



# Grasp Planning

CS 6341 Robotics

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The University of Texas at Dallas

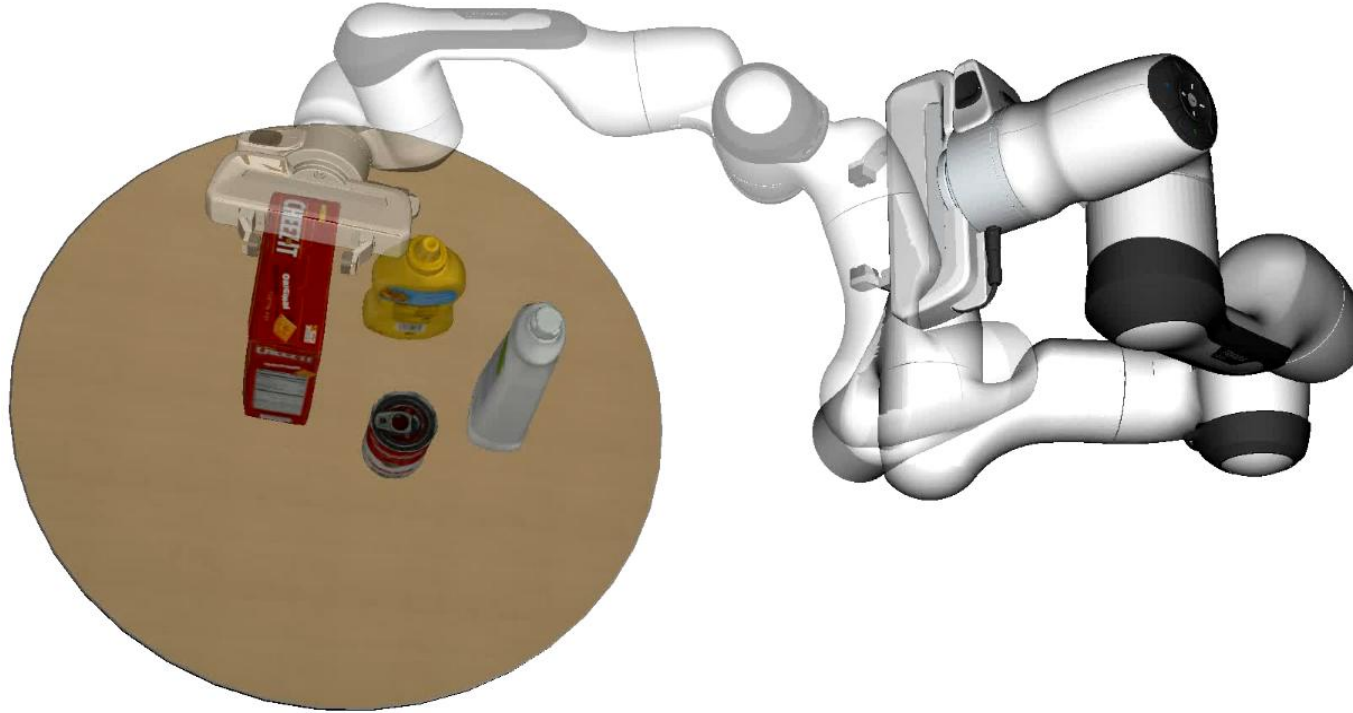
# Introduction



Graspt! <https://graspt-simulator.github.io/>

Andrew Miller and Peter K. Allen. "Graspt!: A Versatile Simulator for Robotic Grasping". IEEE Robotics and Automation Magazine, V. 11, No.4, Dec. 2004, pp. 110-122.

# Grasps are used as the goals for motion planning

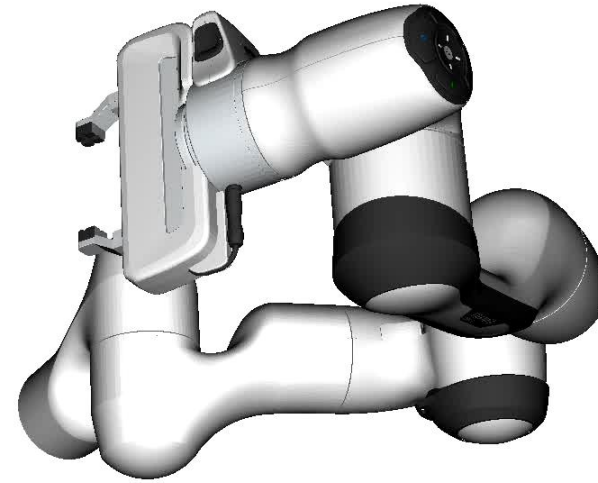
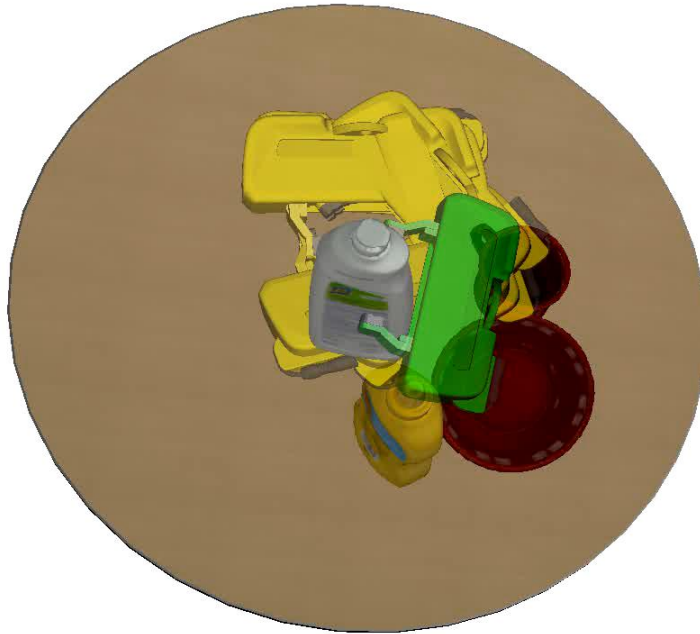


Covariant Hamiltonian Optimization for Motion Planning (CHOMP): Ratliff-Zucker-Bagnell-Srinivasa, ICRA'09

# Grasps are used as the goals for motion planning

OMG Iter: 50

100 grasps



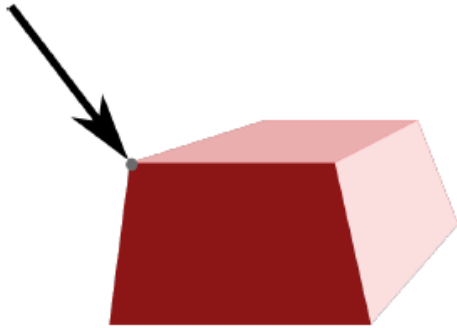
Modeling the goal set distribution

Wang-Xiang-Fox, Manipulation Trajectory Optimization with Online Grasp Synthesis and Selection RSS'20

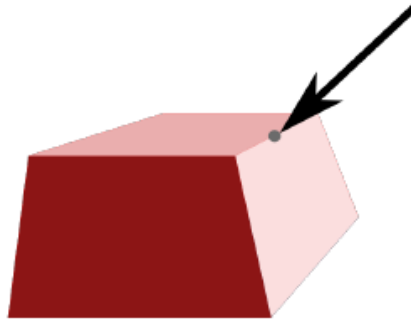
# Grasp Planning

- Model-based methods
  - Force closure analysis
  - Physics simulation
- Learning-based methods
  - Learning generative models with grasping datasets
  - Predicting contact maps, keypoints on object surfaces
  - Predicting hand configurations

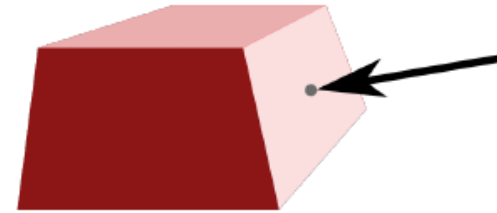
# Contact Modeling



Point-on-point



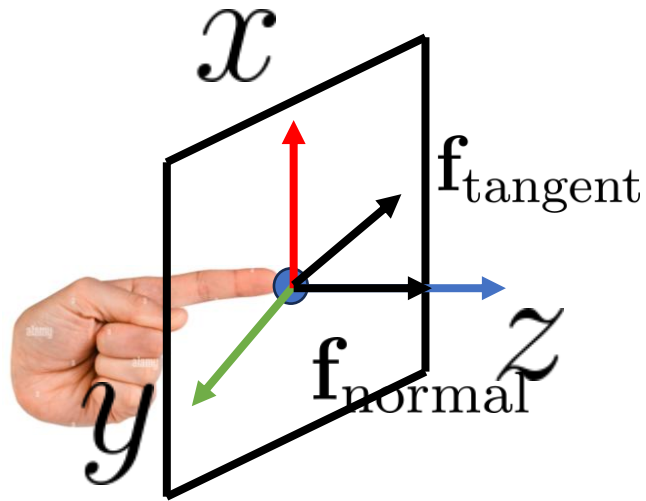
Point-on-line



Point-on-plane

# Point-on-Plane Contact Models

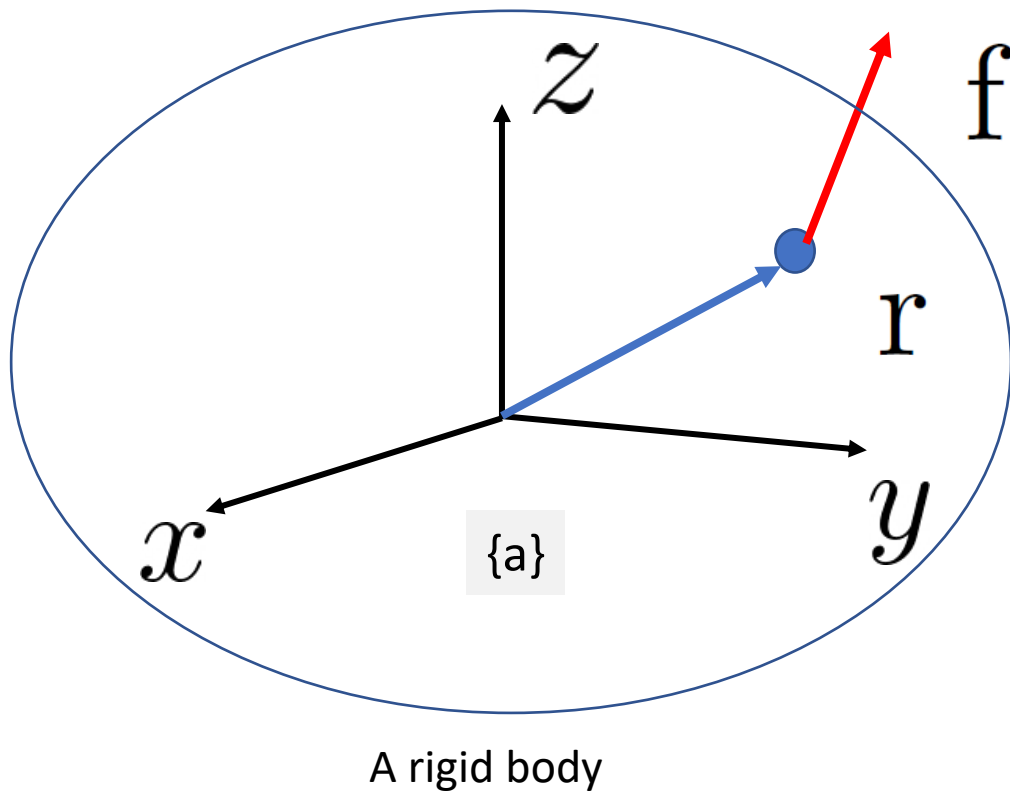
$$\mathbf{f} = \mathbf{f}_{\text{normal}} + \mathbf{f}_{\text{tangent}}$$



$$\mathbf{f}_{\text{normal}} = [0, 0, f_z]^T \quad f_z \geq 0$$

$$\mathbf{f}_{\text{tangent}} = [f_x, f_y, 0]^T$$

# Recall Torque and Wrench



Point  $r_a \in \mathbb{R}^3$

Force  $f_a \in \mathbb{R}^3$

Torque or Moment

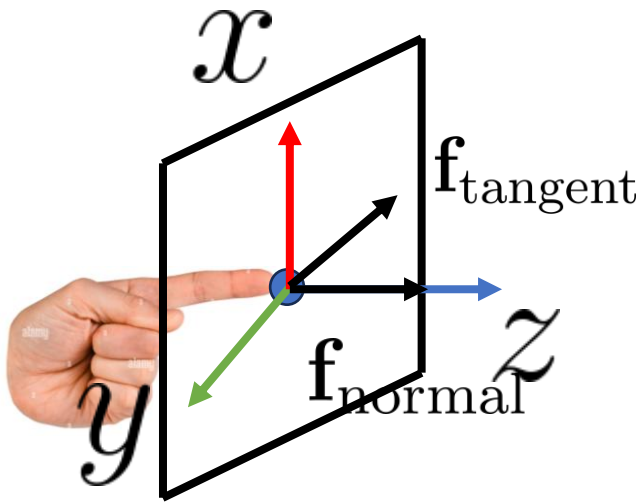
$$m_a \in \mathbb{R}^3$$

$$m_a = r_a \times f_a$$

$$\text{Wrench } \mathcal{F}_a = \begin{bmatrix} m_a \\ f_a \end{bmatrix} \in \mathbb{R}^6$$



# Point-on-Plane Contact Models



## Frictionless Point Contact:

$$\mathcal{F} = \{\mathbf{f}_{\text{normal}} | f_z \geq 0\}$$

Admissible forces (i.e. forces that don't lead to slipping)

## Friction Point Contact:

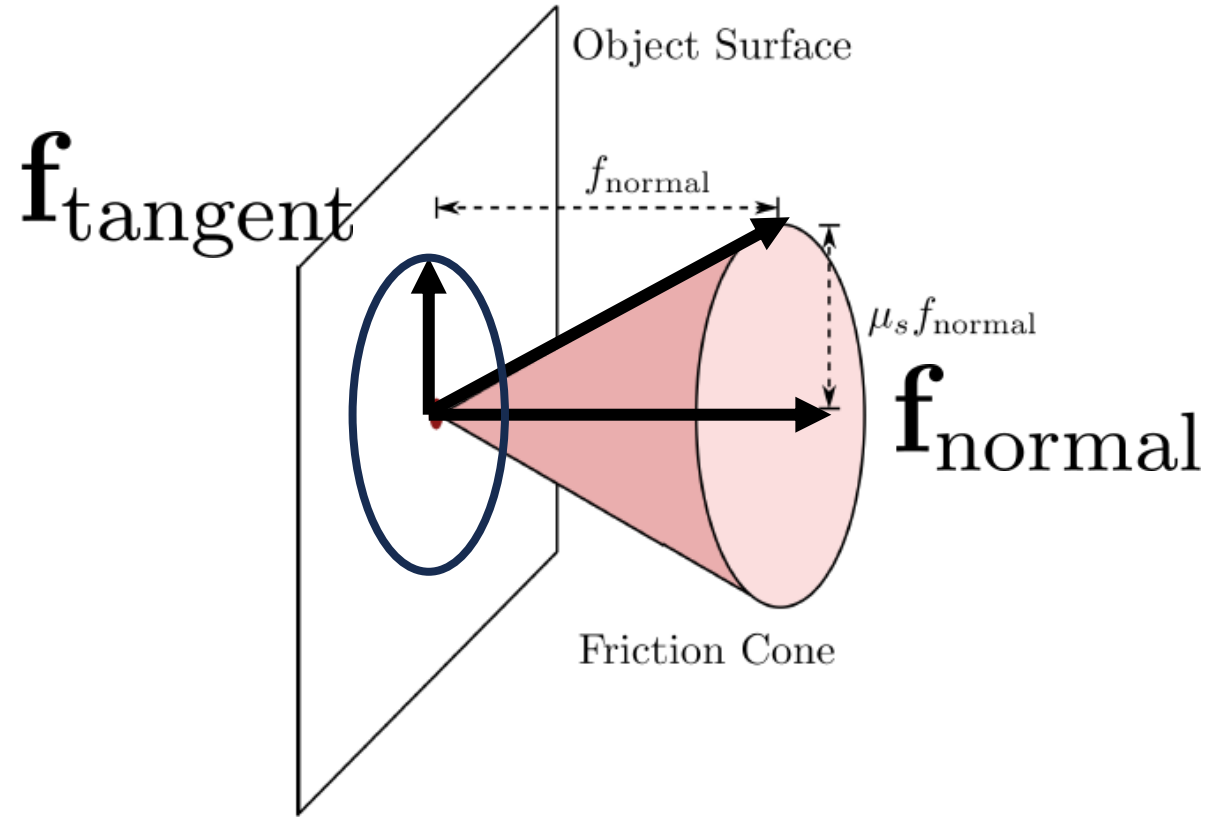
$$\mathcal{F} = \{\mathbf{f} | \|\mathbf{f}_{\text{tangent}}\| \leq \mu_s \|\mathbf{f}_{\text{normal}}\|, f_z \geq 0\}$$

$\mu_s$  static friction coefficient

## Soft-finger Contact Model

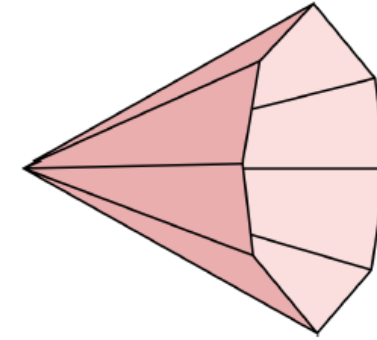
$$\mathcal{F} = \{(\mathbf{f}, \tau_{\text{normal}}) | \|\mathbf{f}_{\text{tangent}}\| \leq \mu_s \|\mathbf{f}_{\text{normal}}\|, f_z \geq 0, |\tau_{\text{normal}}| \leq \gamma f_z\}$$

# Friction Cone



$$\mathcal{F} = \{\mathbf{f} \mid \|\mathbf{f}_{\text{tangent}}\| \leq \mu_s \|\mathbf{f}_{\text{normal}}\|, f_z \geq 0\}$$

$$\tan(\theta) = \mu_s$$



A pyramidal inner-approximation of the friction cone

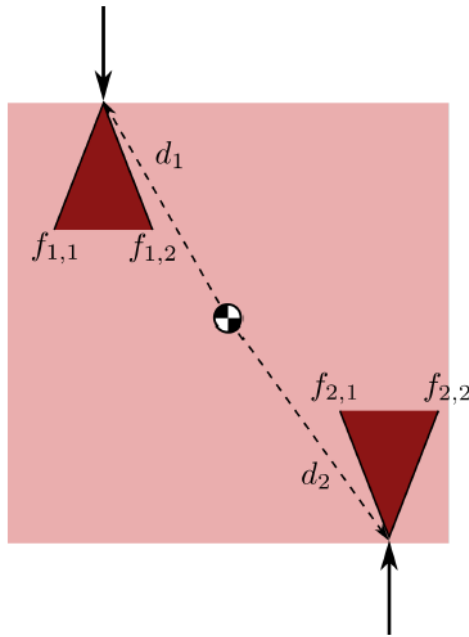
$$\{f_{i,1}, f_{i,2}, \dots, f_{i,m}\}$$

$$f_i = \sum_{j=1}^m \alpha_{i,j} f_{i,j}, \quad \alpha_{i,j} \geq 0$$

$$\sum_{j=1}^m \alpha_{i,j} \leq 1$$

# Grasp Wrench Space

- Wrench: force and torque  $w = \begin{bmatrix} f \\ \tau \end{bmatrix} \in \mathbb{R}^6$
- Each contact point applies a wrench to the object

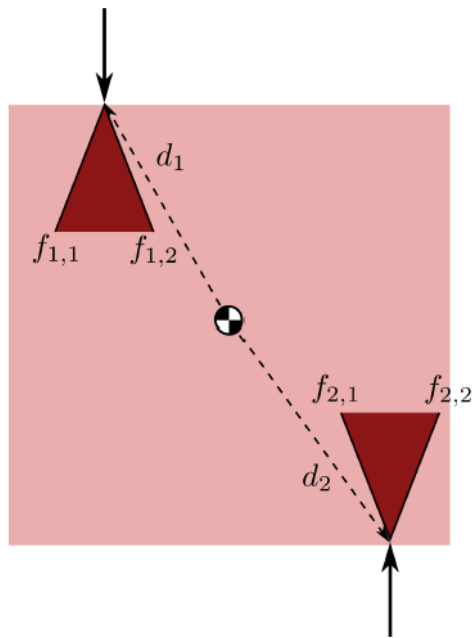


$$w_i = \begin{bmatrix} f_i \\ \lambda(d_i \times f_i) \end{bmatrix}$$

$$\omega_i = \begin{bmatrix} I \\ \lambda[\mathbf{d}_i] \end{bmatrix} \mathbf{f}_i$$

$$\omega_i = G_i \mathbf{f}_i$$

# Grasp Wrench Space



Total wrench on the object

$$w = \sum_{i=1}^k G_i f_i = G \begin{bmatrix} f_1 \\ \vdots \\ f_k \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & \dots & G_k \end{bmatrix}$$

grasp map

k contact points

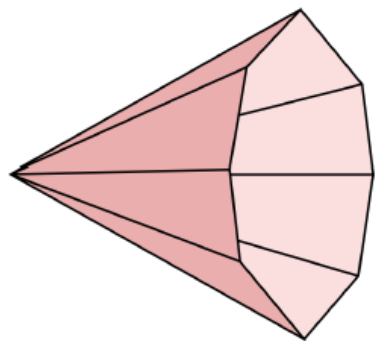
# Grasp Wrench Space

- The grasp wrench space  $W$  for a grasp with  $k$  contact points is the set of all possible wrenches  $\mathbf{w}$  that can be applied to the object through admissible forces:

$$\mathcal{W} := \{ \mathbf{w} \mid \mathbf{w} = \sum_{i=1}^k G_i \mathbf{f}_i, \quad \mathbf{f}_i \in \mathcal{F}_i, \quad i = 1, \dots, k \}.$$

- If the grasp wrench space is large, the grasp can compensate for a bigger set of external wrenches that might be applied to the object, leading to a more robust grasp

# Grasp Wrench Space



$$\mathcal{W} = \{w \mid w = \sum_{i=1}^k w_i, \quad w_i = \sum_{j=1}^m \alpha_{i,j} w_{i,j}, \quad w_{i,j} = \begin{bmatrix} f_{i,j} \\ \lambda(d_i \times f_{i,j}) \end{bmatrix},$$

$$\sum_{j=1}^m \alpha_{i,j} \leq 1, \quad \alpha_{i,j} \geq 0\}.$$

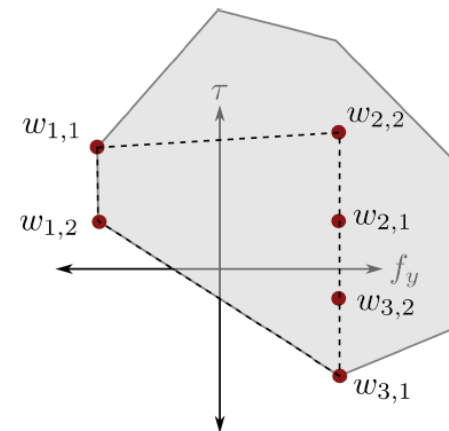
$$\{f_{i,1}, f_{i,2}, \dots, f_{i,m}\}$$

$$f_i = \sum_{j=1}^m \alpha_{i,j} f_{i,j}, \quad \alpha_{i,j} \geq 0.$$

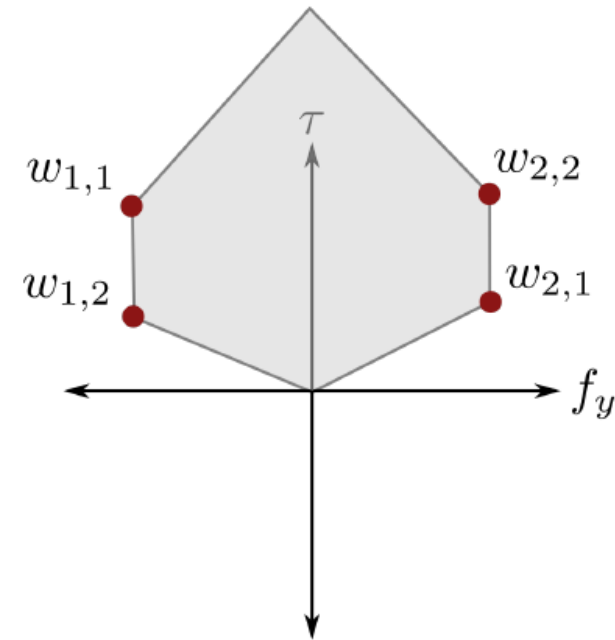
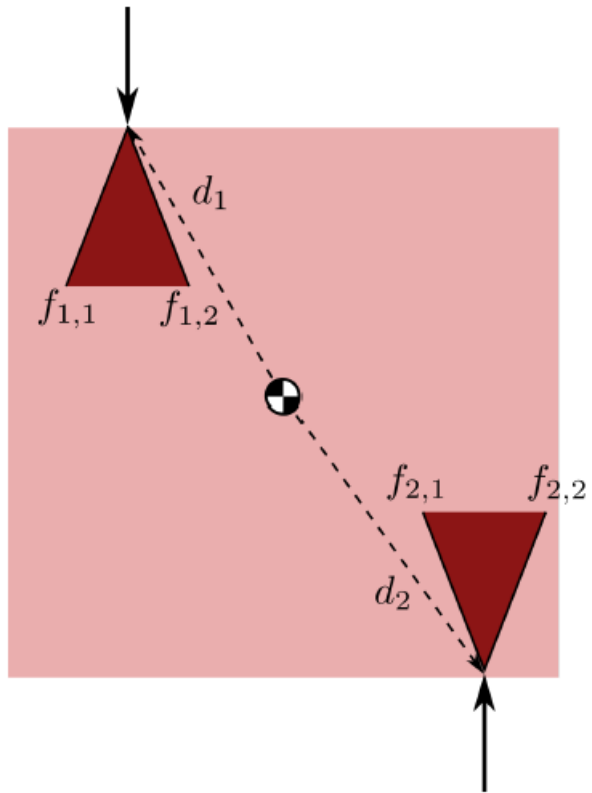
$$\sum_{j=1}^m \alpha_{i,j} \leq 1$$

Grasp Wrench Hull

$$\tilde{\mathcal{W}} = \{w \mid w = \sum_{i=1}^k \sum_{j=1}^m \alpha_{i,j} w_{i,j}, \quad w_{i,j} = \begin{bmatrix} f_{i,j} \\ \lambda(d_i \times f_{i,j}) \end{bmatrix}, \quad \sum_{i=1}^k \sum_{j=1}^m \alpha_{i,j} = 1, \quad \alpha_{i,j} \geq 0\}$$



# Grasp Wrench Space



ignoring the horizontal force components  $f_x$

# Force Closure Grasp

- A grasp is a force closure grasp if for any external wrench  $\mathbf{w}$  there exist contact forces  $\{\mathbf{f}_i\}_{i=1}^k$  such that

$$-\mathbf{w} = \sum_{i=1}^k G_i \mathbf{f}_i, \quad \mathbf{f}_i \in \mathcal{F}_i, \quad i = 1, \dots, k,$$

Or

$$-\mathbf{w} \in \mathcal{W}. \quad \text{resist any external wrench}$$

The grasp wrench space needs to be  $\mathcal{W} = \mathbb{R}^n$  For 2D objects  $n = 3$  and for 3D  $n = 6$



# Force Closure Grasp

$$\tilde{\mathcal{W}} = \{w \mid w = \sum_{i=1}^k \sum_{j=1}^m \alpha_{i,j} w_{i,j}, \quad w_{i,j} = \begin{bmatrix} f_{i,j} \\ \lambda(d_i \times f_{i,j}) \end{bmatrix}, \quad \sum_{i=1}^k \sum_{j=1}^m \alpha_{i,j} = 1, \quad \alpha_{i,j} \geq 0\}$$

- Theorem: In an  $n$ -dimensional vector space with

$$\mathcal{W} := \{w \mid w = \sum_{k=1}^N \beta_k w_k, \quad \beta_k \geq 0\},$$

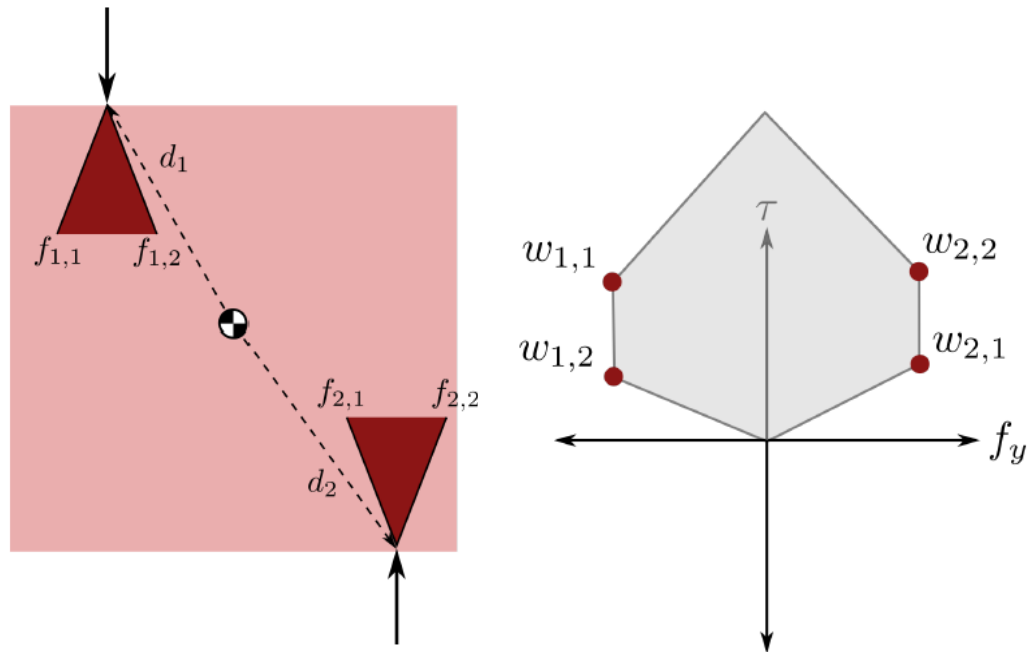
$\mathcal{W} = \mathbb{R}^n$  if and only if the set  $\{w_k\}_{k=1}^N$  contains at least  $n + 1$  vectors,  $n$  of the vectors are linearly independent, and there exists scalars  $\beta_k > 0$  such that:

$$\sum_{k=1}^N \beta_k w_k = 0.$$

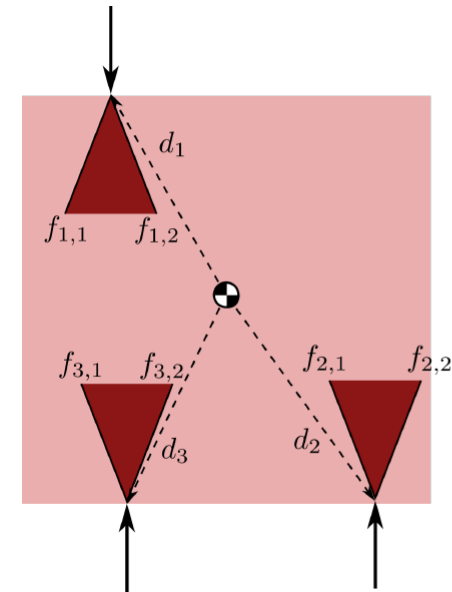
grasp wrench space  $\mathcal{W}$  must contain the origin in its interior

# Force Closure Grasp

$n = 3$



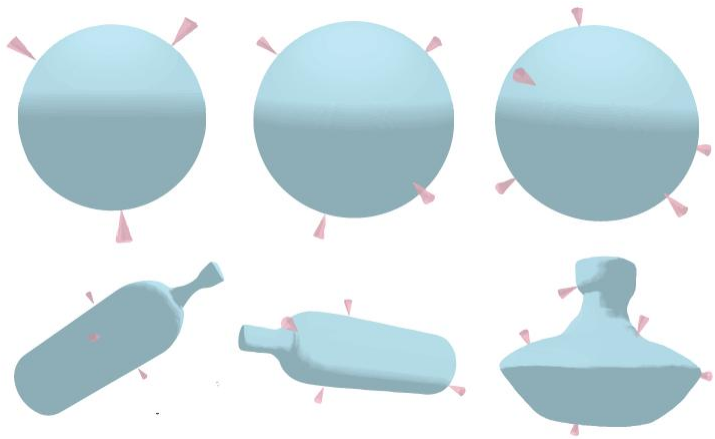
Not force closure



force closure

# Optimization to find force closure grasps

- Find contact points  $\mathbf{X}_1, \dots, \mathbf{X}_k$ , and contact forces  $\mathbf{f}_1, \dots, \mathbf{f}_k$



$$GG' \succeq \epsilon I_{6 \times 6},$$

$$Gf = 0,$$

$$f_i^T c_i > \frac{1}{\sqrt{\mu^2 + 1}} |f_i|,$$

$$x_i \in S,$$

$$G = \begin{bmatrix} I_{3 \times 3} & I_{3 \times 3} & \dots & I_{3 \times 3} \\ [x_1]_{\times} & [x_2]_{\times} & \dots & [x_n]_{\times} \end{bmatrix}$$

**G is full rank**

**grasp wrench hull contains the origin in its interior**

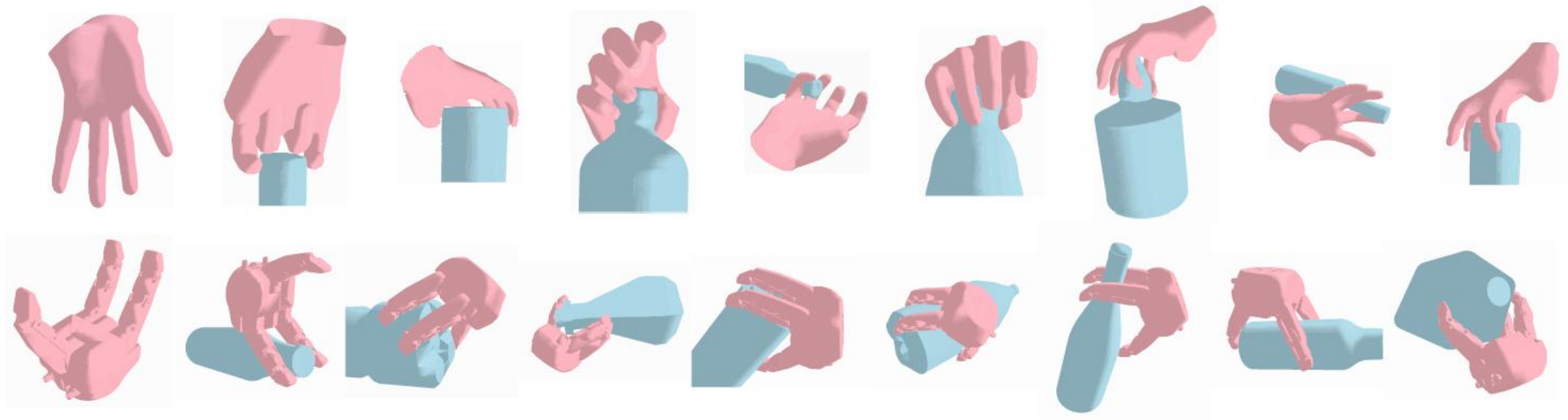
**Friction cone constraint**

**Point on object surface**

[1] Synthesis and Optimization of Force Closure Grasps via Sequential Semidefinite Programming. Hongkai Dai, Anirudha Majumdar and Russ Tedrake, 2015

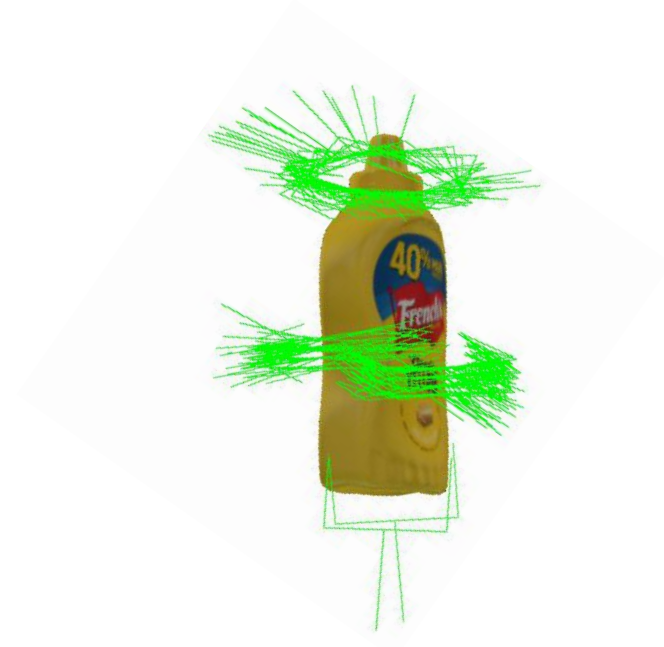
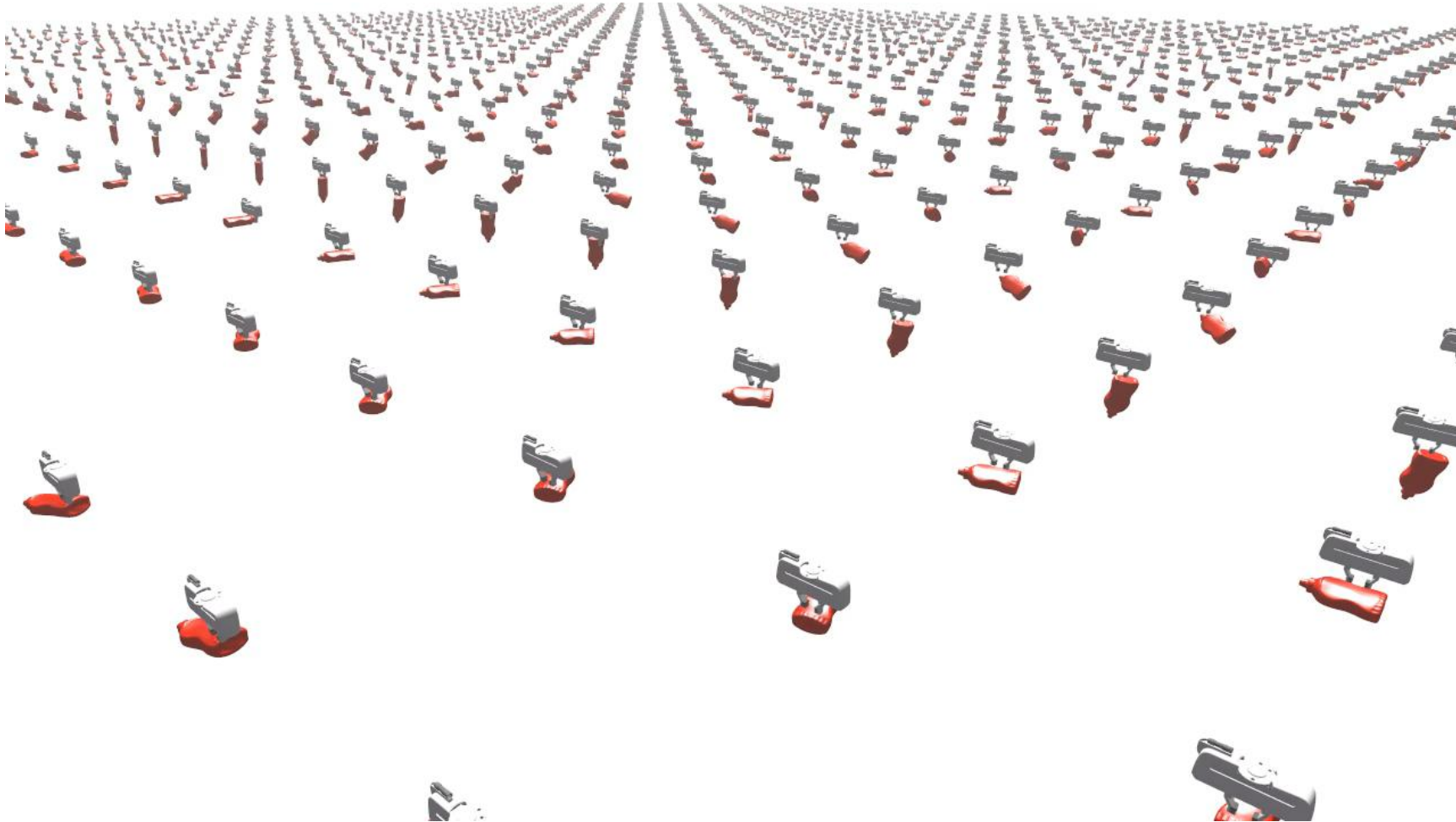
[2] Synthesizing Diverse and Physically Stable Grasps With Arbitrary Hand Structures Using Differentiable Force Closure Estimator. Tengyu Liu, Zeyu Liu, Ziyuan Jiao, Yixin Zhu, and Song-Chun Zhu, 2022

# Optimization to find force closure grasps



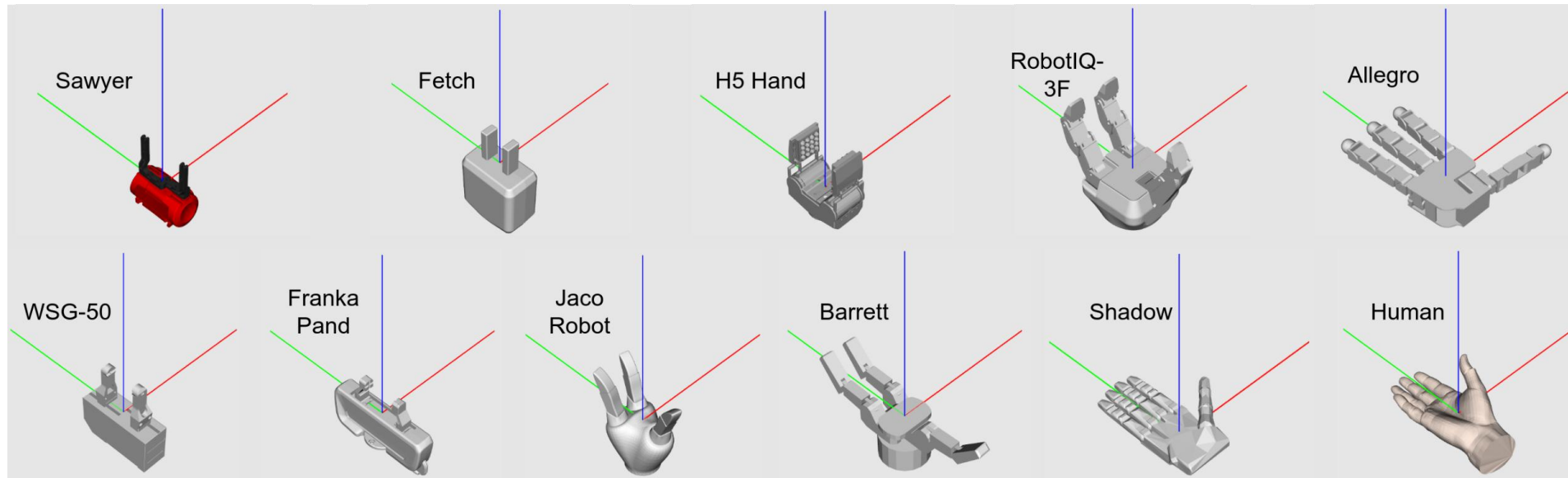
Synthesizing Diverse and Physically Stable Grasps With Arbitrary Hand Structures Using Differentiable Force Closure Estimator. Tengyu Liu , Zeyu Liu, Ziyuan Jiao , Yixin Zhu , and Song-Chun Zhu, 2022

# Use Physics Simulation for Grasp Planning



Eppner-Mousavian-Fox, ISRR'19

# MultiGripperGrasp

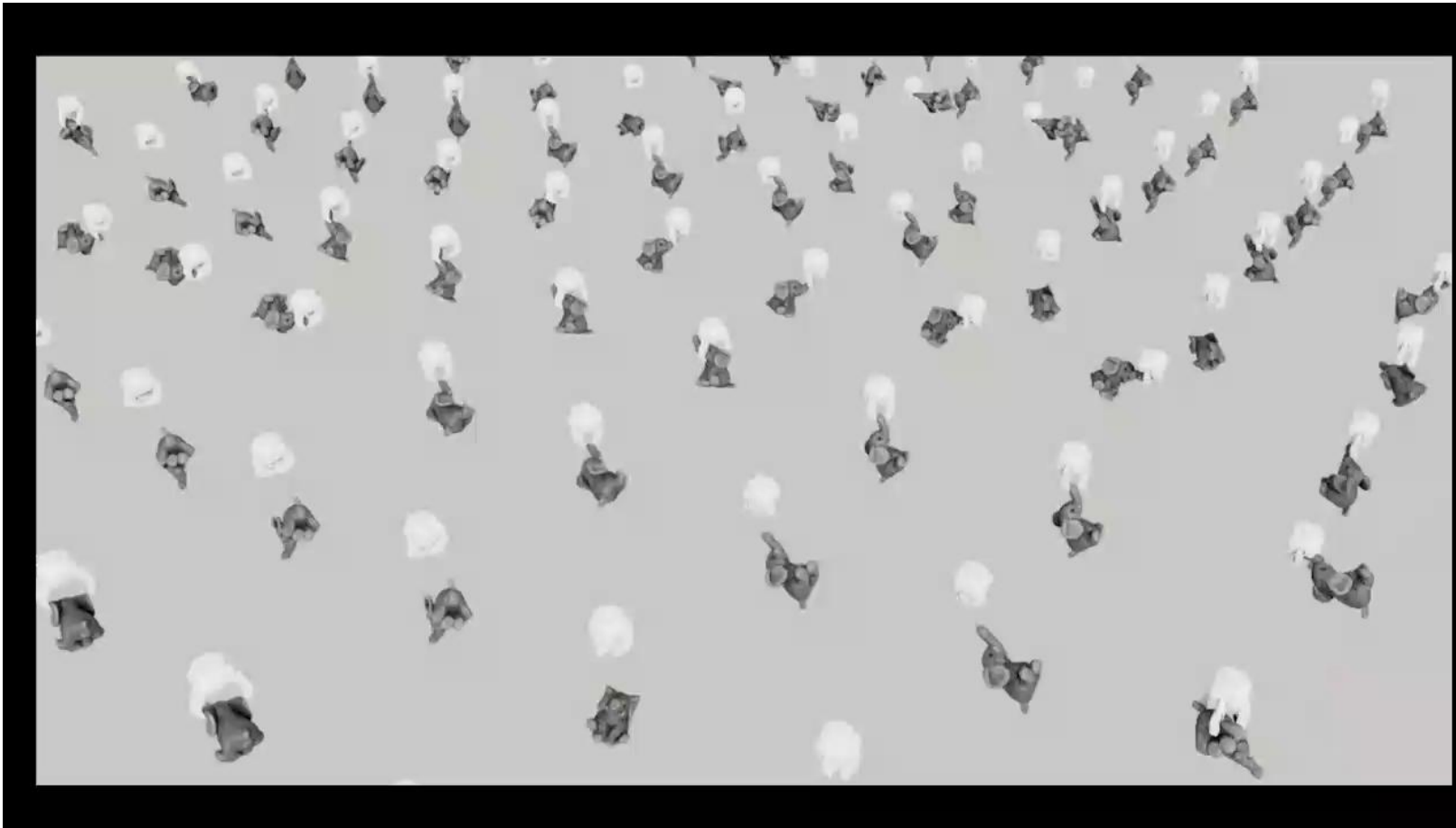


- 11 grippers (aligned with palm directions)
  - 2-finger grippers: Fetch, Franka Panda, WSG50, Sawyer, H5 Hand
  - 3-finger grippers: Barrett, Robotiq-3F, Jaco Robot
  - 4-finger grippers: Allegro
  - 5 finger grippers: Shadow, Human Hand

**MultiGripperGrasp: A Dataset for Robotic Grasping from Parallel Jaw Grippers to Dexterous Hands.** Luis Felipe Casas Murrilo\*, Ninad Khargonkar\*, Balakrishnan Prabhakaran, Yu Xiang (\*equal contribution). In IROS, 2024.

# MultiGripperGrasp

- Generate initial grasps using Grasplt! Ranking grasps in Isaac Sim



**MultiGripperGrasp: A Dataset for Robotic Grasping from Parallel Jaw Grippers to Dexterous Hands.** Luis Felipe Casas Murrilo\*, Ninad Khargonkar\*, Balakrishnan Prabhakaran, Yu Xiang (\*equal contribution). In IROS, 2024.



# Model-based Grasping vs Model-free Grasping

SceneReplica, ICRA'24: <https://irvlutd.github.io/SceneReplica/>



Input real world scene



6D Pose Estimation



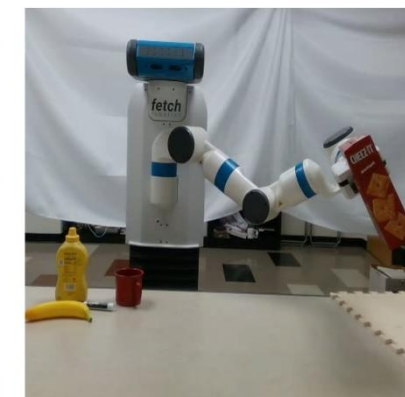
Offline Grasp Database



Motion Planning Setup



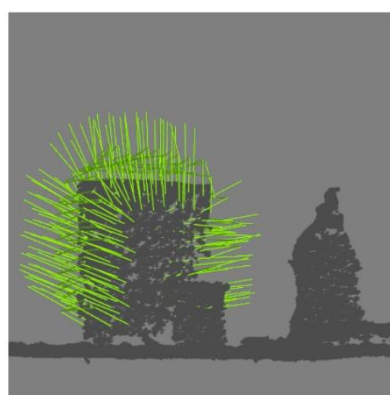
Grasping & Lifting



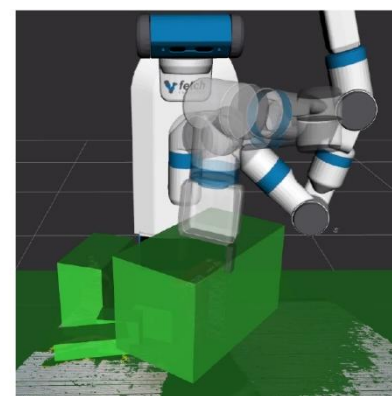
Moving arm for Dropoff



Unseen Object Segmentation



Model-free Grasp Planning



Motion Planning Setup



Grasping & Lifting



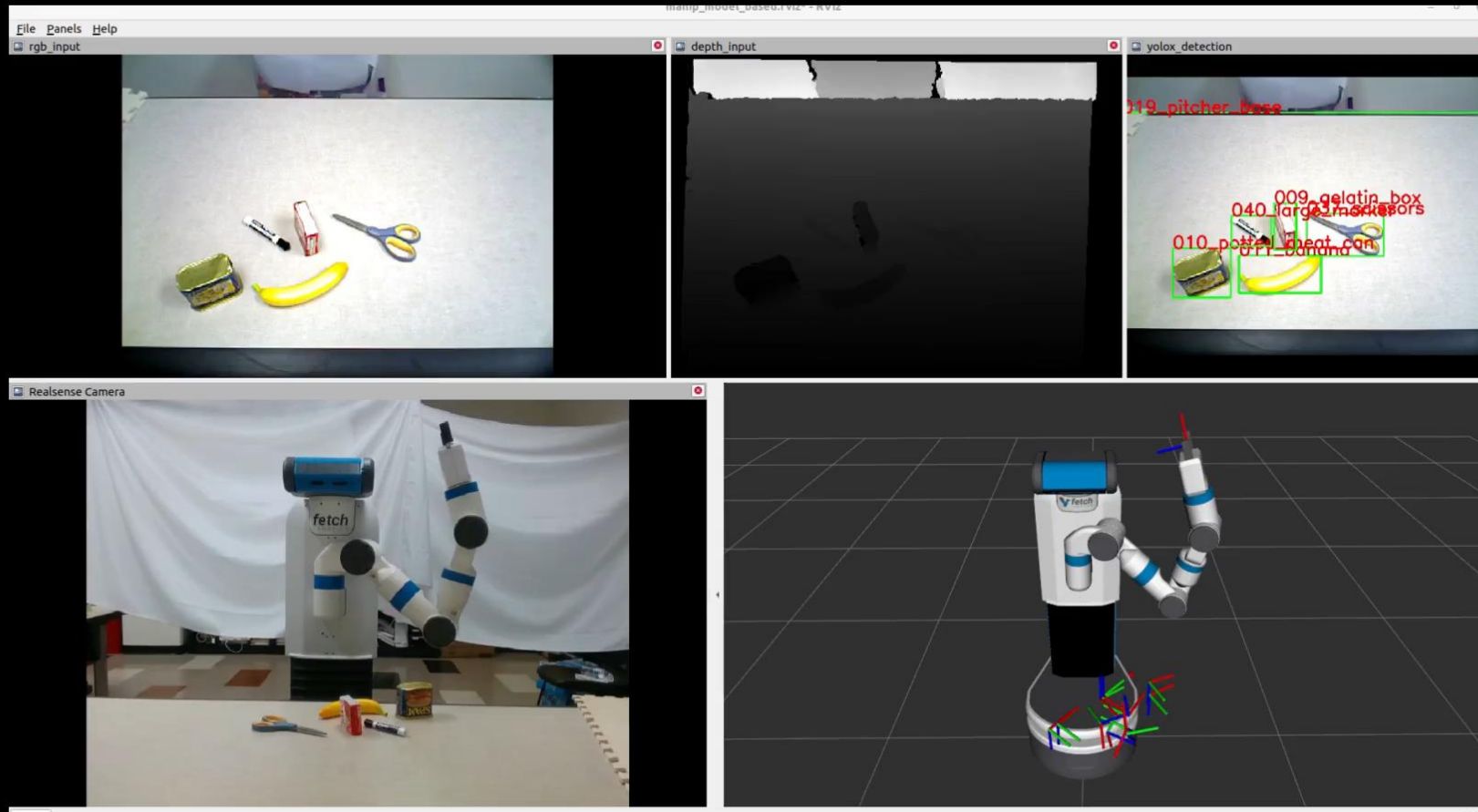
Moving arm for Dropoff



# Model-based Grasping Example

[8X] SceneReplica Benchmark

GDRNPP | Graspit + Top Down | MoveIt



Realsense Capture Scene: 148 | Order: Random Rviz Capture

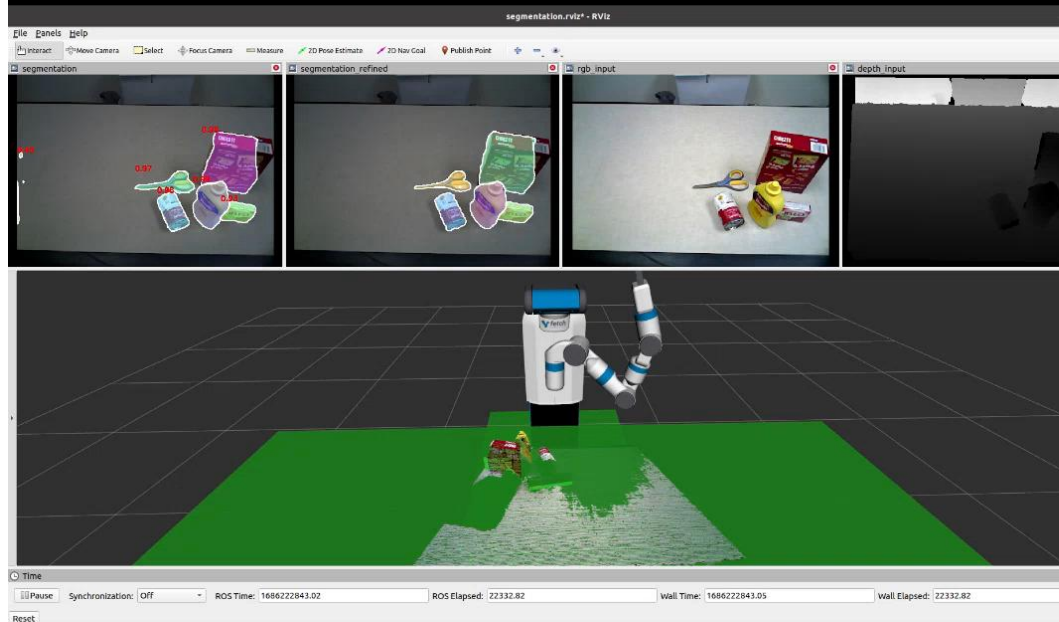
# Model-free Grasping Example

8X

SceneReplica Benchmark

MSMFormer | Contact GraspNet + Top Down | MoveIt

Scene: 130 | Order: Random



Rviz Capture



Realsense Capture

# Summary

- Point-on-Plane Contact Models
- Friction cone
- Grasp Wrench Space
- Force Closure Grasp
- Optimization to find force closure grasps

# Further Reading

- [1] Main reference: [Stanford Principles of Robot Autonomy-II](#): Grasping lectures
- [2] [Friction Cone](#)
- [3] Classical Grasp Planning: [Grasplt Simulator](#)
- [4] [Dex-Net 2.0: Deep Learning to Plan Robust Grasps with Synthetic Point Clouds and Analytic Grasp Metrics](#)
- [5] [6-DOF GraspNet: Variational Grasp Generation for Object Manipulation](#)
- [6] SceneReplica: <https://irvlutd.github.io/SceneReplica/>