Cognitive discounting in Expectaion Formation

Behavioral friction in DSGE

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- Standard model : **full information and rational expectation equilibrium**(FIRE)
- Add behavioral friction : Dampening general equilibrium effect[4](G. Angeletosa& Chen Lian,2023)
- For example,changing amplification & propagation mechanism of model
- Generating a hump-like response to exogenous shocks in DSGE[5](Makowiak&Wiederholt,2015)
- Generating a overreaction to exogenous shocks in DSGE[1](P.Bordalo et al.,2020)

- We are going to be focusing on topics of **bounded rationality**(Agent has full information but deviates from rationality)
- And, in particular, cognitive discounting
- Here I am going to offer an introduction to (how I understand)**cognitive discounting**

Myopic adjustment

- Start with a standard economic model:
- e.g. A Simple Dynamic Consumption Saving Problem

$$\mathrm{Max}\, U(C) = E_0\Big[\sum_{t=0}^{\infty} \beta^t u(C_t)\Big] \quad \text{s.t.}$$

$$C_t + B_{t+1} = R_t B_t + Y_t$$

$$\ln(Y_t/\bar{Y}) = \rho_Y \ln(Y_{t-1}/\bar{Y}) + u_t$$

- Taking derivative against the control variable by Lagrangian:

$$\beta E_t[R_{t+1} C_t / C_{t+1}] = 1$$

$$C_t = Y_t$$

$$\ln(Y_t/\bar{Y}) = \rho_Y \ln(Y_{t-1}/\bar{Y}) + u_t$$

Myopic adjustment

- Log-linearize them:

$$\begin{aligned}\ln(R_{t+1}) &= -\ln(\beta) - \hat{C}_t + E_t(\widehat{C_{t+1}}) \\ &= -\ln(\beta) - \hat{Y}_t + E_t(\widehat{Y_{t+1}}) \\ &= -\ln(\beta) - \hat{Y}_t + \rho_Y \hat{Y}_t\end{aligned}$$

- $\hat{Y}_t$ increases by one unit, interest rate $\ln R_t$ decreases by $1-\rho$.
- Here we use solution concept of **full information rational equilibrium** to solve this model:

Myopic adjustment

- Here we use solution concept of **full information rational equilibrium** to solve this model:
- Agent is aware of the true structure of this dynamic system (e.g. the $In(Y_t/\bar{Y})$ follows an AR1 process).
- Agent is aware of the true parameters of this dynamic system (e.g. persistence of income shocks ρ_Y,even interest rates R_t)
- Agent is aware of the shocks realized in each period (e.g., at time t, the agent knows Y_t)
- Agent is aware of the general equilibrium effects(level-k thinking)
- Agent possesses perfect information processing capabilities(rational inattention)
- Agent's decisions are not influenced by sentiments(e.g., higher income in this period does not lead to retaliatory consumption)
- ......

Myopic adjustment

- Let $E_t(\hat{Y_{t+1}})$ denote a correct forecasting rule under FIRE.
- $E_t^B(\hat{Y_{t+1}})$ could be associated with the forecast about future income - **in deviation from its steady state with myopic adjustment**.
- Then we specify expectation in our model: $E_t^B(\hat{Y_{t+h}}) = m^h E_t(\hat{Y_{t+h}})$, which means agent have cognitive discounting for future impact,as the forecasting horizon h increases, the cognitive discounting becomes more severe.[3]
- Log-linearize them:

$$
\begin{aligned}
\ln(R_{t+1}) &= -\ln(\beta) - \hat{Y}_t + E_t^B(\hat{Y_{t+1}}) \\
&= -\ln(\beta) - \hat{Y}_t + mE_t(\hat{Y_{t+1}}) \\
&= -\ln(\beta) - \hat{Y}_t + m\rho_Y \hat{Y}_t
\end{aligned}
$$

- $\hat{Y}_t$ increases by one unit, interest rate $\ln R_t$ decreases by $1 - m\rho$.
- Introducing cognitive discounting changes agent's consumption-saving problem,and changes interest rates/asset prices.

Expectation formation

- There is a state vector X_t that will evolve in equilibrium as:

$$\begin{aligned} X_{t+1} &= G^X(X_t, \epsilon_{t+1}) \\ &= \Gamma X_t + \epsilon_{t+1} \quad \text{linearizing} \end{aligned}$$

- The agent perceives that the state vector evolves as:

$$\begin{aligned} E_t^B[X_{t+1}] &= E_T^B[G^X(X_t, \epsilon_{t+1})] \\ &= E_t^B[\Gamma X_t + \epsilon_{t+1}] = m\Gamma E_t[X_t] \end{aligned}$$

- For any variable $Z(X_t)$with$Z(0) = 0$:(Linearizing,we have $Z(X_t) = b^Z X_t$)

$$\begin{aligned} E_t^B[Z(X_{t+1})] &= E_t^B[b^Z X_{t+1}] \\ &= b^Z E_t^B[X_{t+1}] = mb^Z E_t[X_{t+1}] = mE_t[b^Z X_{t+1}] \\ &= mE_t[Z(X_{t+1})] \end{aligned}$$

Household problem

- Representative agent:

$$MaxU = E_0^B \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$$

$$u(C_t, N_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\Phi} - 1}{1+\Phi}$$

$$C_t = (\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}}$$

$$\int_0^1 P_{it} C_{it} \, di + B_{t+1} = (1+i_t)B_t + W_t N_t + T_t$$

- refer to appendix for the solving process
- BDIS:$\widetilde{y_t} + \frac{1}{1+h} m\widetilde{y_{t+1}} - \frac{1}{\sigma}(i_{t+1} - mE_t ln\Pi_{t+1} - r_{t+1}^n)$

Firm problem

- Firms with monopolistic competetion and calvo-pricing friction:

$$Y_{it} = A_t N_{it}$$
$$ln(A_{t+1}) = \rho_A ln(A_t) + u_{t+1}$$
$$P_{it+1} = \begin{cases} P_{it}\Pi_t^{\omega} & \text{with Prob.}\theta \\ P^*_{it+1} & \text{with Prob.}1-\theta \end{cases}$$

- **Dynamic pricing**:Firms choose optimal price to maximize the sum of the present value of all future real profits

$$Max\text{Profit} = E_t^B \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}[P_i t Y_i t - W_t N_{it}]$$

$$Q_{t,t+k} = \beta^k \frac{C_{t+k}^{-\gamma}}{C_t^{-\gamma}} \frac{P_t}{P_{t+k}} \quad Q_{t,t+k} \text{ is stochastic discounting factor}$$

Firm problem

- Firms with monopolistic competetion and calvo-pricing friction:
- refer to appendix for the solving process
- BNKPC:$ln(\Pi_t) = Aln(\Pi_{t-1}) + BE_t ln(\Pi_{t+1}) + C\widetilde{y}_t$
- $A = \frac{\omega}{1+\omega\beta m[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta m}]}$

  $B = \frac{\beta m[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta m}]}{1+\omega\beta m[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta m}]}$

  $C = \frac{(1-\theta)(1-\theta\beta)}{\theta[1+\omega\beta m[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta m}]]}$

Monetary policy

- We adopt a price-based monetary policy rule:

$$i_t = (1-\rho_i)i^{ss} + \rho_i i_{t-1} + (1-\rho_i)(\Phi_\Pi lnE_t\Pi_{t+1} + \Phi_y \widetilde{y}_t)$$

- combining monetary policy rule, BNKPC and BDIS to form the three-equation NK model

$$\begin{cases} \widetilde{y}_t + \frac{1}{1+h} m\widetilde{y_{t+1}} - \frac{1}{\sigma}(i_{t+1} - mE_t ln\Pi_{t+1} - r^n_{t+1}) \\ ln(\Pi_t) = Aln(\Pi_{t-1}) + BE_t ln(\Pi_{t+1}) + C\widetilde{y}_t \\ i_t = (1-\rho_i)i^s s + \rho_i i_{t-1} + (1-\rho_i)(\Phi_\Pi lnE_t\Pi_{t+1} + \Phi_y \widetilde{y}_t) \end{cases}$$

Endogenizing Cognitive Discounting

- In our baseline model, cognitive discounting parameters are exogenously given, lacking micro mechanisms similar to Calvo pricing.
- Here I am going to introduce a simple behavioral setup
- Endogenizing cognitive discounting as an individual attention optimization problem!

A possible foundation: Dispersed Information

- Previously, we assumed that agents have **full information** about X_t at period t.
- However, it is unreasonable.
- We relax this unreasonable assumption, modeling it as an signal extraction problem for the agent.

$$X_t = G^X(X_{t-1}, \epsilon_t) \quad \text{ture law of motion}$$
$$X_t^B = mG^X(X_{t-1}, \epsilon_t) \quad \text{perceived law of motion}$$

A possible foundation: Dispersed Information

- Now, we assume that the agent cannot directly access X_t but instead gains information about X_t through a signal Y_t.
- At period t, agent observes the signal Y_t and infers X_t based on Y_t.

$$Y_t = \begin{cases} X_t & \text{with Prob.}\theta \\ X'_t & \text{with Prob.}1-\theta \end{cases}$$

- X'_t is drawed from the unconditional distribution of X_t
- Normalizing throughout the unconditional mean of X_t to be $\bar{X} = 0$

A possible foundation: Dispersed Information

$$E^B[X_t|Y_t = y] = E[X_t|Y_t = y] = \theta y$$
$$E[Y_t|X_t] = \theta X_t + (1-\theta)E[X_t'] = \theta X_t + (1-\theta)\bar{X} = \theta X_t$$
$$E^B[X_t|X_t] = E[E[X_t|Y_t]|X_t] = E[\theta Y_t|X_t] = \theta^2 X_t$$

- Denoting $m = \theta^2$,we have $X_t^B = mX_t = mG^X X_{t-1}$
- θ represents how precise the signal is,**the less precise the signal, the larger the cognitive discount**
- Using the simple signal extraction model, we provide a microfoundation for cognitive discount
- In more general scenario, the signal consists of X_t and a noise variable ϵ_t.Agent infers X_t through Kalman filtering

More general algorithm: Sparse Dynamic Programming

- When we were tackling the NK model with cognitive discount, our usual approach was to start by linearizing the system and then plugging it into Dynare to solve it using algorithms like the BK method.
- However,doing linearization is error-prone
- More general approach: Sparse Dynamic Programming

$$v(a_t, X_t, m) = u(a) + \beta E[V(GS(X_t, a_t, m, \epsilon_{t+1}), m)]$$
$$\max_m Ev(a(m, X_t), X_t, 1) - \kappa g(m - m^d)$$

- refer to (Gabaix,2016) [2]

[1] P.Bordalo et al.
Overreaction in macroeconomic expectations.
AMERICAN ECONOMIC REVIEW, 2020.

[2] Xavier Gabaix.
Behavioral macroeconomics via sparse dynamic programming.
NBER working paper, 2016.

[3] Ina Hajdini.
Mis-specified forecasts and myopia in an estimated new keynesian model.
NBER working paper, 2023.

[4] G.Angeletosa; Chen Lian.
Dampening general equilibrium:incomplete information and bounded rationality.
Handbook of Economic Expectations, 2023.

[5] Mackowiak;Wiederholt.
Business cycle dynamics under rational inattention.
Review of Economic Studies, 2015.

Thanks!