



Image Filtering and Convolution

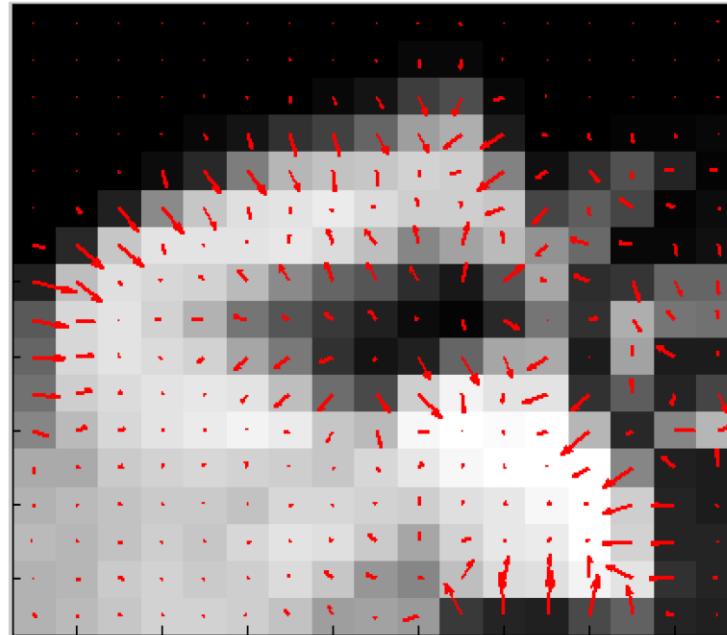
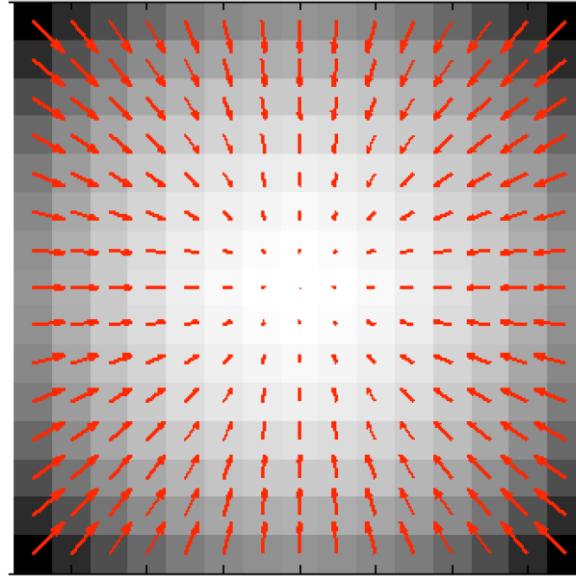
CS 4391 Introduction Computer Vision

Instructor: Yu Xiang

The University of Texas at Dallas

Recall: Image Gradient

- Gradient = Vector of partial derivatives of image
- Gradient direction is the steepest direction to increase the function value



Numerical Derivatives

Finite forward difference

$$\frac{f(x+h) - f(x)}{h} = f'(x) + O(h)$$

Finite backward difference

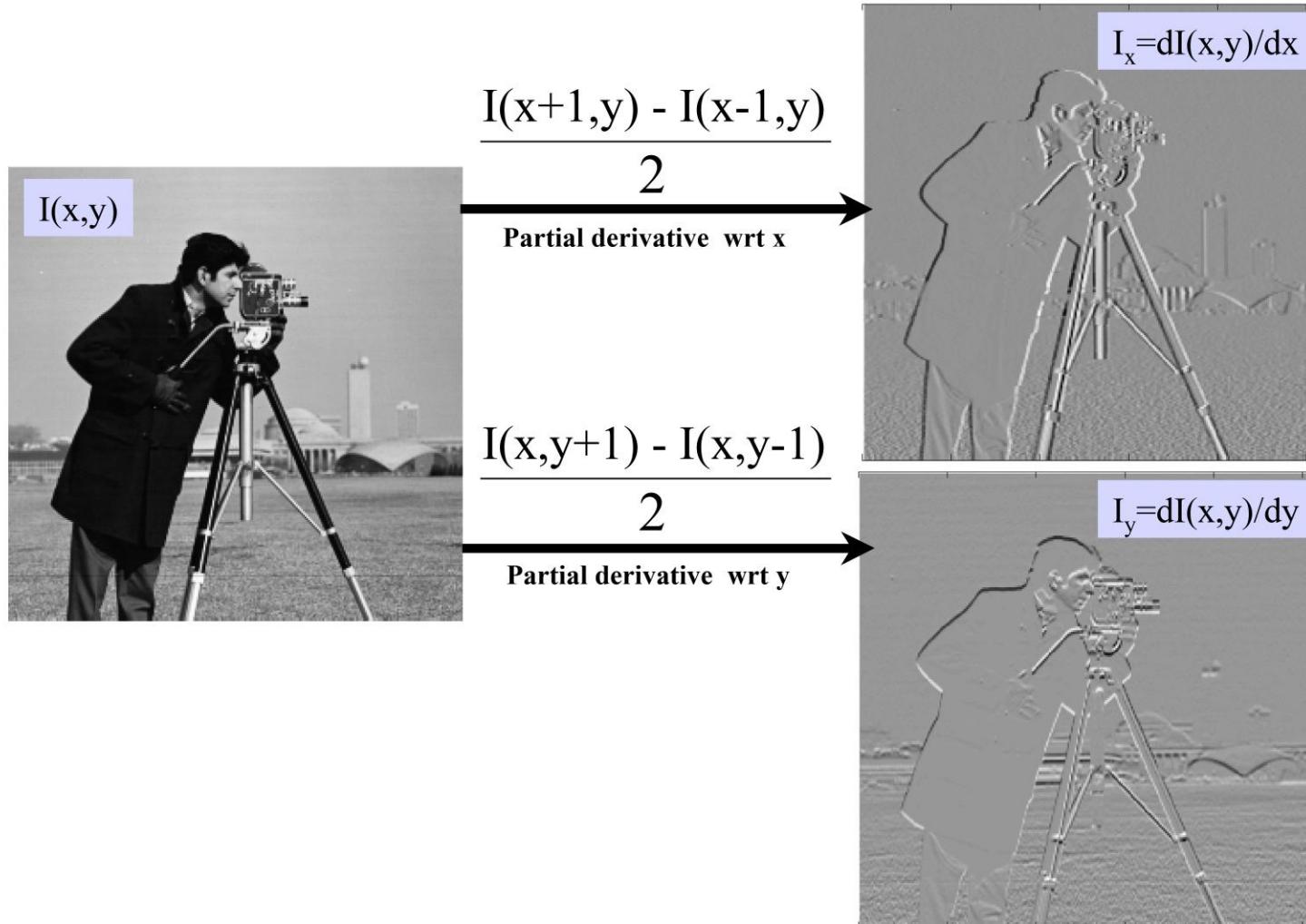
$$\frac{f(x) - f(x-h)}{h} = f'(x) + O(h)$$

Finite central difference

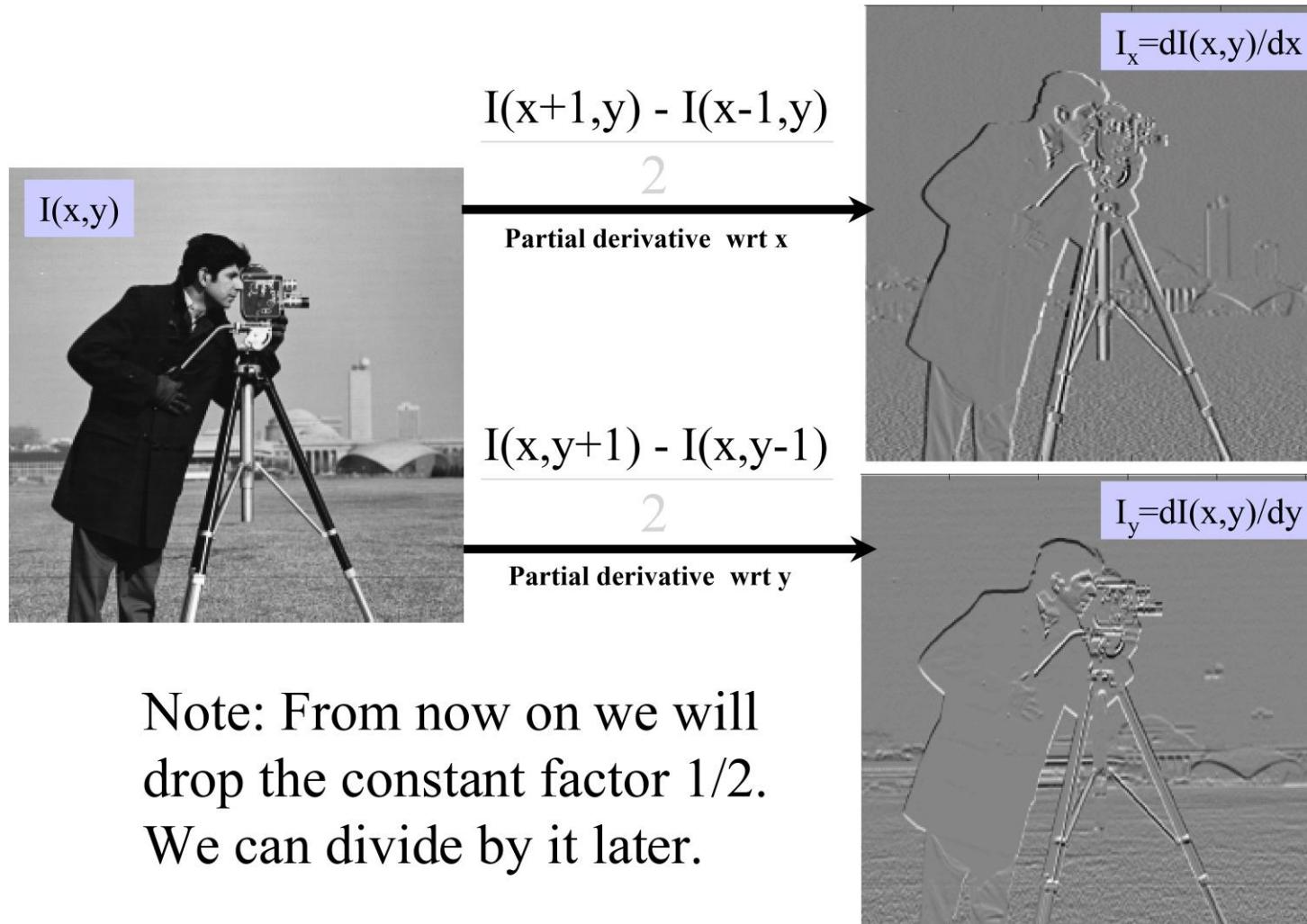
$$\frac{f(x+h) - f(x-h)}{2h} = f'(x) + O(h^2)$$

} **More accurate**

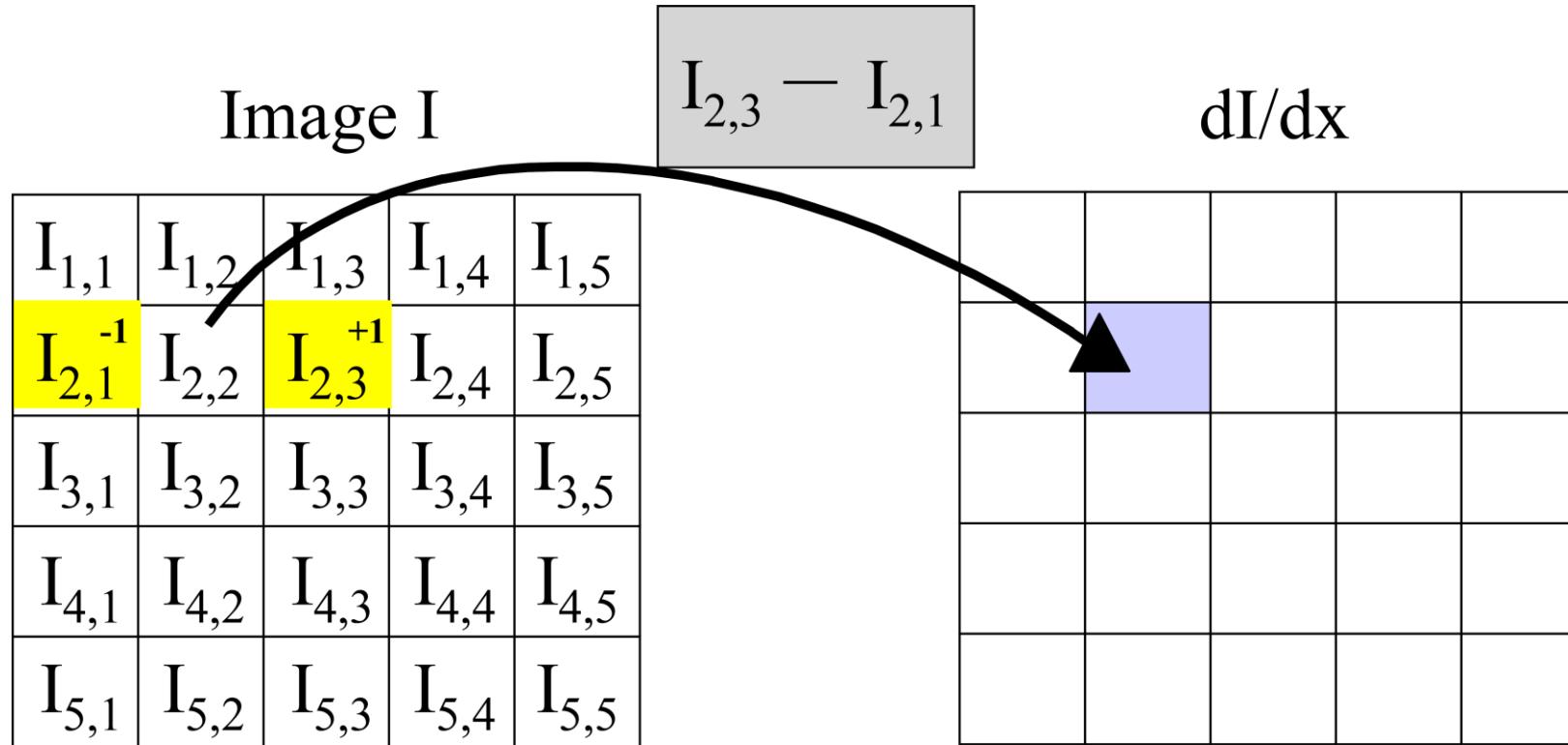
Example: Spatial Image Gradient



Example: Spatial Image Gradient



Example: Spatial Image Gradient



Example: Spatial Image Gradient

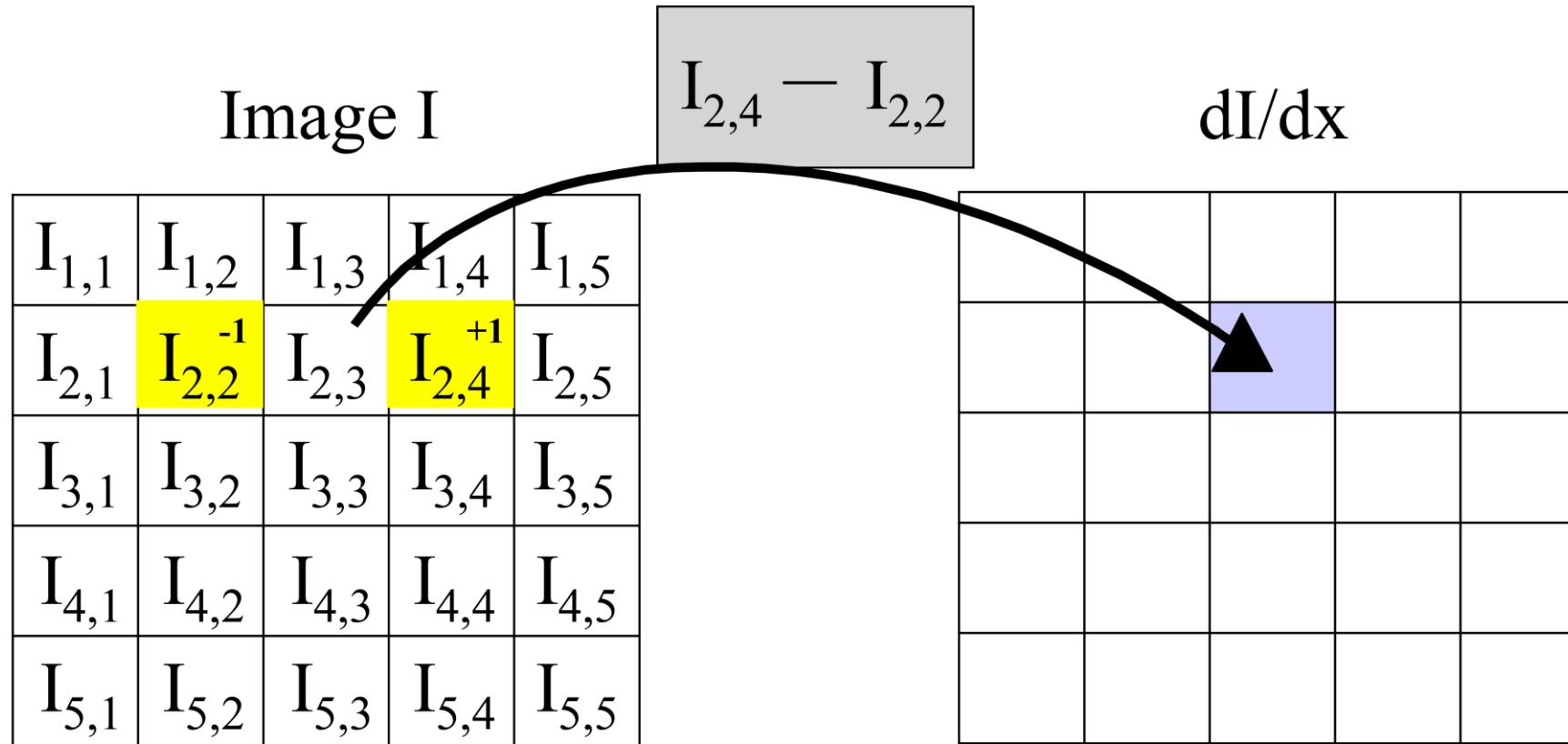


Image Filtering

- Modifying the pixels in an image based on some function of a local neighborhood of pixels

10	5	3
4	5	1
1	1	7

Local image data

Some function
→

		7

Modified image data

Linear Filtering

- Using linear combination of the neighborhood of a pixels (weighted sum)

10	5	3
4	5	1
1	1	7

Local image data

0	0	0
0	0.5	0
0	1	0.5

kernel

	7	

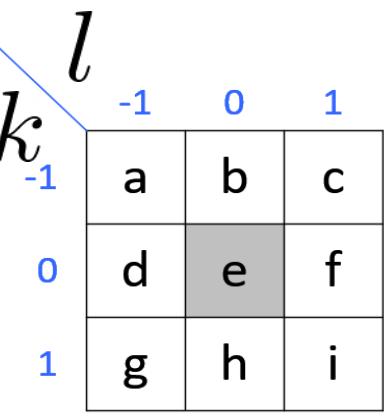
Modified image data

$$10*0+5*0+3*0+4*0+5*.5+1*0+1*0+1*1+7*.5 = 7$$

Linear Filtering

45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120

$f(x,y)$



$h(x,y)$

69	95	116	125	129	132
68	92	110	120	126	132
66	86	104	114	124	132
62	78	94	108	120	129
57	69	83	98	112	124
53	60	71	85	100	114

$g(x,y)$

Correlation $g(i, j) = \sum_{k,l} f(i + k, j + l)h(k, l)$

$$g = f \otimes h$$

Correlation vs. Convolution

- Correlation $g(i, j) = \sum_{k,l} f(i + k, j + l)h(k, l)$

$$g = f \otimes h$$



What is the difference?

- Convolution $g(i, j) = \sum_{k,l} f(i - k, j - l)h(k, l)$

$$g = f * h$$

k	l	-1	0	1
-1	a	b	c	
0	d	e	f	
1	g	h	i	

Filter flipped
vertically and
horizontally

$$h(x, y)$$

0.1	0.1	0.1
0.1	0.2	0.1
0.1	0.1	0.1

Properties of Convolution

Commutative

$$a \star b = b \star a$$

Associative

$$(((a \star b_1) \star b_2) \star b_3) = a \star (b_1 \star b_2 \star b_3)$$

Distributes over addition

$$a \star (b + c) = (a \star b) + (a \star c)$$

Scalars factor out

$$\lambda a \star b = a \star \lambda b = \lambda(a \star b)$$

Derivative Theorem of Convolution

$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

Convolution Example

h

1	-1	-1
1	2	-1
1	1	1

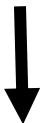
f

2	2	2	3
2	1	3	3
2	2	1	2
1	3	2	2

$h * f$

?	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

Filter flipped vertically
and horizontally



1	1	1
-1	2	1
-1	-1	1

Apply

adapted from C. Rasmussen, U. of Delaware

1	-1	-1
1	2	-1
1	1	1

1	1	1			
-1	2	1	2	2	3
-1	-1	1	2	1	3
	2	1	2	1	2
1	3	2			

5	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

Filter flipped vertically
and horizontally

1	1	1
-1	2	1
-1	-1	1

Apply

$$2*2 + 1*2 + (-1)*2 + 1*1 = 5$$

1	-1	-1
1	2	-1
1	1	1

-

1	1	1	
-1	2	1	2
-1	-1	1	3
2	1	3	3
2	2	1	2
1	3	2	2

5	4	?	?
?	?	?	?
?	?	?	?
?	?	?	?

Filter flipped vertically
and horizontally

1	1	1
-1	2	1
-1	-1	1

Apply

$$2*2+1*2+(-1)*2+1*1 = 5$$

$$-1*2+2*2+1*2-1*2-1*1+1*3=4$$

1	-1	-1
1	2	-1
1	1	1

Filter flipped vertically
and horizontally

1	1	1	2	1		
-1	2	2	1	1	3	3
-1	-1	2	1	2	1	2
1	3	2	1	2	2	2
1	3	2	2	2	2	2

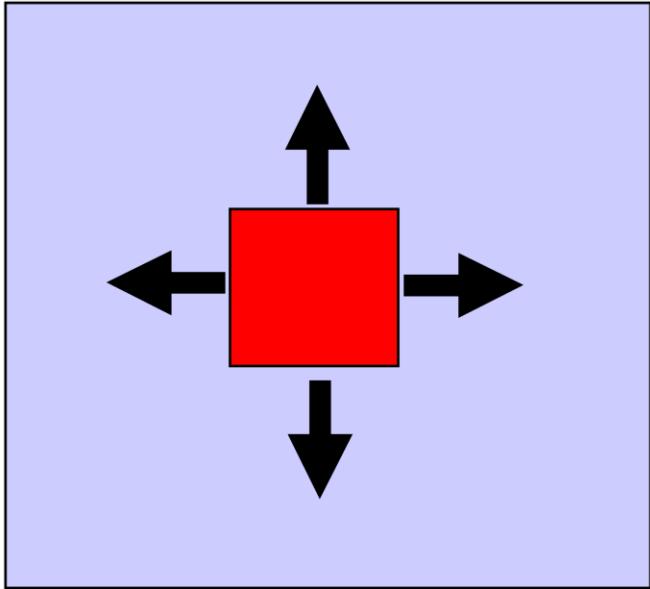
5	4	4	-2
9	?	?	?
?	?	?	?
?	?	?	?

1	1	1
-1	2	1
-1	-1	1

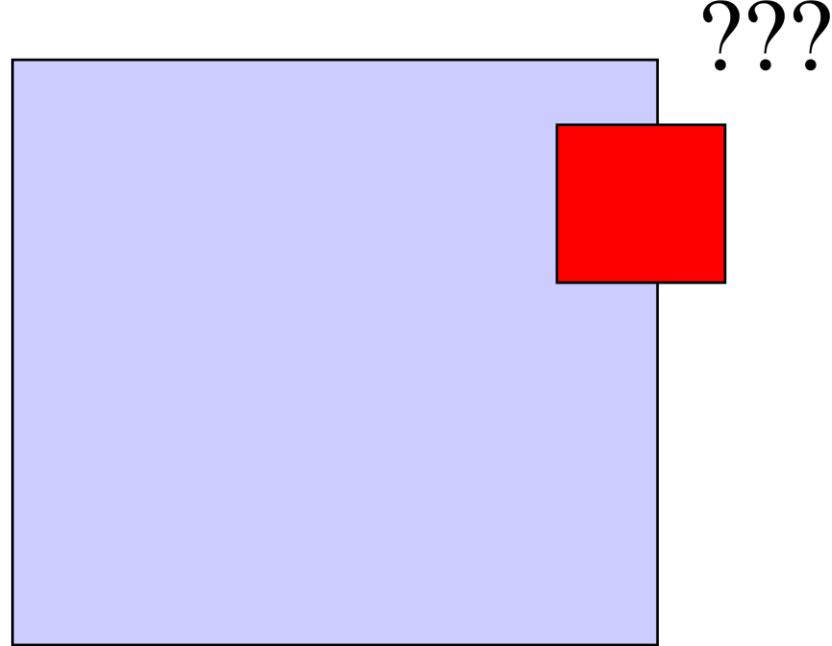
Apply

$$\begin{aligned}
 & 2*2 + 1*2 + (-1)*2 + 1*1 = 5 \\
 & -1*2 + 2*2 + 1*2 - 1*2 - 1*1 + 1*3 = 4 \\
 & -1*2 + 2*2 + 1*3 - 1*1 - 1*3 + 1*3 = 4 \\
 & -1*2 + 2*3 - 1*3 - 1*3 = -2 \\
 & 1*2 + 1*2 + 2*2 + 1*1 - 1*2 + 1*2 = 9
 \end{aligned}$$

Border Handling



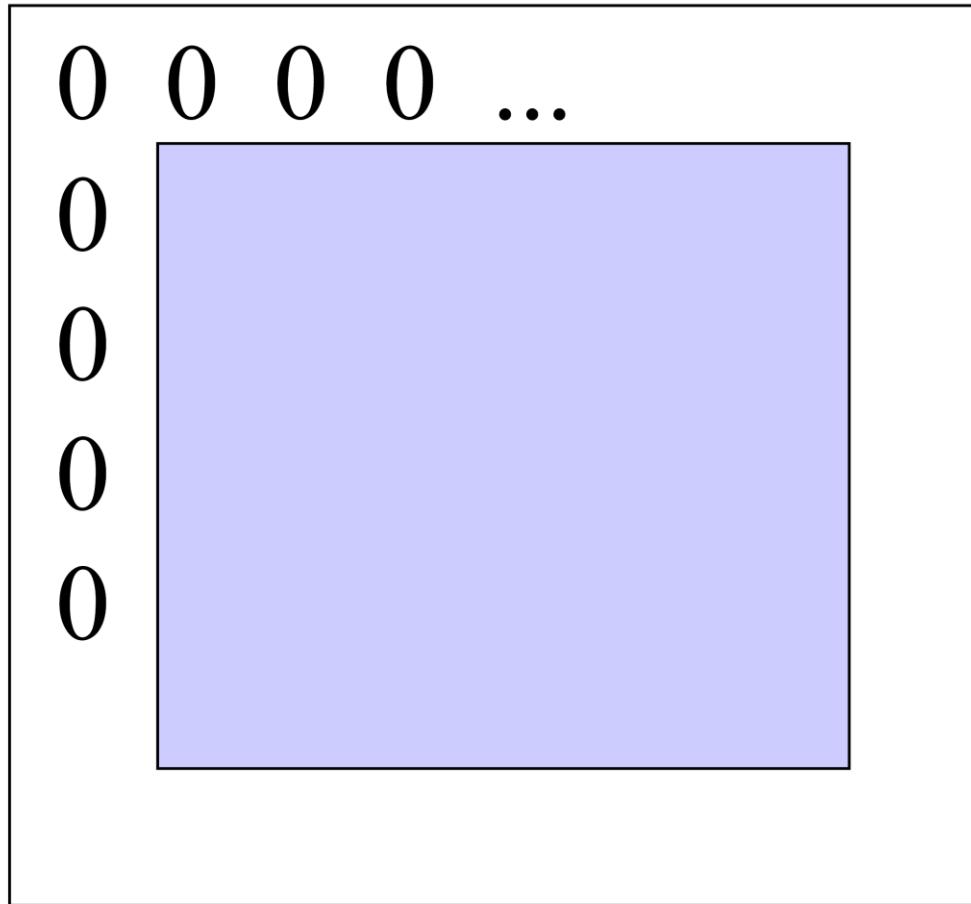
for interior pixels where
there is full overlap, we
know what to do.



but what values do we
use for pixels that are
“off the image” ?

Border Handling

- Zero padding



Border Handling

- Replication

		1 2 3																																						
	1s	1 2 3	3s																																					
		1 2 3																																						
<table border="1"><tbody><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>5</td><td>6</td></tr><tr><td>7</td><td>8</td><td>9</td></tr></tbody></table>	1	2	3	4	5	6	7	8	9	<table border="1"><tbody><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>4</td><td>4</td><td>4</td></tr><tr><td>7</td><td>7</td><td>7</td></tr></tbody></table>	1	1	1	4	4	4	7	7	7	<table border="1"><tbody><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>5</td><td>6</td></tr><tr><td>7</td><td>8</td><td>9</td></tr></tbody></table>	1	2	3	4	5	6	7	8	9	<table border="1"><tbody><tr><td>3</td><td>3</td><td>3</td></tr><tr><td>6</td><td>6</td><td>6</td></tr><tr><td>9</td><td>9</td><td>9</td></tr></tbody></table>	3	3	3	6	6	6	9	9	9	
1	2	3																																						
4	5	6																																						
7	8	9																																						
1	1	1																																						
4	4	4																																						
7	7	7																																						
1	2	3																																						
4	5	6																																						
7	8	9																																						
3	3	3																																						
6	6	6																																						
9	9	9																																						
		7 8 9																																						
	7s	7 8 9	9s																																					
		7 8 9																																						

Border Handling

- Reflection

1	2	3
4	5	6
7	8	9

9	8	7	7	8	9	9	8	7
6	5	4	4	5	6	6	5	4
3	2	1	1	2	3	3	2	1
3	2	1	1	2	3	3	2	1
6	5	4	4	5	6	6	5	4
9	8	7	7	8	9	9	8	7
9	8	7	7	8	9	9	8	7
6	5	4	4	5	6	6	5	4
3	2	1	1	2	3	3	2	1

Back to Image Gradient

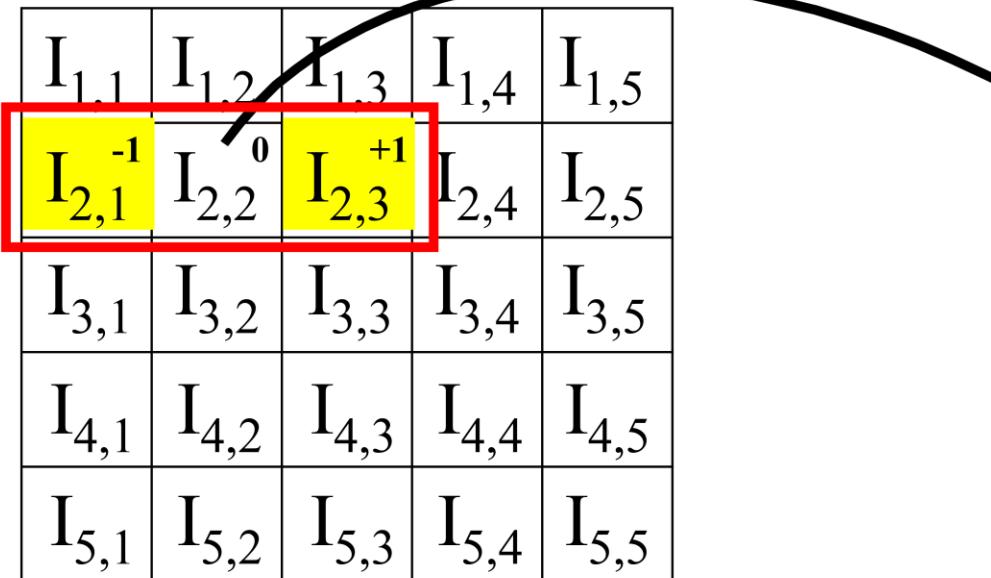
$$(-1) * I_{2,1} + (0) * I_{2,2} + (+1) * I_{2,3}$$

Image I

$I_{1,1}$	$I_{1,2}$	$I_{1,3}$	$I_{1,4}$	$I_{1,5}$
$I_{2,1}^{-1}$	$I_{2,2}^0$	$I_{2,3}^{+1}$	$I_{2,4}$	$I_{2,5}$
$I_{3,1}$	$I_{3,2}$	$I_{3,3}$	$I_{3,4}$	$I_{3,5}$
$I_{4,1}$	$I_{4,2}$	$I_{4,3}$	$I_{4,4}$	$I_{4,5}$
$I_{5,1}$	$I_{5,2}$	$I_{5,3}$	$I_{5,4}$	$I_{5,5}$

$$I_{2,3} - I_{2,1}$$

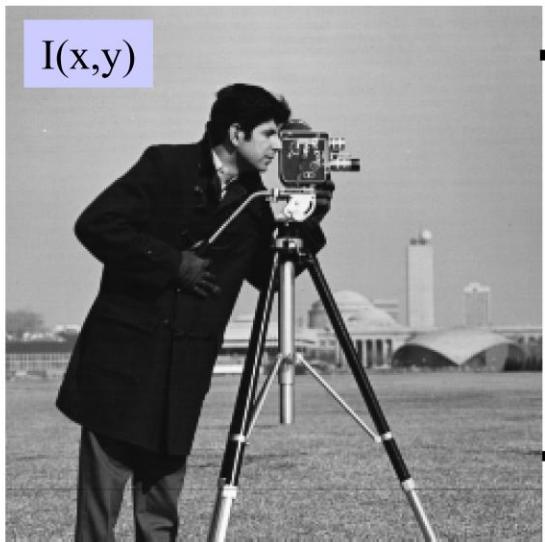
$$dI/dx$$

| | | | | |
| --- | --- | --- | --- | --- |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

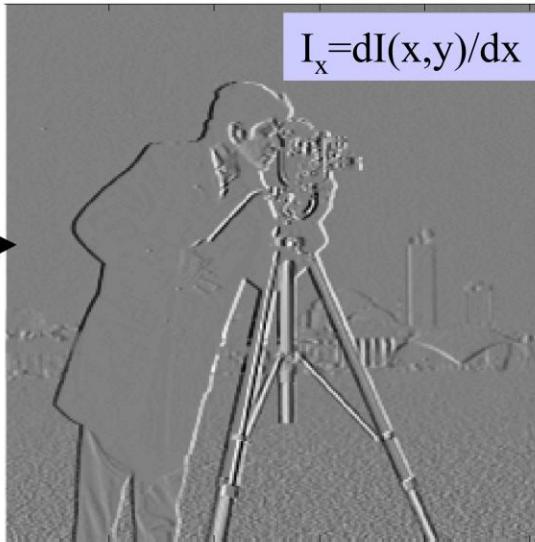
What filter is this?

Back to Image Gradient

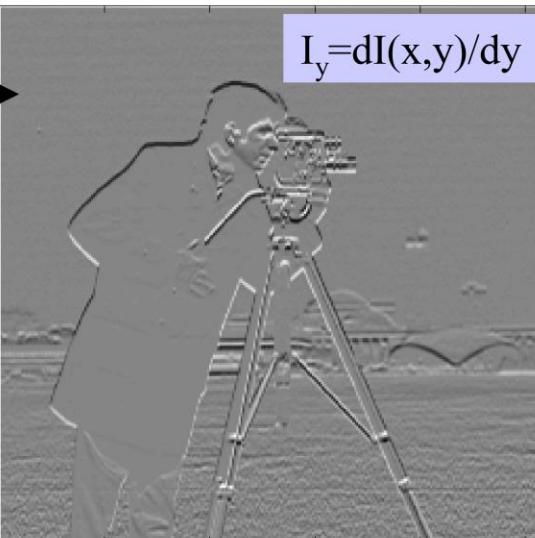
$$I_x = [1 \ 0 \ -1] * I$$



Partial derivative wrt x



Partial derivative wrt y



Note that there
is a difference between
convolving with a $1 \times n$ row
filter and an $n \times 1$ col filter.

$$I_y = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * I$$

Further Reading

- Chapter 3.2, Richard Szeliski
- Carlo Tomasi, Image Correlation, Convolution and Filtering,
<https://courses.cs.duke.edu/fall15/cps274/notes/convolution-filtering.pdf>