Doubly Robust Estimators with Weak Overlap

A Robust Strategy for Causal Inference

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Outline

The Dilemma: Doubly Robustness vs. Weak Overlap

The Appeal of Doubly Robust (DR) Estimators

DR estimators (e.g., AIPW) are popular in causal inference because:

- They remain consistent if **either** the Propensity Score Model **or** the Outcome Regression Model is correctly specified.
- They offer "double protection" against misspecification.

The Problem: Weak Overlap

However, when propensity scores $P(X)$ are close to 0 or 1:

- The IPW weights ($\frac{1}{P(X)}$ or $\frac{1}{1-P(X)}$) become extremely large.
- **Consequence:** The estimator suffers from **Variance Explosion** and instability.

The Cost of Existing Solutions

How do researchers typically handle weak overlap?

Current Practice: Fixed Trimming

- *Method:* Drop observations with scores outside $[0.1, 0.9]$.
- *Literature:* Crump et al. (2009), Yang & Ding (2018).

The Hidden Cost

- It **changes the estimand**.
- You are no longer estimating the ATE for the whole population, but only for the sub-population with good overlap.
- Risk: You might exclude the individuals who need the intervention the most.

Goal: Solve the variance explosion without changing the target parameter.

Core Intuition: "Trim-then-Bias-Correct"

The authors propose a two-step strategy to handle extreme weights:

1. **Dynamic Trimming:**
   - Use a moving threshold h_n that shrinks to 0 as sample size $n \to \infty$.
   - Temporarily remove observations causing variance explosion.
   - *Function: Stabilizes Variance.*
2. **Bias Correction:**
   - Acknowledge that trimming introduces bias.
   - Use **Sieve Regression** to estimate the contribution of the trimmed tails based on the behavior of the remaining data.
   - *Function: Recovers the Original Estimand (ATE/ATT).*

The Proposed Estimator

The new estimator $\hat{\theta}$ consists of two components:

$$\hat{\theta} = \underbrace{\text{Trimmed Mean}}_{\text{Stabilization}} + \underbrace{\text{Bias Correction}}_{\text{Recovery}}$$

- **Part 1: The Trimmed Mean**

$$E_n\left[\frac{B_l(\gamma)}{A_l(\gamma)}\mathbb{I}\{|A_l(\gamma)| \geq h\}\right]$$

  Only keeps observations where the denominator $|A|$ is larger than threshold h.

- **Part 2: The Bias Correction Term**

$$\sum_{\kappa=1}^{k} \frac{E_n[A^{\kappa-1}\mathbb{I}\{|A| < h\}]}{\kappa!} \cdot \hat{m}^{(\kappa)}(0)$$

  Uses polynomial approximation (Taylor expansion) to impute the missing contribution of the trimmed data.

Theoretical Properties

The authors prove that this estimator is well-behaved:

Asymptotic Normality

Provided the trimming threshold h converges to 0 at a specific rate:

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V)$$

Implications:

- Despite the "weak overlap" and the trimming procedure, the final estimator is normally distributed.
- We can validly construct **Confidence Intervals** and perform **t-tests**.

Application: Difference-in-Differences (DiD)

This framework extends the DR-DiD method by Sant'Anna & Zhao (2020).

The Challenge in DR-DiD

- The estimator relies on weights like $\frac{1}{1-P(X)}$.
- If some control units look very similar to treated units ($P(X) \approx 1$), the weights explode.

The Improvement

- Applies the trimming + correction strategy to the ATT estimator.
- Allows for robust estimation of ATT even when covariate overlap between treated and control groups is poor.

It acts as a "safety net" for DR-DiD against data irregularity.

Simulation Results

Comparing the **Conventional (CON)** method vs. the **New (NEW)** method under weak overlap (DGP2):

Statistic	Conventional (CON)	**New (NEW)**	Interpretation
Bias	0.000	**-0.087**	Both have acceptable bias
Std. Dev (SD)	268.8	**0.257**	**Variance reduced by ∼1000x!**
RMSE	268.8	**0.257**	Massive gain in precision
95% Coverage	0.947	**0.958**	More accurate inference

Table: Performance under Severe Weak Overlap (df=10)

* Note: CON refers to the standard Sant'Anna & Zhao (2020) estimator.

Conclusion

1. **Awareness:** Be cautious of variance explosion in IPW/DR methods when propensity scores are extreme.
2. **Innovation:** The **"Trim-then-Bias-Correct"** strategy solves instability without sacrificing the target parameter (Estimand).
3. **Practical Value:**
   - Provides a robust alternative when reviewers question "Common Support".
   - Allows researchers to retain the full sample for causal inference.

Thank You!