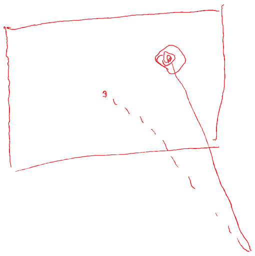
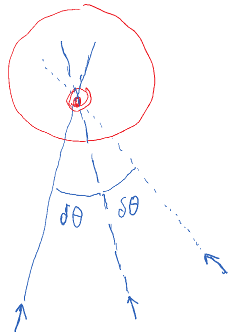
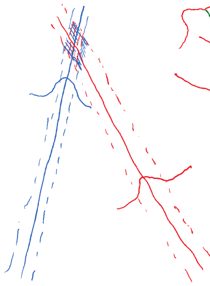
Possibly useful https://iopscience.iop.org/article/10.1088/1361-6560/abe75f/pdf
http://learnwebgl.brown37.net/08_projections/projections_perspective.html
A Kalman filtering approach to particle track filtering and track uncertainty quantification for 3D PTV measurements
and
Flow Enhancement of Tomographic Particle Image Velocimetry Measurements Using Sequential Data Assimilation

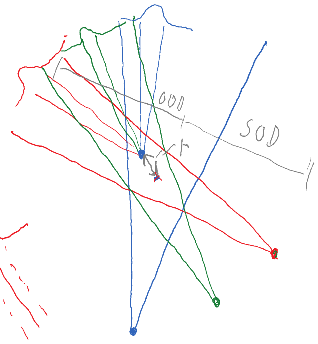
Saturday, July 2, 2022 5:51 PM

Just like in tomo piv for any one particle from projection(s)

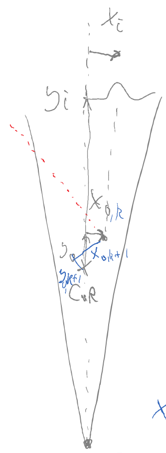








Use SW?



magnifications

$y_i = ODD$

$M \equiv \frac{SOD+ODD}{SOD}$, but $M_{particle,k}$ for angle k, $M_{p,k} = \frac{SOD + \cancel{y_o} + ODD - \cancel{y_o}}{SOD + y_{o,k}} \Rightarrow M_{p,k} = \frac{SOD+ODD}{SOD+y_{o,k}}$

$X_{i,k} = X_{o,k} M_{p,k} \rightarrow x_o = \frac{x_i}{M_p}$, and uncertainty is in x_i from localization and SOD & ODD from setup, & y_o from localization

Similarly after $\delta\theta$ turn, but now getting info of y_o, call point at θ k, $\theta+\delta\theta$ k+1 etc.

$X_{i,k+1} = X_{o,k+1} M_{p,k+1}$ / Next lets relate $\vec{x}_{o,k}$ to $\vec{x}_{o,k+1}$ after a $\delta\theta$ turn

IF particle stationary $\vec{X}^{(1)}_{k+1} = \vec{X}^{(1)}_k$ (coordinate system), and $\vec{X}^{(2)}_k$ = find rotation transformation

$$\vec{X}^{(2)} = R_{12}\vec{X}^{(1)} = \begin{bmatrix} x^{(1)}\cos(\delta\theta) - y^{(1)}\sin(\delta\theta) \\ x^{(1)}\sin(\delta\theta) + y^{(1)}\cos(\delta\theta)\end{bmatrix}$$

IF particle is not stationary, then

$$\vec{X}^{(1)}_{k+1} = \vec{X}^{(1)}_k + \underbrace{\frac{\vec{V}^{(1)}_k + \vec{V}^{(1)}_{k+1}}{2}}_{\text{note, if small steps could approximate with } \vec{V}^{(1)}_k}, \text{ and } \vec{X}^{(2)}_{k+1} = R_{12}\left(\vec{X}^{(1)} + \vec{V}^{(1)}_k\right)$$

$$x^{(1)}_{o,k} = \frac{X_{i,k}}{M_{p,k}} = \frac{X_{i,k}\,SOD + X_{i,k}\,y^{(1)}_{o,k}}{SOD+ODD}$$

$$x^{(2)}_{o,k+1} = \frac{X_{i,k+1}}{M_{p,k+1}} = \frac{X_{i,k+1}\,SOD + X_{i,k+1}\,y^{(2)}_{o,k+1}}{SOD+ODD}$$

$$x^{(2)}_k = x^{(1)}_k\cos(\delta\theta) - y^{(1)}_k\sin(\delta\theta)$$

$$y^{(2)}_k = x^{(1)}_k\sin(\delta\theta) + y^{(1)}_k\cos(\delta\theta)$$

4 unknown, 4 eq.
if $\vec{x}^{(any)}_{o,k} = \vec{x}_{o,k+1}$ (any)

In stationary case ... then could take average of $\frac{1}{N_k}\sum_{k=1}^{N_k} \vec{X}^{(1)}_k = \overline{\vec{X}^{(1)}}$ as best guess for $\vec{X}^{(1)}$ or some norm to be more outlier tolerant

weighted mean

- further from mean the less weight
- median
- remove outliers and take new mean

what about when motion (ASSUMED STEDY), when $\vec{x}_{k+1} = \vec{x}_k + \vec{V}_k$, then 6 eq. & 6 unknowns

FROM ABOVE

$x^{(1)}_{o,k} = \frac{X_{i,k}}{M_{p,k}} = \frac{X_{i,k}\,SOD + X_{i,k}\,y^{(1)}_{o,k}}{\ldots}$

$X^{(2)}_{o,k+1}\left(SOD+ODD\right) - X_{i,k+1}\,SOD \ldots = y^{(2)} = x^{(1)}\sin(\delta\theta) + y^{(1)} \ldots$

$$x^{(1)}_{o,k} = \frac{x_{i,k}}{M_{p,k}} = \frac{x_{i,k}\,SOD + x_{i,k}\,y^{(1)}_{o,k}}{SOD + ODD}$$

$$x^{(2)}_{o,k+1} = \frac{x_{i,k+1}}{M_{p,k+1}} = \frac{x_{i,k+1}\,SOD + x_{i,k+1}\,y^{(2)}_{o,k+1}}{SOD + ODD}$$

$$x^{(2)}_{k} = x^{(1)}_{k}\cos(\delta\theta) - y^{(1)}_{k}\sin(\delta\theta)$$

$$y^{(2)}_{k} = x^{(1)}_{k}\sin(\delta\theta) + y^{(1)}_{k}\cos(\delta\theta)$$

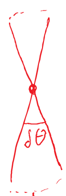
(1) or

4 unknown, 4 eq. if $\vec{x}^{(ang)}_{o,k} = \vec{x}_{o,k+1}(ang)$

$$\frac{x^{(2)}_{o,k+1}(SOD + ODD) - x_{i,k+1}\,SOD}{x_{i,k+1}} = y^{(2)}_{o,k+1} = x^{(1)}_{k+1}\sin(\delta\theta) + y^{(1)}_{k+1}\cos(\delta\theta)$$

etc...

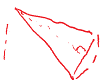
Could try to match all particles & accept those w/ small error, but can use fact $\delta\theta$ small to advantage



Magnification limits for object in domain with radius R

$$\frac{SOD + ODD}{SOD + R} \leq M \leq \frac{SOD + ODD}{SOD - R}$$

$d_S = \frac{\delta\theta}{360} 2\pi\, SOD$ though!



not /, / is $d_S = 2\tan\left(\frac{\delta\theta}{2}\right) SOD$ CHECK

and max $x_{i,k} - x_{i,k+1} = M\, d_S$

i.e. old search radius

$\max(M d_S) + \varepsilon$

$\min(M d_S) + \varepsilon$