

The result is shown as follows:

par	AB	RMS	par	AB	RMS
$\mu_1$	0.033	0.079	$\lambda_{21}$	0.036	0.245
$\mu_2$	0.043	0.094	$\lambda_{31}$	0.192	0.214
$\mu_3$	0.006	0.085	$\lambda_{52}$	0.004	0.023
$\mu_4$	0.054	0.032	$\lambda_{62}$	0.032	0.149
$\mu_5$	0.010	0.040	$\lambda_{72}$	0.102	0.179
$\mu_6$	0.001	0.145	$\lambda_{93}$	0.010	0.086
$\mu_7$	0.078	0.096	$b$	0.095	0.163
$\mu_8$	0.039	0.056	$\gamma_1$	0.017	0.038
$\mu_9$	0.028	0.029	$\gamma_2$	0.075	0.112

[illegible]

2. a. Let  $y_i^* = (y_{i1}^*, y_{i2}^*, y_{i3}^*)^T$  be the latent continuous random vector corresponds to the dichotomous random vector  $(z_{i1}, z_{i2}, z_{i3})^T$ . ~~For~~ <sup>Define</sup>  $y_i = (y_{i1}, y_{i2}, y_{i3}, \frac{1}{1-p_{i4}})^T$ , and ~~define~~  
 And for  $k=4, \dots, 7$ ,  $y_{ik} \propto \exp\{y_{ik}v_{ik} - \log(1+e^{v_{ik}})\}$  with  $b(v_{ik}) = \log(1+e^{v_{ik}})$  and  $v_{ik} = \log \frac{p_{ik}}{1-p_{ik}}$ .  
 Then the measurement equation is,

$$y_{ik}^{(g)} = \mu_k^{(g)} + \lambda_k^{(g)T} w_i^{(g)} + \varepsilon_i^{(g)}, \quad k=1, 2, 3, 4, 5$$

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And the structural equation is,

$$\eta_i^{(g)} = \Gamma^{(g)} \xi_i^{(g)} + \delta_i^{(g)}$$

To tackle the identification problem, we fix  $\alpha_{m,1}^{(g)}$  and  $\alpha_{m,bm}^{(g)}$  as preassigned values. And we can select the first group as the reference group, and identify its ordered categorical variables by fixing both end thresholds as above. For any  $m$ , and  $g \neq 1$ , we have impose the restriction:

$$\alpha_{m,k}^{(g)} = \alpha_{m,k}^{(1)}, \quad k=1, \dots, bm$$

Also, we ~~can~~ <sup>can</sup> preassign some fixed value to some appropriate portions of  $\lambda_k^{(g)}$ ,  
 $\lambda_k^{(g)}$ , ~~for example~~ <sup>for example</sup>, the (1,1) elements of  $\lambda_k^{(g)}$  is 1.

b. ① Prior distribution for unconstrained parameters in different groups are naturally assumed to be independent. So, in estimating the unconstrained parameters, we need to specify its own prior distribution, and the data in the corresponding group are used.

② For constrained parameters, only one prior distribution for these constrained parameters is ~~used~~ <sup>needed</sup>, and all the data in the groups should be combined in the estimation. Under this situation, we may not take a joint prior distribution for the factor loading matrix and the unique variance of the error measurement.

C. In Bayesian analysis, testing invariance over the groups is equivalent to model comparison. Therefore, we can use Bayes factor or DIC for testing the invariance. Take Bayes factors as example.

$$B_{12} = \frac{p(X, z | M_1)}{p(X, z | M_2)}$$

where  $(X, z)$  is the observed data set. Let  $t$  be a path in  $[0, 1]$  to link  $M_1$  and  $M_2$ , and  $0 = t_{(0)} < t_{(1)} < \dots < t_{(S)} < t_{(S+1)} = 1$  be fixed grids in  $[0, 1]$ .

$$U(\theta, \alpha, Y, \Omega, X, z, t) = d \log p(Y, \Omega, X, z | \theta, \alpha, t) / dt$$

Then,

$$\log \hat{B}_{12} = \frac{1}{2} \sum_{s=0}^S (t_{(s+1)} - t_{(s)}) (\bar{U}_{(s+1)} + \bar{U}_{(s)}),$$

$$\text{where } \bar{U}_{(s)} = \frac{1}{J} \sum_{j=1}^J U(\theta^{(j)}, \alpha^{(j)}, Y^{(j)}, X, z, t_{(s)})$$