$\min\{1, R_{gk}\}$, where

$$R_{gk} = \frac{p(\boldsymbol{\alpha}_{gk}, \mathbf{Y}_{gk}|\boldsymbol{\theta}, \mathbf{Z}_{gk}, \boldsymbol{\Omega}_{1g})p(\boldsymbol{\alpha}_{gk}^{(m)}, \mathbf{Y}_{gk}^{(m)}|\boldsymbol{\alpha}_{gk}, \mathbf{Y}_{gk}, \boldsymbol{\theta}, \mathbf{Z}_{gk}, \boldsymbol{\Omega}_{1g})}{p(\boldsymbol{\alpha}_{gk}^{(m)}, \mathbf{Y}_{gk}^{(m)}|\boldsymbol{\theta}, \mathbf{Z}_{gk}, \boldsymbol{\Omega}_{1g})p(\boldsymbol{\alpha}_{gk}, \mathbf{Y}_{gk}|\boldsymbol{\alpha}_{gk}^{(m)}, \mathbf{Y}_{gk}^{(m)}, \boldsymbol{\theta}, \mathbf{Z}_{gk}, \boldsymbol{\Omega}_{1g})}.$$

To search for a new observation via the proposal density (6.A9), we first generate a vector of thresholds $(\alpha_{gk,2}, \cdots, \alpha_{gk,b_k-1})$ from the following truncated normal distribution

$$\alpha_{gk,z} \stackrel{D}{=} N[\alpha_{gk,z}^{(m)}, \sigma_{\alpha gk}^2] I_{(\alpha_{gk,z-1}, \alpha_{gk,z+1}^{(m)}]}(\alpha_{gk,z}), \quad z = 2, \cdots, b_k - 1,$$

where $\sigma_{\alpha gk}^2$ is a preassigned value to give an approximate acceptance rate 0.44, see Cowles (1996). It follows from (6.A4) and the above result that

$$R_{gk} = \prod_{z=2}^{b_k-1} \frac{\Phi^*\{(\alpha_{gk,z+1}^{(m)} - \alpha_{gk,z}^{(m)})/\sigma_{\alpha gk}\} - \Phi^*\{(\alpha_{gk,z-1} - \alpha_{gk,z}^{(m)})/\sigma_{\alpha gk}\}}{\Phi^*\{(\alpha_{gk,z+1} - \alpha_{gk,z})/\sigma_{\alpha gk}\} - \Phi^*\{(\alpha_{gk,z-1}^{(m)} - \alpha_{gk,z})/\sigma_{\alpha gk}\}} \times$$
$$\prod_{i=1}^{N_g} \frac{\Phi^*\{\psi_{1gyk}^{-1/2}(\alpha_{gk,z_{gik}+1} - v_{gyk} - \boldsymbol{\Lambda}_{1gyk}^T \boldsymbol{\omega}_{1gi})\} - \Phi^*\{\psi_{1gyk}^{-1/2}(\alpha_{gk,z_{gik}} - v_{gyk} - \boldsymbol{\Lambda}_{1gyk}^T \boldsymbol{\omega}_{1gi})\}}{\Phi^*\{\psi_{1gyk}^{-1/2}(\alpha_{gk,z_{gik}+1}^{(m)} - v_{gyk} - \boldsymbol{\Lambda}_{1gyk}^T \boldsymbol{\omega}_{1gi})\} - \Phi^*\{\psi_{1gyk}^{-1/2}(\alpha_{gk,z_{gik}}^{(m)} - v_{gyk} - \boldsymbol{\Lambda}_{1gyk}^T \boldsymbol{\omega}_{1gi})\}},$$

where Φ^* is the distribution function of $N[0,1]$. Since R_{gk} only depends on the old and new values of $\boldsymbol{\alpha}_{gk}$ but not on $\mathbf{Y}_{gk}$, it only requires to generate a new $\mathbf{Y}_{gk}$ for an accepted $\boldsymbol{\alpha}_{gk}$. This new $\mathbf{Y}_{gk}$ is simulated from the truncated normal distribution $p(\mathbf{Y}_{gk}|\boldsymbol{\alpha}_{gk}, \cdot)$ via the algorithm given in Robert (1995).

Appendix 6.3: PP p-value for Two-level Nonlinear SEM with Mixed Continuous and Ordered Categorical Variables

Suppose null hypothesis H_0 is that the model defined in (6.1) and (6.2) is plausible, the PP p-value is defined as

$$p_B(\mathbf{X}, \mathbf{Z}) = Pr\{D(\mathbf{U}^{\text{rep}}|\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{Y}, \mathbf{V}, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) \geq D(\mathbf{U}|\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{Y}, \mathbf{V}, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2)|\mathbf{X}, \mathbf{Z}, H_0\},$$

where $\mathbf{U}^{\text{rep}}$ denotes a replication of $\mathbf{U} = \{\mathbf{u}_{gi},\ i = 1, \cdots, N_g,\ g = 1, \cdots, G\}$, with $\mathbf{u}_{gi}$ satisfies the model defined by equation (6.3) that involves structural parameters and latent variables satisfying (6.4) and (6.5), and $D(\cdot|\cdot)$ is a discrepancy variable. Here, the following χ^2 discrepancy variable is used:

$$D(\mathbf{U}^{\mathrm{rep}}|\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{Y}, \mathbf{V}, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2) = \sum_{g=1}^{G} \sum_{i=1}^{N_g} (\mathbf{u}_{gi}^{\mathrm{rep}} - \mathbf{v}_g - \boldsymbol{\Lambda}_{1g}\boldsymbol{\omega}_{1gi})^T \boldsymbol{\Psi}_{1g}^{-1} (\mathbf{u}_{gi}^{\mathrm{rep}} - \mathbf{v}_g - \boldsymbol{\Lambda}_{1g}\boldsymbol{\omega}_{1gi}),$$

which is distributed as $\chi^2(pn)$, a χ^2 distribution with pn degrees of freedom. Here, $n = N_1 + \cdots + N_g$. The PP p-value on the basis of this discrepancy variable is

$$P_B(\mathbf{X}, \mathbf{Z}) = \int Pr\{\chi^2(pn) \geq D(\mathbf{U}|\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{Y}, \mathbf{V}, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2)\} \times$$

$$p(\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{Y}, \mathbf{V}, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2 | \mathbf{X}, \mathbf{Z}) d\boldsymbol{\theta} d\boldsymbol{\alpha} d\mathbf{Y} d\mathbf{V} d\boldsymbol{\Omega}_1 d\boldsymbol{\Omega}_2.$$

A Rao-Blackwellized type estimate of the PP p-value is equal to

$$\hat{P}_B(\mathbf{X}, \mathbf{Z}) = T^{-1} \sum_{t=1}^{T} Pr\{\chi^2(pn) \geq D(\mathbf{U}|\boldsymbol{\theta}^{(t)}, \boldsymbol{\alpha}^{(t)}, \mathbf{Y}^{(t)}, \mathbf{V}^{(t)}, \boldsymbol{\Omega}_1^{(t)}, \boldsymbol{\Omega}_2^{(t)})\}$$

where $D(\mathbf{U}|\boldsymbol{\theta}^{(t)}, \boldsymbol{\alpha}^{(t)}, \mathbf{Y}^{(t)}, \mathbf{V}^{(t)}, \boldsymbol{\Omega}_1^{(t)}, \boldsymbol{\Omega}_2^{(t)})$ is calculated at each iteration and the tail-area of a χ^2 distribution which can be obtained via standard statistical software. The hypothesized model is rejected if $\hat{P}_B(\mathbf{X}, \mathbf{Z})$ is not close to 0.5.

Appendix 6.4: WinBUGS Code

```
model {
for (g in 1:G) {
#second level
for (j in 1:P) { vg[g,j]~dnorm(u2[g,j],psi2[j]) }

u2[g,1]<-1.0*xi2[g,1]
u2[g,2]<-lb[1]*xi2[g,1]
u2[g,3]<-lb[2]*xi2[g,1]

u2[g,4]<-1.0*xi2[g,2]
u2[g,5]<-lb[3]*xi2[g,2]
u2[g,6]<-lb[4]*xi2[g,2]

u2[g,7]<-1.0*xi2[g,3]
u2[g,8]<-lb[5]*xi2[g,3]
```