1 Results

1.1 Sampling

(1) if $k_1 \neq k_2$:

$$P(Z(m,n) = k|Z^{-(m,n)}, D, O, T) \tag{1}$$

$$=(n_u^-(m,k) + \alpha_k) \cdot \frac{n_w^-(k_1,w_1) + \beta_{w_1}}{\sum_{v_w=1}^{V_w} n_w^-(k_1,v_w) + \beta_{v_w}} \cdot \frac{n_w^-(k_2,w_2) + \beta_{w_2}}{\sum_{v_w=1}^{V_w} n_w^-(k_2,v_w) + \beta_{v_w}} \cdot \frac{n_t^-(k_3,w_3) + \gamma_{w_3}}{\sum_{v_t=1}^{V_t} n_t^-(k_3,v_t) + \gamma_{v_t}} \tag{2}$$

(2) if $k_1 = k_2$ and $w_1 \neq w_2$:

$$P(Z(m,n) = k|Z^{-(m,n)}, D, O, T) \tag{3}$$

$$=(n_u^-(m,k) + \alpha_k) \cdot \frac{n_w^-(k_1,w_1) + \beta_{w_1}}{\sum_{v_w=1}^{V_w} n_w^-(k_1,v_w) + \beta_{v_w}} \cdot \frac{n_w^-(k_1,w_2) + \beta_{w_2}}{\sum_{v_w=1}^{V_w} n_w^-(k_1,v_w) + \beta_{v_w} + 1} \cdot \frac{n_t^-(k_3,w_3) + \gamma_{w_3}}{\sum_{v_t=1}^{V_t} n_t^-(k_3,v_t) + \gamma_{v_t}} \tag{4}$$

(3) if $k_1 = k_2$ and $w_1 = w_2$:

$$P(Z(m,n) = k|Z^{-(m,n)}, D, O, T) \tag{5}$$

$$=(n_u^-(m,k) + \alpha_k) \cdot \frac{n_w^-(k_1,w_1) + \beta_{w_1}}{\sum_{v_w=1}^{V_w} n_w^-(k_1,v_w) + \beta_{v_w}} \cdot \frac{n_w^-(k_1,w_1) + \beta_{w_1} + 1}{\sum_{v_w=1}^{V_w} n_w^-(k_1,v_w) + \beta_{v_w} + 1} \cdot \frac{n_t^-(k_3,w_3) + \gamma_{w_3}}{\sum_{v_t=1}^{V_t} n_t^-(k_3,v_t) + \gamma_{v_t}} \tag{6}$$

1.2 Parameter estimation

Same as cLDA, for $P(\phi_k|Z, W, \beta) = Dirichlet(\phi_k|n_w + \beta)$, which can be derived from Equation 18.

$$\theta_{m,i} = \frac{n_u(m,i) + \alpha}{\sum_{i'=1}^{KKL}(n_u(m,i') + \alpha)} \tag{7}$$

$$\phi_{k,v_w} = \frac{n_w(k,v_w) + \beta}{\sum_{v_w'=1}^{V_w}(n_w(k,v_w') + \beta)} \tag{8}$$

$$\psi_{l,v_t} = \frac{n_t(l,v_t) + \gamma}{\sum_{v_t'=1}^{V_t}(n_t(l,v_t') + \gamma)} \tag{9}$$

2 Inference

$$P(D,O,Z,\theta,\phi;\alpha,\beta) \tag{10}$$

$$= \prod_{i=1}^{R} P(\phi_i;\beta) \prod_{j=1}^{M} P(\theta_j;\alpha) \prod_{t=1}^{N} P(Z_{j,t}|\theta_j) P(D_{j,t}|\phi_{Z_{j,t,1}}) P(O_{j,t}|\phi_{j,t,2}) \tag{11}$$

$$P(D,O,Z;\beta) \tag{12}$$

$$= \int_{\phi} \prod_{i=1}^{R} P(\phi_i;\beta) \prod_{j=1}^{M} \prod_{t=1}^{N} P(D_{j,t}|\phi_{Z_{j,t,1}}) P(O_{j,t}|\phi_{Z_{j,t,2}}) d\phi \cdot \int_{\theta} \prod_{j=1}^{M} P(\theta_j;\alpha) \prod_{t=1}^{N} P(Z_{j,t}|\theta_j) d\theta \tag{13}$$

$$= \prod_{i=1}^{R} \int_{\phi_i} \left(\frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \prod_{r=1}^{V} \phi_{i,r}^{\beta_r - 1} \right) \left(\prod_{j=1}^{M} \prod_{t=1}^{N} P(D_{j,t}|\phi_{Z_{j,t,1}}) \right) \left(\prod_{j=1}^{M} \prod_{t=1}^{N} P(O_{j,t}|\phi_{Z_{j,t,2}}) \right) d\phi_i \tag{14}$$

$$\cdot \left(\prod_{j=1}^{M} \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \frac{\prod_{i=1}^{K} \Gamma(n_u(i,j) + \alpha_i)}{\Gamma(\sum_{i=1}^{K} (n_u(i,j) = \alpha_i))} \right) (= A) \tag{15}$$

$$= \prod_{i=1}^{R} \int_{\phi_i} \left(\frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \prod_{r=1}^{V} \phi_{i,r}^{\beta_r - 1} \right) \left(\prod_{r=1}^{V} \phi_{i,r}^{n_d(i,r)} \right) \left(\prod_{r=1}^{V} \phi_{i,r}^{n_o(i,r)} \right) d\phi_i \cdot A \tag{16}$$

$$(n_w = n_d + n_o) = \prod_{i=1}^{R} \int_{\phi_i} \left(\frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \prod_{r=1}^{V} \phi_{i,r}^{\beta_r - 1} \right) \left(\prod_{r=1}^{V} \phi_{i,r}^{n_w(i,r)} \right) d\phi_i \cdot A \tag{17}$$

$$= \prod_{i=1}^{R} \frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \frac{\prod_{r=1}^{V} \Gamma(n_w(i,r) + \beta_r)}{\Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \cdot A \tag{18}$$

if $w_1 \neq w_2$ and $k_1 \neq k_2$

$$P(Z_{m,n} = (k_1, k_2) | Z_{-m,n}, D, O; \alpha, \beta) \tag{19}$$

$$\propto P(Z_{m,n} = (k_1, k_2), Z_{-m,n}, D, O; \alpha, \beta) \tag{20}$$

$$= \left(\frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \right)^M \cdot \prod_{j \neq m}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_u(i,j) + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_u(i,j) + \alpha_i)} \tag{21}$$

$$\times \left(\frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \right)^R \cdot \prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w(i,r) + \beta_r) \tag{22}$$

$$\times \frac{\prod_{i=1}^{K} \Gamma(n_u(i,m) + \alpha_i)}{\Gamma(\sum_{i=1}^{k} n_u(i,m) + \alpha_i)} \cdot \frac{\prod_{i=1}^{R} \Gamma(n_w(i,w_1) + \beta_{w_1}) \prod_{i=1}^{R} \Gamma(n_w(i,w_2) + \beta_{w_2})}{\prod_{i=1}^{R} \Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \tag{23}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \tag{24}$$

$$\cdot \Gamma(n_w(k_1, w_1) + \beta_{w_1}) \prod_{i \neq k_1}^{R} \Gamma(n_w(i, w_1) + \beta_{w_1}) \tag{25}$$

$$\cdot \Gamma(n_w(k_2, w_2) + \beta_{w_2}) \prod_{i \neq k_2}^{R} \Gamma(n_w(i, w_2) + \beta_{w_2}) \tag{26}$$

$$\text{(sampling)} \tag{27}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w^-(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{28}$$

$$\cdot \Gamma(n_w^-(k_1, w_1) + \beta_{w_1} + 1) \prod_{i \neq k_1}^{R} \Gamma(n_w^-(i, w_1) + \beta_{w_1}) \tag{29}$$

$$\cdot \Gamma(n_w^-(k_2, w_2) + \beta_{w_2} + 1) \prod_{i \neq k_2}^{R} \Gamma(n_w^-(i, w_2) + \beta_{w_2}) \tag{30}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w^-(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{31}$$

$$\cdot (n_w^-(k_1, w_1) + \beta_{w_1}) \Gamma(n_w^-(k_1, w_1) + \beta_{w_1}) \prod_{i \neq k_1}^{R} \Gamma(n_w^-(i, w_1) + \beta_{w_1}) \tag{32}$$

$$\cdot (n_w^-(k_2, w_2) + \beta_{w_2}) \Gamma(n_w^-(k_2, w_2) + \beta_{w_2}) \prod_{i \neq k_2}^{R} \Gamma(n_w^-(i, w_2) + \beta_{w_2}) \tag{33}$$

$$= (\cdot) \cdot \frac{(n_w^-(k_1, w_1) + \beta_{w_1})(n_w^-(k_2, w_2) + \beta_{w_2})}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{34}$$

if $w_1 \neq w_2$ and $k_1 = k_2$

$$P(Z_{m,n} = (k_1, k_1)|Z_{-m,n}, D, O; \alpha, \beta) \tag{35}$$

$$\propto P(Z_{m,n} = (k_1, k_1), Z_{-m,n}, D, O; \alpha, \beta) \tag{36}$$

$$= \left(\frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \right)^M \cdot \prod_{j \neq m}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_u(i,j) + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_u(i,j) + \alpha_i)} \tag{37}$$

$$\times \left(\frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \right)^R \cdot \prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w(i,r) + \beta_r) \tag{38}$$

$$\times \frac{\prod_{i=1}^{K} \Gamma(n_u(i,m) + \alpha_i)}{\Gamma(\sum_{i=1}^{k} n_u(i,m) + \alpha_i)} \cdot \frac{\prod_{i=1}^{R} \Gamma(n_w(i,w_1) + \beta_{w_1}) \prod_{i=1}^{R} \Gamma(n_w(i,w_2) + \beta_{w_2})}{\prod_{i=1}^{R} \Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \tag{39}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \tag{40}$$

$$\cdot \Gamma(n_w(k_1, w_1) + \beta_{w_1}) \prod_{i \neq k_1}^{R} \Gamma(n_w(i, w_1) + \beta_{w_1}) \tag{41}$$

$$\cdot \Gamma(n_w(k_1, w_2) + \beta_{w_2}) \prod_{i \neq k_2}^{R} \Gamma(n_w(i, w_2) + \beta_{w_2}) \tag{42}$$

$$\text{(sampling)} \tag{43}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w^-(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{44}$$

$$\cdot \Gamma(n_w^-(k_1, w_1) + \beta_{w_1} + 1) \prod_{i \neq k_1}^{R} \Gamma(n_w^-(i, w_1) + \beta_{w_1}) \tag{45}$$

$$\cdot \Gamma(n_w^-(k_1, w_2) + \beta_{w_2} + 1) \prod_{i \neq k_2}^{R} \Gamma(n_w^-(i, w_2) + \beta_{w_2}) \tag{46}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1, w_2}^{V} \Gamma(n_w^-(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{47}$$

$$\cdot (n_w^-(k_1, w_1) + \beta_{w_1}) \Gamma(n_w^-(k_1, w_1) + \beta_{w_1}) \prod_{i \neq k_1}^{R} \Gamma(n_w^-(i, w_1) + \beta_{w_1}) \tag{48}$$

$$\cdot (n_w^-(k_1, w_2) + \beta_{w_2}) \Gamma(n_w^-(k_1, w_2) + \beta_{w_2}) \prod_{i \neq k_2}^{R} \Gamma(n_w^-(i, w_2) + \beta_{w_2}) \tag{49}$$

$$= (\cdot) \cdot \frac{(n_w^-(k_1, w_1) + \beta_{w_1})(n_w^-(k_1, w_2) + \beta_{w_2})}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{50}$$

if $w_1 = w_2$ and $k_1 \neq k_2$

$$P(Z_{m,n} = (k_1,k_2)|Z_{-m,n}, D, O; \alpha, \beta) \tag{51}$$

$$\propto P(Z_{m,n} = (k_1,k_2), Z_{-m,n}, D, O; \alpha, \beta) \tag{52}$$

$$= \left(\frac{\Gamma(\sum_{i=1}^{K}\alpha_i)}{\prod_{i=1}^{K}\Gamma(\alpha_i)}\right)^M \cdot \prod_{j\neq m}^{M} \frac{\prod_{i=1}^{K}\Gamma(n_u(i,j)+\alpha_i)}{\Gamma(\sum_{i=1}^{K} n_u(i,j)+\alpha_i)} \tag{53}$$

$$\times \left(\frac{\Gamma(\sum_{r=1}^{V}\beta_r)}{\prod_{r=1}^{V}\Gamma(\beta_r)}\right)^R \cdot \prod_{i=1}^{R}\prod_{r\neq w_1}^{V} \Gamma(n_w(i,r)+\beta_r) \tag{54}$$

$$\times \frac{\prod_{i=1}^{K}\Gamma(n_u(i,m)+\alpha_i)}{\Gamma(\sum_{i=1}^{k} n_u(i,m)+\alpha_i)} \cdot \frac{\prod_{i=1}^{R}\Gamma(n_w(i,w_1)+\beta_{w_1})}{\prod_{i=1}^{R}\Gamma(\sum_{r=1}^{V} n_w(i,r)+\beta_r)} \tag{55}$$

$$=(\cdot) \cdot \frac{\prod_{i=1}^{R}\prod_{r\neq w_1}^{V}\Gamma(n_w(i,r)+\beta_r)}{\prod_{i=1}^{R}\Gamma(\sum_{r=1}^{V} n_w(i,r)+\beta_r)} \prod_{i\neq k_1,k_2}^{R} \Gamma(n_w(i,w_1)+\beta_{w_1}) \tag{56}$$

$$\cdot \Gamma(n_w(k_1,w_1)+\beta_{w_1})\Gamma(n_w(k_2,w_1)+\beta_{w_1}) \tag{57}$$

$$\text{(sampling)} \tag{58}$$

$$=(\cdot) \cdot \frac{\prod_{i=1}^{R}\prod_{r\neq w_1,}^{V}\Gamma(n_w^-(i,r)+\beta_r)}{\prod_{i=1}^{R}\Gamma((\sum_{r=1}^{V} n_w^-(i,r)+\beta_r)+2)} \prod_{i\neq k_1,k_2}^{R} \Gamma(n_w^-(i,w_1)+\beta_{w_1}) \tag{59}$$

$$\cdot \Gamma(n_w^-(k_1,w_1)+\beta_{w_1}+1)\Gamma(n_w^-(k_2,w_1)+\beta_{w_1}+1) \tag{60}$$

$$=(\cdot) \cdot \frac{\prod_{i=1}^{R}\prod_{r\neq w_1}^{V}\Gamma(n_w^-(i,r)+\beta_r)}{\prod_{i=1}^{R}\Gamma((\sum_{r=1}^{V} n_w^-(i,r)+\beta_r)+2)} \prod_{i\neq k_1,k_2}^{R} \Gamma(n_w^-(i,w_1)+\beta_{w_1}) \tag{61}$$

$$\cdot (n_w^-(k_1,w_1)+\beta_{w_1})(n_w^-(k_2,w_1)+\beta_{w_1})\Gamma(n_w^-(k_1,w_1)+\beta_{w_1})\Gamma(n_w^-(k_2,w_1)+\beta_{w_1}) \tag{62}$$

$$=(\cdot) \cdot \frac{(n_w^-(k_1,w_1)+\beta_{w_1})(n_w^-(k_2,w_1)+\beta_{w_1})}{\prod_{i=1}^{R}\Gamma((\sum_{r=1}^{V} n_w^-(i,r)+\beta_r)+2)} \tag{63}$$

if $w_1 = w_2$ and $k_1 = k_2$

$$P(Z_{m,n} = (k_1, k_2) | Z_{-m,n}, D, O; \alpha, \beta) \tag{64}$$

$$\propto P(Z_{m,n} = (k_1, k_2), Z_{-m,n}, D, O; \alpha, \beta) \tag{65}$$

$$= \left(\frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \right)^M \cdot \prod_{j \neq m}^{M} \frac{\prod_{i=1}^{K} \Gamma(n_u(i,j) + \alpha_i)}{\Gamma(\sum_{i=1}^{K} n_u(i,j) + \alpha_i)} \tag{66}$$

$$\times \left(\frac{\Gamma(\sum_{r=1}^{V} \beta_r)}{\prod_{r=1}^{V} \Gamma(\beta_r)} \right)^R \cdot \prod_{i=1}^{R} \prod_{r \neq w_1}^{V} \Gamma(n_w(i,r) + \beta_r) \tag{67}$$

$$\times \frac{\prod_{i=1}^{K} \Gamma(n_u(i,m) + \alpha_i)}{\Gamma(\sum_{i=1}^{k} n_u(i,m) + \alpha_i)} \cdot \frac{\prod_{i=1}^{R} \Gamma(n_w(i,w_1) + \beta_{w_1})}{\prod_{i=1}^{R} \Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \tag{68}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1}^{V} \Gamma(n_w(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma(\sum_{r=1}^{V} n_w(i,r) + \beta_r)} \tag{69}$$

$$\cdot \Gamma(n_w(k_1, w_1) + \beta_{w_1}) \prod_{i \neq k_1}^{R} \Gamma(n_w(i, w_1) + \beta_{w_1}) \tag{70}$$

$$\text{(sampling)} \tag{71}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1}^{V} \Gamma(n_w^-(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{72}$$

$$\cdot \Gamma(n_w^-(k_1, w_1) + \beta_{w_1} + 2) \prod_{i \neq k_1}^{R} \Gamma(n_w^-(i, w_1) + \beta_{w_1}) \tag{73}$$

$$= (\cdot) \cdot \frac{\prod_{i=1}^{R} \prod_{r \neq w_1}^{V} \Gamma(n_w^-(i,r) + \beta_r)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{74}$$

$$\cdot (n_w^-(k_1, w_1) + \beta_{w_1})(n_w^-(k_1, w_1) + \beta_{w_1} + 1)\Gamma(n_w^-(k_1, w_1) + \beta_{w_1}) \prod_{i \neq k_1}^{R} \Gamma(n_w^-(i, w_1) + \beta_{w_1}) \tag{75}$$

$$= (\cdot) \cdot \frac{(n_w^-(k_1, w_1) + \beta_{w_1})(n_w^-(k_1, w_1) + \beta_{w_1} + 1)}{\prod_{i=1}^{R} \Gamma((\sum_{r=1}^{V} n_w^-(i,r) + \beta_r) + 2)} \tag{76}$$