```
}

#Caluate log Bayes factor
logBF=0
for (i in 1:20) { logBF=logBF+(u[i+1]+u[i])*0.05/2 }
print(logBF)
```

Appendix 4.3 PP p-value for Model Assessment

Based on the posterior predictive assessment as discussed in Rubin (1984), Gelman, Meng and Stern (1996) introduced a Bayesian counterpart of the classical p-value by defining a posterior predictive (PP) p-value for model checking. To apply the approach for establishing a goodness-of-fit assessment of a hypothesized model M_0 with parameter vector $\boldsymbol{\theta}$, observed data $\mathbf{Y}$ and latent data $\boldsymbol{\Omega}$, we consider a discrepancy variable $D(\mathbf{Y}|\boldsymbol{\theta}, \boldsymbol{\Omega})$ for measuring the discrepancy between $\mathbf{Y}$ and the generated hypothetical replicate data $\mathbf{Y}^{\text{rep}}$. Then, the PP p-value is defined as

$$\begin{aligned} p_B(\mathbf{Y}) &= Pr\{D(\mathbf{Y}^{\text{rep}}|\boldsymbol{\theta}, \boldsymbol{\Omega}) \geq D(\mathbf{Y}|\boldsymbol{\theta}, \boldsymbol{\Omega})|\mathbf{Y}, M_0\}, \\ &= \int I\{D(\mathbf{Y}^{\text{rep}}|\boldsymbol{\theta}, \boldsymbol{\Omega}) \geq D(\mathbf{Y}|\boldsymbol{\theta}, \boldsymbol{\Omega})\} p(\mathbf{Y}^{\text{rep}}, \boldsymbol{\theta}, \boldsymbol{\Omega}|\mathbf{Y}, M_0) d\mathbf{Y}^{\text{rep}} d\boldsymbol{\theta} d\boldsymbol{\Omega}. \end{aligned}$$

where $I(\cdot)$ is an indicator function. The probability is taken over the following joint posterior distribution of $(\mathbf{Y}^{\text{rep}}, \boldsymbol{\theta}, \boldsymbol{\Omega})$ given $\mathbf{Y}$ and M_0:

$$p(\mathbf{Y}^{\text{rep}}, \boldsymbol{\theta}, \boldsymbol{\Omega}|\mathbf{Y}, M_0) = p(\mathbf{Y}^{\text{rep}}|\boldsymbol{\theta}, \boldsymbol{\Omega}) p(\boldsymbol{\theta}, \boldsymbol{\Omega}|\mathbf{Y}).$$

In almost all our applications to SEMs considered in this book, we take the chi-square discrepancy variable such that $D(\mathbf{Y}^{\text{rep}}|\boldsymbol{\theta}, \boldsymbol{\Omega})$ has a chi-squared distribution with d^* degrees of freedom. Thus, the PP p-value is equal to

$$\int p\{\chi^2(d^*) \geq D(\mathbf{Y}|\boldsymbol{\theta}, \boldsymbol{\Omega})\} p(\boldsymbol{\theta}, \boldsymbol{\Omega}|\mathbf{Y}) d\boldsymbol{\theta} d\boldsymbol{\Omega}.$$

A Rao-Blackwellized type estimate of this PP p-value is:

$$\hat{p}_B(\mathbf{Y}) = J^{-1} \sum_{j=1}^{J} Pr\{\chi^2(d^*) \geq D(\mathbf{Y}|\boldsymbol{\theta}^{(j)}, \boldsymbol{\Omega}^{(j)})\},$$

where $\{(\boldsymbol{\theta}^{(j)}, \boldsymbol{\Omega}^{(j)}),\ j = 1, \cdots, J\}$ are observations simulated during the estimation. The computational burden for obtaining this $\hat{p}_B(\mathbf{Y})$ is light.