Intensity Surfaces and Gradients

CS 4391 Introduction Computer Vision

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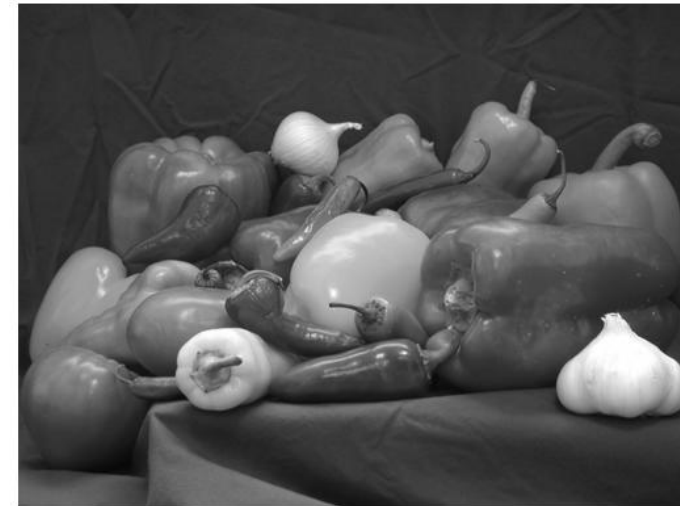
Some slides of this lecture are courtesy Robert Collins (PSU)

Image Data

width

height

$$H \times W \times 3$$

$$H \times W$$

Grayscale
[0, 255]

RGB color space
[0, 255]

0.2989 * R + 0.5870 * G + 0.1140 * B

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OpenCV for Images

```python
import cv2

# Load a color image. By default, it is loaded in BGR format.
img_color = cv2.imread('path/to/your/image.jpg')
```

```python
# Convert the BGR image to grayscale
img_gray = cv2.cvtColor(img_color, cv2.COLOR_BGR2GRAY)
```

```python
# Display the images
cv2.imshow('Original Color', img_color)
cv2.imshow('Grayscale', img_gray)
cv2.waitKey(0)
cv2.destroyAllWindows()

# Save the grayscale image
cv2.imwrite('path/to/save/grayscale_image.jpg', img_gray)
```
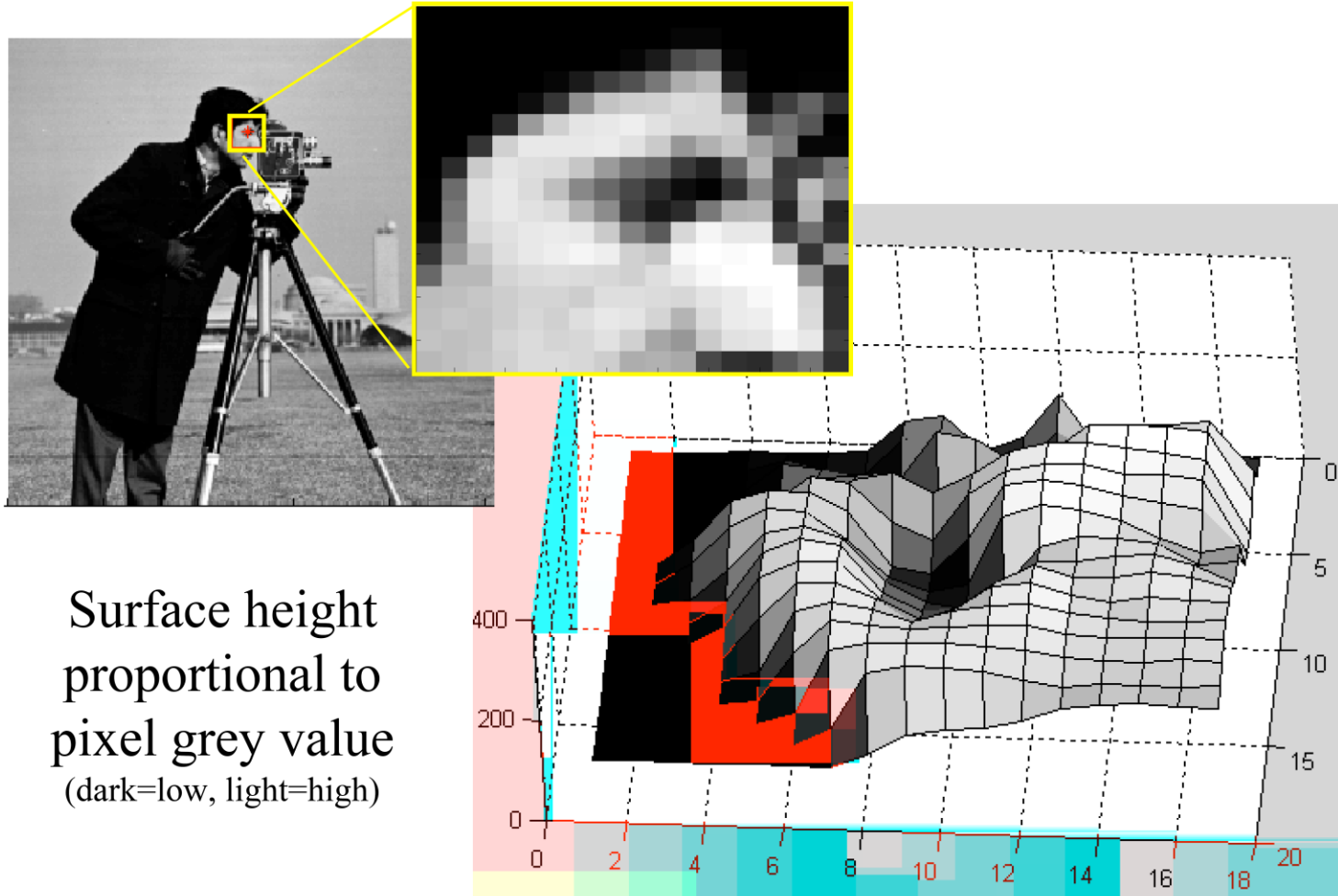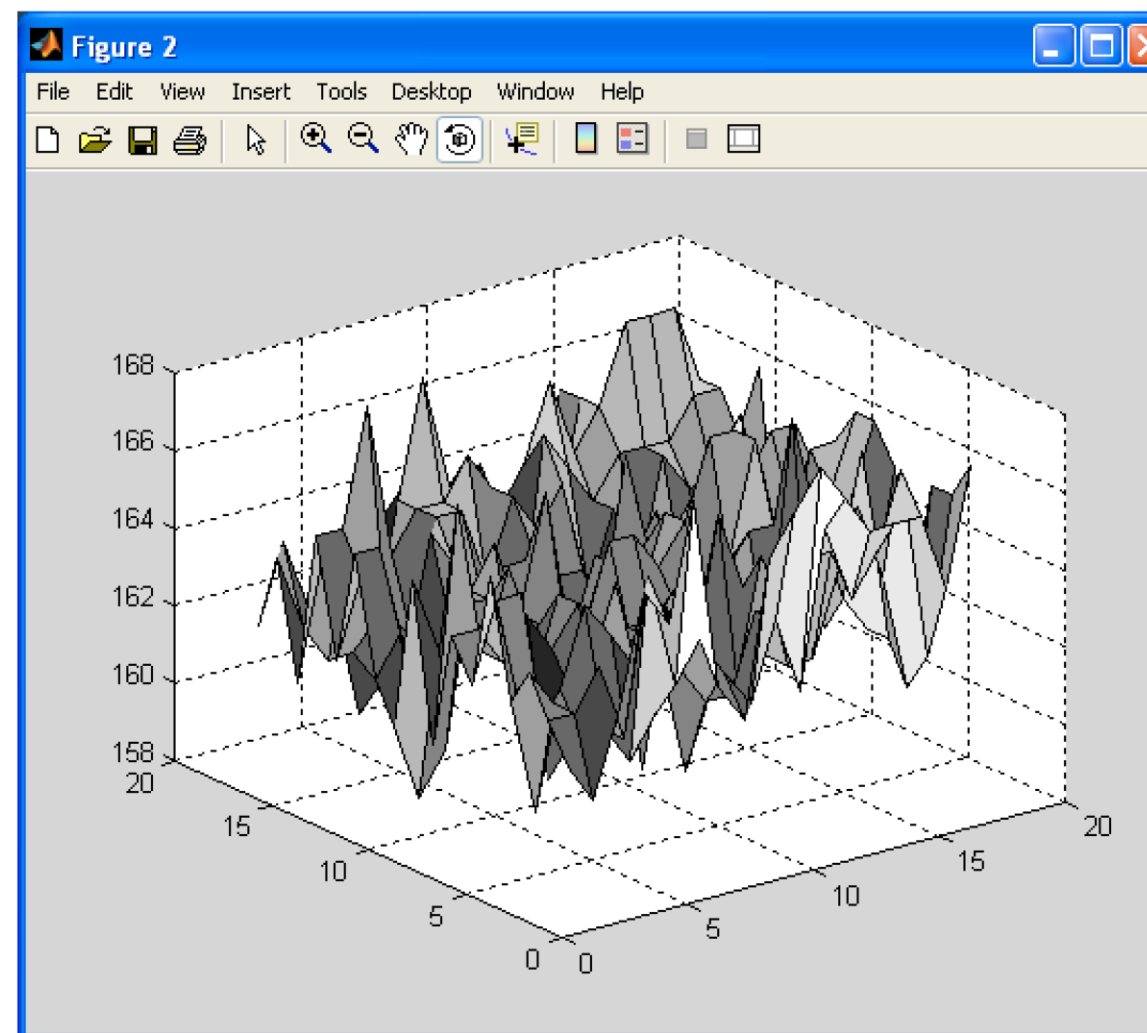
Images as Surfaces

Surface height
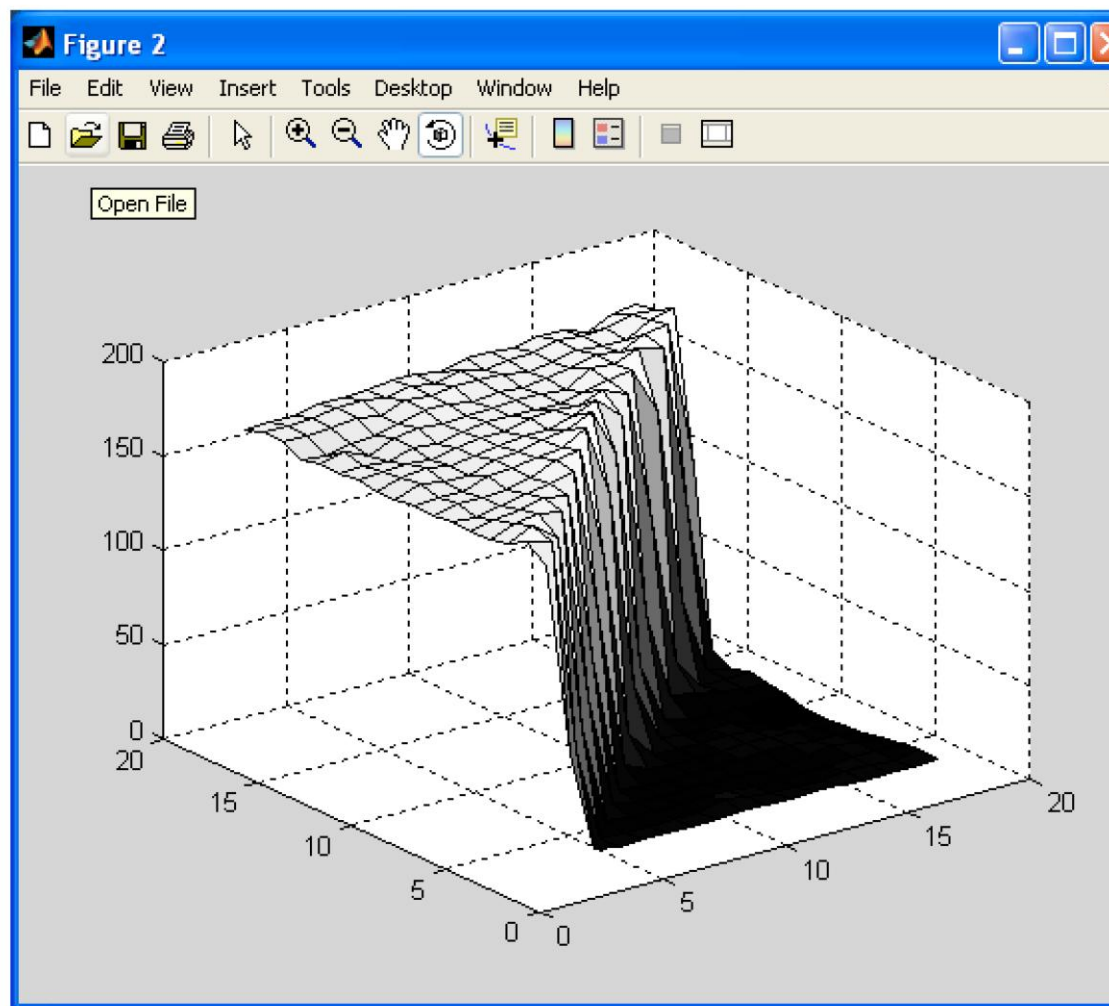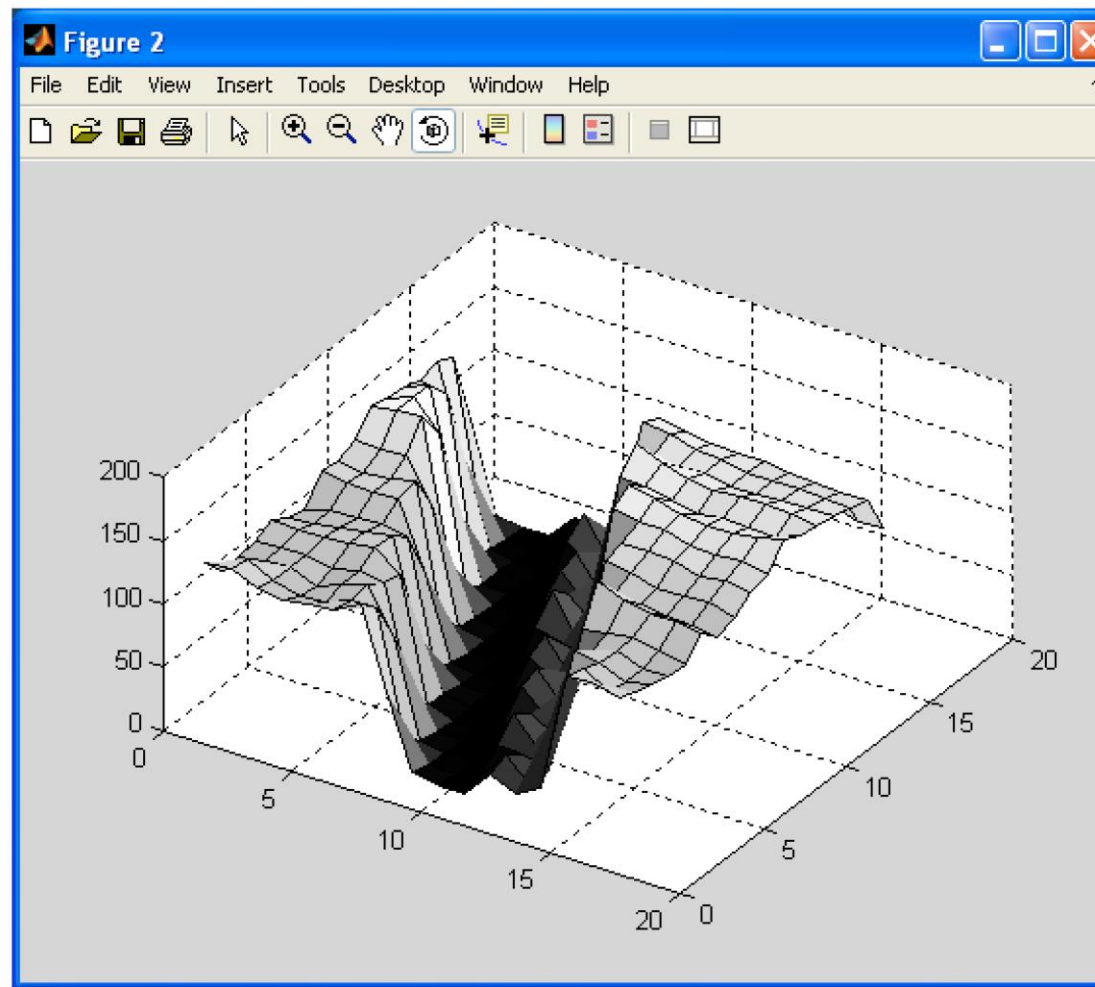proportional to
pixel grey value
(dark=low, light=high)

Examples

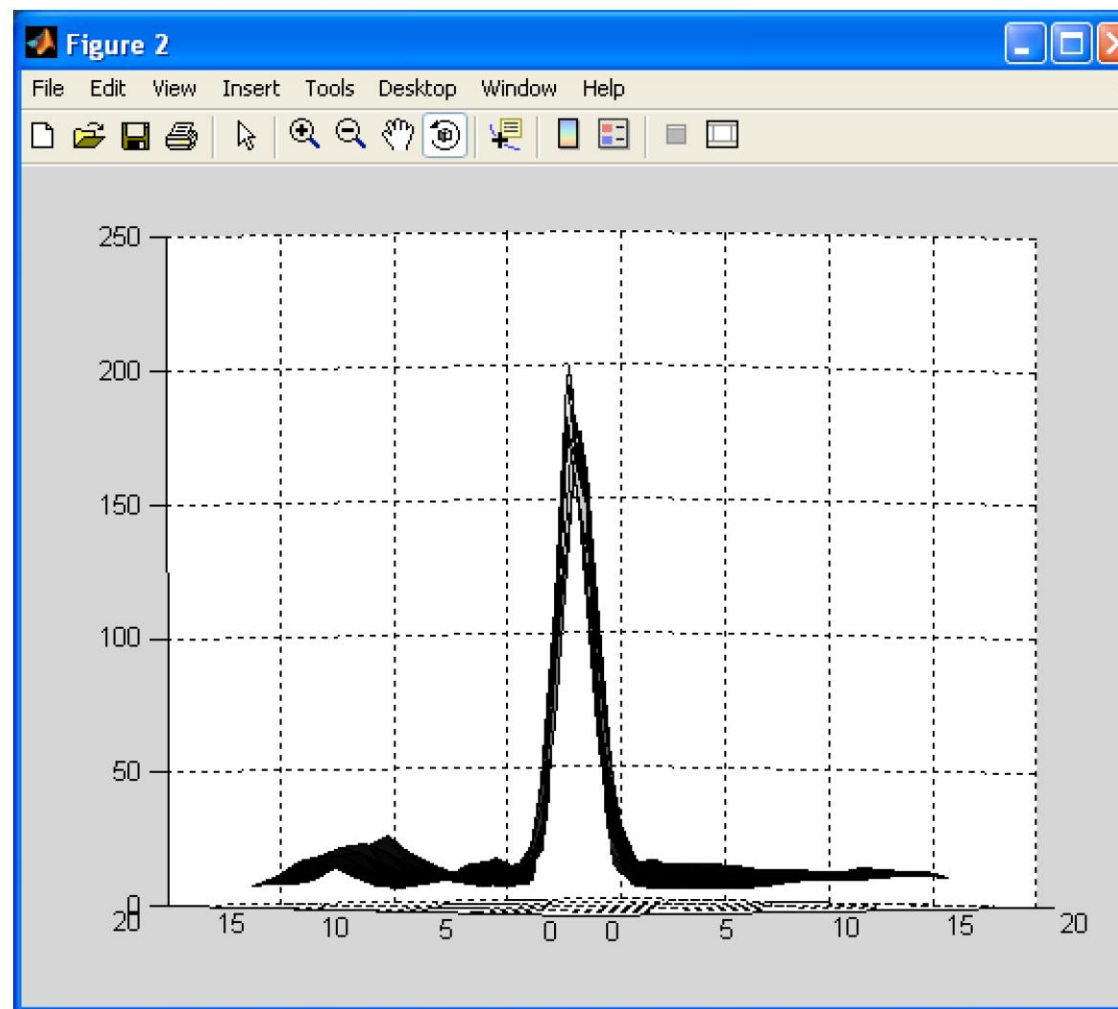
Mean = 164   Std = 1.8

Examples

Examples

Examples

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Examples

Mean = 111   Std = 15.4

Examples

Images as Functions

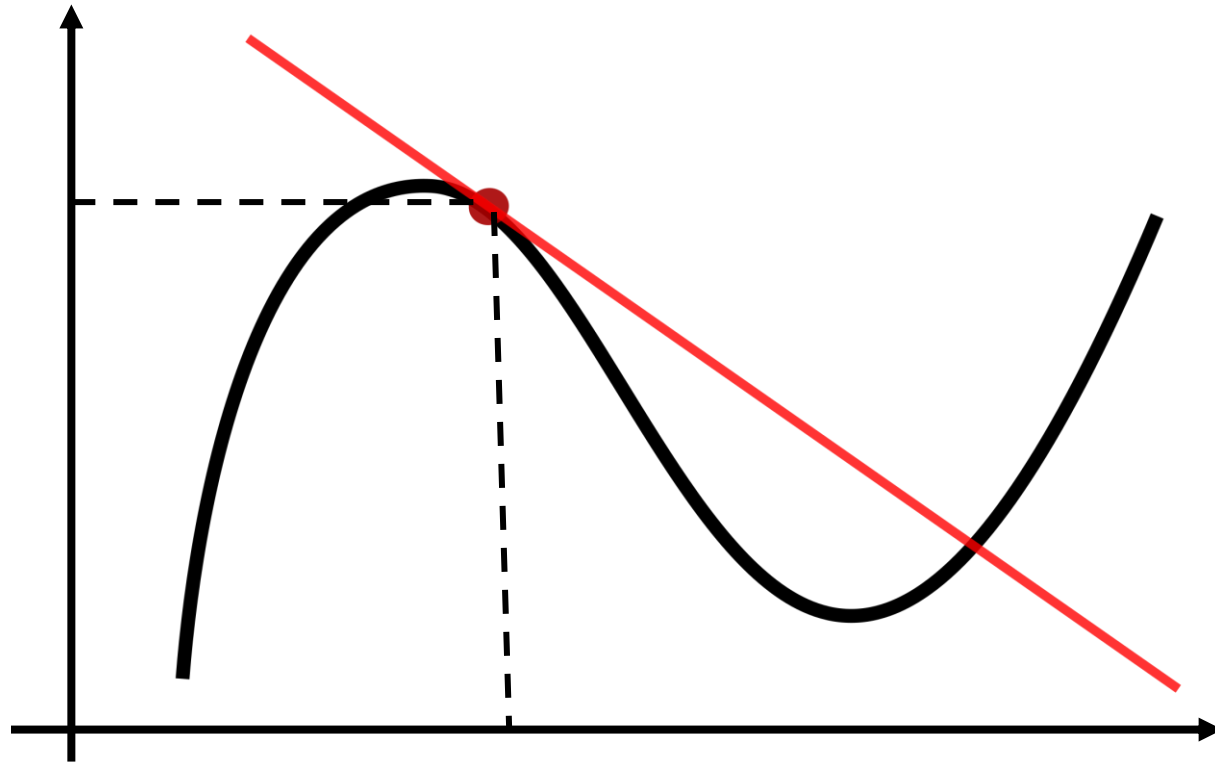| 45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120

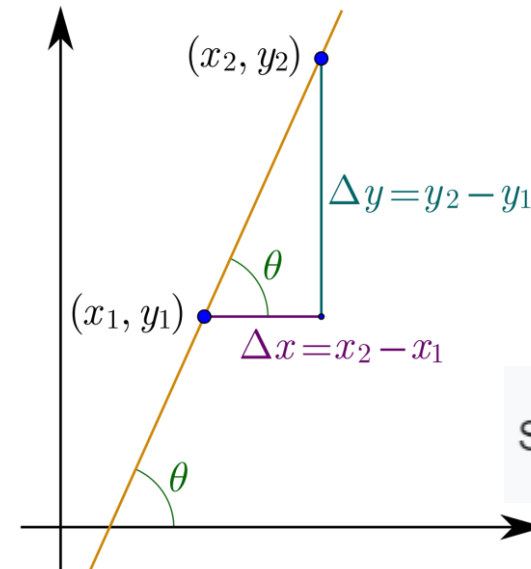Function $I(\mathbf{x})\; f(\mathbf{x})$

$I(x,y)\; f(x,y)$

Derivative

- Derivative of a 1D function
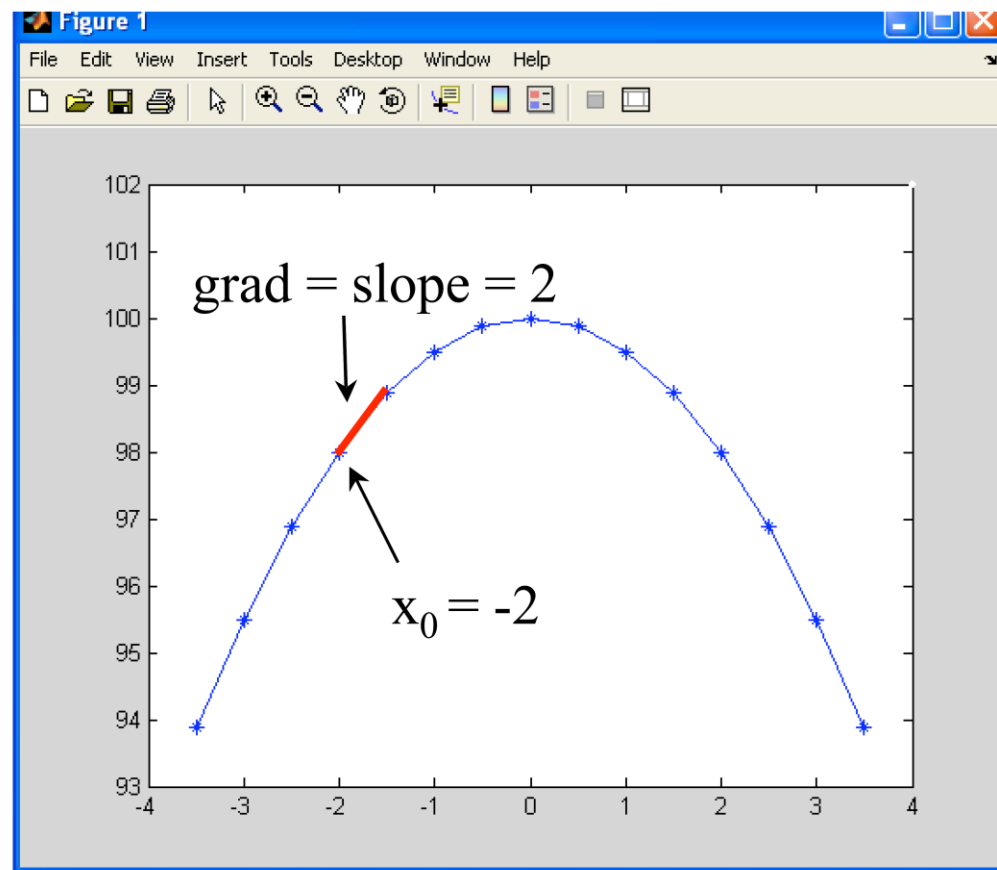
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The [slope](#) of the tangent line is equal to the derivative of the function at the marked point.

Slope: $m = \dfrac{\Delta y}{\Delta x} = \tan(\theta)$

Derivative

$$f(x) = 100 - 0.5 * x\text{^}2 \qquad df(x)/dx = -x$$

Partial Derivative

- Multivariable function

$$\frac{\partial}{\partial x_i} f(\mathbf{a}) = \lim_{h \to 0} \frac{f(a_1, \ldots, a_{i-1}, a_i + h, a_{i+1}, \ldots, a_n) - f(a_1, \ldots, a_i, \ldots, a_n)}{h}$$

$$= \lim_{h \to 0} \frac{f(\mathbf{a} + h\mathbf{e_i}) - f(\mathbf{a})}{h}.$$

- Example $\quad f(x, y) = x^2 + xy + y^2$

- Total differential $\quad dy = \frac{\partial y}{\partial x_1} dx_1 + \cdots + \frac{\partial y}{\partial x_n} dx_n$

Gradient

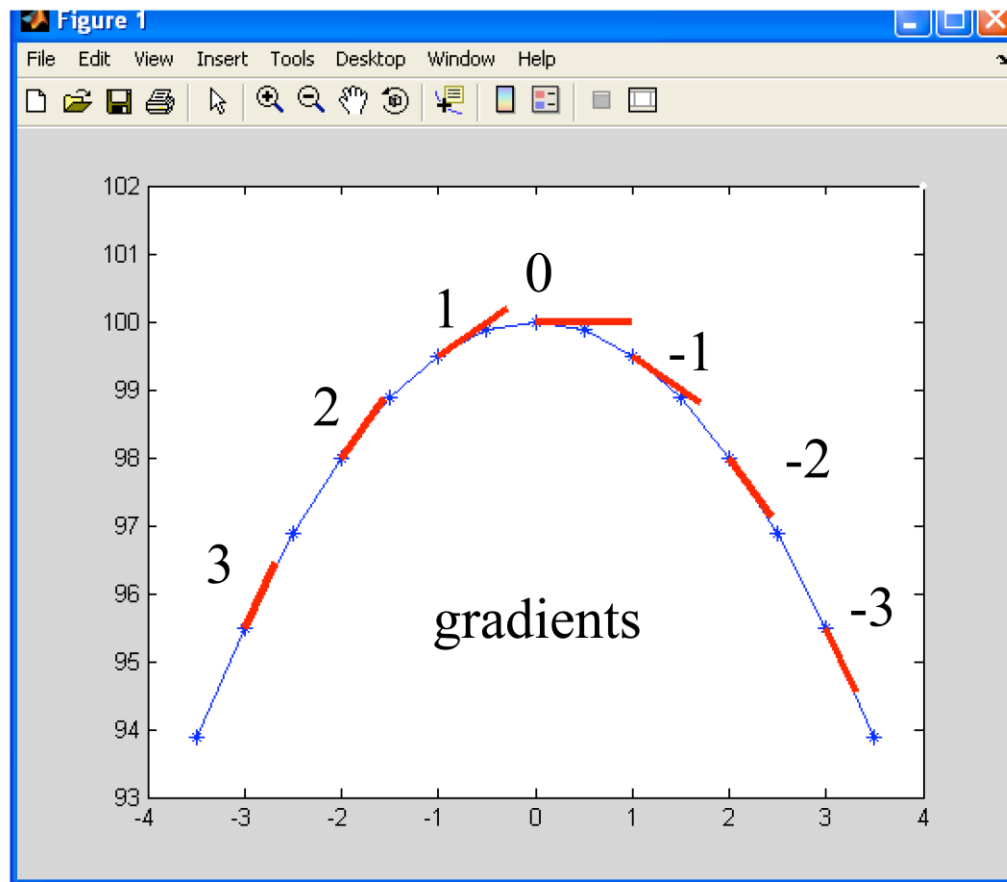- For a function $f: \mathbb{R}^n \to \mathbb{R}$, gradient at a point $p = (x_1, \ldots, x_n)$

$$\nabla f(p) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(p) \\ \vdots \\ \frac{\partial f}{\partial x_n}(p) \end{bmatrix}$$

- Gradient vs. total differential

$$df_p = \begin{bmatrix} \frac{\partial f}{\partial x_1}(p) & \cdots & \frac{\partial f}{\partial x_n}(p) \end{bmatrix}$$
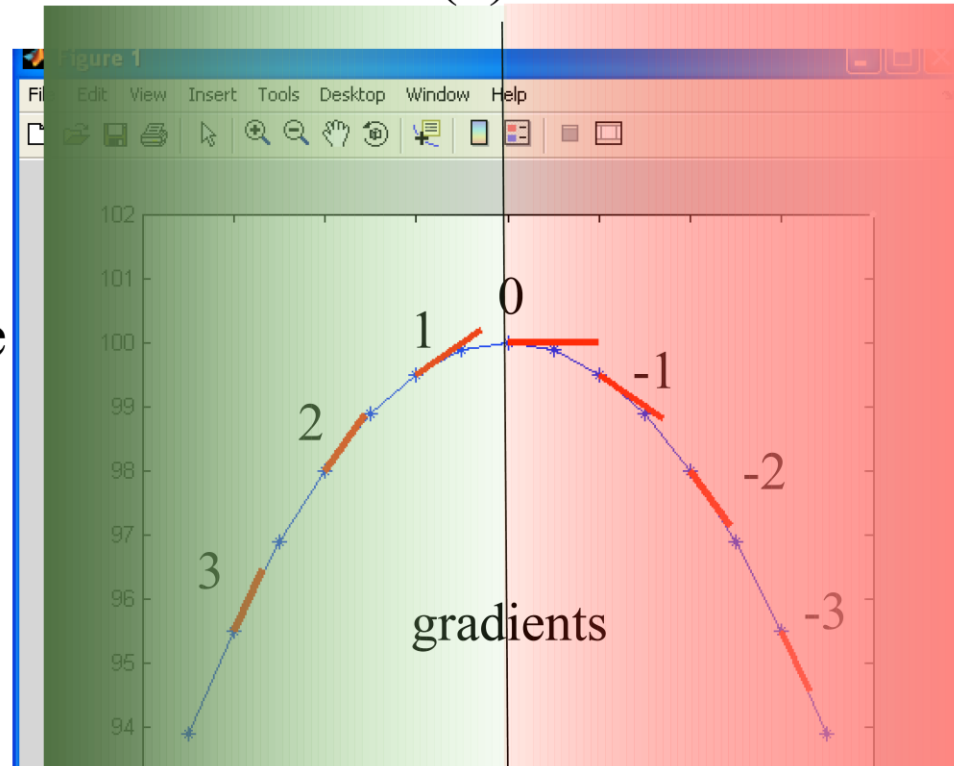
1D Gradient

$$f(x) = 100 - 0.5 * x^2 \qquad df(x)/dx = -x$$

1D Gradient

$f(x) = 100 - 0.5 * x^2$ $df(x)/dx = -x$

Gradients on this side of peak are positive

Gradients on this side of peak are negative

Note: Sign of gradient at point tells you what direction to go to travel "uphill"
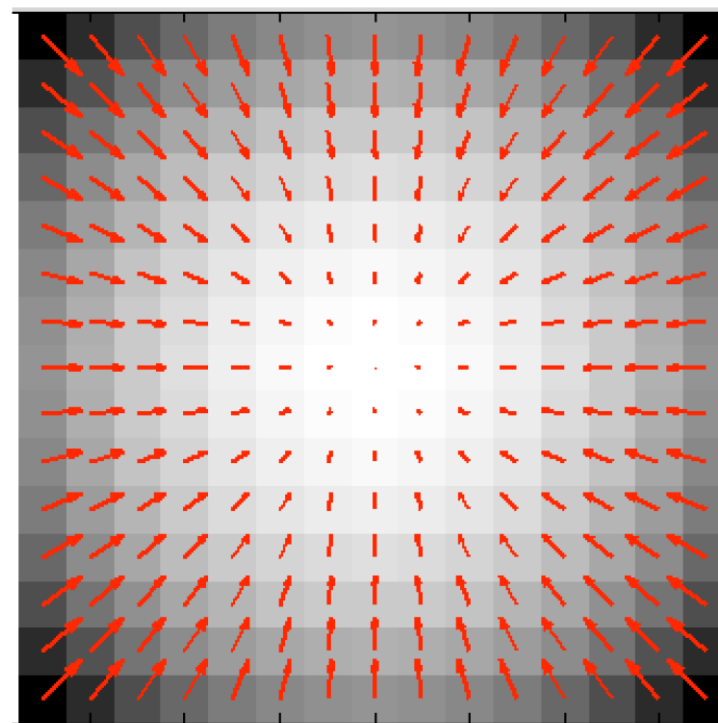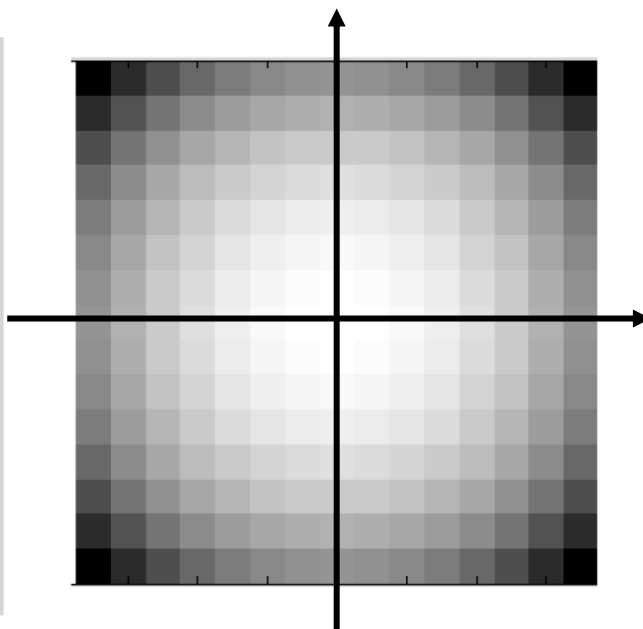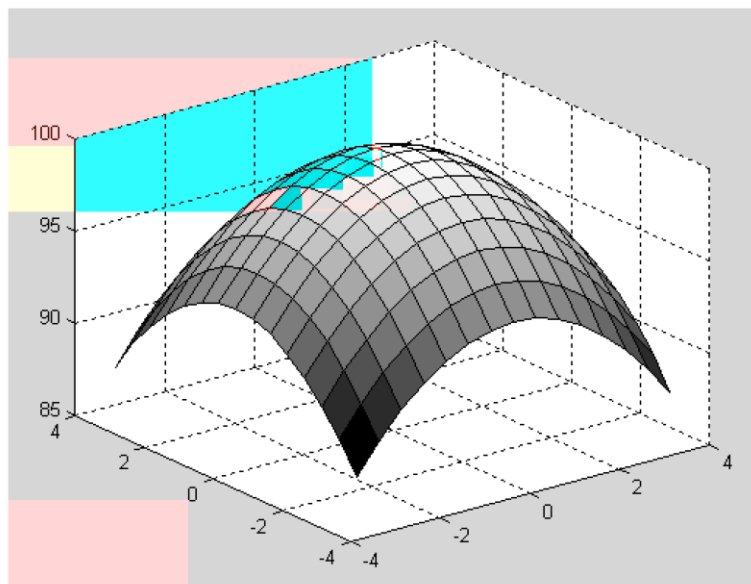
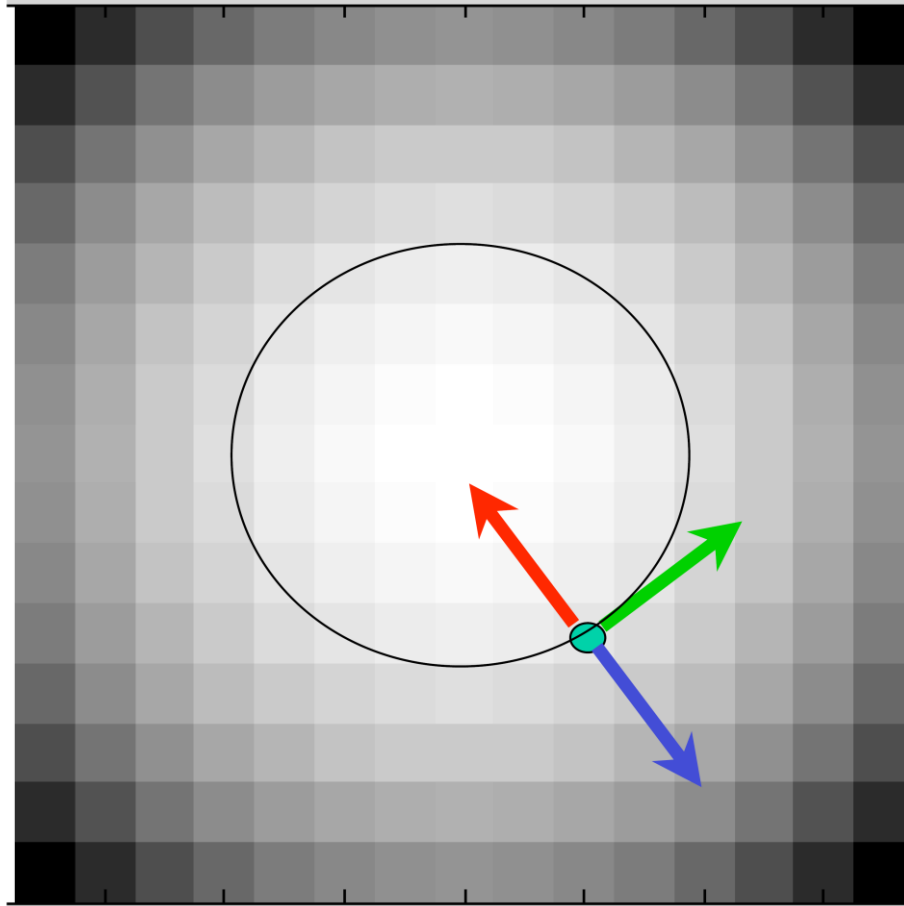# 2D Gradient

$$f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2$$

$$df(x,y)/dx = -x \qquad df(x,y)/dy = -y$$

$$\text{Gradient} = [df(x,y)/dx, \ df(x,y)/dy] = [-x, -y]$$

The gradient indicates the direction of steepest ascent.

2D Gradient

Let $g=[g_x, g_y]$ be the gradient vector at point/pixel (x_0, y_0)

Vector g points uphill
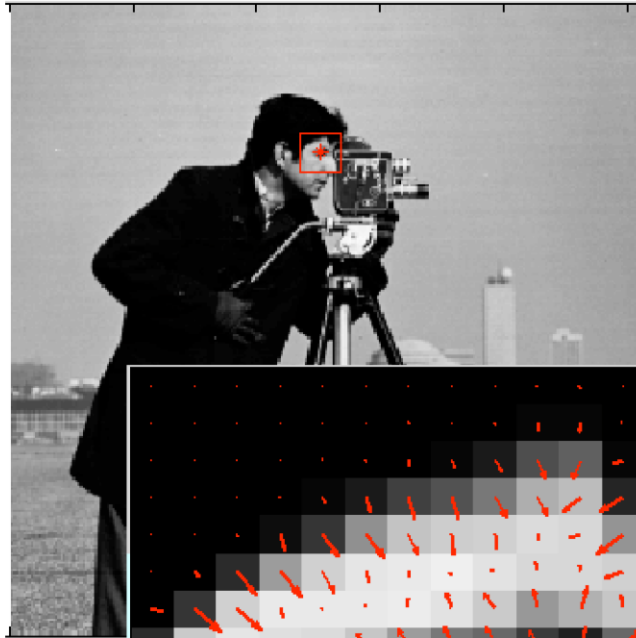(direction of steepest ascent)

Vector - g points downhill
(direction of steepest descent)

Vector $[g_y, -g_x]$ is perpendicular, and denotes direction of constant elevation. i.e. normal to contour line passing through point (x_0, y_0)

Image Gradient

The same is true of 2D image gradients.

The underlying function is numerical (tabulated) rather than algebraic. So need numerical derivatives.
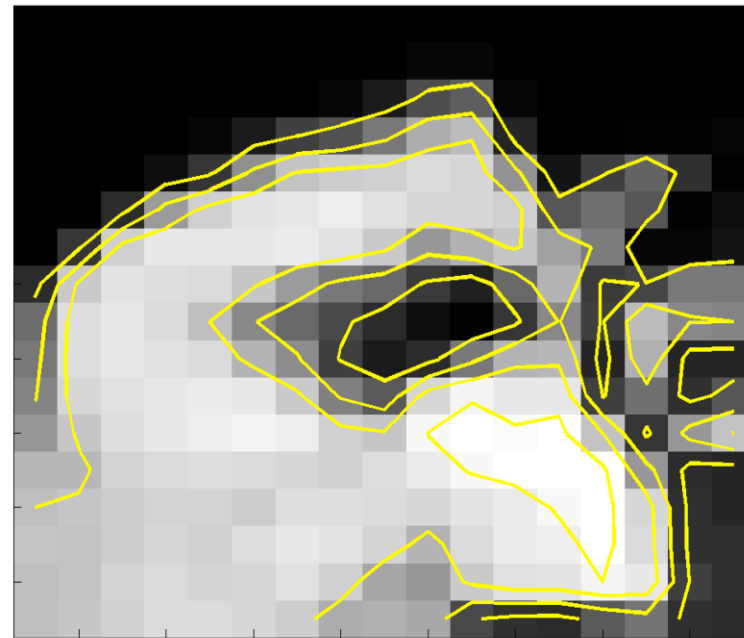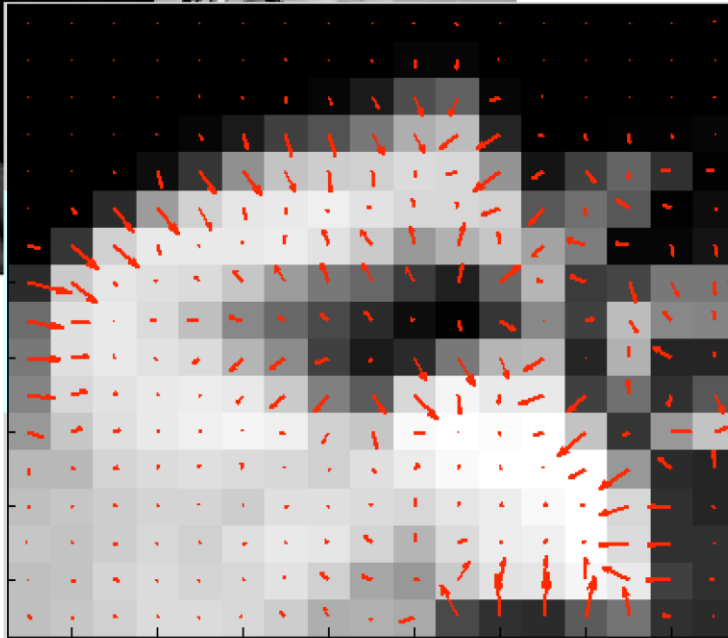
1/22/2026
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Image Gradient

- Derivative of a function
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Central difference is more accurate

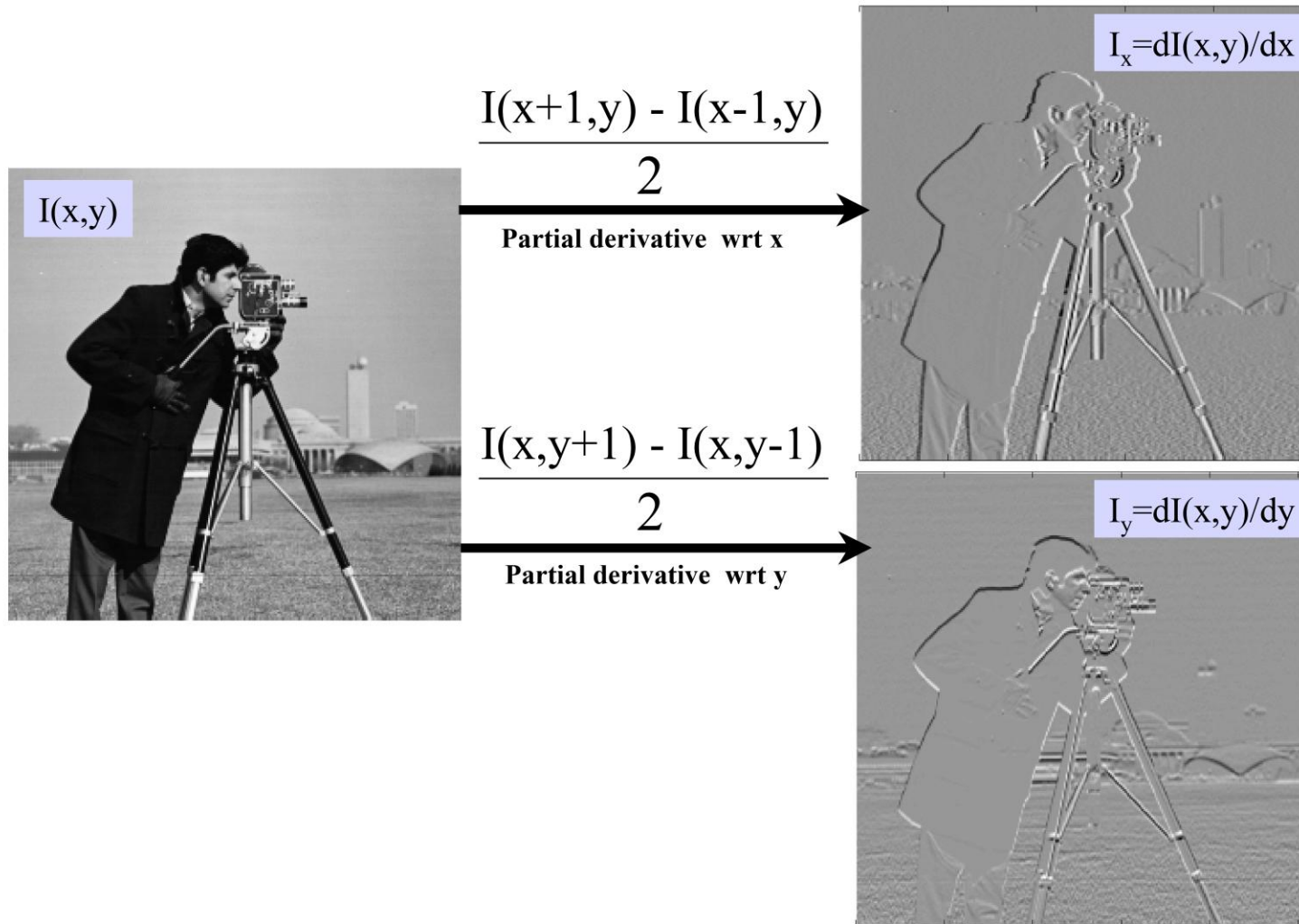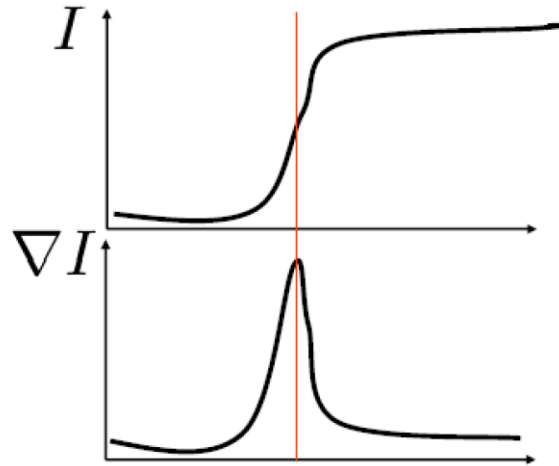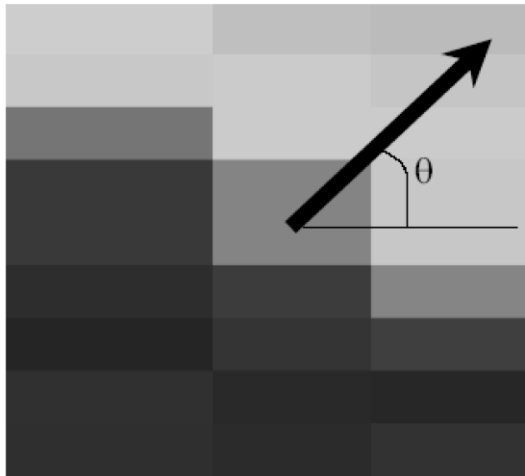$$f'(x) = \lim_{h \to 0} \frac{f(x+0.5h) - f(x-0.5h)}{h}$$

Image Gradient

$$\frac{I(x+1,y) - I(x-1,y)}{2}$$

Partial derivative wrt x

$I_x = dI(x,y)/dx$

$$\frac{I(x,y+1) - I(x,y-1)}{2}$$

Partial derivative wrt y

$I_y = dI(x,y)/dy$

$I(x,y)$

Image Gradient

Gradient Vector: $\nabla I = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}]^{\mathbf{T}}$

$$|\nabla I| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

Magnitude:

$$\theta = atan2(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x})$$

Orientation

Further Reading

- Chapter 3.1, Richard Szeliski

- Slope https://en.wikipedia.org/wiki/Slope

- Gradient https://en.wikipedia.org/wiki/Gradient

- Matplotlib 3D surface: https://matplotlib.org/stable/gallery/mplot3d/surface3d.html