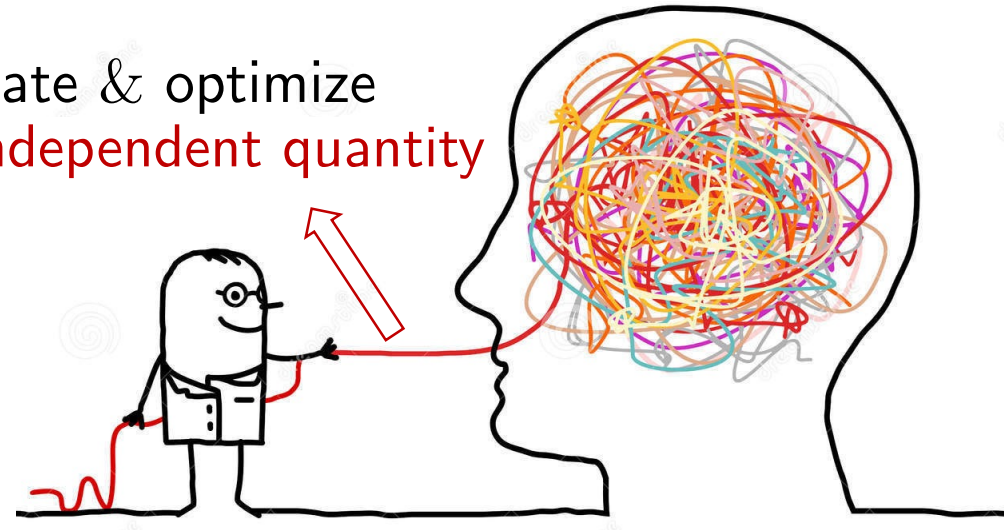
LESSON 1
Offline RL

isolate & optimize
reward-independent quantity



isolate & optimize
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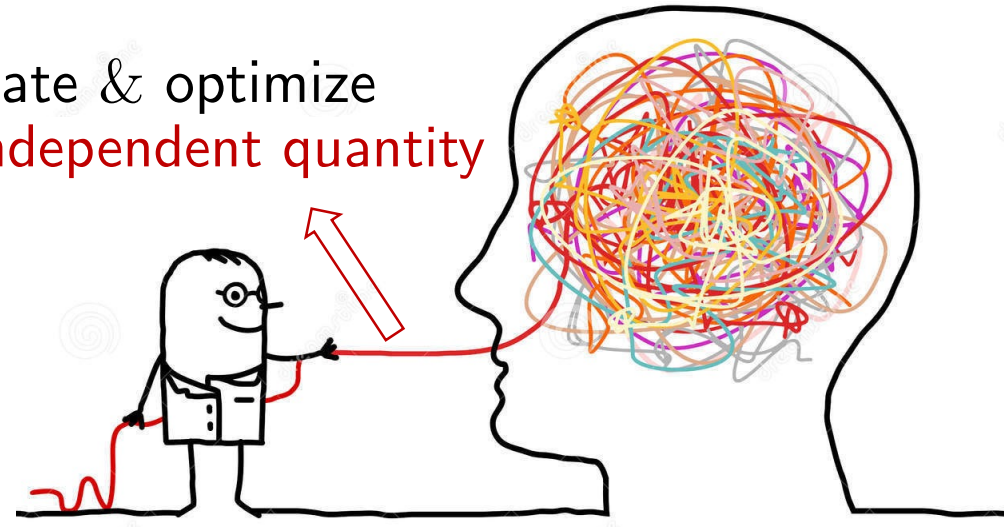
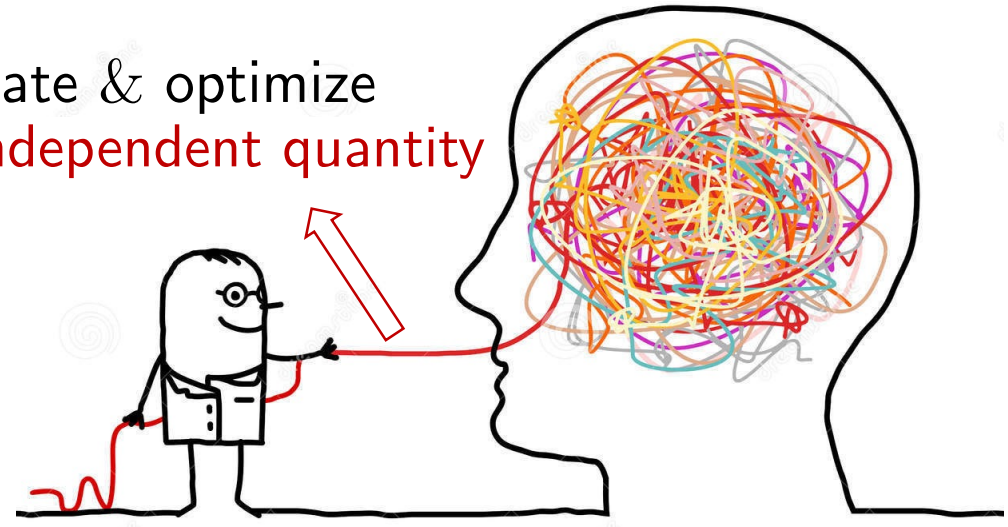


lessons learned from offline RL: offline model-based alg. gives

$$V_1^\star(\rho) - V_1^{\widehat{\pi}}(\rho) \lesssim \frac{1}{\sqrt{K}} \sum_{h,s,a} d_h^{\pi^\star}(s,a) \min\left\{ \sqrt{\frac{\mathsf{Var}_{h,s,a}(V_{h+1}^\star)}{d_h^{\mathsf{behavior}}(s,a)}},\ H \right\}$$

isolate & optimize
reward-independent quantity



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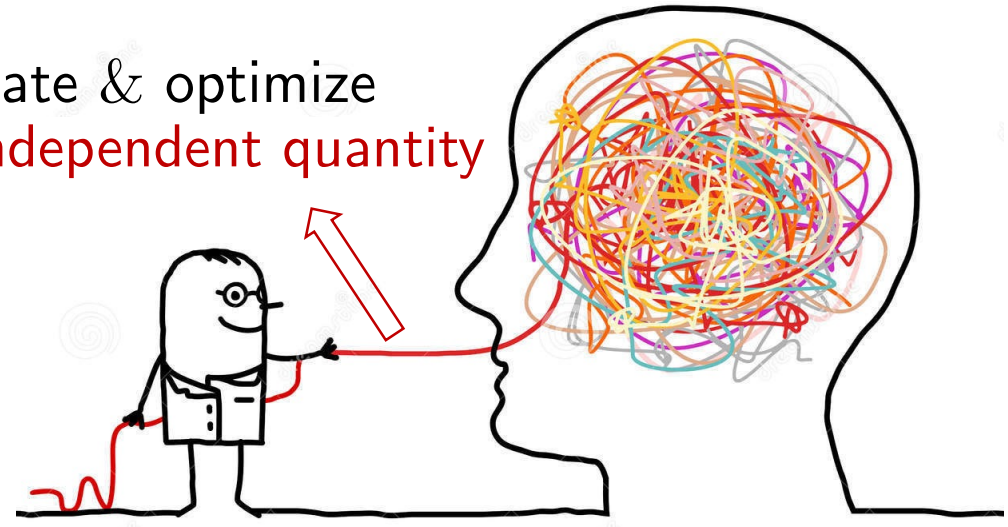
$$\lesssim \frac{1}{\sqrt{K}} \underbrace{\left(\max_\pi \sum_{h,s,a} \frac{d_h^\pi(s,a)}{\frac{1}{KH} + d_h^{\mathsf{behavior}}(s,a)} \right)^{\frac{1}{2}}}_{\text{reward-independent}} \underbrace{\left(\sum_{h,s,a} d_h^{\pi^\star}(s,a) \mathsf{Var}_{h,s,a}(V_{h+1}^\star) + H \right)^{\frac{1}{2}}}_{\text{reward-dependent}}$$

isolate & optimize
reward-independent quantity



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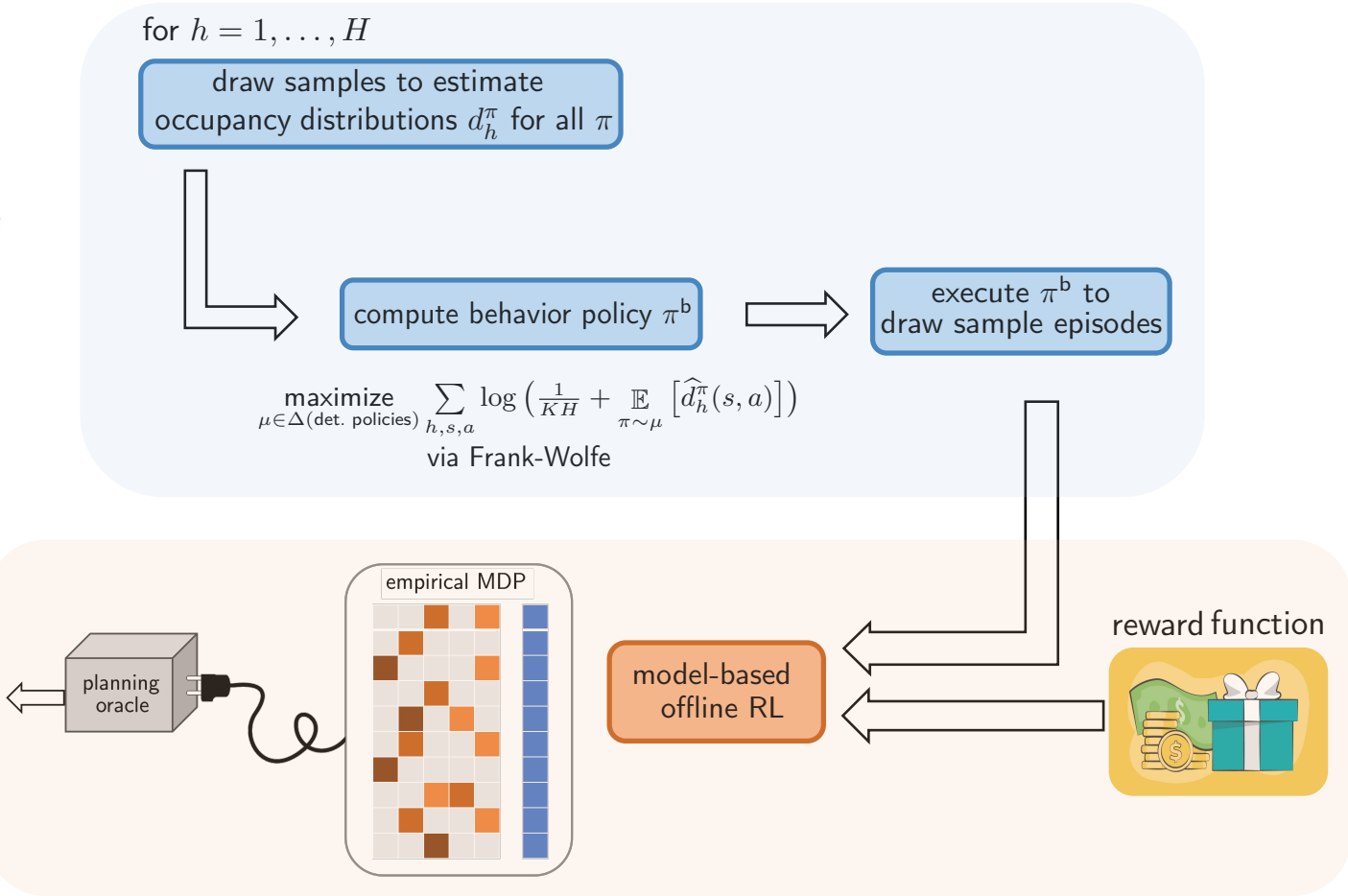
key: find behavior policy to optimize reward-independent quantity

Our algorithm

exploration stage (w/o rewards)

policy learning (w/ rewards)



Main results

Theorem 2 (Li, Yan, Chen, Fan '23)

Suppose there are poly(H, S, A) *fixed reward functions of interest, and suppose* ε *is small enough. Using the same batch of samples w/*

$$\widetilde{O}\left(\frac{H^3SA}{\varepsilon^2}\right) \text{ episodes},$$

our algorithm can find, for each reward function, a policy $\widehat{\pi}$ *obeying*

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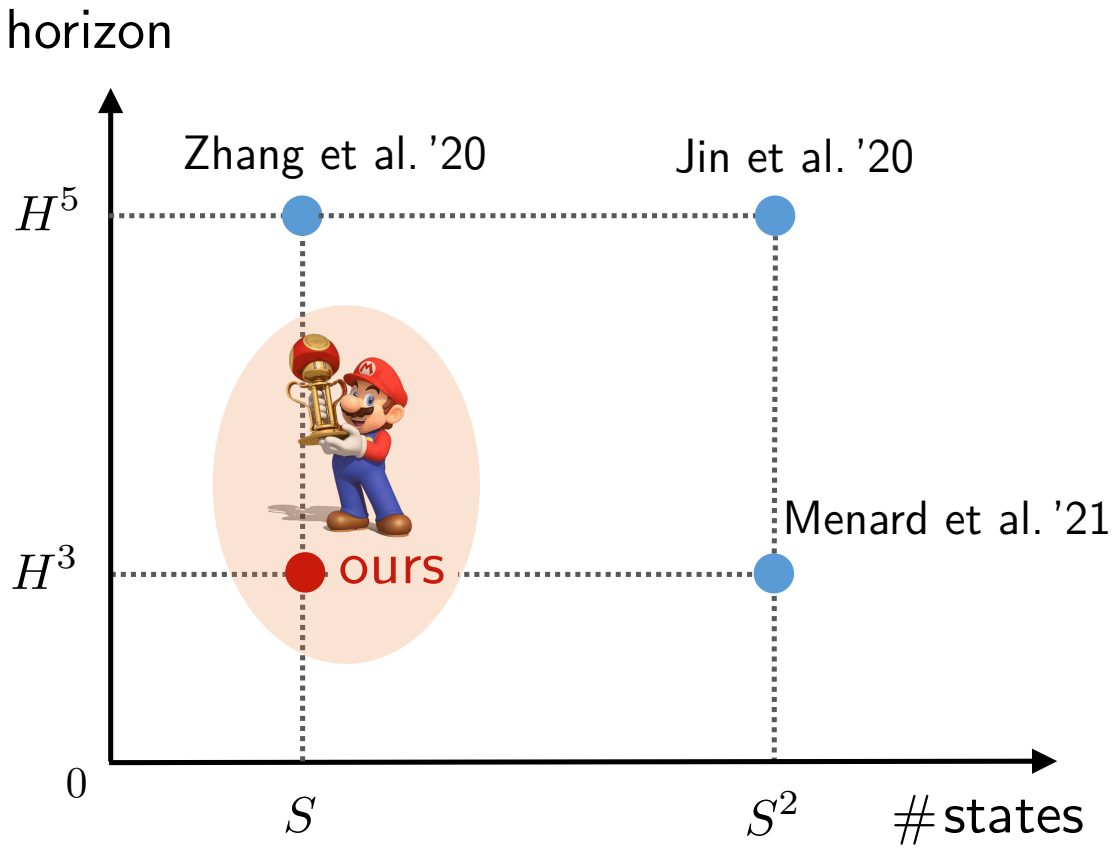
- optimal sample complexity
- collect data once $\longrightarrow$ work for poly(H, S, A) reward functions



The studies of offline RL inspire optimal reward-agnostic exploration!

Concluding remarks

Theoretical studies of offline RL shed light on data-efficient algorithm designs for other RL scenarios:

- online exploration
- hybrid RL
- ...

"Minimax-optimal reward-agnostic exploration in reinforcement learning," G. Li, Y. Yan, Y. Chen, J. Fan, *COLT* 2024

"Settling the sample complexity of model-based offline reinforcement learning," G. Li, L. Shi, Y. Chen, Y. Chi, Y. Wei, *Annals of Statistics*, 2024

"Reward-agnostic fine-tuning: provable statistical benefits of hybrid reinforcement learning," G. Li, W. Zhan, J. Lee, Y. Chi, Y. Chen, *NeurIPS* 2023