Stat 5005 Homework Assignment #1 (Due 9/~~21~~) 22

1. Let $\mathcal{A}$ be a collection of subsets of Ω. Prove that $\mathcal{A}$ is a λ-system if

   Ignore the question

   (i) $\Omega \in \mathcal{A}$

   (ii) If $A, B \in \mathcal{A}$ and $A \subset B$, then $B - A \in \mathcal{A}$,

   (iii) If $A_i \in \mathcal{A}$ for $i \geq 1$ and $A_i \uparrow$, then $\cup_{i=1}^{\infty} A_i \in \mathcal{A}$.

2. Prove that for any events $A_i,\ i = 1, 2, ..., n$, where $n \geq 3$

$$P(\bigcup_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i A_j)$$

   and

$$P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i A_j) + \sum_{1 \leq i < j < k \leq n} P(A_i A_j A_k)$$

3. Show that, the so called countable additivity or σ-additivity, is equivalent to finite additivity plus continuity (if $A_n \downarrow \emptyset$, then $P(A_n) \to 0$).

4. If X_1 and X_2 are random variables, so is $X_1 + X_2$.

- **Textbook:**

  1.1.5, 1.2.3, 1.2.5, 1.2.6, 1.2.7, 1.3.1, 1.3.7, 1.3.8