Chapter 3. Image processing

Contents

Chapter 3. Image processing

3.1 Pixel Processing

We can transform the brightness value of a pixel based on the value itself, independently of its location or the values of other pixels in the image.

It is basically a mapping of one brightness value to another brightness value or one color to another color.

$$g(x, y) = T(f(x, y))$$

Chapter 3. Image processing

3.1 Pixel Processing

The following figures illustrate an example of such transformations.

Original (f)

f-128

f+128

255-f

Chapter 3. Image processing

3.1 Pixel Processing

Original (f)

f/2

f*2

0.3*f_R+0.6*f_G+0.1*f_B

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

We will present this concept using one-dimensional signals before extending to multiple dimensions.
Figure 1 shows an LSIS system with input *f(x)* and output *g(x)*.

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Linearity:
If the transformations (see below) are verified, then, the system is linear.

$$f_1 \longrightarrow \boxed{\text{LSIS}} \longrightarrow g_1 \qquad f_2 \longrightarrow \boxed{\text{LSIS}} \longrightarrow g_2$$

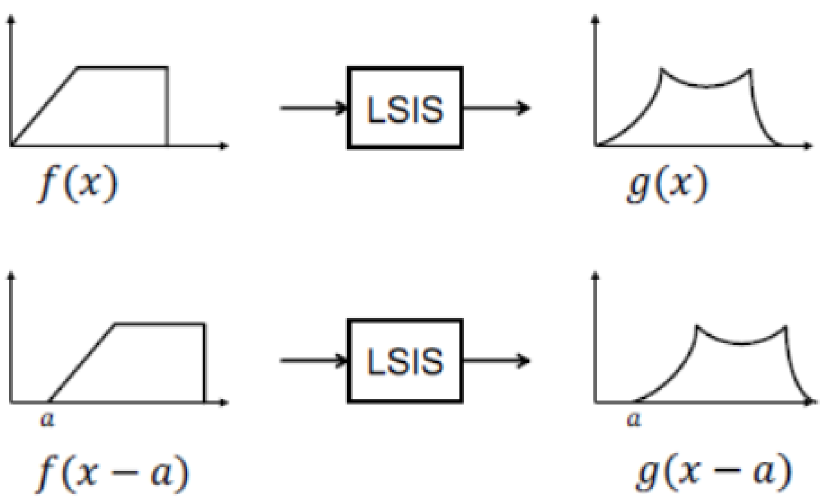
$$\alpha f_1 + \beta f_2 \longrightarrow \boxed{\text{LSIS}} \longrightarrow \alpha g_1 + \beta g_2$$

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Shift Invariance:

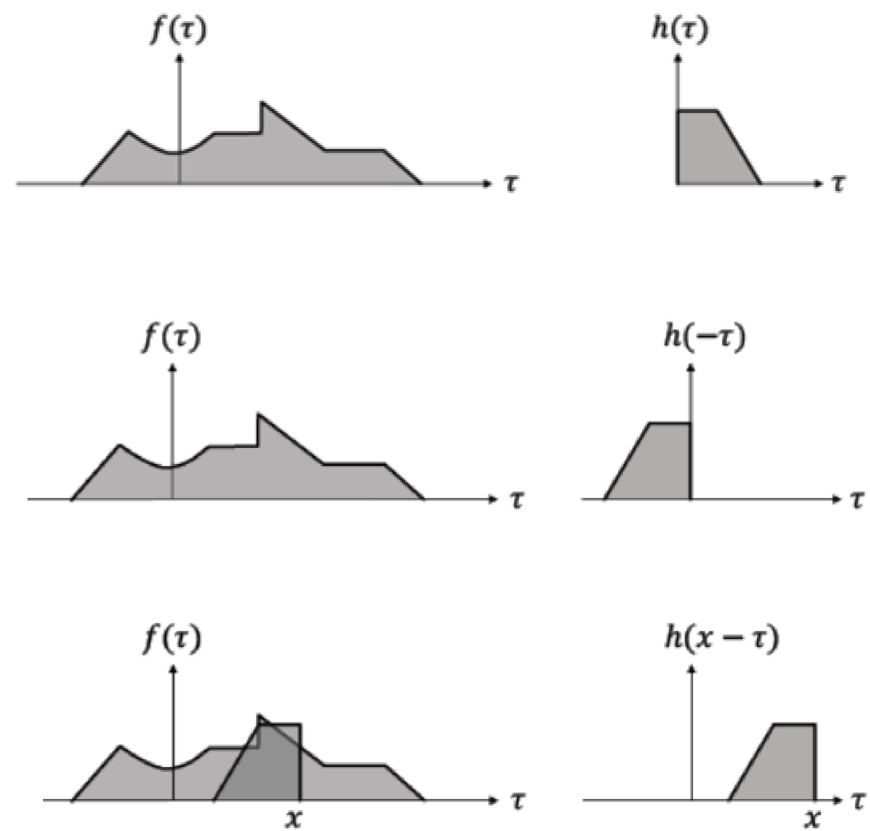
Any system that satisfies linearity and shift invariance is a linear shift invariant system. The following figure illustrates that f(x) is shift invariant.



3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Convolution:

$$g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau$$



3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Convolution:

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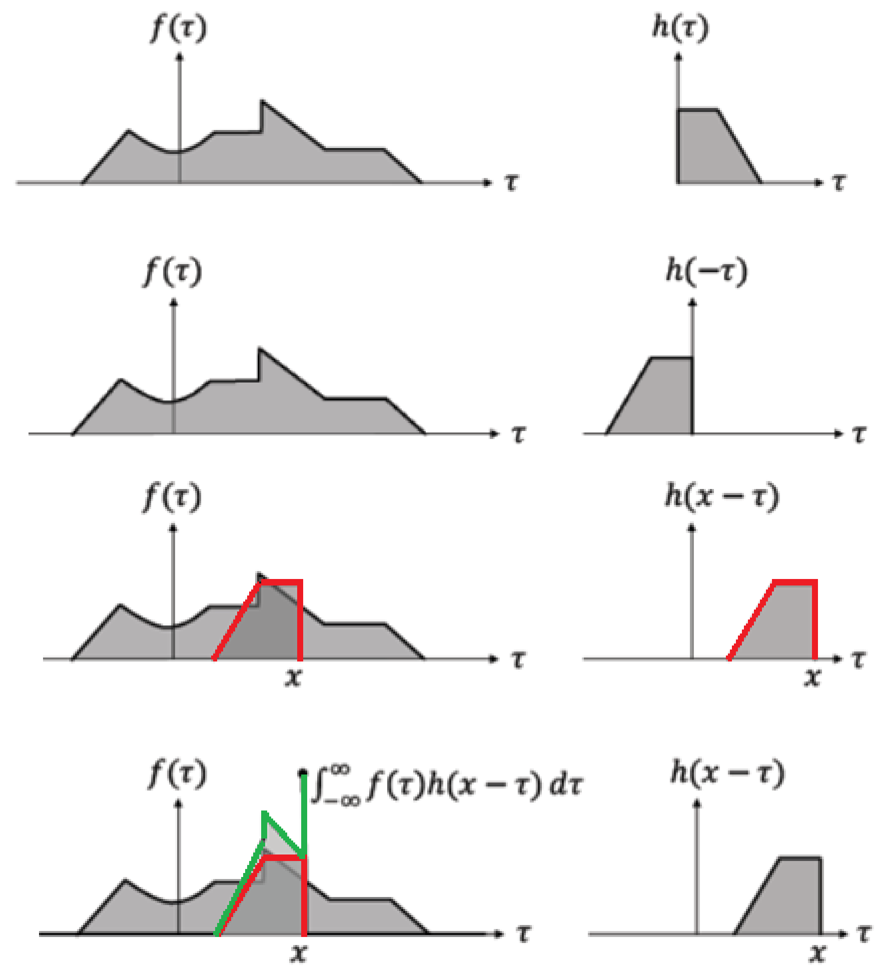


If we take the product $f(\tau)h(x-\tau)$ of these two overlapping functions and integrate it from minus infinity to infinity, this gives us a single number, which is the result of the convolution at the point x.

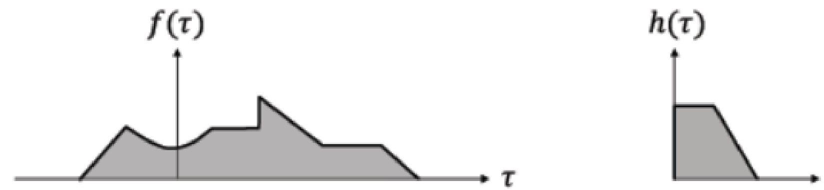
3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

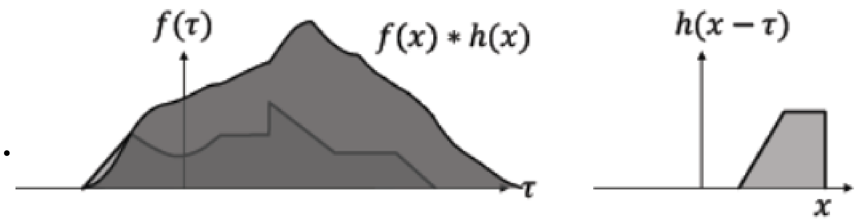
$$\boxed{g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau}$$

Convolution:

To find the entire function g(x), we would flip the function h(τ) and then move it to minus infinity, that is the shift x in h(x- τ) equals minus infinity.



We then vary the shift from minus infinity to plus infinity by sliding the function h(-τ) over f(τ) from left to right. For each shift value x we find the product of the two functions and then the integral of the product. This gives us the entire function g(x), which is the result of the convolution.

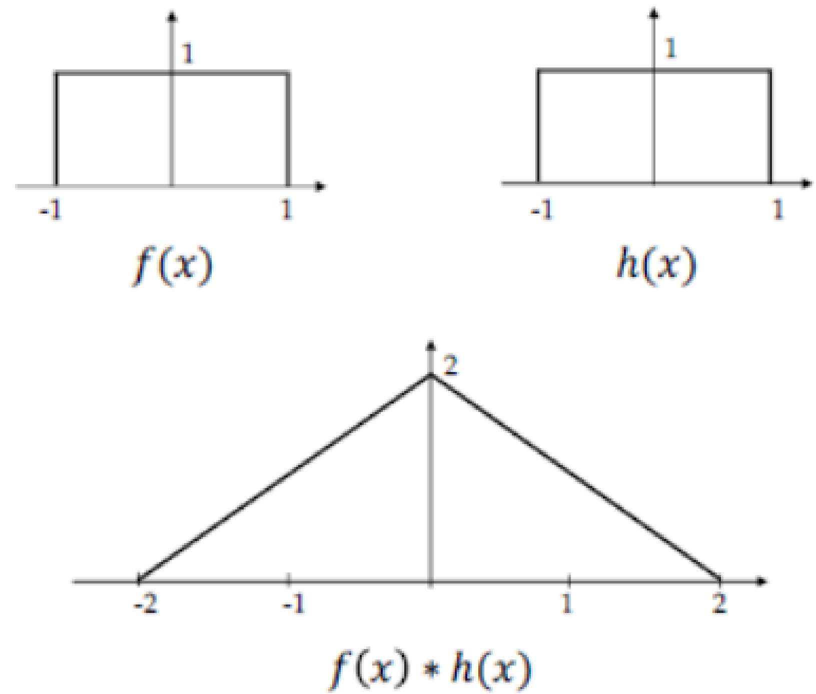


3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Convolution:

Example of convolution.

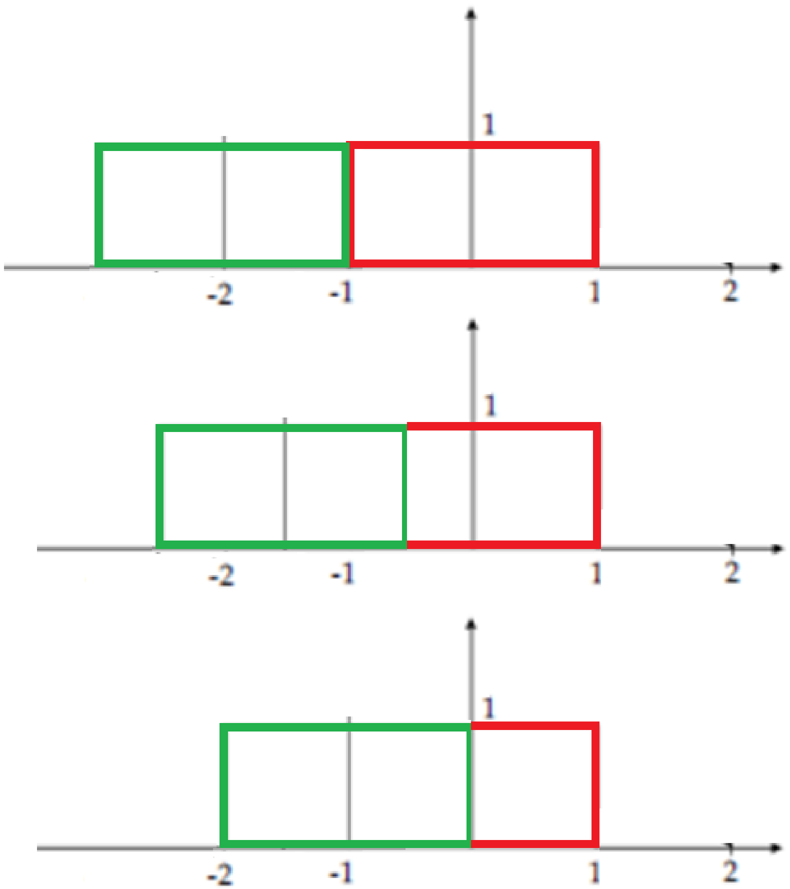


3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Convolution:

Example of convolution.



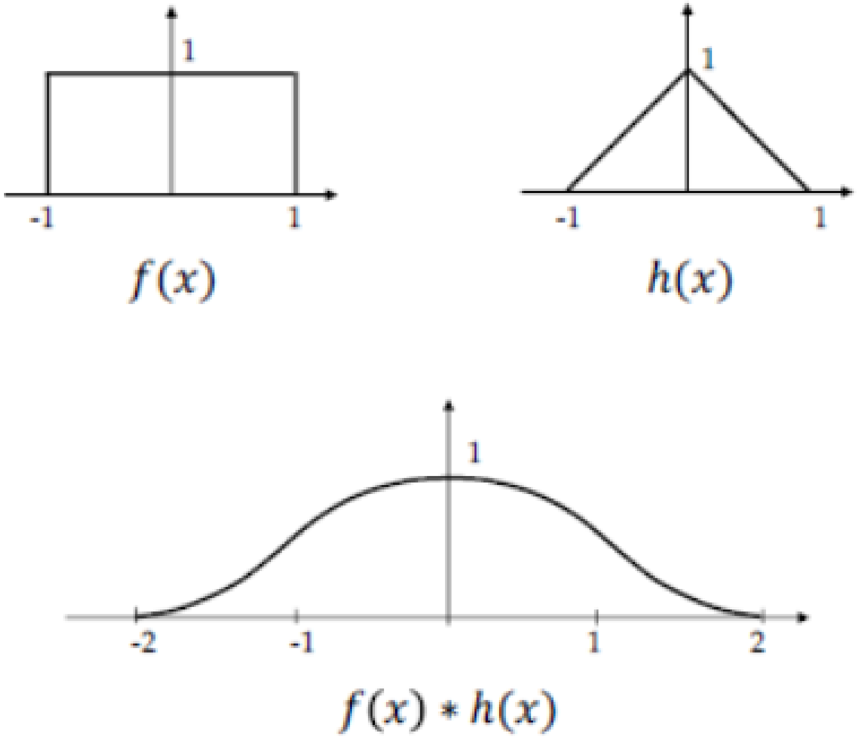
$f(x) * h(x)$

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Convolution:

Example of convolution.



3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Convolution is LSIS :
Linearity:
We assume that: $g_1(x) = \int_{-\infty}^{\infty} f_1(\tau)h(x-\tau)\,d\tau$, $g_2(x) = \int_{-\infty}^{\infty} f_2(\tau)h(x-\tau)\,d\tau$

$$\int_{-\infty}^{\infty} (\alpha f_1(\tau) + \beta f_2(\tau))h(x-\tau)\,d\tau$$

$$= \alpha \int_{-\infty}^{\infty} f_1(\tau)h(x-\tau)\,d\tau + \beta \int_{-\infty}^{\infty} f_2(\tau)h(x-\tau)\,d\tau$$

$$= \alpha g_1(x) + \beta g_2(x)$$

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Convolution is LSIS :
Shift Invariance

We assume that: $$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau$$

We shift the function ***f*** by the value *a*

$$\int_{-\infty}^{\infty} f(\tau-a)h(x-\tau)\,d\tau \quad = \int_{-\infty}^{\infty} f(\mu)h(x-a-\mu)\,d\mu \quad = g(x-a)$$

Where: $\mu = \tau - a$

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

Can we find f, such as g=h?
This is the case of black box doing a unknown convolution filter.

$$f \longrightarrow \boxed{h} \longrightarrow g \qquad g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau$$

What input f will produce output $g = h$?

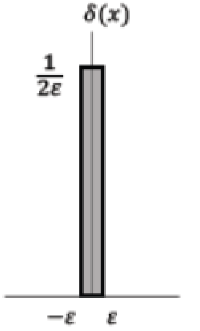
$$h(x) = \int_{-\infty}^{\infty} ?\,(\tau)h(x-\tau)\,d\tau$$

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

f(x) is equal to δ(x) such that :

$$\delta(x) = \begin{cases} 1/2\varepsilon, & |x| \le \varepsilon \\ 0, & |x| > \varepsilon \end{cases} \qquad \varepsilon \to 0$$



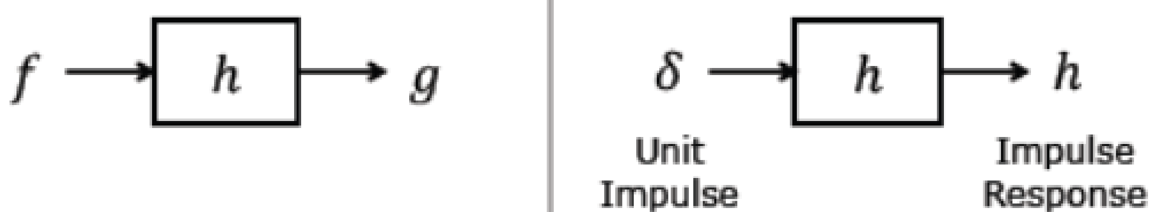
$$\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = \frac{1}{2\varepsilon} . 2\varepsilon = 1$$

$$\int_{-\infty}^{\infty} \delta(\tau)\, \mathbf{h}(x-\tau)\, d\tau = \mathbf{h}(x)$$

3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of an LSIS

f(x) is equal to δ(x) such that :



$$g(x) = f(x) * h(x) \qquad h(x) = \delta(x) * h(x)$$

$$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau \qquad h(x) = \int_{-\infty}^{\infty} \delta(\tau)h(x-\tau)\,d\tau$$

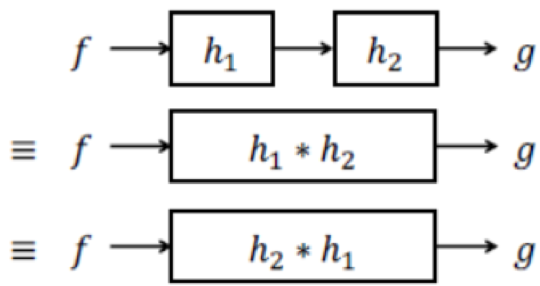
3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Properties of convolution

Commutative $a * b = b * a$

Associative $(a * b) * c = a * (b * c)$

Cascaded System



3.2 LSIS (Linear Shift Invariant Systems) and Convolution

Convolution 2D

LSIS:

$$f(x,y) \longrightarrow \boxed{h(x,y)} \longrightarrow g(x,y)$$

Convolution:

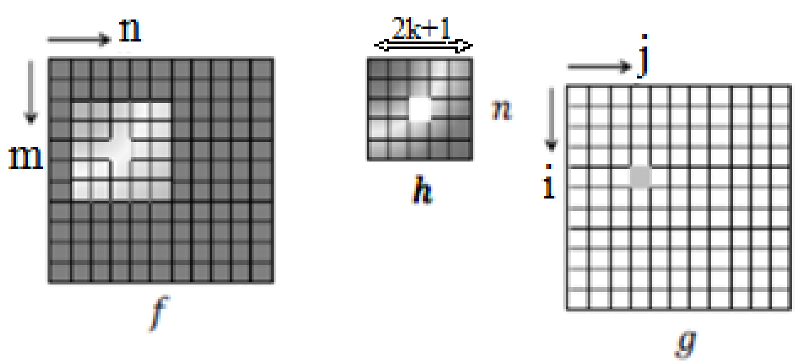
$$g(x,y) = \iint_{-\infty}^{\infty} f(\tau,\mu)h(x-\tau,y-\mu)\,d\tau d\mu$$

3.3 Linear Image Filters

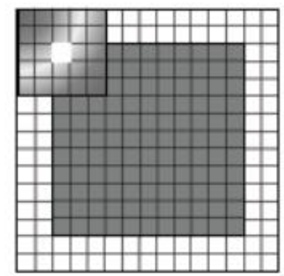
Convolution with Discrete Images

$$f[i,j] \longrightarrow \boxed{h[i,j]} \longrightarrow g[i,j]$$

$$g[i,j] = \sum_{m=-k}^{k} \sum_{n=-k}^{k} f(i+m,j+n)h(m,n)$$



Border Problem



Solution:

- Ignore border
- Pad with constant value
- Pad with reflection

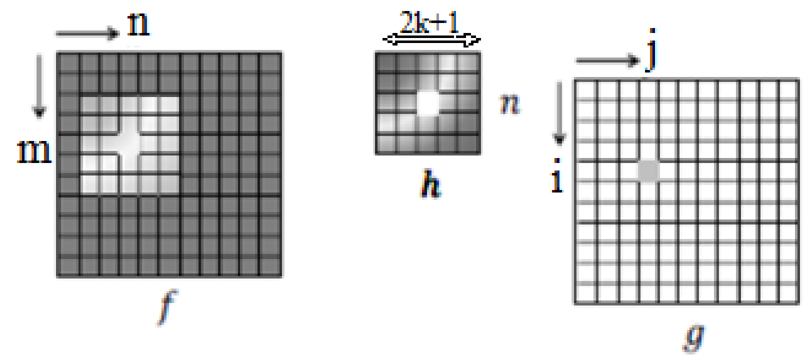
In image processing, the impulse response $h[i,j]$ is referred to as a mask, a kernel, or a filter.

3.3 Linear Image Filters

Convolution with Discrete Images

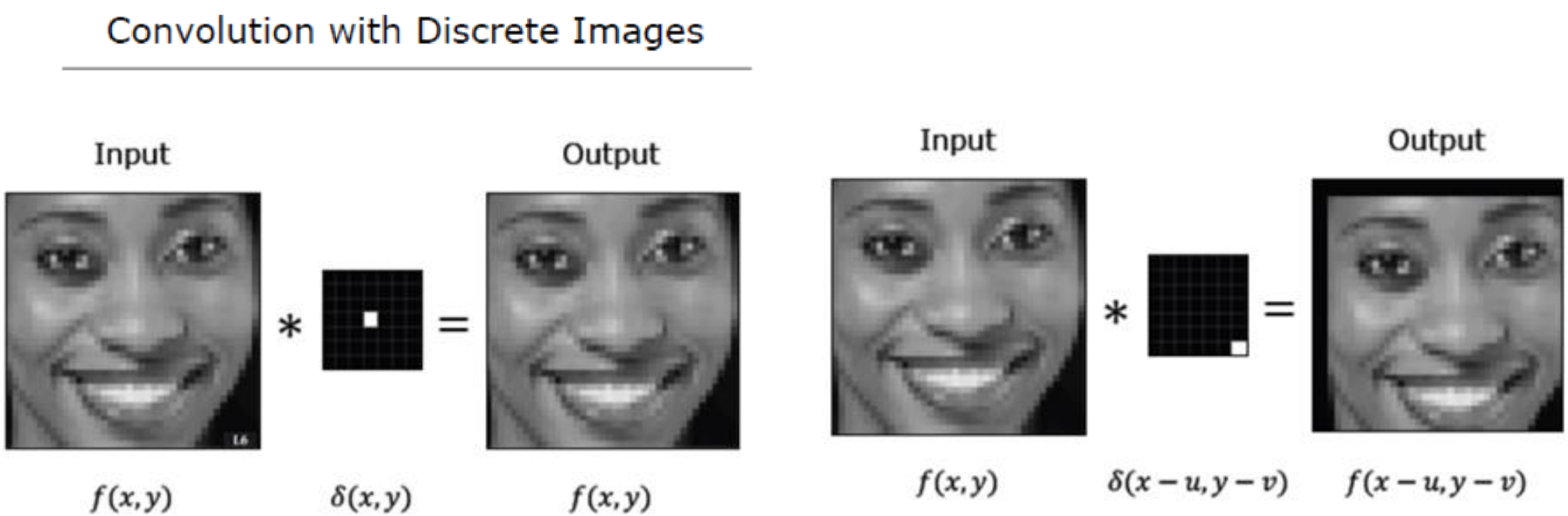
$$f[i,j] \longrightarrow \boxed{h[i,j]} \longrightarrow g[i,j]$$

$$g[i,j] = \sum_{m=-k}^{k} \sum_{n=-k}^{k} f(i+m,j+n)h(m,n)$$



the value of the output image g at pixel location [i,j] is obtained by flipping the filter h twice, overlaying it on the image f with the center of the filter at [i,j], and finding the sum of the product of the pixel values of the image and the filter in the overlap region

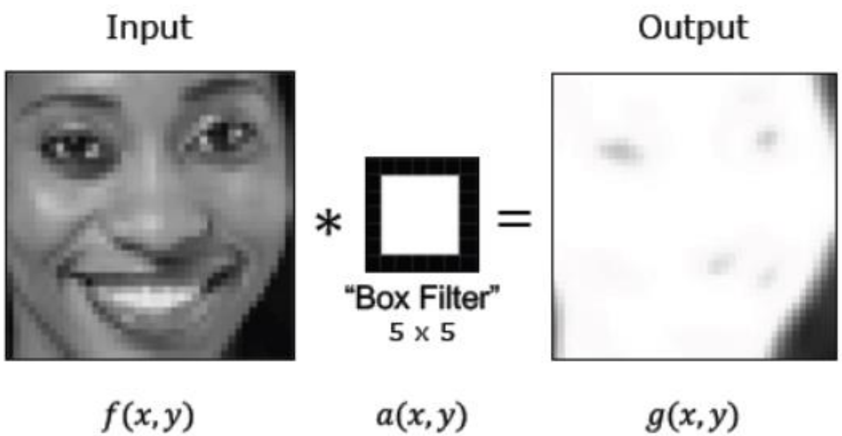
3.3 Linear Image Filters



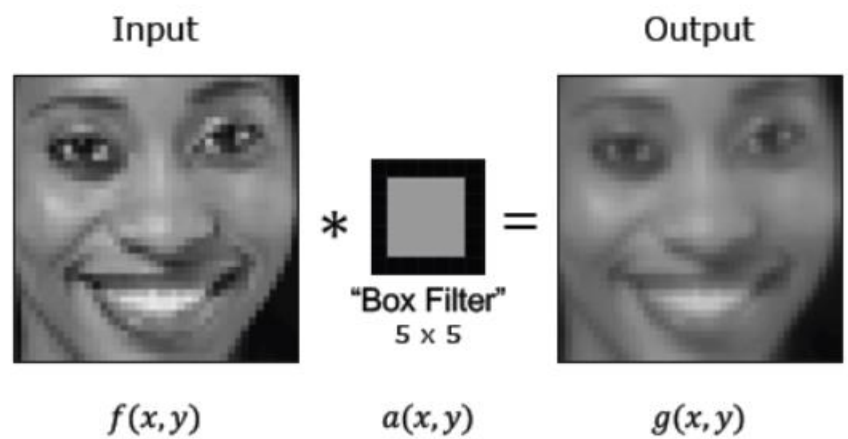
3.3 Linear Image Filters

Convolution with Discrete Images

Example: Averaging



Example: Averaging



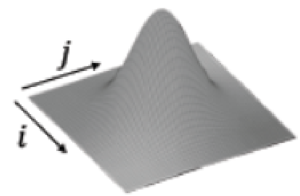
Sum of all the filter (kernel) weights should be 1.

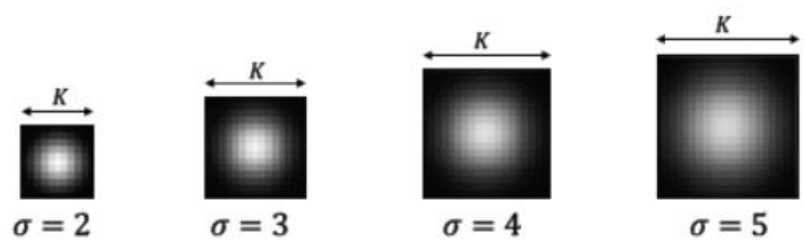
3.3 Linear Image Filters

Gaussian Kernel: A Fuzzy Filter

$$n_\sigma[i,j] = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}\left(\frac{i^2+j^2}{\sigma^2}\right)}$$

σ^2: Variance





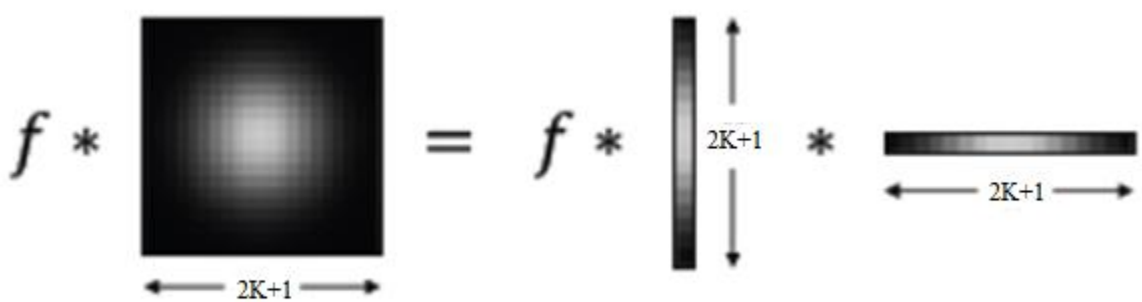
Rule of thumb: Set kernel size $K \approx 2\pi\sigma$

3.3 Linear Image Filters

Gaussian Kernel: is separable.

$$g[i,j] = \frac{1}{2\pi\sigma^2} \sum_{m=-k}^{+k} \sum_{n=-k}^{+k} e^{-\frac{1}{2}\left(\frac{m^2+n^2}{\sigma^2}\right)} f\,[i+m,j+n]$$

$$g[i,j] = \frac{1}{2\pi\sigma^2} \sum_{m=-k}^{K} e^{-\frac{1}{2}\left(\frac{m^2}{\sigma^2}\right)} . \sum_{n=-k}^{K} e^{-\frac{1}{2}\left(\frac{n^2}{\sigma^2}\right)} f\,[i+m,j+n]$$



3.3 Linear Image Filters

Gaussian Kernel: is separable.

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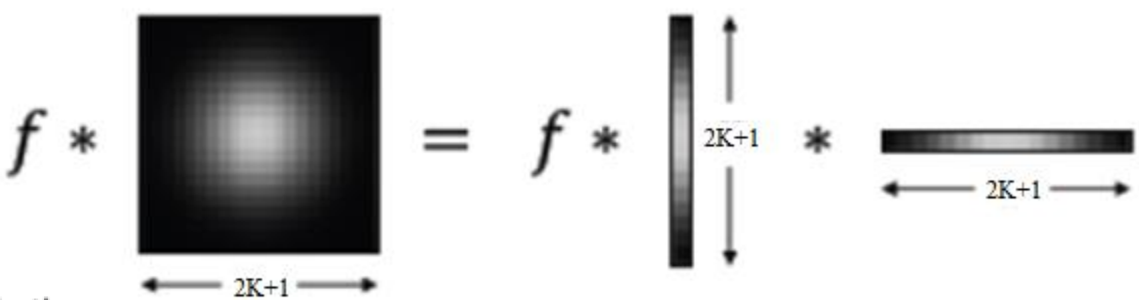

p^2 Multiplications

$p^2 - 1$ Additions

$p = 2k+1$

$2p$ Multiplications

$2(p-1)$ Additions