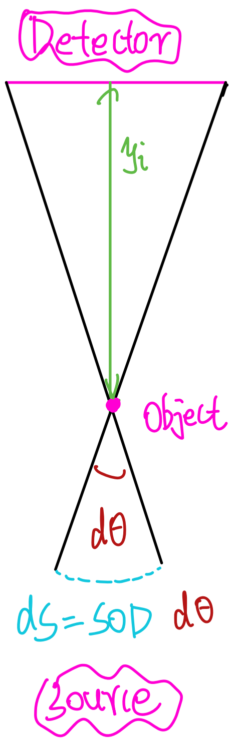
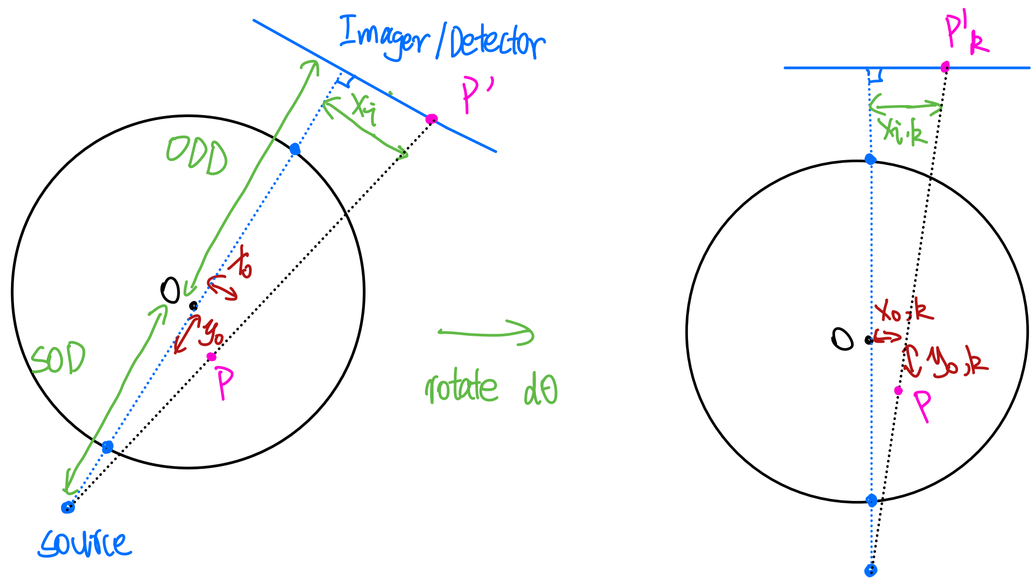


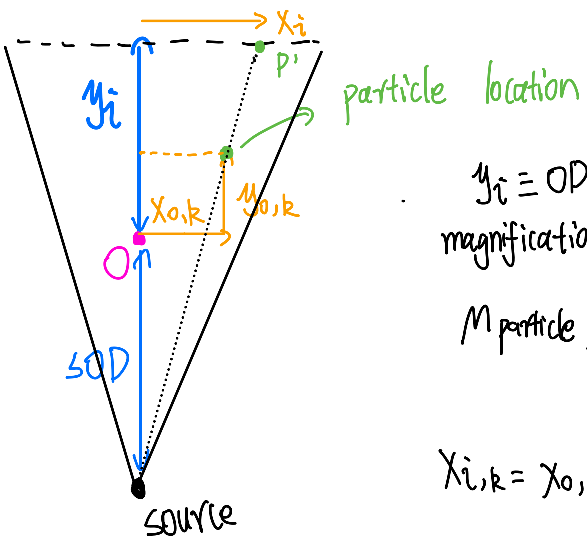
$y_i \equiv ODD$

$$M \equiv \frac{SOD + ODD}{SOD}$$

If we have K frames, then $d\theta = \frac{2\pi}{K}$

Assume the particle is in 2-D plane :





$y_i = ODD =$ object - detector distance

magnification $= M = \dfrac{SOD+ODD}{SOD}$

$M_{particle,k} = M_{p,k} = \dfrac{SOD + y_{o,k} + ODD - y_{o,k}}{SOD + y_{o,k}} = \dfrac{SOD+ODD}{SOD+y_{o,k}}$

$x_{i,k} = x_{o,k}\, M_{p,k} \longrightarrow x_{o,k} = \dfrac{x_{i,k}}{M_{p,k}} = \dfrac{x_{i,k}(SOD+y_{o,k})}{SOD+ODD}$

uncertainty arises from ① localization of x_o + y_o
② SOD + ODD variation from mechanical imperfection
(distance sensor can solve the problem — by Hubert)

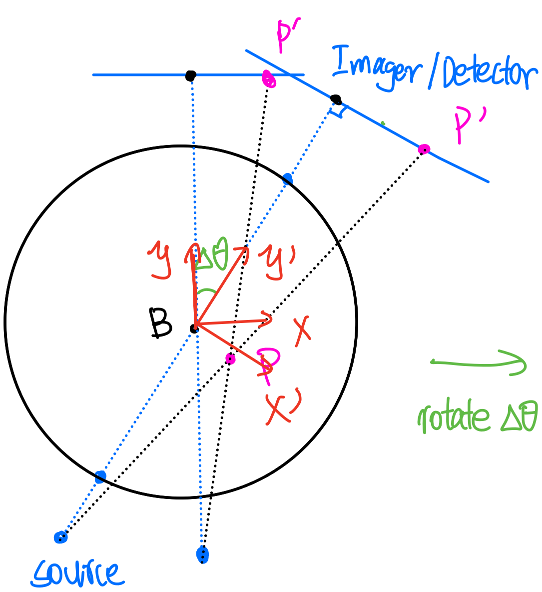
unknown: $x_{o,k}$, $y_{o,k}$ known: SOD, ODD, $x_{i,k}$ # of eq = 1

∴ we need rotation

Matrix of transformation $= A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore \begin{bmatrix} x^{(2)} \\ y^{(2)} \end{bmatrix} = A \begin{bmatrix} x^{(1)} \\ y^{(1)} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x^{(1)} \\ y^{(1)} \end{bmatrix} = \begin{bmatrix} x^{(1)}\cos(\Delta\theta) + y^{(1)}\sin(\Delta\theta) \\ -x^{(1)}\sin(\Delta\theta) + y^{(1)}\cos(\Delta\theta) \end{bmatrix}$

coordinates after rotation



equations:

① $x_o^{(2)} = x_o^{(1)}\cos(\Delta\theta) + y_o^{(1)}\sin(\Delta\theta)$

② $y_o^{(2)} = -x_o^{(1)}\sin(\Delta\theta) + y_o^{(1)}\cos(\Delta\theta)$

③ $x_o^{(2)} = \dfrac{x_i^{(2)}(SOD+y_o^{(2)})}{SOD+ODD}$

④ $x_o^{(1)} = \dfrac{x_i^{(1)}(SOD+y_o^{(1)})}{SOD+ODD}$

Unknowns: $x_o^{(2)}$, $y_o^{(2)}$, $x_o^{(1)}$, $y_o^{(1)}$

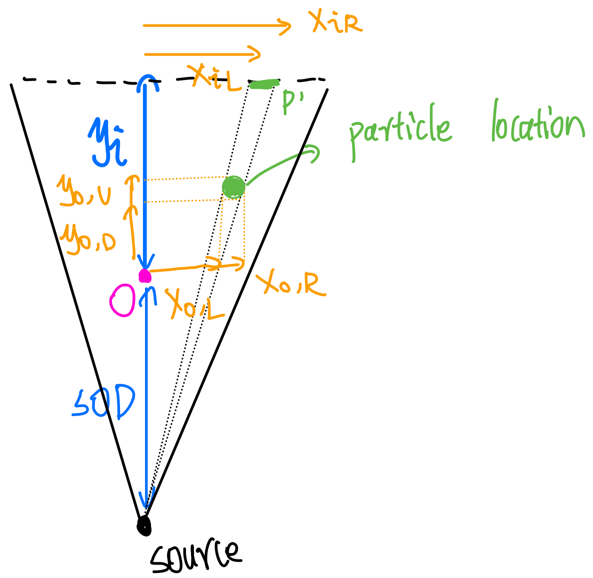
For uncertainty → the particle is essentially stationary but moves a little over time

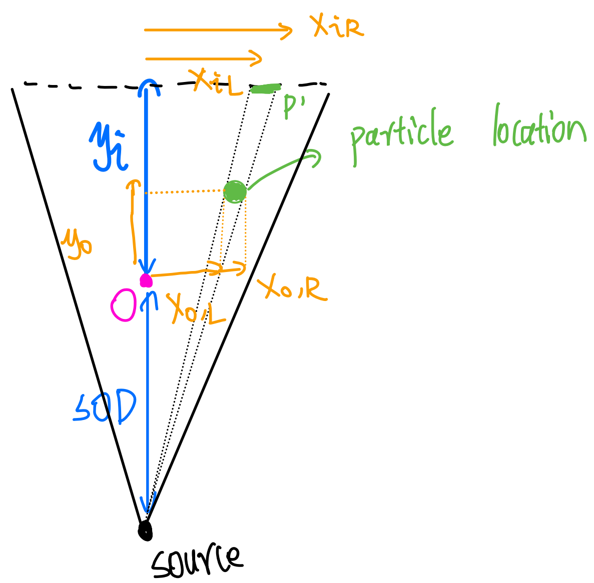
sol : take the average of $\begin{bmatrix} x_0^{(i)} \\ y_0^{(i)} \end{bmatrix}$ → $x_0^{true} \approx \frac{1}{N}\sum_{i=1}^{N} x_0^{(i)}$, same goes with y_0^{true}

calculated from coordinate i and coordinate $i+1$ system of eqs (Cyrus' idea)

Step 1 of phase 1 done

If particle is a 2d circle :





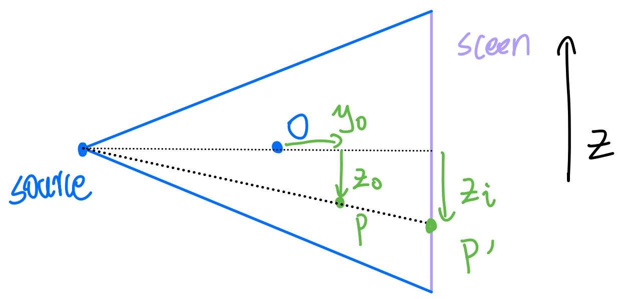
$M_{p,k} = \frac{SOD + y_{o,k} + ODD - y_{o,k}}{SOD + y_{o,k}} = \frac{SOD+ODD}{SOD+y_{o,k}}$ is same for both L and R ends of the particle

$x_{o,R} - x_{o,L}$ = particle diameter

Step 2 done!

Now work in 3d space

- the z-coordinate



$$z_i = z_0 \cdot M_p = z_0 \cdot \frac{ODD+SOD}{SOD+y_0}$$

$$\therefore \boxed{z_0 = z_i \cdot \frac{SOD+y_0}{ODD+SOD}}$$

$\therefore$ once we get y_0 from Step 1, we can get z_0 even w.l.o. rotation!

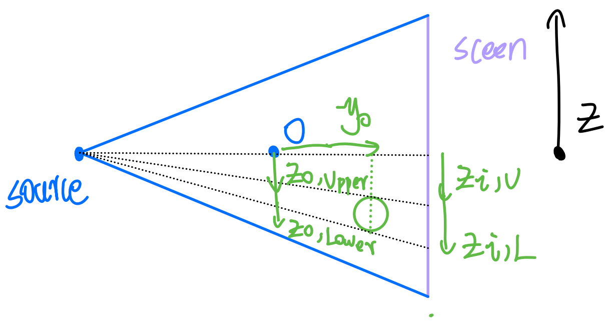
$$z_0 = z_{i1} \cdot \frac{SOD+y_0^{(1)}}{ODD+SOD} \qquad z_0 = z_{i2} \cdot \frac{SOD+y_0^{(2)}}{ODD+SOD}$$

with z_i alone, the rotation doesn't add more eqs than unknowns $\longrightarrow$ we rely on the y_0 calculated from x_i

Step 3 done!

Full Form of Phase 1: 3d particle in 3d space

• the z-coordinate



similarly:

$$z_{o,U} = \frac{z_{i,U}}{M_p} = z_{i,U} \cdot \frac{SOD + y_0}{ODD + SOD}$$

① $x_o^{(2)} = x_o^{(1)} \cos(\Delta\theta) + y_o^{(1)} \sin(\Delta\theta)$

② $y_o^{(2)} = -x_o^{(1)} \sin(\Delta\theta) + y_o^{(1)} \cos(\Delta\theta)$

③ $x_o^{(2)} = \dfrac{x_i^{(2)}(SOD + y_o^{(2)})}{SOD+ODD}$ $\overset{or}{\Rightarrow}$ $\dfrac{x_o^{(2)}(SOD+ODD)}{x_i^{(2)}} - SOD = y_o^{(2)}$

④ $x_o^{(1)} = \dfrac{x_i^{(1)}(SOD + y_o^{(1)})}{SOD+ODD}$

⑤ $z_o = z_{i1} \cdot \dfrac{SOD + y_o^{(1)}}{ODD+SOD} \rightarrow z_o - \dfrac{z_i^{(1)} y_o^{(1)}}{ODD+SOD} = z_i^{(1)}\left(\dfrac{SOD}{ODD+SOD}\right)$

⑥ $z_o = z_{i2} \cdot \dfrac{SOD + y_o^{(2)}}{ODD+SOD} \rightarrow z_o - \dfrac{z_i^{(2)} y_o^{(2)}}{ODD+SOD} = z_i^{(2)}\left(\dfrac{SOD}{ODD+SOD}\right)$

Unknowns: $x_o^{(1)}$, $y_o^{(1)}$, $x_o^{(2)}$, $y_o^{(2)}$, z_o

$$A = \begin{array}{c} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \end{array} \begin{bmatrix} \cos\theta & \sin\theta & -1 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & -1 & 0 \\ 0 & 0 & -1 & \dfrac{x_i^{(2)}}{SOD+ODD} & 0 \\ -1 & \dfrac{x_i^{(1)}}{SOD+ODD} & 0 & 0 & 0 \\ 0 & -\dfrac{z_i^{(1)}}{ODD+SOD} & 0 & 0 & 1 \\ 0 & 0 & 0 & -\dfrac{z_i^{(2)}}{ODD+SOD} & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{x_i^{(2)} SOD}{SOD+ODD} \\ -\dfrac{x_i^{(1)} SOD}{SOD+ODD} \\ z_i^{(1)} \dfrac{SOD}{ODD+SOD} \\ z_i^{(2)} \dfrac{SOD}{ODD+SOD} \end{bmatrix}$$

Phase 2 → moving particle w.o. acc. → $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

1 rotation:

$$x_{k+1}^{(1)} = x_k^{(1)} + u\Delta t \qquad y_{k+1}^{(1)} = y_k^{(1)} + v\Delta t$$

actual position

$$z_{k+1}^{(1)} = z_k^{(1)} + w\Delta t$$

unknowns : $u, v, w, x_k^{(1)}, y_k^{(1)}, z_k^{(1)}, x_k^{(2)}, y_k^{(2)}$ $= 9$

eqs : # = 6 , same as phase 1

2 rotations:

$$x_{k+2}^{(1)} = x_{k+1}^{(1)} + u\Delta t = x_k^{(1)} + 2u\Delta t \quad \ldots \text{ etc.}$$

new unknowns : $x_k^{(3)}, y_k^{(3)}$ # = 2

new eqs : # = 4, (2 from transformation, 2 from x and z magnification)