LLM.int8(): 8-bit Matrix Multiplication for Transformers at Scale

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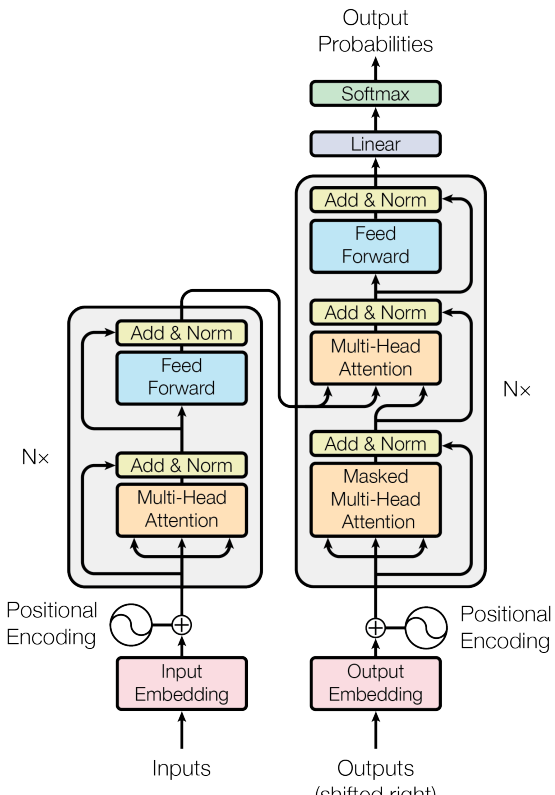
2025.03.28

Outline

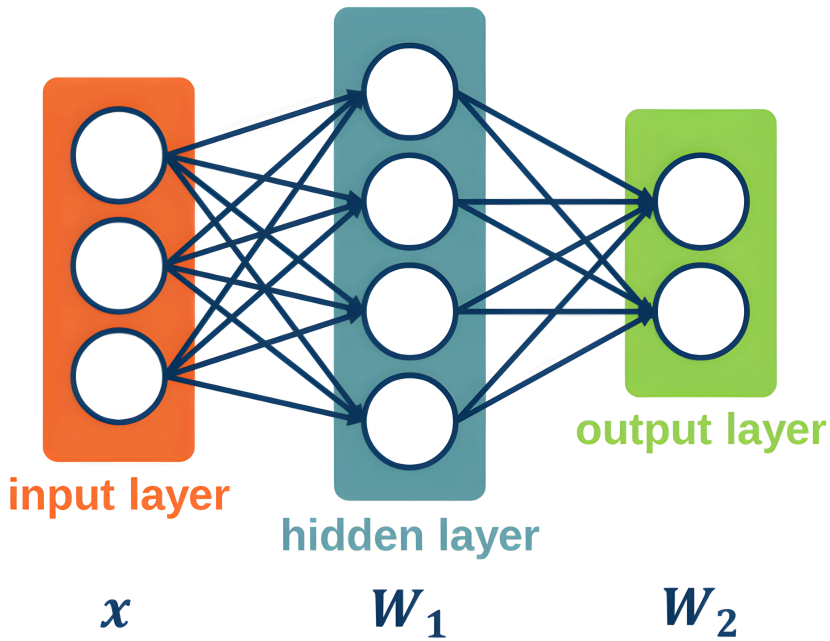
1. Background
2. Quantization
3. Evaluation
4. Outlier Features
5. Discussions

Background


Output Probabilities
Softmax
Linear
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Add & Norm
Masked Multi-Head Attention
Nx
Add & Norm
Feed Forward
Add & Norm
Multi-Head Attention
Nx
Positional Encoding
Positional Encoding
Input Embedding
Output Embedding
Inputs
Outputs (shifted right)

Layers

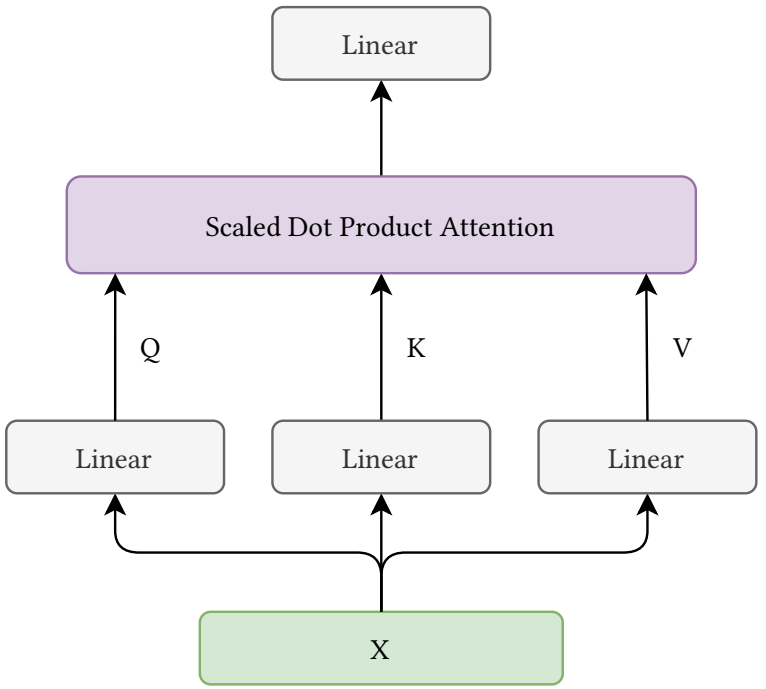
- **MLP**[1]
  - Expansion Layer
  - Contraction Layer
- Attention Block
- Transformer Block



[1]Figure adapted from slides by Zsolt Kira: https://faculty.cc.gatech.edu/~zk15/teaching/AY2024_cs7643_spring/assets/L4_GradientDescent_NNs.pdf

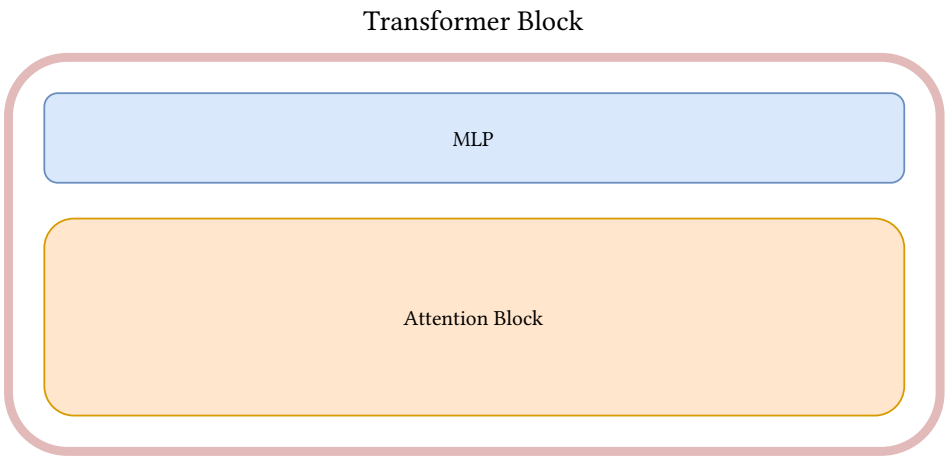
Layers

- MLP
- **Attention Block**
  - 4 Projection (Linear) Layers
  - Key, Value, Query, Output
- Transformer Block



Layers

- MLP
- Attention Block
- **Transformer Block**
  - Attention Block
  - MLP



Scenarios

- Training
  - Forward Pass (value) + Backward Pass (gradient)
  - Forward Pass results need to be stored in memory
- Finetune
  - Similar to training, but sometimes with most of the weights freezed
- Inference
  - Forward Pass only

Scenarios

- Training
- Finetune
- **Inference**

Motivation

- So, what's the problem with the inference?
  - Model is large: OPT-175B/BLOOM
  - Weights need to be loaded into GPU memory
  - 175B parameters (16 bit) -> 8x A100 (80G) -> 💰150000+

Motivation

- So, what's the problem with the inference?
  - Model is large: OPT-175B/BLOOM
  - Weights need to be loaded into GPU memory
  - 175B parameters (16 bit) -> 8x A100 (80G) -> 💰150000+
- Solution: **load "less" weights -> Quantization**

Previous Attempts

- Degrade performance
- Require further tuning
- Only been studied for models with less than 350M parameters

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So, how to do **degradation-free quantization** up to **billions of parameters?**

Quantization

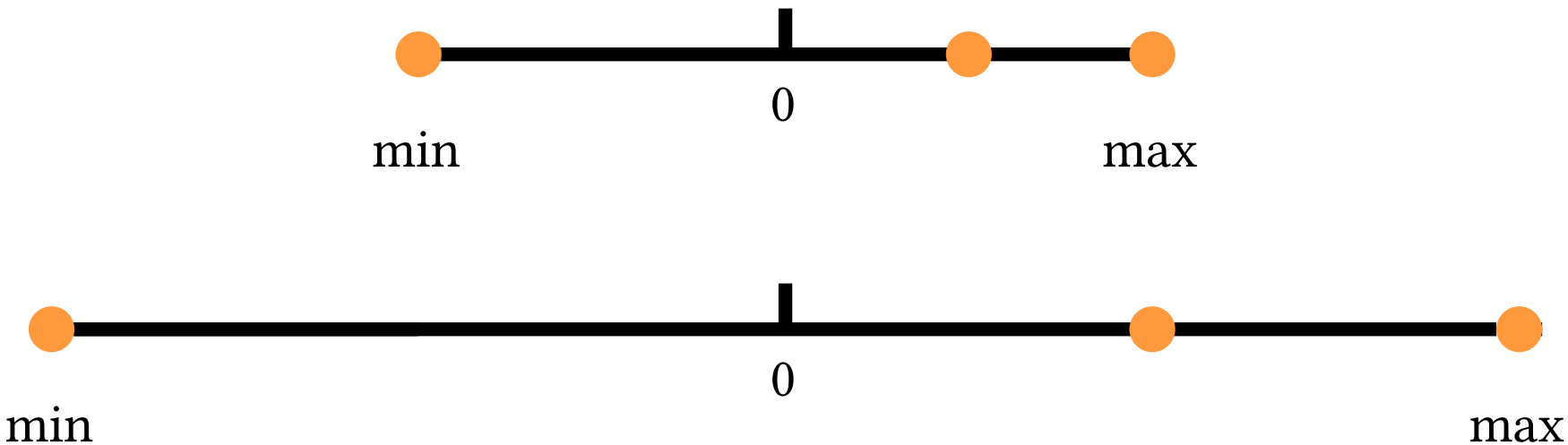
- Quantize fp16 matrices to int8 matrices
- Support matrix multiplication

Absmax

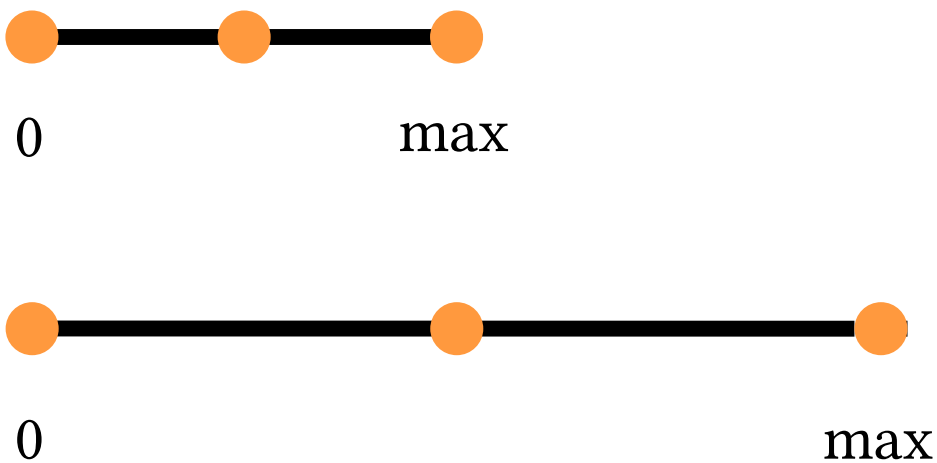
- Absolute Maximum Quantization
- Scale input into $[-127, 127]$ ranges

$$\boldsymbol{X}_{i8} = \left\lfloor \frac{127 \cdot \boldsymbol{X}_{f16}}{\max_{ij}\left(\left|\boldsymbol{X}_{f16}^{ij}\right|\right)} \right\rceil = \left\lfloor s_{x_{f16}} \boldsymbol{X}_{f16} \right\rceil$$

Absmax



Absmax



Zeropoint

- Absmax is bad when the data is asymmetric.
- Shift the input into the range $[-127, 127]$ by **scaling** with the normalized dynamic range nd_x and **shifting** by the zeropoint zp_x

$$nd_{x_{f16}} = \frac{2 \cdot 127}{\max_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right) - \min_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right)}$$

$$zp_{x_{i16}} = \left\lfloor X_{f16} \cdot \min_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right) \right\rceil$$

$$X_{i8} = \left\lfloor nd_{x_{f16}} X_{f16} \right\rceil$$

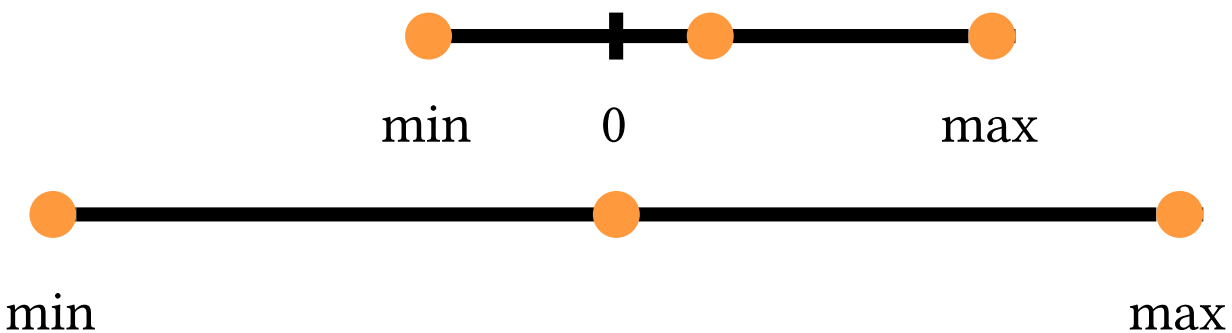
Zeropoint

- However, the formula in the paper is wrong.[2]

[2] Their code for calculating zeropoint is also wrong, but in a different way: see e1f515. A few hours after making the slides, the wrong test was removed from the main branch.

Zeropoint

- However, the formula in the paper is wrong.
- Let's recall our goal:



Zeropoint

$$nd_{x_{f16}} \cdot \min_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right) + zp_{x_{i16}} = -127$$

$$nd_{x_{f16}} \cdot \max_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right) + zp_{x_{i16}} = 127$$

Solving the above equations gives us

$$nd_{x_{f16}} = \frac{2 \cdot 127}{\max_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right) - \min_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right)}$$

$$zp_{x_{i16}} = -\left\lfloor nd_{x_{f16}} \cdot \min_{ij}\left(\boldsymbol{X}_{f16}^{ij}\right) \right\rceil - 127$$

Matrix Multiplication - Naive

Given input $\boldsymbol{X}_{f16} \in R^{s \times h}$ and weights $\boldsymbol{W}_{f16} \in R^{h \times o}$ with sequence dimension s, feature dimension h, and output dimension o,

$$\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} \approx \frac{1}{s_{x_{f16}} s_{w_{f16}}} \cdot \boldsymbol{X}_{i8}\boldsymbol{W}_{i8} \qquad \text{(absmax)}$$

$$\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} \approx \frac{1}{nd_{x_{f16}} nd_{w_{f16}}} \cdot \left(\boldsymbol{X}_{i8} - zp_{x_{i16}}\right) \cdot \left(\boldsymbol{W}_{i8} - zp_{w_{i16}}\right) \qquad \text{(zp)}$$

- zeropoint is not used due to the need to add zp to quantized values[3]

[3] some GPUs/TPUs don't support this

Matrix Multiplication - Naive

$$\boldsymbol{X}_{f16} = \boldsymbol{W}_{f16} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \xrightarrow{\text{absmax quantization}} \begin{pmatrix} 64 & 127 \\ -127 & 64 \end{pmatrix}$$

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$$\boldsymbol{X}_{i8}\boldsymbol{W}_{i8} = \begin{pmatrix} -12033 & 16256 \\ -16256 & -12033 \end{pmatrix} \xrightarrow{\text{dequantization}} \begin{pmatrix} -2.98 & 4.03 \\ -4.03 & -2.98 \end{pmatrix}$$

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$$\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} = \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix}$$

Matrix Multiplication - Naive

Let's consider a different input matrix

$$\boldsymbol{X}_{f16} = \begin{pmatrix} 3 & 508 \\ 1 & 1 \end{pmatrix}$$

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Matrix Multiplication - Rowwise

- Quantize each row of $\boldsymbol{X}_{f16}$ independently

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$$\boldsymbol{X}_{f16} = \begin{pmatrix} 3 & 508 \\ 1 & 1 \end{pmatrix} \xrightarrow{\text{rowwise quantization}} \begin{pmatrix} 1 & 127 \\ 127 & 127 \end{pmatrix}, \quad S_{x_{f16}} = \begin{pmatrix} 0.25 \\ 127 \end{pmatrix}$$

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$$\boldsymbol{W}_{f16} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \xrightarrow{\text{absmax quantization}} \begin{pmatrix} 64 \\ 127 \end{pmatrix}, \quad S_{w_{f16}} = 63.5$$

Matrix Multiplication - Rowwise

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$$\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} = \begin{pmatrix} 1019 \\ 3 \end{pmatrix} \quad \text{and} \quad \boldsymbol{X}_{i8}\boldsymbol{W}_{i8} = \begin{pmatrix} 16193 \\ 24257 \end{pmatrix} \xrightarrow[\text{div by } S_{x_{f16}} \times S_{w_{f16}}]{\text{dequantization}} \begin{pmatrix} 1020.03 \\ 3.01 \end{pmatrix}$$

Matrix Multiplication - Rowwise

Let's consider a different weight matrix

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Matrix Multiplication - Vectorwise

Observations: matmul are a sequence of independent inner products

- $XW_{ab} = \sum_h X_{ah} W_{hb} = \langle X_a, W_b^T \rangle$

Matrix Multiplication - Vectorwise

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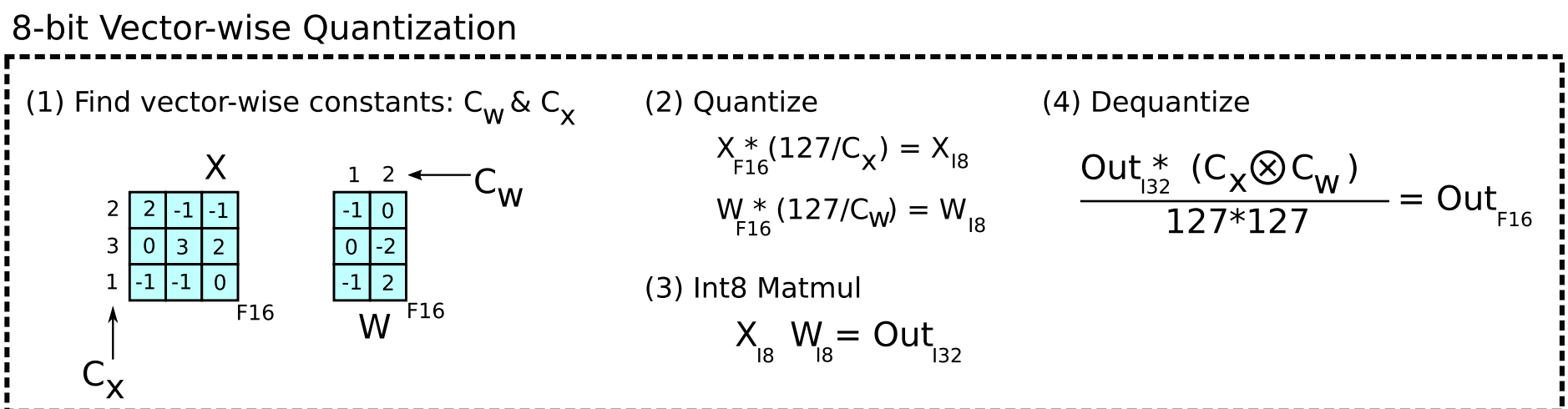
- $XW_{ab} = \sum_h X_{ah} W_{hb} = \langle X_a, W_b^T \rangle$

Quantize each *column* of $\boldsymbol{W}$ independently



Matrix Multiplication - Vectorwise

$$\mathrm{Out}_{\mathrm{I32}_{11}} = \left\langle X_{\mathrm{I8}_1}, W^T_{\mathrm{I8}_1} \right\rangle \approx C_{X_1} C_{W_1} \left(X_{\mathrm{F16}_1} W_{\mathrm{F16}_1} \right)$$

Matrix Multiplication - Vectorwise

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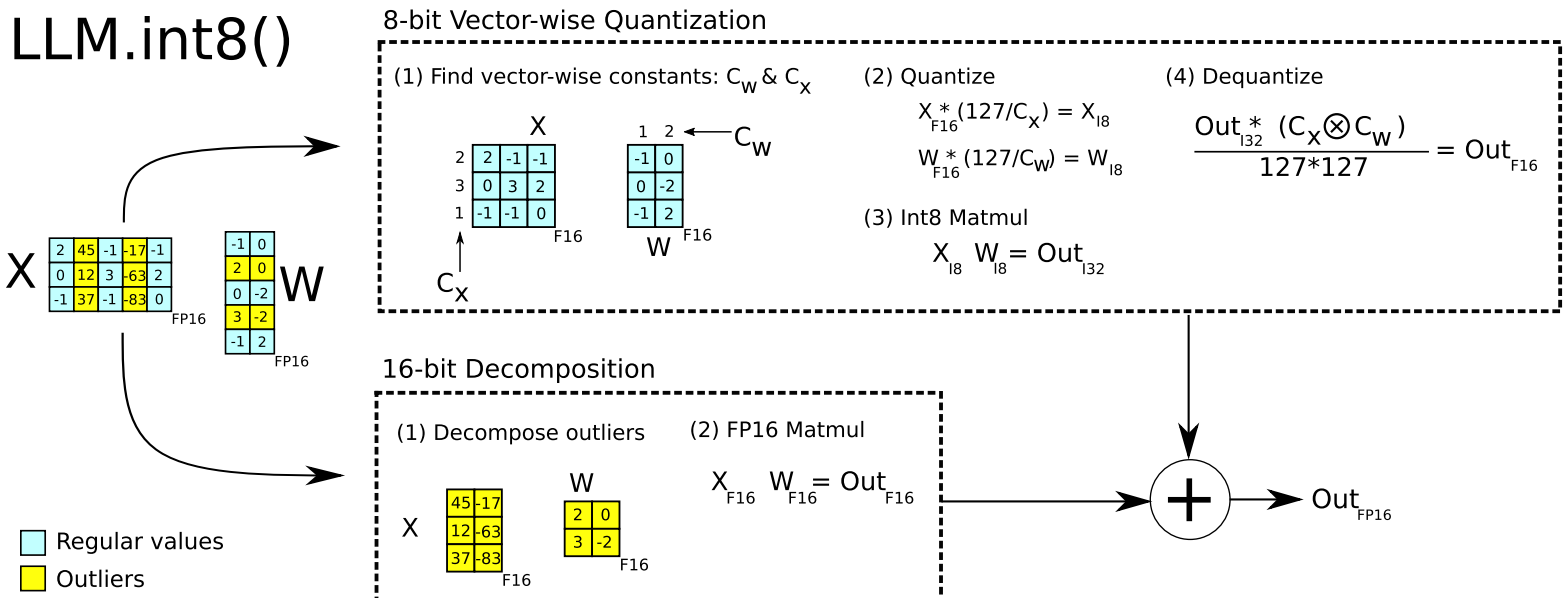
$$C_X C_W = \begin{pmatrix} 2 & 4 \\ 3 & 6 \\ 1 & 2 \end{pmatrix}$$

- $C_{X_1} C_{W_1}$ is the topleft element in the outer product

Mixed-precision Decomposition



- Consistent outlier values in columns of X + Do separate matmul in fp16
- Luckily, they are sparse (only 0.1%). Is LLM.int8() actually int8 🤔?

LLM.int8 in Practice

- Load the weight from fp16/32 checkpoints
- Quantize it to int8 and send to GPUs
- When fp16 matmul is needed, weight matrix is dequantized to fp16

Evaluation

- Question: how well `LLM.int8()` performs as the model size **scales**.
  - Models: 125M - 125B
- Two Settings: Language Modelling + End Tasks
- Language Modelling
  - Dataset: C4 corpus
  - Metric: Perplexity (robust, sensitive to quantization degradation)
- End Tasks
  - Dataset: EleutherAI language model evaluation harness
  - Metric: Zero-shot Accuracy

Evaluation Results

Parameters	125M	1.3B	2.7B	6.7B	13B
32-bit Float	25.65	15.91	14.43	13.30	12.45
Int8 absmax	87.76	16.55	15.11	14.59	19.08
Int8 zeropoint	56.66	16.24	14.76	13.49	13.94
Int8 absmax row-wise	30.93	17.08	15.24	14.13	16.49
Int8 absmax vectorwise	35.84	16.82	14.98	14.13	16.48
Int8 zeropoint vectorwise	25.72	15.94	14.36	13.38	13.47
Int8 absmax rowwise + decomp	30.76	16.19	14.65	13.25	12.46
Absmax LLM.int8() vectorwise + decomp	25.83	15.93	14.44	13.24	12.45
Zp LLM.int8() vectorwise + decomp	25.69	15.92	14.43	13.24	12.45

Table 1: C4 validation perplexities of quantization methods for different transformer sizes.

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- Performance degrades as model scales + LLM.int8 has comparable perf

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- With decomposition, zeropoint is no longer advantageous

Outlier Features

- Given a hidden state $X \in R^{s \times h}$ where s is the sequence/token dimension and h the hidden/feature dimension, a **feature** is a particular dimension h_i.

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How to determine if a feature is an outlier?

Outlier Features

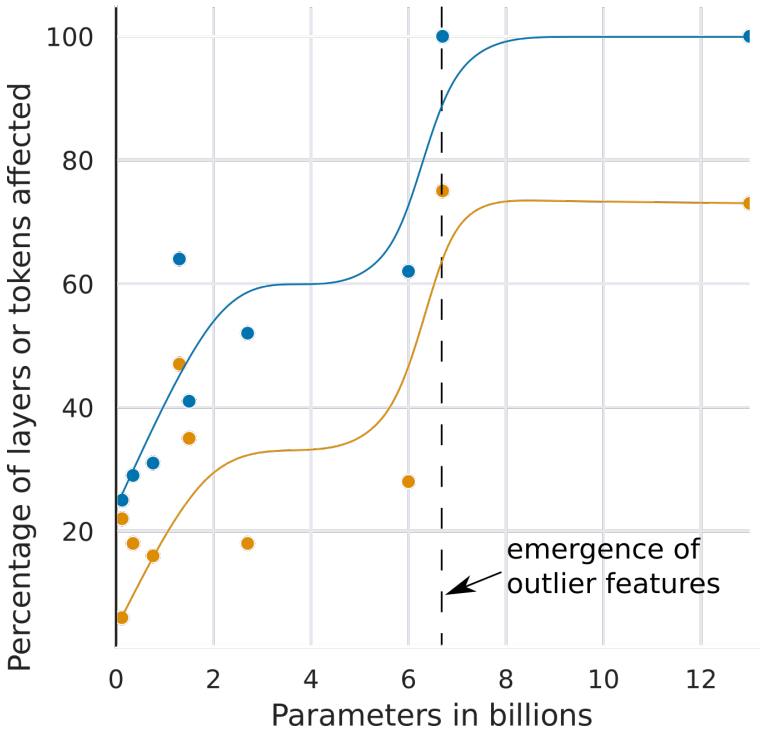
- 4 Projection layers in Attention + Expansion layer in MLP
- Outliers
  - Treat features ≥ 6 as outliers $\Rightarrow$ no more perplexity degradation
  - *Systematic* in large models: either in most layers or not at all
  - *Probabilistic* in small models: sometimes in some layers
- Find thresholds to limit detection to a single outlier in the smallest model with 125M parameters

Outlier Features - Threshold

- The feature value is at least 6.0
- It occurs in the same h_i in at least 25% of layers
- It appears in at least 6% of the sequence dimensions

Understanding Outlier Features

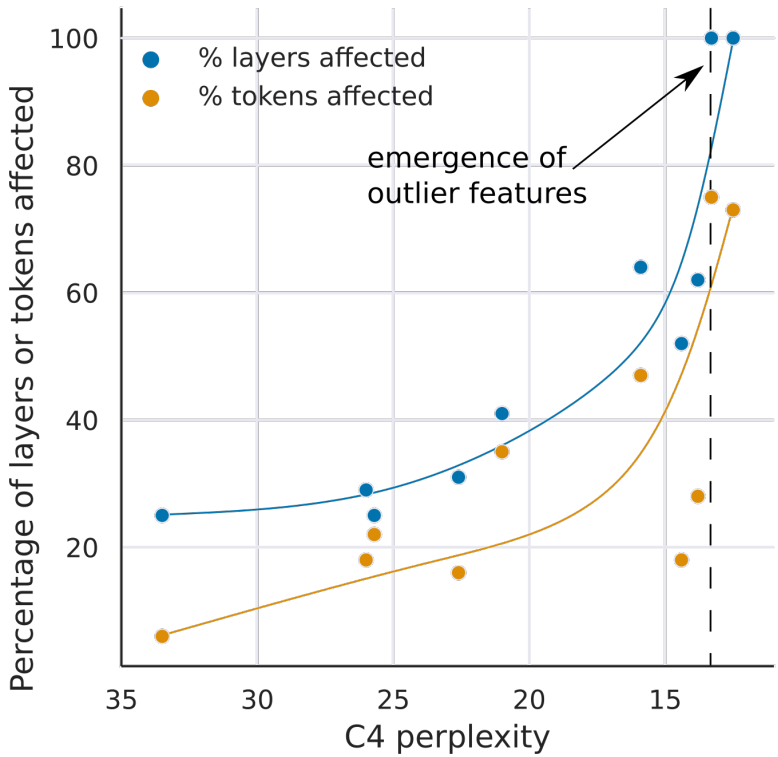
- **Emergence**[4]
  - Sudden increase of presence in feature (blue) and sequence dimensions (orange) as model size increases
- Systematic after phase shift
- Critical



[4]The following studies are done in a series of models up to 13B parameters.

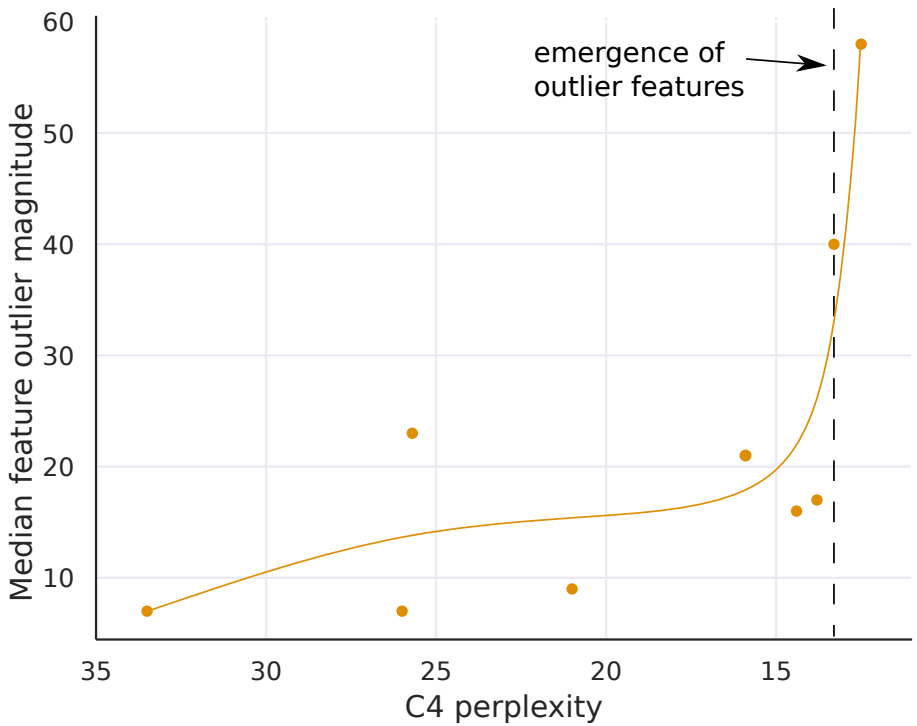
Understanding Outlier Features

- **Emergence**
  - Sudden increase of presence in feature (blue) and sequence dimensions (orange) as model size increases
  - Nothing sudden in terms of perplexity
- Systematic after phase shift
- Critical



Understanding Outlier Features

- **Emergence**
  - Sudden increase of presence in feature (blue) and sequence dimensions (orange) as model size increases
  - Nothing sudden in terms of perplexity
  - Median rapidly increases
- Systematic after phase shift
- Critical



Understanding Outlier Features

- Emergence
- **Systematic** after phase shift
  - For a 6.7B transformer with a sequence length of 2048,
    - 150k outlier features per sequence
    - Concentrated in only *6* different hidden dimensions
- Critical

Understanding Outlier Features

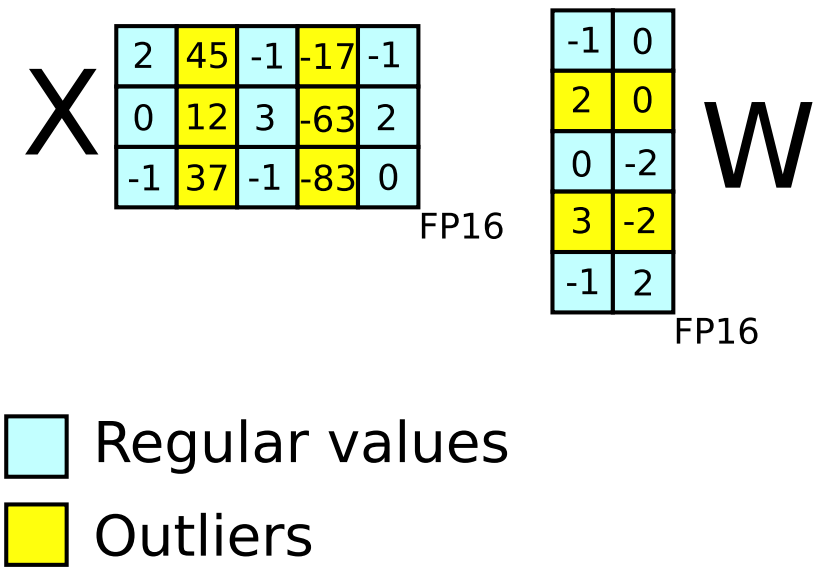
- Emergence
- Systematic after phase shift
- **Critical**
  - Even if there are only 7 outliers, after removing them,
    - the mean top-1 softmax probability is reduced from 40% to 20%
    - the perplexity increases by 600-1000%
  - Remove 7 random features,
    - the top-1 probability decreases only between 0.02-0.3%
    - the perplexity increases by 0.1%

Outlier Features in Practice

- Any problems in outlier detections? 👀

Outlier Features in Practice

- Outlier features are context-dependent[5]
  - Different outliers can show at different positions for different input
- In practice, outlier detection works on weight matrix instead[6]



[5]The author uses this word in his blog: https://timdettmers.com/2022/08/17/llm-int8-and-emergent-features/

[6]See their code here: https://github.com/bitsandbytes-foundation/bitsandbytes/blob/8b6fe9eef1a2f93af701e020b9a757e43b18a42f/bitsandbytes/utils.py#L8-L103

Discussions

- Limitations
  - Focus on int8 only (no fp8)
  - Only study models up to 175B params
- Int8 training and finetuning performance are not ideal

- Small-scale language models is close to baseline performance
- Performance degrades when int8 is used in attention projection layers even with decomposition

Params	Is 8-bit MLP	Out Proj	Attn	Decomp	PPL
209M				0%	16.74
209M	✓			0%	16.77
209M	✓	✓		0%	16.83
209M	✓	✓		2%	16.78
209M	✓	✓		5%	16.77
209M	✓	✓		10%	16.80
209M	✓	✓	✓	2%	24.33
209M	✓	✓	✓	5%	20.00
209M	✓	✓	✓	10%	19.00
1.1B				0%	9.99
1.1B	✓			0%	9.93
1.1B	✓	✓		0%	10.52
1.1B	✓	✓		1%	10.41

Discussions - Speedup

GPT-3 Size	Small	Medium	Large	XL	2.7B	6.7B	13B	175B
Model dimension	768	1024	1536	2048	2560	4096	5140	12288
FP16-bit baseline	1.00x	1.00x	1.00x	1.00x	1.00x	1.00x	1.00x	1.00x
Int8 no overhead	0.99x	1.08x	1.43x	1.61x	1.63x	1.67x	2.13x	2.29x
Vectorwise	0.43x	0.49x	0.74x	0.91x	0.94x	**1.18x**	**1.59x**	**2.00x**
LLM.int8() vectorwise decomp	0.14x	0.20x	0.36x	0.51x	0.64x	0.86x	**1.22x**	**1.81x**

- Ideal Speedup: the 8-bit without overhead assumes no quantization or dequantization is performed.
- Speed up only for models of large sizes. Why?

Thank you