Expectaion Formation

Behavioral Macro

赵宇轩

北京大学元培学院

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- Standard model: full information and rational expectation equilibrium(FIRE)
- Add behavioral friction: cognitive discounting in expectation formation of state variables
- For example,changing amplification & propagation mechanism of model

  Generating a hump-like response to exogenous shocks in DSGE[9](Makowiak&Wiederholt,2015)

  Generating a overreaction to exogenous shocks in DSGE[5](P.Bordalo et al.,2020)

- Generating totally different policy effects in HANK[6](Tobias Broer et al.,2021)

  "While in the model without information choice a wealth tax reduces wealth inequality,in our framework it reduces information acquired in the economy, leading to increased volatility and higher wealth inequality in equilibrium..."[6]

Myopic adjustment

- Start with a standard economic model:
- e.g. A Simple Dynamic Consumption Saving Problem

$$
\begin{aligned}
\mathrm{Max}\, U(C) &= E_0[\sum_{t=0}^{\infty} \beta^t u(C_t)] \quad \text{s.t.} \\
C_t + B_{t+1} &= R_t B_t + Y_t \\
\ln(Y_t/\bar{Y}) &= \rho_Y \ln(Y_{t-1}/\bar{Y}) + u_t
\end{aligned}
$$

- Taking derivative against the control variable by Lagrangian:

$$
\begin{aligned}
\beta E_t[R_{t+1} C_t / C_{t+1}] &= 1 \\
C_t &= Y_t \\
\ln(Y_t/\bar{Y}) &= \rho_Y \ln(Y_{t-1}/\bar{Y}) + u_t
\end{aligned}
$$

Myopic adjustment

- Log-linearize them:

$$\begin{aligned}\ln(R_{t+1}) &= -\ln(\beta) - \hat{C}_t + E_t(\hat{C_{t+1}}) \\ &= -\ln(\beta) - \hat{Y}_t + E_t(\hat{Y_{t+1}}) \\ &= -\ln(\beta) - \hat{Y}_t + \rho_Y \hat{Y}_t\end{aligned}$$

- $\hat{Y}_t$ increases by one unit, interest rate $\ln R_t$ decreases by $1-\rho$.
- Here we use solution concept of **full information rational equilibrium** to solve this model:

Myopic adjustment

- Here we use solution concept of **full information rational equilibrium** to solve this model:
- Agent is aware of the true structure of this dynamic system (e.g. the $ln(Y_t/\bar{Y})$ follows an AR1 process).
- Agent is aware of the true parameters of this dynamic system (e.g. persistence of income shocks ρ_Y,even interest rates R_t)
- Agent is aware of the shocks realized in each period (e.g., at time t, the agent knows Y_t)
- Agent is aware of the general equilibrium effects(level-k thinking)
- Agent possesses perfect information processing capabilities(rational inattention)
- Agent's decisions are not influenced by sentiments
- ......

Myopic adjustment

- Let $E_t(\hat{Y_{t+1}})$ denote a correct forecasting rule under FIRE.
- $E_t^B(\hat{Y_{t+1}})$ could be associated with the forecast about future income - **in deviation from its steady state with myopic adjustment**.
- Then we specify expectation in our model: $E_t^B(\hat{Y_{t+h}}) = m^h E_t(\hat{Y_{t+h}})$, which means agent have cognitive discounting for future impact,as the forecasting horizon h increases, the cognitive discounting becomes more severe.[7]
- Log-linearize them:

$$
\begin{aligned}
\ln(R_{t+1}) &= -\ln(\beta) - \hat{Y}_t + E_t^B(\hat{Y_{t+1}}) \\
&= -\ln(\beta) - \hat{Y}_t + mE_t(\hat{Y_{t+1}}) \\
&= -\ln(\beta) - \hat{Y}_t + m\rho_Y \hat{Y}_t
\end{aligned}
$$

- $\hat{Y}_t$ increases by one unit, interest rate $\ln R_t$ decreases by $1 - m\rho$.
- Introducing cognitive discounting changes agent's consumption-saving problem,and changes interest rates/asset prices.

Myopic adjustment

- Here we use another solution concept to solve the behavioral version of the model.
- ture law of motion of state valiable:
  $X_{t+1} = G_X(X_t, \epsilon_{t+1}) = \Gamma X_t + \epsilon_{t+1}$(linearizing)
- $\Rrightarrow$ perceived law of motion: $X_{t+1} = G_X^B(X_t, \epsilon_{t+1})$

$$
\begin{aligned}
E_t^B[X_{t+1}] &= E_T^B[G^X(X_t, \epsilon_{t+1})] \\
&= E_t^B[\Gamma X_t + \epsilon_{t+1}] = m\Gamma E_t[X_t]
\end{aligned}
$$

- $\Rrightarrow$ optimal action under behavioral expectation:
  $a^B = g(X_t, E_t^B[X_{t+1}], ...)$
- $\Rrightarrow$ ture law of motion of state valiable: $X_{t+1} = G_X(X_t, \epsilon_{t+1})$ ....
- find a fixed point $\Rrightarrow$ euqilibrium!

Microfoundation of Cognitive Discounting

- In our baseline model, cognitive discounting parameters are exogenously given, lacking microfoundation.
- I am going to introduce some behavioral setup
- Provide a microfoundation for cognitive discounting

Passive learning

- Previously, we assumed that agents have **full information** about X_t at period t.
- However, it is unreasonable.
- We relax this unreasonable assumption, modeling it as an passive learning problem for the agent.
- Expectation formation in the passive learning model is an exogenous process， which does not include agent's informative choices.
- For expample,there are signal extraction models, and calvo pricing-like information update models.

Signal extraction models

- Suppose there is a true value of state valiable x_t, drawn from a Gaussian distribution $\mathcal{N}(0, \sigma_x^2)$
- However, the agent does not know this true value, and instead he receives a signal: $s_t = x_t + \varepsilon_t$, ε_t is drawn from $\mathcal{N}(0, \sigma_\varepsilon^2)$
- Each period agent learns x_t by **Kalman filter**:

$$\mathbb{E}[x_t \mid s_t] = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2} s_t$$

$$\mathbb{E}^B[x_t|s_t = s] = \mathbb{E}[x_t \mid s_t = s] = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2} s_t$$

$$\mathbb{E}[s_t|x_t] = x_t + \mathbb{E}[\varepsilon_t \mid x_t] = x_t$$

$$\mathbb{E}^B[x_t \mid x_t] = \mathbb{E}[\mathbb{E}[x_t|s_t]|x_t] = \mathbb{E}[\frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2} s_t|x_t] = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2} x_t$$

- Denoting $m = \frac{\sigma_x^2}{\sigma_x^2+\sigma_\varepsilon^2}$,we have $\mathbb{E}^B[x_t] = mX_t = mG^X X_{t-1}$
- $\frac{\sigma_x^2}{\sigma_x^2+\sigma_\varepsilon^2}$ represents how precise the signal is,**the less precise the signal, the larger the cognitive discount**

Sticky information model

- x_{it} unknown.
- Each period agent learns x_{it} by Kalman filter from an unbiased signals $s_{it} = x_{it} + \eta_{it}$ with noise $\eta_{it} \sim \text{iid} N(0, \sigma_\eta^2)$.
- Assume x_{it} follow a random walk:$x_{it} = x_{it-1} + \epsilon_{it}$,$\epsilon_{it} \sim \text{iid} N(0, \sigma_\epsilon^2)$
- Only random fraction $1 - \lambda$ of agents receive latest information in any given period[3](D.Carroll et al.,2020)

$$\bar{E}_{t+1}[x_{it+1}] = (1-\lambda)E^*_{t+1}[x_{it+1} \mid s^{it+1}] + \lambda \tilde{E}_t[x_{it+1}]$$

- $E^*_{t+1}[x_{it+1} \mid s^{it+1}]$ represents the conditional expectation of x_{it+1} for the agent in period $t+1$ after acquiring information.
- $\tilde{E}_t[x_{it+1}]$ represents the sticky expectation of x_{it+1}in period t. Due to the updated agent being randomly selected, $\tilde{E}_t[x_{it+1}] = \bar{E}_t[x_{it}]$.

Application in passive learning model

- Two different signal channel: Each agent receives a public signal z_t and private signal s_{it} that reveal additional information about the state

$$z_t = x_t + \eta_t^z \quad , \eta_t^z \sim N(0, \sigma_z^2)$$
$$s_{it} = x_t + \eta_{it}^s \quad , \eta_{it}^s \sim N(0, \sigma_s^2)$$

  The signals' precisions σ_z^2 and σ_s^2 are equal across agents.
- Assume optimal action:$a_{it}^* = (1-r)E[x_t \mid I_{it}] + rE[a|I_{it}] = (1-r)\bar{E}[x_t] + r\bar{E}[a]$ infered public signal become more important when r is raised.[8](G.Angeletosa; Chen Lian,2023)

Application in passive learning model

- It's a quantitative sovereign default model with bayesian-learning agent.
- Evolution of output:$y_t = \mu_j * (1-\rho) + \rho y_{t-1} + \eta\epsilon_t$
- $\{\mu_j\}_{\mathrm{j=L,H}}$ are economic environment parameters of the two regimes which changes according to a Markov process
- The real regime u_j is drawn, agents observe y and form a new belief p_t about the regime by bayesian rule.(p is agents' perceived probability of being in the High regime,i.e. $p_t = Prob(\mu_j = \mu_H)$)

$$p'(y,p,y') = \frac{[p\pi(\mu_H \mid \mu_H) + (1-p)\pi(\mu_H \mid \mu_L)]f_{\mu_H}(y',y)}{\sum_{\mu' - \mu_L,\mu_H}[p\pi(\mu' \mid \mu_H) + (1-p)\pi(\mu' \mid \mu_L)]f_{\mu'}(y',y)}$$

- This model matches the delayed reaction of bond spreads after the 2008 European debt crisis[12](Paluszynski,2023)

Active learning

- Active learning means that agents make choices to influence their future information sets,which means trade-off(information cost) need to be introduced into model.
- The two most commonly used learning technologies: Sticky information model & Rational inattention model.

Sticky information model with information acquisition cost

- Most of the time agents get no information flow,occasionally,however,agents observe the entire history of events.

$$L = \text{Min} E_0[\sum_{t=0}^{\infty}(\beta^t[U(a^*) - U(a)] + D_{it}\kappa_{it})]$$

$$\text{optimal action } a^* = g(x_t|I_{it})$$

$$\kappa_{it} = \begin{cases} \kappa & \text{with prob. } 1-\lambda \\ 0 & \text{with prob. } \lambda \end{cases}$$

- Setting $\kappa = \infty$ and $\lambda > 0$,this information cost structure generates the (passive learning) sticky information model
- Sticky information reduces information cost from the amount of information to the number of times information is obtained, which significantly simplifies the computational problem of dynamic information choice.

Rational inattention model

- Unlike the simplification of the sticky information model, the rational inattention model measures the amount of information in terms of information entropy.[9](Mackowiak;Wiederholt,2015)
- Mutual information(reduction of information entropy):

$$I(x,s) = H(x) - H(x \mid s)$$
$$H(x) = \frac{1}{2}\log_2(2\pi e \sigma_x^2)$$
$$H(x \mid s) = \frac{1}{2}\log_2(2\pi e \sigma_{x|s}^2)$$

$$L = \text{Min} E_0[\sum_{t=0}^{\infty}(\beta^t[U(a^*) - U(a)] + c * \kappa_{it})]$$
$$\text{such that } 0 \leq I(x,s) \leq \kappa$$
$$\text{optimal action } a^* = g(x_t | I_{it})$$

- The computational problem of dynamic information choice is more complicated,(Makowiak;Wiederholt,2018)[10](Miao et al.,2019)[4](Afrouzi;Yang,2021)[1]propose algorithms to solve them.

calvo-pricing in NK model

- standard model:

$$\text{opyimal pricing:} p_{jt}^{*} = E_{jt}^{\text{Rational}}[\sum_{h=0}^{\infty}(\alpha\beta)^{h}((1-\alpha\beta)\hat{mc}_{t+h} + \alpha\beta\hat{\pi}_{t+h})]$$

$$\text{standard phillips curve:} \pi_t = \kappa\hat{mc}_t + \beta E_t\hat{\pi}_{t+1}$$

Mis-specified forecast rule:Extrapolation

- Mis-specified learning model:firms form inflation forecasts based on simple autoregressive rules./citename4(Ina Hajdini,2023)

  mis-specified inflation forecasts rule(δ, γ unknown):

  $\pi_t = \delta + \gamma(\hat{\pi}_{t-1} - \delta) + \epsilon_t$

  inflation expectation formation:

  $E_t^{\text{mis-specified}} \hat{\pi}_{t+h} = \delta(1 - \gamma^{h+1}) + \gamma^{h+1} \hat{\pi}_{t-1}$

  opyimal pricing:

  $$p_{jt}^* = E_{jt}^{\text{mis-specified}} [\sum_{h=0}^{\infty} (\alpha\beta)^h ((1 - \alpha\beta)\hat{mc}_{t+h} + \alpha\beta\hat{\pi}_{t+h})]$$

  actual law of motion for inflation:

  $$\pi_t = \beta\delta(\frac{1-\alpha}{1-\alpha\beta} - \frac{(1-\alpha)\gamma^2}{1-\alpha\beta\gamma}) + \frac{\kappa}{1-\alpha\beta\rho}\hat{mc}_t + \frac{\beta(1-\alpha)\gamma^2}{1-\alpha\beta\gamma}\hat{\pi}_{t-1}$$

- δ, γ will be pinned down using the solution concept of a CE equilibrium

Mis-specified forecast rule:Extrapolation

- Full information rational equilibrium:perceived distribution of inflation euqals its actual/realized distribution.
- Consistent expectations equilibrium: perceived unconditional moments of the distribution with the actual unconditional moments.
- Then,there exists a unique consistent expectations equilibrium(δ^*,γ^*),where $\delta^* = 0$ and $\gamma^* \geq \rho$.

Adaptive learning model

- Agent chooses parametersγ_t in learning strategy:$\beta_t = \beta_{t-1} + \gamma_t(\pi_t - \beta_{t-1})$
  to minimize the expected error:Min$E(\pi_t - \beta_{t-1})^2$,$\beta_t = E_t^s(\pi_{t+1})$
- Two common learning strategies:/citename13(W.Evans;Garey Ramey,2006)
  - RLS(Recursive Least Squares Learning):$\gamma_t = t^{-1}$,$\beta_t = \beta_{t-1} + t^{-1} * (\pi_t - \beta_{t-1})$
  - CG-LS(Constant Gain Least Squares Learning):$\gamma_t = \gamma$,$\beta_t = \beta_{t-1} + \gamma * (\pi_t - \beta_{t-1})$,$\gamma$ will be pinned down by Min$E(\pi_t - \beta_{t-1})^2$.

/

Level-k thinking

- [13](Emmanuel Farhi;Ivan Werning,2019)

more and more behavioral friction

- fading memory
- sentiment
- overconfidence
- projection bias
- ....

More general algorithm: Sparse Dynamic Programming

- When we were tackling the model with cognitive discount, our usual approach was to start by linearizing the system and then plugging it into Dynare to solve it using algorithms like the BK method.
- However,doing linearization is error-prone
- More general approach: Sparse Dynamic Programming
- The agent’ s rational problem is:

$$\max_{\{a_t\}} \sum_{t=0}^{\infty} \beta^t u(a_t, z_t) \text{ s.t. } z_{t+1} = F^z(a_t, z_t, \varepsilon_{t+1})$$

$$V^r(z) = \max_a \{u(a, z) + \beta \mathbb{E}[V^r(F^z(a, z, \varepsilon_{t+1}))]\}$$

- refer to (Gabaix,2016) [?]

More general algorithm: Sparse Dynamic Programming

- an attention-dependent extension of the utility function: $v(a, x, m) := v(a, m_1 x_1, \ldots, m_n x_n)$

$$\max_{\{a_t\},\{m_t\}} \sum_{t=0}^{\infty} \beta^t [u(a_t, z_t, m_t) - \mathscr{C}(m_t)] \text{ s.t. } z_{t+1} = F^z(a_t, z_t, \varepsilon_{t+1}, m_t)$$

$$V^r(z) = \max_{a,m} \{u(a, z, m) - \mathscr{C}(m) + \beta \mathbb{E}[V^r(F^z(a, z, \varepsilon_{t+1}, m))]\}$$

- This problem is much more complex than the original problem (we are threatened by "infinite regress" problem)
- In that formulation, the BR agent is very sophisticated about his own future behavior: he sees how much he will see how much he will see (etc. iterated times) future inattention.

More general algorithm: Sparse Dynamic Programming

- The sparse dynamic programing is defined by the following procedure:

Step 1: Choose the attention vector m^* :

$$m^* = \arg\min_{m\in[0,1]^n} \sum_i [\frac{1}{2}\Lambda_{ii}(1-m_i)^2 + C(m_i)]$$

with the cost-of-inattention factors$\Lambda_{ii} := -\mathbb{E}[a_{m_i} v_{aa} a_{m_i}]$

Step 2: Choose the optimal action

$$a^s = \arg\max_a \{u(a, m^* z) + \beta\mathbb{E}[V^r(F^z(a, z, \varepsilon_{t+1}))]\}$$

Generational heterogeneity

- Generational heterogeneity arises naturally due to differences in experience between generations.[11](Ulrike Malmendier;Stefan Nagel,2016)

perceived law of motion:$\pi_{t+1} = \alpha + \Phi\pi_t + \eta_{t+1}$

estimate parameters of perceived law by adaptive learning:$b = [\alpha, \Phi]^{\mathsf{T}}$

$$b_{ts} = b_{t-1s} + \gamma_{ts} R_{ts}^{-1} x_{t-1}(\pi_t - b_{t-1s}^{\mathsf{T}} x_{t-1})$$

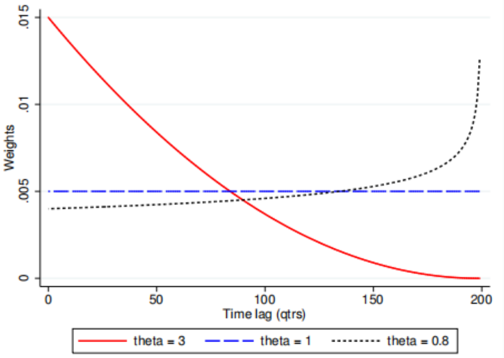
$$R_{ts} = R_{t-1s} + \gamma_{ts}(x_{t-1}^{\mathsf{T}} x_{t-1} - R_{t-1s})$$

decreasing-gain least squares learning specification:

$$\gamma_{ts} = \begin{cases} \frac{\theta}{t-s} & t-s \geq \theta \\ 1 & t-s < \theta \end{cases}$$

- θ is flexible in accommodating monotonically increasing, decreasing, and flat.

Generational heterogeneity



Sticky information in HANK

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Thanks!