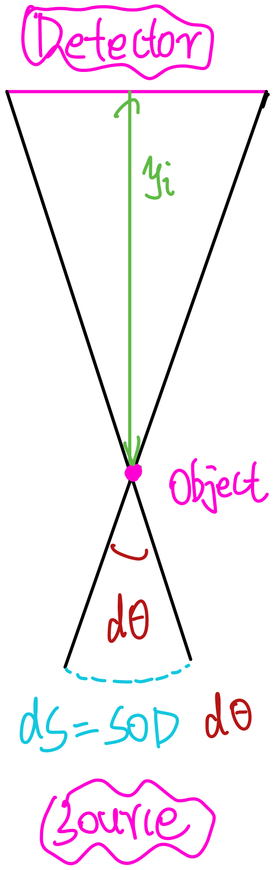
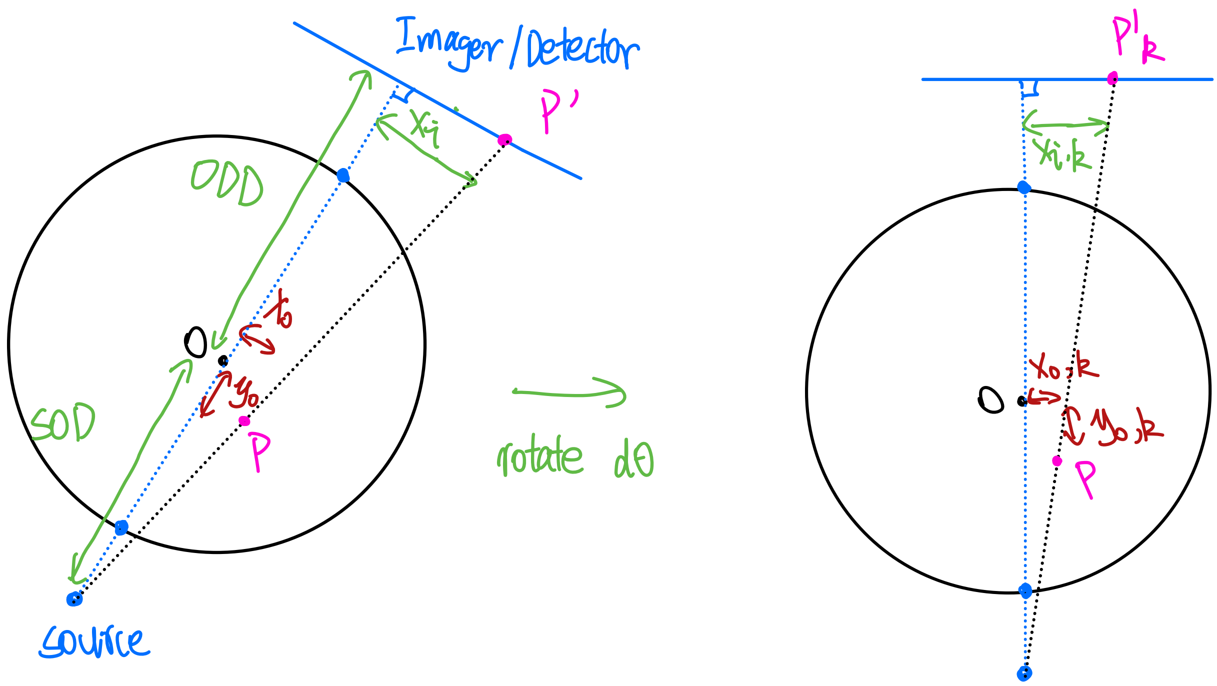


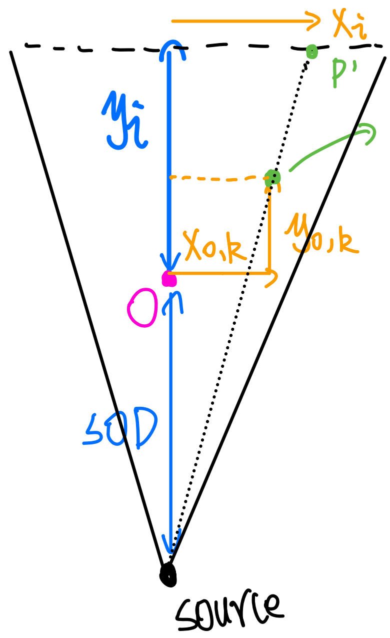
$y_i \equiv ODD$

$M \equiv \dfrac{SOD + ODD}{SOD}$

If we have K frames, then $d\theta = \dfrac{2\pi}{K}$

Assume the particle is in 2-D plane:





particle location

$y_i = ODD =$ object - detector distance

magnification $= M = \dfrac{SOD+ODD}{SOD}$

$$M_{particle,k} = M_{p,k} = \frac{SOD + y_{o,k} + ODD - y_{o,k}}{SOD + y_{o,k}} = \frac{SOD+ODD}{SOD+y_{o,k}}$$

$$x_{i,k} = x_{o,k}\, M_{p,k} \longrightarrow x_{o,k} = \frac{x_{i,k}}{M_{p,k}} = \frac{x_{i,k}(SOD+y_{o,k})}{SOD+ODD}$$

uncertainty arises from ① localization of x_o + y_o

② SOD + ODD variation from mechanical imperfection
(*distance sensor can solve the problem — by Hubert)

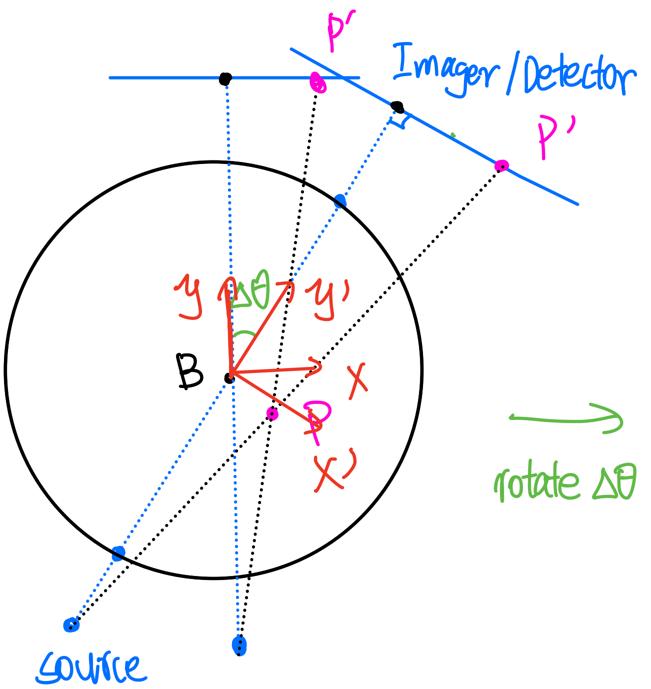
unknown: $x_{o,k}$, $y_{o,k}$ known: SOD, ODD, $x_{i,k}$ # of eq = 1

∴ we need rotation

$$\text{Matrix of transformation} = A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \begin{bmatrix} x^{(2)} \\ y^{(2)} \end{bmatrix} = A \begin{bmatrix} x^{(1)} \\ y^{(1)} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x^{(1)} \\ y^{(1)} \end{bmatrix} = \begin{bmatrix} x^{(1)}\cos(\Delta\theta) - y^{(1)}\sin(\Delta\theta) \\ x^{(1)}\sin(\Delta\theta) + y^{(1)}\cos(\Delta\theta) \end{bmatrix}$$

coordinates after rotation



rotate Δθ

equations:

① $x_o^{(2)} = x_o^{(1)}\cos(\Delta\theta) - y_o^{(1)}\sin(\Delta\theta)$

② $y_o^{(2)} = x_o^{(1)}\sin(\Delta\theta) + y_o^{(1)}\cos(\Delta\theta)$

③ $x_o^{(2)} = \dfrac{x_i^{(2)}(SOD+y_o^{(2)})}{SOD+ODD}$

④ $x_o^{(1)} = \dfrac{x_i^{(1)}(SOD+y_o^{(1)})}{SOD+ODD}$

Unknowns: $x_o^{(2)}$, $y_o^{(2)}$, $x_o^{(1)}$, $y_o^{(1)}$

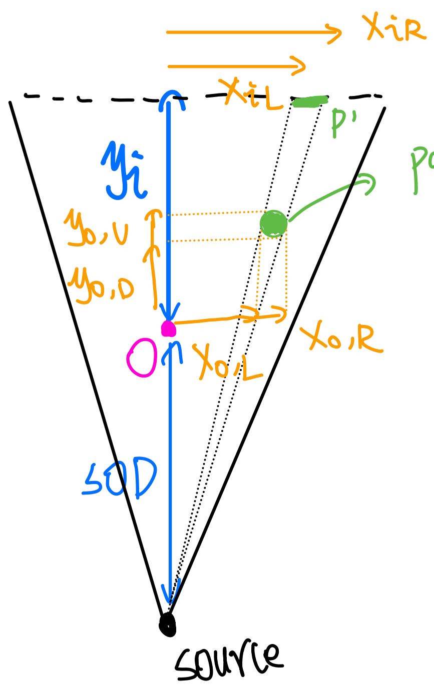
For uncertainty → the particle is essentially stationary but moves a little over time

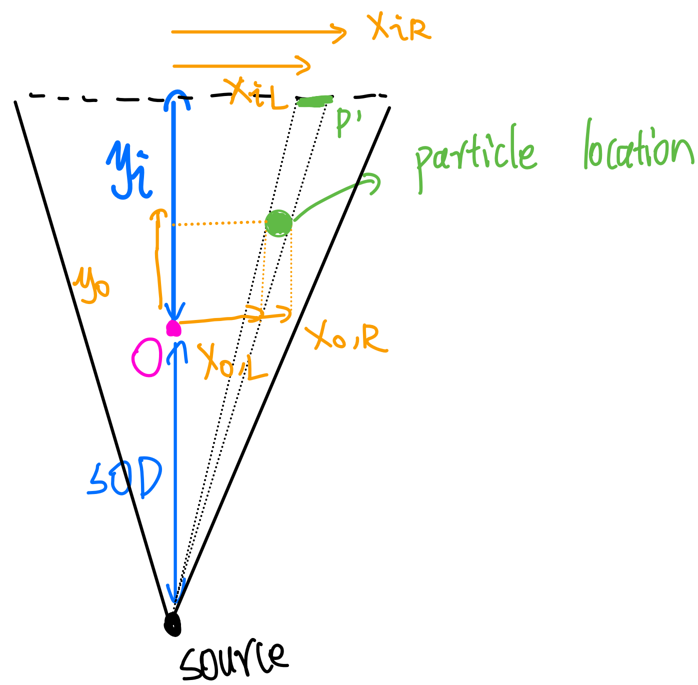
sol: take the average of $\begin{bmatrix} x_0^{(i)} \\ y_0^{(i)} \end{bmatrix}$ → $x_0^{true} \approx \frac{1}{N}\sum_{i=1}^{N} x_0^{(i)}$, same goes with y_0^{true}

calculated from coordinate i and coordinate i+1 system of eqs (Cyrus' idea)

☆ Step 1 of phase 1 done

If particle is a 2d circle:





$$M_{p,R} = \frac{SOD + y_{0,R} + ODD - y_{0,R}}{SOD + y_{0,R}} = \frac{SOD+ODD}{SOD+y_{0,R}}$$

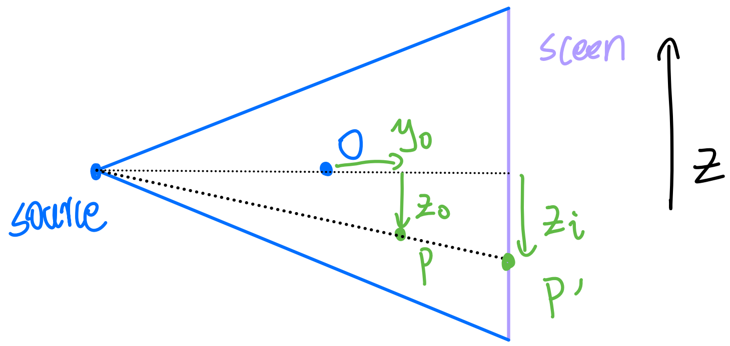
is same for both L and R ends of the particle

$x_{0,R} - x_{0,L}$ = particle diameter

Step 2 done!

Now work in 3d space

- the z-coordinate



$$z_i = z_0 \cdot M_p = z_0 \cdot \frac{ODD+SOD}{SOD+y_0}$$

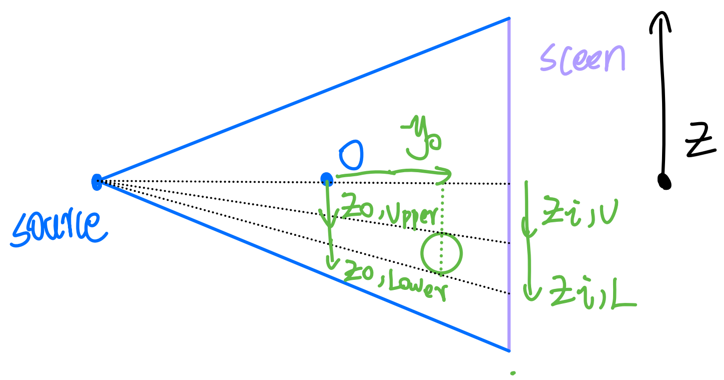
$$\therefore \boxed{z_0 = z_i \cdot \frac{SOD+y_0}{ODD+SOD}}$$

$\therefore$ once we get y_0 from Step 1, we can get z_0 even w/o. rotation!

Step 3 done!

Full Form of Phase 1: 3d particle in 3d space

- the z-coordinate



similarly:

$$z_{0,U} = \frac{z_{i,U}}{M_p} = z_{i,U} \cdot \frac{SOD+y_0}{ODD+SOD}$$