Corner Detection

CS 4391 Introduction Computer Vision

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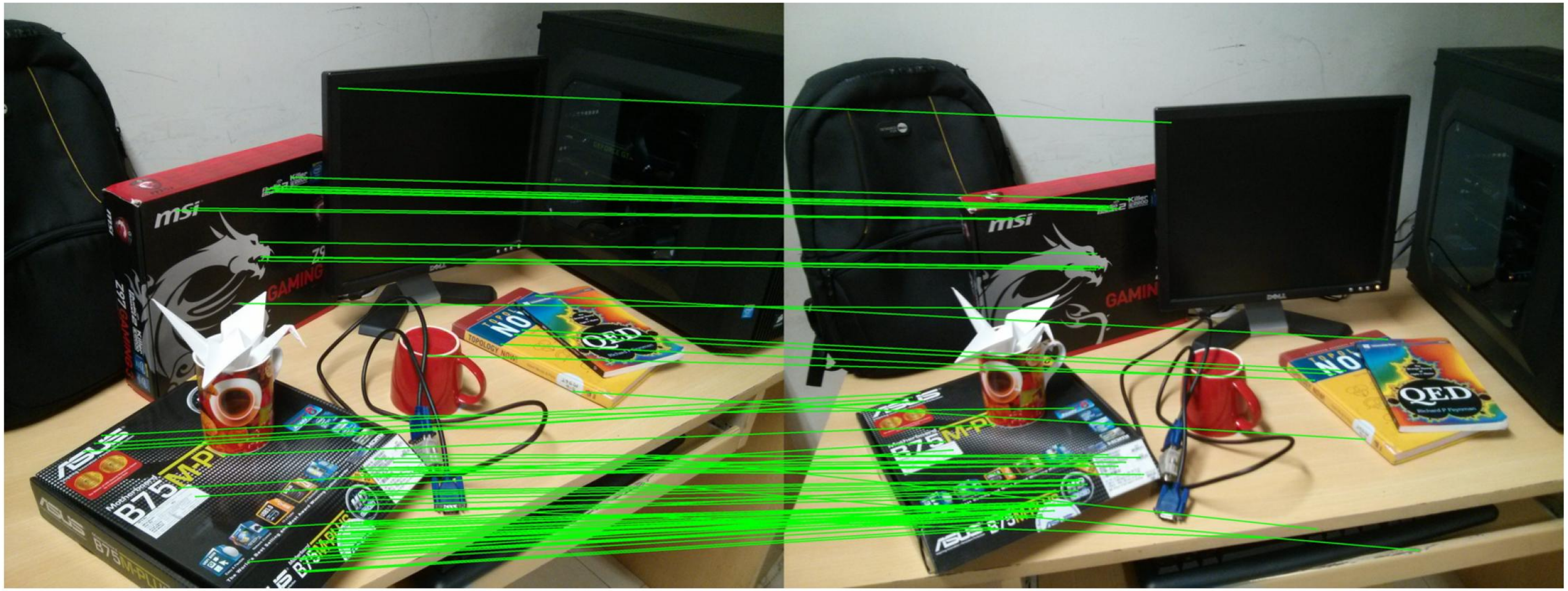
Feature Detection and Matching

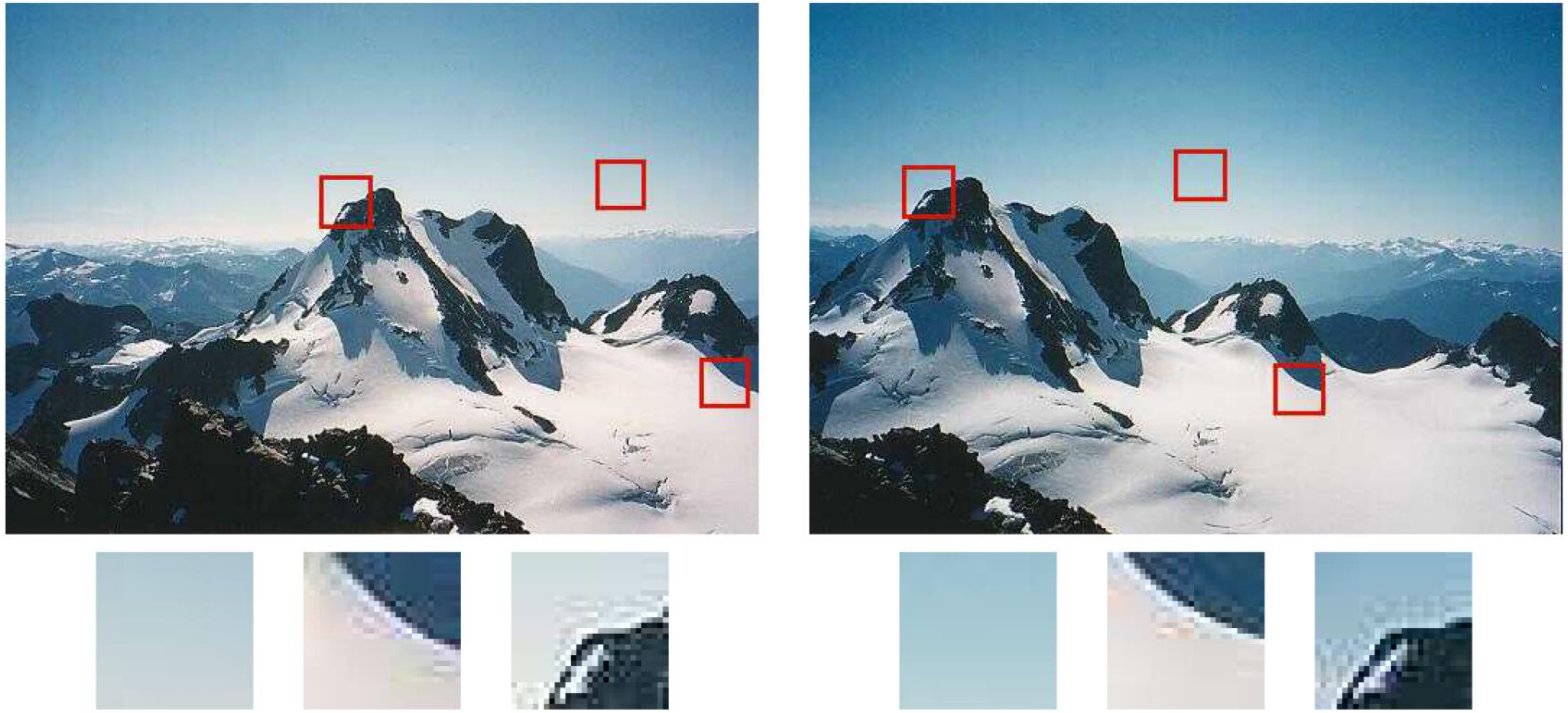


Geometry-aware Feature Matching for Structure from Motion Applications. Shah et al, WACV'15

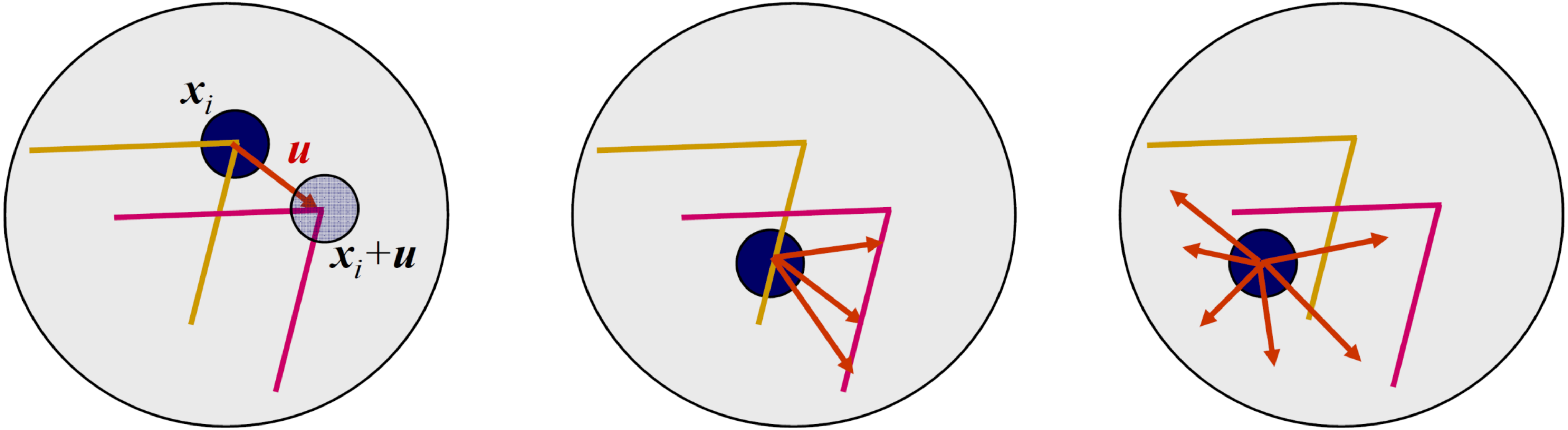
Applications: stereo matching, image stitching, 3D reconstruction, camera pose estimation, object recognition

Feature Detectors

- How to find image locations that can be reliably matched with images?



Feature Detectors



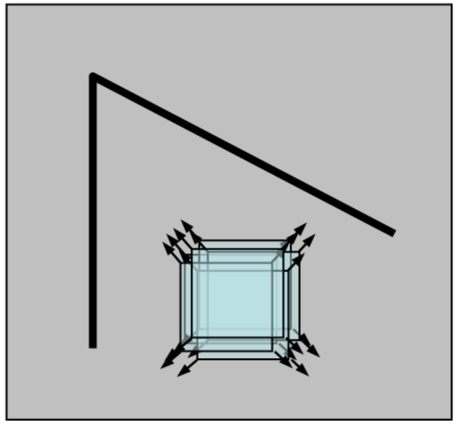
(a) Corner

(b) Edge

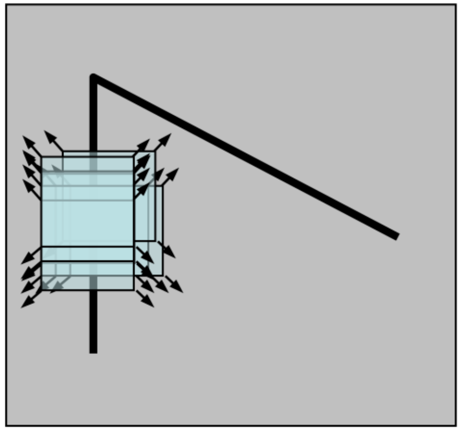
(c) Textureless region

Harris Corner Detector

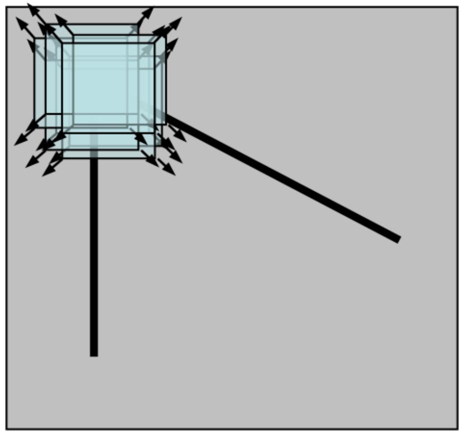
- Corners are regions with large variation in intensity in all directions



"flat" region:
no change in
all directions

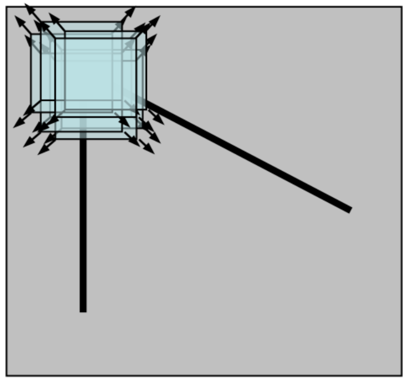


"edge":
no change
along the edge
direction



"corner":
significant
change in all
directions

Harris Corner Detector



Grayscale image $I(x, y)$

Window function

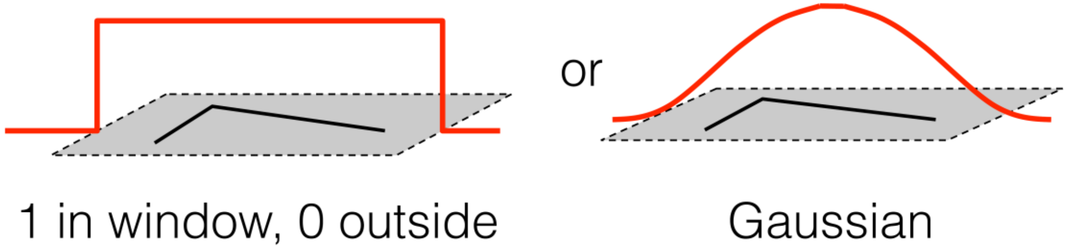


Image patch inside the window

$$f(\Delta x, \Delta y) = \sum_{x_k, y_k} w(x_k, y_k)(I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$$

sum of squared differences (SSD)

Shift (offset)

Idea: if $f(\Delta x, \Delta y)$ is large for all $(\Delta x, \Delta y)$, the patch has a corner

Harris Corner Detector

- Taylor series

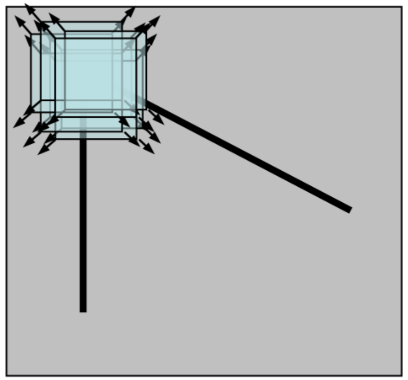
One dimension about x_0

$$f(x_0 + \Delta x) = f(x_0) + \Delta x\, f'(x_0) + \frac{1}{2!}(\Delta x)^2 f''(x_0) + \ldots$$

Two dimension about (x, y)

$$f(x + \Delta x, y + \Delta y) = f(x, y) + \left[f_x(x, y)\,\Delta x + f_y(x, y)\,\Delta y\right] + \frac{1}{2!}\left[(\Delta x)^2 f_{xx}(x, y) + 2\,\Delta x\,\Delta y\, f_{xy}(x, y) + (\Delta y)^2 f_{yy}(x, y)\right] +$$
$$\frac{1}{3!}\left[(\Delta x)^3 f_{xxx}(x, y) + 3\,(\Delta x)^2\,\Delta y\, f_{xxy}(x, y) + 3\,\Delta x\,(\Delta y)^2 f_{xyy}(x, y) + (\Delta y)^3 f_{yyy}(x, y)\right] + \ldots$$

Harris Corner Detector



Sum of squared differences $f(\Delta x, \Delta y) = \sum_{x_k, y_k} w(x_k, y_k)(I(x_k, y_k) - I(x_k + \Delta x, y_k + \Delta y))^2$

First order approximation $I(x + \Delta x, y + \Delta y) \approx I(x, y) + I_x(x, y)\Delta x + I_y(x, y)\Delta y$

X derivative Y derivative

$$f(\Delta x, \Delta y) \approx \sum_{x,y} w(x, y)(I_x(x, y)\Delta x + I_y(x, y)\Delta y)^2$$

$$f(\Delta x, \Delta y) \approx (\Delta x \quad \Delta y) M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \qquad M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{x,y} w(x, y) I_x^2 & \sum_{x,y} w(x, y) I_x I_y \\ \sum_{x,y} w(x, y) I_x I_y & \sum_{x,y} w(x, y) I_y^2 \end{bmatrix}$$

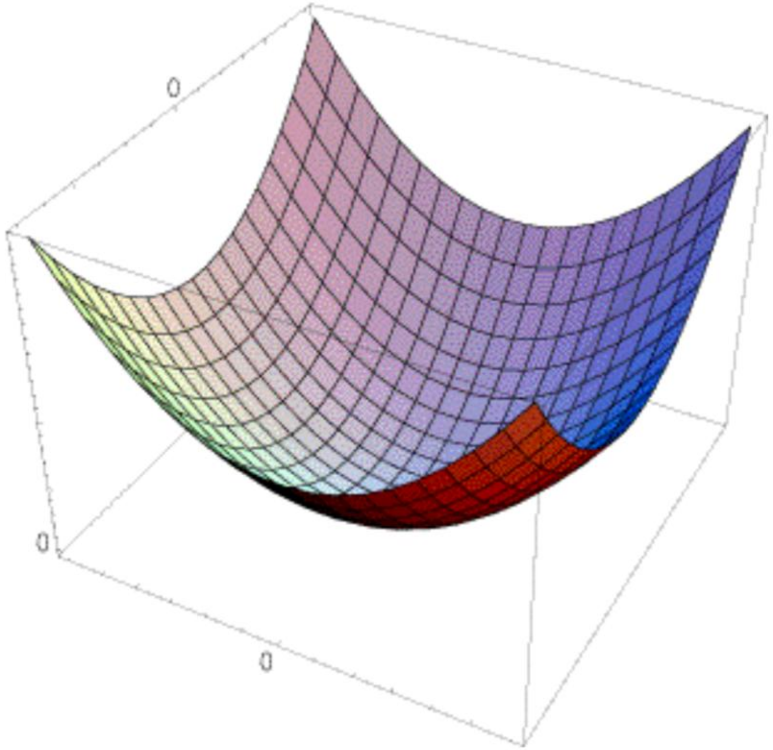
Idea: if $f(\Delta x, \Delta y)$ is large for all $(\Delta x, \Delta y)$, the patch has a corner

Harris Corner Detector

- A quadratic function

$$f(\Delta x, \Delta y) \approx \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

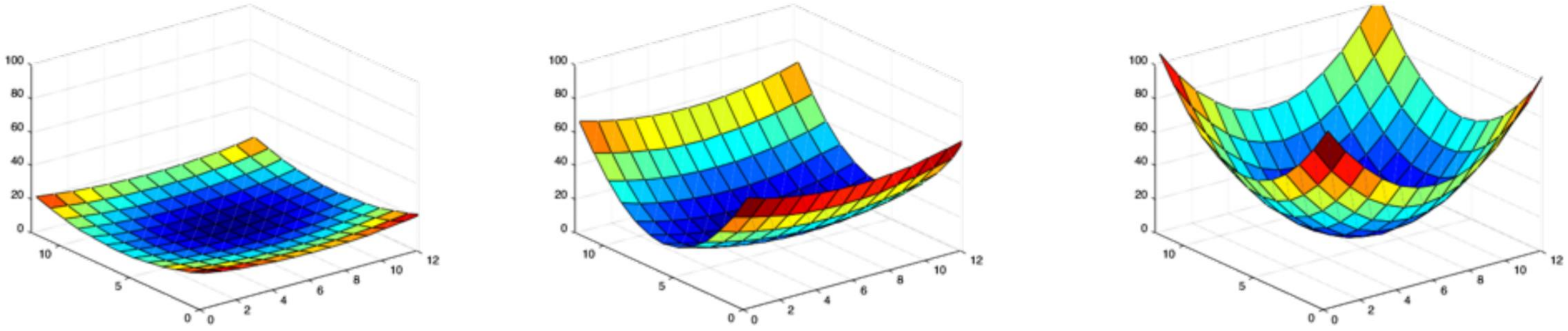
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$



Gradient covariance matrix

Harris Corner Detector

- A quadratic function $$f(\Delta x, \Delta y) \approx \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$



Flat Edge Corner

Idea: if $f(\Delta x, \Delta y)$ is large for all $(\Delta x, \Delta y)$, the patch has a corner

Harris Corner Detector

- Compute the eigenvalues and eigenvectors of M

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

Eigenvalues: find the roots of $$\det(M - \lambda I) = 0$$

Eigenvectors: for each eigenvalue, solve $$(M - \lambda I)\boldsymbol{e} = 0$$

Harris Corner Detector

- Real symmetric matrices
  - All eigenvalues of a real symmetric matrix are real
  - Eigenvectors corresponding to distinct eigenvalues are orthogonal

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

- Since M is symmetric, we have

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

R is a 2D rotation matrix

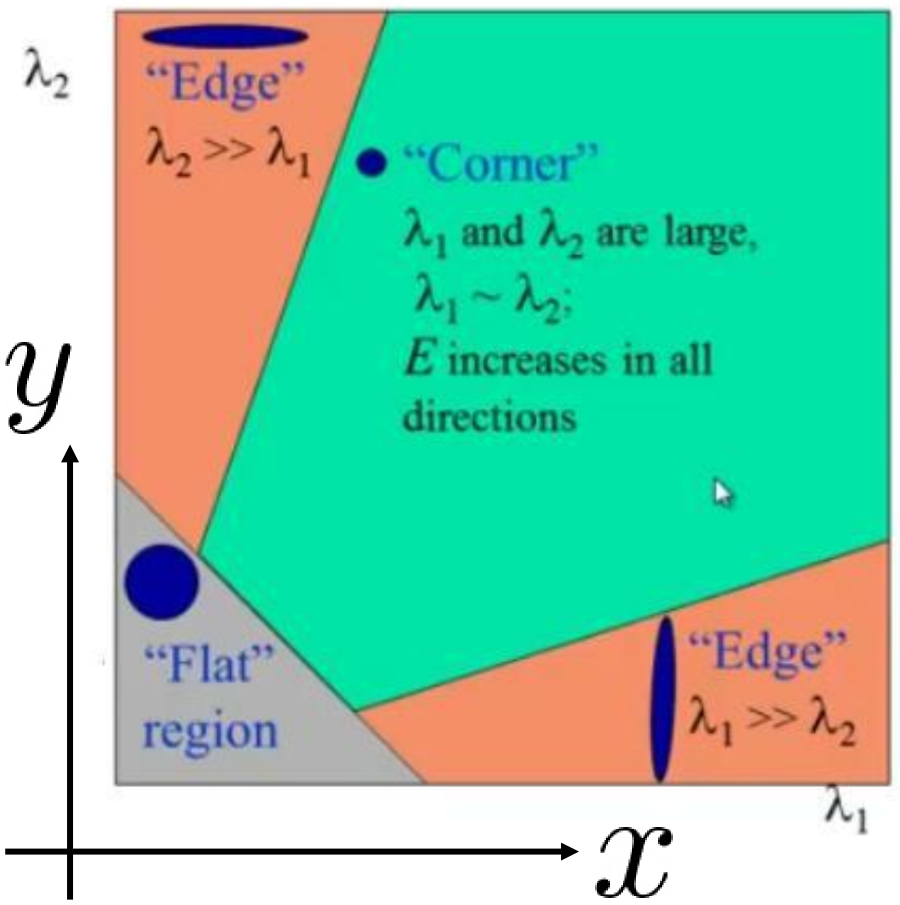
Harris Corner Detector

- Interpreting Eigenvalues

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

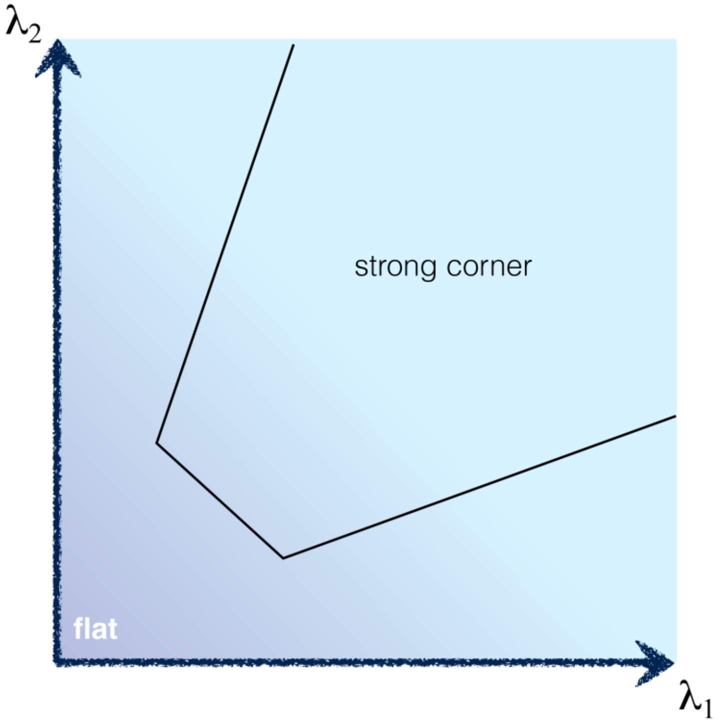
$$f(\Delta x, \Delta y) \approx (\Delta x \quad \Delta y) M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

λ_1 X direction gradient λ_2 Y direction gradient



Harris Corner Detector

- Define a score to detect corners



Option 1   Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Option 2   Harris & Stephens (1988)

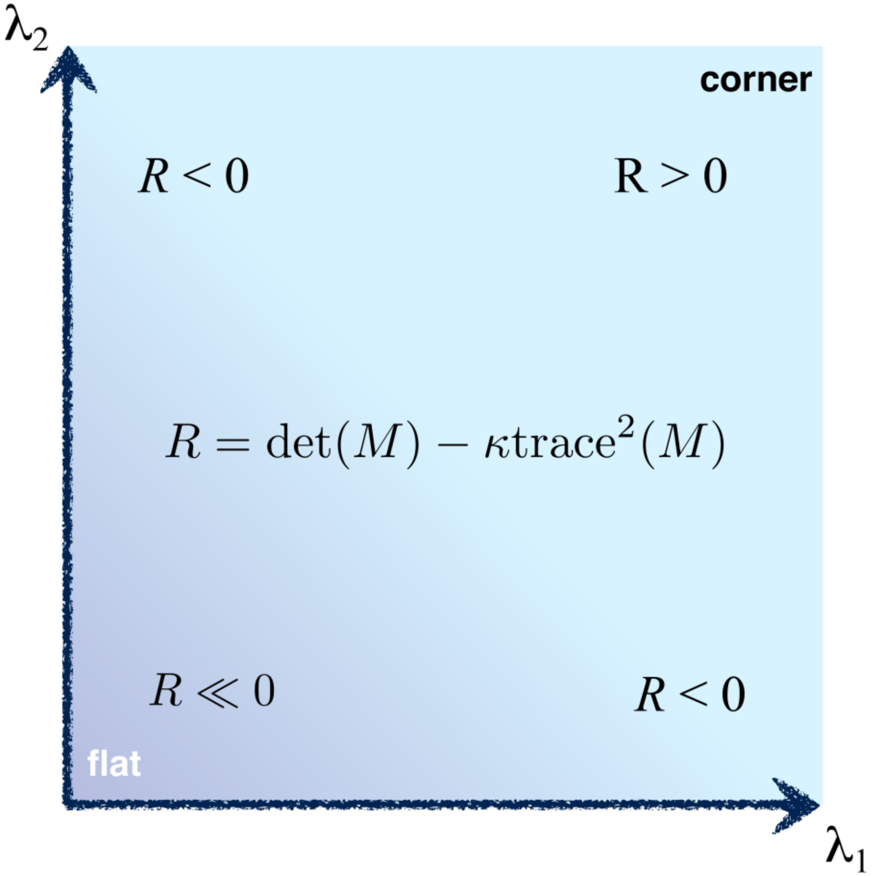
$$R = \lambda_1\lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently…

Harris Corner Detector

- Define a score to detect corners

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$



$$\det M = \lambda_1 \lambda_2$$
$$\text{trace}\, M = \lambda_1 + \lambda_2$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

$$\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$$
$$\operatorname{tr}(\mathbf{P}^{-1}\mathbf{AP}) = \operatorname{tr}(\mathbf{APP}^{-1}) = \operatorname{tr}(\mathbf{A})$$

Harris Corner Detector

$$\begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

Sobel filter

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of products of derivatives at each pixel

Gaussian window

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

Harris Corner Detector

3. Determine the matrix at every pixel

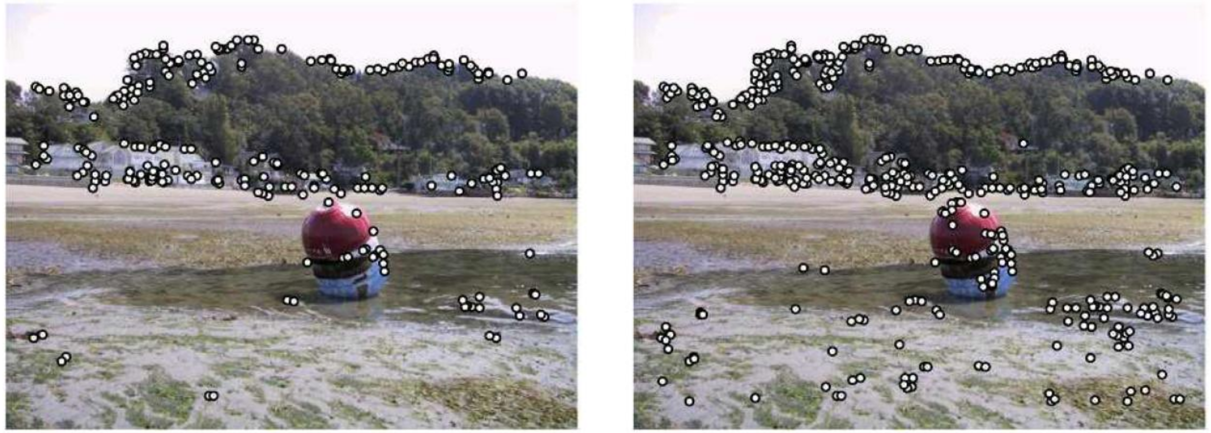
$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

4. Compute the response of the detector at each pixel

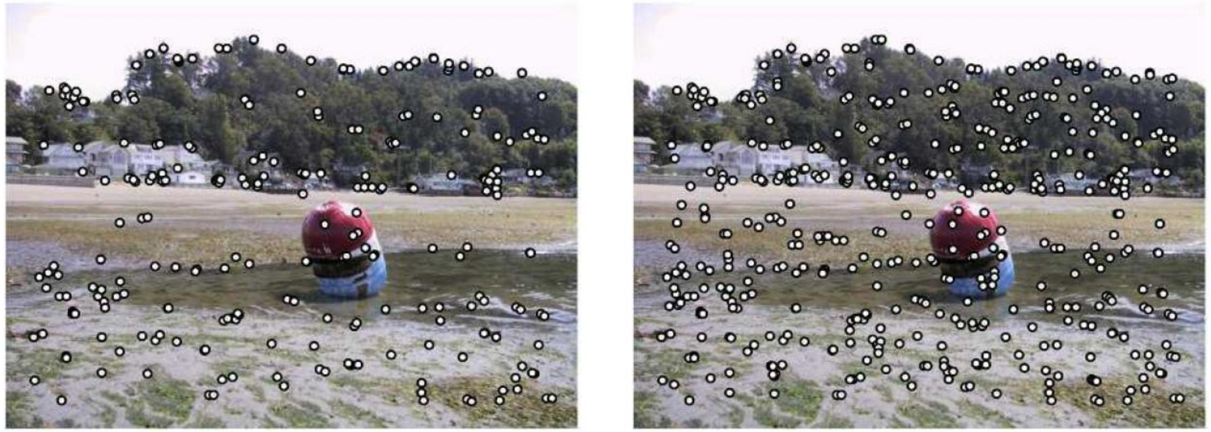
$$R = \det M - k(\mathrm{trace} M)^2$$

5. Threshold on R and perform non-maximum suppression

Non-Maximum Suppression (NMS)

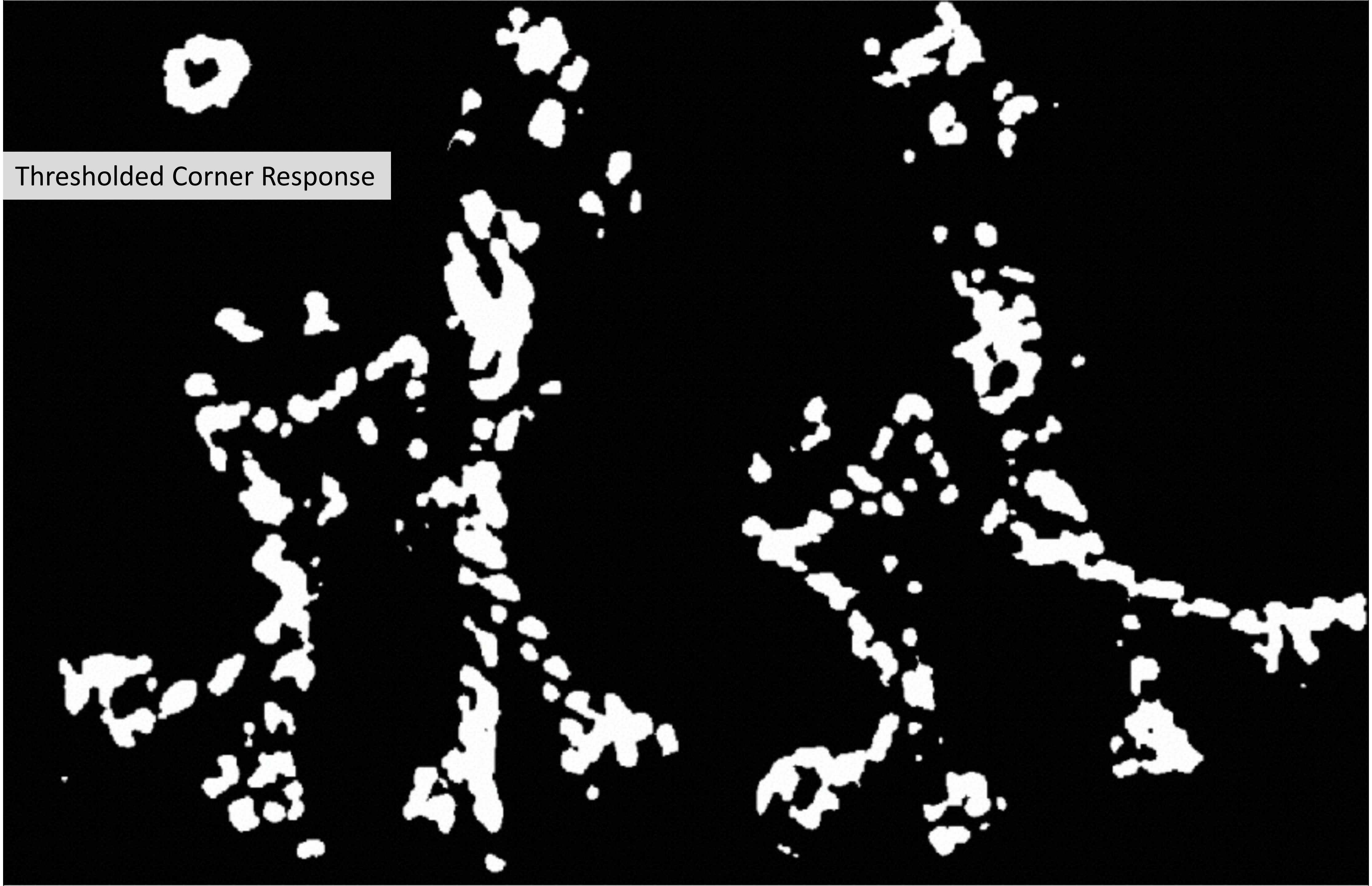


(a) Strongest 250 (b) Strongest 500



(c) ANMS 250, $r = 24$ (d) ANMS 500, $r = 16$

Adaptive non-maximal suppression
Suppression radius r


Thresholded Corner Response


NMS

Further Reading

- Chapter 7.1, Richard Szeliski

- Harris corner detector
  https://en.wikipedia.org/wiki/Harris_corner_detector