Consecutive

$$\begin{array}{ccccccccc} 1 & & 2 & & 3 & & 4 & & 5 \\ & i=1 & & 2 & & 3 & & 4 & \end{array}$$

$n-1$ results

Big Matrix

① $x_0^{(2)} = x_0^{(1)} \cos(\Delta\theta) + y_0^{(1)} \sin(\Delta\theta)$

② $y_0^{(2)} = -x_0^{(1)} \sin(\Delta\theta) + y_0^{(1)} \cos(\Delta\theta)$

③ $x_0^{(2)} = \dfrac{x_i^{(2)}(SOD + y_0^{(2)})}{SOD + ODD}$

④ $x_0^{(1)} = \dfrac{x_i^{(1)}(SOD + y_0^{(1)})}{SOD + ODD}$

⑤ $z_0 = z_{i1} \cdot \dfrac{SOD + y_0^{(1)}}{ODD + SOD} \rightarrow z_0 - \dfrac{z_i^{(1)} y_0^{(1)}}{ODD + SOD} = z_i^{(1)} \left(\dfrac{SOD}{ODD + SOD}\right)$

⑥ $z_0 = z_{i2} \cdot \dfrac{SOD + y_0^{(2)}}{ODD + SOD} \rightarrow z_0 - \dfrac{z_i^{(2)} y_0^{(2)}}{ODD + SOD} = z_i^{(2)} \left(\dfrac{SOD}{ODD + SOD}\right)$

Unknowns: z_0, $x_0^{(1)}$, $y_0^{(1)}$, $x_0^{(2)}$, $y_0^{(2)}$

$$A = \begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{array} \begin{array}{c} \begin{array}{ccccc} z_0 & x_0^{(1)} & y_0^{(1)} & x_0^{(2)} & y_0^{(2)} \end{array} \\ \begin{bmatrix} 1 & 0 & -\dfrac{z_i^{(1)}}{ODD+SOD} & 0 & 0 \\ 0 & -1 & \dfrac{x_i^{(1)}}{SOD+ODD} & 0 & 0 \\ 1 & 0 & 0 & 0 & -\dfrac{z_i^{(2)}}{ODD+SOD} \\ 0 & 0 & 0 & -1 & \dfrac{x_i^{(2)}}{SOD+ODD} \\ 0 & \cos\theta & \sin\theta & -1 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 & -1 \end{bmatrix} \end{array}$$

$$b = \begin{bmatrix} z_i^{(1)} \dfrac{SOD}{ODD+SOD} \\ \dfrac{-x_i^{(1)} SOD}{SOD+ODD} \\ z_i^{(2)} \dfrac{SOD}{ODD+SOD} \\ \dfrac{-x_i^{(2)} SOD}{SOD+ODD} \\ 0 \\ 0 \end{bmatrix}$$

Unknowns : $z_0,\ x_0^{(1)},\ y_0^{(1)},\ x_0^{(2)},\ y_0^{(2)},\ x_0^{(3)},\ y_0^{(3)}$

2 shots: size(A) = 6 x 5

3 : size = 10 x 7

4 : size = 14 x 9

z will give 1 more eqs → magnification

xy will give 3 more eqs → 2 from Transformation, 1 from magnification

col position of $x_0^{(j)} = 2*j$ of $y_0^{(j)} = 2*j+1$

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} z_0 & x_0^{(1)} & y_0^{(1)} & x_0^{(2)} & y_0^{(2)} & x_0^{(3)} & y_0^{(3)} \end{array} \\
\begin{array}{c} \mathrm{I.}\ ① \\ ② \\ \mathrm{II.}\ ③ \\ ④ \\ ⑤ \\ ⑥ \\ \mathrm{III.}\ ⑦ \\ ⑧ \\ ⑨ \\ ⑩ \end{array} &
A=\left[\begin{array}{ccccccc}
1 & 0 & -\frac{z_i^{(1)}}{ODD+SOD} & 0 & 0 & 0 & 0 \\
0 & -1 & \frac{x_i^{(1)}}{SOD+ODD} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -\frac{z_i^{(2)}}{ODD+SOD} & 0 & 0 \\
0 & 0 & 0 & -1 & \frac{x_i^{(2)}}{SOD+ODD} & 0 & 0 \\
0 & \cos\theta & \sin\theta & -1 & 0 & 0 & 0 \\
0 & -\sin\theta & \cos\theta & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & -\frac{z_i^{(3)}}{SDD} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & \frac{x_i^{(3)}}{SDD} \\
0 & 0 & 0 & \cos\theta & \sin\theta & -1 & 0 \\
0 & 0 & 0 & -\sin\theta & \cos\theta & 0 & -1
\end{array}\right]
\end{array}
$$

Big Matrix 2

Unknowns : z_0, $x_0^{(1)}$, $y_0^{(1)}$, $x_0^{(2)}$, $y_0^{(2)}$, $x_0^{(3)}$, $y_0^{(3)}$

$$A = \begin{array}{l} \text{shot ① magnification equation} \\ \\ \text{shot ② mag} \\ \\ \text{shot ③ mag} \\ \\ \text{trans ①->②} \\ \\ \text{trans ②->③} \\ \\ \text{trans ①->③} \\ \\ \end{array} \begin{bmatrix} 1 & 0 & \frac{-z_i^{(1)}}{ODD+SOD} & 0 & 0 & 0 & 0 \\ 0 & -1 & \frac{x_i^{(1)}}{SOD+ODD} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{-z_i^{(2)}}{ODD+SOD} & 0 & 0 \\ 0 & 0 & 0 & -1 & \frac{x_i^{(2)}}{SOD+ODD} & 0 & 0 \\ 1 & 0 & 0 & 0 & -\frac{z_i^{(3)}}{SDD} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & \frac{x_i^{(3)}}{SDD} \\ 0 & \cos\theta & \sin\theta & -1 & 0 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & -1 & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 & -1 \\ 0 & \cos 2\theta & \sin 2\theta & 0 & 0 & -1 & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 & 0 & 0 & -1 \end{bmatrix}$$

angle I II III IV

If N angles, then

C_2^N combinations

$$= \frac{N!}{(N-2)!\,2}$$

for $k = 2 : NOS$

if now it's kth shot, then we have $k-1$ transformations

$\therefore 2(k-1) =$ the # of rows taken up

Phase 3

Big Matrix Sections:

Sec 1: magnifications, # = $NOS \times 2$ (one from x, one from z)

- original code from phase 1
- moving z coordinate equation

Sec 2: Transformation, # = ${}_{NOS}C_2$

- original code from phase 1
- added columns for u, v and a_x, a_y

$\boxed{\text{moving } z \text{ coordinate equation}}$

$$z_{0,k+1} = z_{i2} \cdot \frac{SOD + y_0^{(2)}}{ODD + SOD} \rightarrow z_{0,k+1} - \frac{z_{i2}\, y_0^{(2)}}{ODD+SOD} = z_{i,2}\left(\frac{SOD}{ODD+SOD}\right) \quad ①$$

$$z_{0,k+1} = z_{0,k} + wt + \frac{1}{2} a_z t^2 \quad ②$$

Substitute ② to ①:

$$\boxed{wt + \frac{1}{2} a_z t^2 + \underbrace{z_{0,k} - \frac{z_{i2}\, y_{0,k+1}^{(2)}}{ODD+SOD} = z_{i,2}\left(\frac{SOD}{ODD+SOD}\right)}_{\text{same as phase 1}}}$$

If $\vec{a} = \vec{0}$:

Unknowns : $z_0 \quad x_0^{(1)}, \; y_0^{(1)}, \; x_0^{(2)}, \; y_0^{(2)}, \; u \quad v \quad w$

$$A = \begin{array}{l} \text{shot ① magnification equation} \\ \\ \text{shot ② mag} \\ \\ \text{trans ①→②} \\ \\ \end{array} \left\{ \begin{array}{cccccccc} 1 & 0 & \frac{-z_i^{(1)}}{ODD+SOD} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & \frac{x_i^{(1)}}{SOD+ODD} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{-z_i^{(2)}}{ODD+SOD} & 0 & 0 & \Delta t \\ 0 & 0 & 0 & -1 & \frac{x_i^{(2)}}{SOD+ODD} & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & -1 & 0 & \Delta t\cos\theta & \Delta t\sin\theta & \\ 0 & -\sin\theta & \cos\theta & 0 & -1 & \Delta t(-\sin\theta) & \Delta t\cos\theta & \end{array} \right.$$

① $x^{(2)}_{0,k+1} = \Big(\underbrace{x_0^{(1)} + ut + \tfrac{1}{2}a_x t^2}_{x^{(1)}_{k+1}}\Big)\cos(\Delta\theta) + \Big(\underbrace{y_0^{(1)} + Vt + \tfrac{1}{2}a_y t^2}_{y^{(1)}_{k+1}}\Big)\sin(\Delta\theta)$

② $y^{(2)}_{0,k+1} = x^{(1)}_{k+1} - \sin(\Delta\theta) + y^{(1)}_{k+1}\cos(\Delta\theta)$

③ $x^{(2)}_{0,k+1} = \dfrac{x_i^{(2)}(SOD + y^{(2)}_{0,k+1})}{SOD + ODD}$ or $\Rightarrow \dfrac{x_0^{(2)}(SOD+ODD)}{x_i^{(2)}} - SOD = y_0^{(2)}$

④ $x^{(1)}_{0,k} = \dfrac{x_i^{(1)}(SOD + y^{(1)}_{0,k})}{SOD + ODD}$

⑤ $z_{0,k} = z_{i1} \cdot \dfrac{SOD + y_0^{(1)}}{ODD + SOD} \rightarrow z_{0,k} - \dfrac{z_i^{(1)} y_0^{(1)}}{ODD+SOD} = z_i^{(1)}\left(\dfrac{SOD}{ODD+SOD}\right)$

⑥ $z_{0,k+1} = z_{i2} \cdot \dfrac{SOD + y_0^{(2)}}{ODD + SOD} \rightarrow z_{0,k+1} - \dfrac{z_{i2}\, y_0^{(2)}}{ODD+SOD} = z_{i,2}\left(\dfrac{SOD}{ODD+SOD}\right)$