

WFF Guide

There are three types of cubes in WFF N Proof: Sentence variables, which are the main part of well formed formulas and are similar to (but not exactly like) nouns in English; Connectives, which hold the sentence variables together; and Blanks which are combined with connectives to form words.

Sentence Variables: p, q, r, and s

Connectives: C, A, K, E, and N,

Blanks: R, i, and o

The sentence variables (p, q, r, and s) are all well formed formulas (WFFs) by themselves. Two WFFs joined by a connective (excluding N) **in front** of them is also a WFF.

N is a leech letter. Any WFF with N in front of it is still a WFF. There is no way to take an N out of a WFF or put an N into a WFF, in Basic WFF N' Proof.

Examples of WFFs: p, q, Kpq, Asr, Ars, KArsErp, ECCrpqp, KKKKKKKKAAAAAApppppssssssrrrr, Np, Ns, NNKKpqs, NNNNNNNNKKKrspq

Not WFFs: pq, qps, Eqps, ssEErr, KKKKssqqKs, KKAECKKKsq

The goal of WFF N Proof is to transform a supposition(sometimes known as a "Said-So"), your starting point or given, into the goal, which has to be a WFF. You do this by using rules, which help you change around WFFs.

These rules are a connective followed by an i or an o. The rules used in basic WFF are, Ko, Ki, Ai, Co, Eo, Ei, Rp. o stands for out and i stands for in. The R cube can stand for any rule (CAKE letter and blank) and R doesn't have to be the same thing everyplace you use it. R can also stand for the R in Rp.

Ki: Ki says if you have two WFFs, then you can connect those two WFFs with a K. This rule can be stated $s, r \rightarrow Ksr$ or Krs .

Example proofs and solutions: \rightarrow

| | $s, p \rightarrow Kps$ | | |
|---|------------------------|--------|---------|
| 1 | s | s | s, p/Ki |
| 2 | p | s | |
| 3 | Kps | Ki 1,2 | |

| | $p, q, r \rightarrow KKqpKrp$ | | |
|---|-------------------------------|--------|------------|
| 1 | p | s | p, q, r/Ki |
| 2 | q | s | |
| 3 | r | s | |
| 4 | Kqp | Ki 1,2 | |
| 5 | Krp | Ki 1,3 | |
| 6 | KKqpKrp | Ki 4,5 | |

| | $p, s \rightarrow KKKpsKspKps$ | | |
|---|--------------------------------|--------|---------|
| 1 | p | s | p, s/Ki |
| 2 | s | s | |
| 3 | Kps | Ki 1,2 | |
| 4 | Ksp | Ki 1,2 | |
| 5 | KKpsKsp | Ki 3,4 | |
| 6 | KKKpsKspKps | Ki 3,5 | |

Practice proofs:

1. $p, q \rightarrow Kpq$
2. $s, p, q \rightarrow KKqsKKspKpq$
3. $s, r, p, q \rightarrow KKKspqr$
4. $r, p \rightarrow KKrpKKrpr$
5. $s, r, q \rightarrow KKsrKKqsr$

Rp: Repeat. Rp lets you repeat a WFF. The rule can be stated as $p \rightarrow p$
 Example proofs and solutions:

| $r \rightarrow r$ | | |
|-------------------|-----|------|
| 1 | r | s |
| 2 | r | Rp 1 |

r/Rp

| $p, s \rightarrow KKssp$ | | |
|--------------------------|---------|--------|
| 1 | p | s |
| 2 | s | s |
| 3 | s | Rp 2 |
| 4 | Kss | Ki 2,3 |
| 5 | $KKssp$ | Ki 1,4 |

$p, s/Rp, Ki$

| $Ksp, q \rightarrow KKKspKspq$ | | |
|--------------------------------|-------------|--------|
| 1 | Ksp | s |
| 2 | q | s |
| 3 | Ksp | Rp 1 |
| 4 | $KKspKsp$ | Ki 1,3 |
| 5 | $KKKspKspq$ | Ki 2,4 |

$Ksp, q / Rp, Ki$

Practice proofs:

1. $s \rightarrow Kss$
2. $p \rightarrow KKKpppKpp$
3. $s, q \rightarrow KsKKssKqqq$
4. $r, p \rightarrow KKrpKpp$
5. $s, p, r \rightarrow KpKKpsKKrrKss$

Ko: “When in doubt, Kout.” Ko says that if you have a K WFF (any WFF in which K is the left most connective) you can get either of the two smaller WFFs contained in the K WFF. This rule can be stated as $Ksr \rightarrow r$ or s . Example proofs and solution:

| | $Krs \rightarrow Ksr$ | | |
|---|-----------------------|--------|--------------|
| 1 | Krs | s | |
| 2 | s | Ko 1 | $Krs/Ko, Ki$ |
| 3 | r | Ko 1 | |
| 4 | Ksr | Ki 1,2 | |

| | $KKspq \rightarrow KKKsqKqpKKspq$ | | |
|---|-----------------------------------|--------|----------------|
| 1 | $KKspq$ | s | |
| 2 | q | Ko 1 | |
| 3 | Ksp | Ko 1 | $KKspq/Ki, Ko$ |
| 4 | s | Ko 3 | |
| 5 | p | Ko 3 | |
| 5 | Ksq | Ki 2,3 | |
| 6 | Kqp | Ki 2,4 | |
| 7 | $KKsqKqp$ | Ki 5,6 | |
| 8 | $KKKsqKqpKKspq$ | Ki 1,7 | |

| | $Ksp, KAprq \rightarrow KpApr$ | | |
|---|--------------------------------|--------|---------------------|
| 1 | Ksp | s | |
| 2 | $KAprq$ | s | |
| 3 | p | Ko 1 | $Ksp, KAprq/Ko, Ki$ |
| 4 | Apr | Ko 2 | |
| 5 | $KpApr$ | Ki 3,4 | |

Practice Proofs

1. $KsApr \rightarrow KAprs$
2. $KCrpq, KEqps \rightarrow Ksq$
3. $KECrpsp \rightarrow KKECrpspp$
4. $KKpsr \rightarrow KKrps$
5. $KpKrs, Kqs \rightarrow KKpqKsr$

Ai: “Buy one, get one free.” Ai says if you have a WFF, you can get an A WFF with the WFF you had and a new WFF. Ai can be stated $r \rightarrow Arp$ or Apr.

Example proofs and solutions:

| | | | |
|--|----------|--------|-----------------|
| <u>KpNs \rightarrow Arp</u> | | | |
| 1 | KpNs | s | KpNs/Ko, Ai |
| 2 | p | Ko 1 | |
| 3 | Arp | Ai 2 | |
| <u>p \rightarrow AKppr</u> | | | |
| 1 | p | s | p/Rp, Ki, Ai |
| 2 | p | Rp 1 | |
| 3 | Kpp | Ki 1,2 | |
| 4 | AKppr | Ai 3 | |
| <u>KKsrNCsr \rightarrow AEprAsr</u> | | | |
| 1 | KKsrNCsr | s | KKsrNcsr/Ko, Ai |
| 2 | Ksr | Ko 1 | |
| 3 | s | Ko 2 | |
| 4 | Asr | Ai 3 | |
| 5 | AEprAsr | Ai 4 | |

Practice Proofs:

1. $s \rightarrow Ars$
2. $p \rightarrow ArKKppp$
3. $Krs \rightarrow Arp$
4. $KKprs \rightarrow AKrsq$
5. $Kqp \rightarrow KKppAKpps$

Co: “Cancel Out” Co says that if you have a C WFF and the first little WFF of the C WFF, then you can get the second little WFF of the C WFF.

This can be stated as $Cqs, q \rightarrow s$

Example proofs and solutions:

| <u>KCrpr \rightarrow p</u> | | | |
|---|-------|--------|--------------|
| 1 | KCrpr | s | |
| 2 | Crp | Ko 1 | KCrpr/Ko, Co |
| 3 | r | Ko 1 | |
| 4 | p | Co 3,4 | |

| <u>KCCrpsCrp \rightarrow s</u> | | | |
|---|-----------|--------|------------------|
| 1 | KCCrpsCrp | s | |
| 2 | CCrps | Ko 1 | KCCrpsCrp/Ko, Co |
| 3 | Crp | Ko 1 | |
| 4 | s | Co 3,4 | |

| <u>KCKqpsp, Krq \rightarrow s</u> | | | |
|--|---------|--------|-------------------------|
| 1 | KCKqpsp | s | |
| 2 | Krq | s | |
| 3 | CKqps | Ko 1 | KCKqpsp, Krq/Ko, Ki, Co |
| 4 | p | Ko 1 | |
| 5 | q | Ko 2 | |
| 6 | Kqp | Ki 4,5 | |
| 7 | s | Co 3,6 | |

| <u>CArsp, Kqs \rightarrow Kqp</u> | | | |
|--|-------|--------|---------------------------|
| 1 | CArsp | s | |
| 2 | Kqs | s | |
| 3 | s | Ko 2 | CArsp, Kqs/Ko, Ai, Co, Ki |
| 4 | Ars | Ai 3 | |
| 5 | p | Co 1,4 | |
| 6 | q | Ko 2 | |
| 7 | Kqp | Ki 5,6 | |

Practice Proofs:

1. $r, CAAssrq \rightarrow AqNp$
2. $Krp, CAsKprEps \rightarrow KEpsp$
3. $CArps, Cqr, Krq \rightarrow Ksr$
4. $Crp, Csr, Ksq \rightarrow KApEqrq$
5. $CpCrq, Csp, Ksr \rightarrow KKsrq$

Ei: E in says Cqs, Csq \rightarrow Eqs or Esq. Some other examples of Ei are CrErq, CErqr \rightarrow CrErq or CErqr and Cpq, Cqp \rightarrow Cqp or Cpq. Both C WFFs are needed to get the E WFF.

Example Proofs and Solutions:

| Cqr, KCrqs \rightarrow Erq | | |
|------------------------------|-------|--------|
| 1 | Cqr | s |
| 2 | KCrqs | s |
| 3 | Crq | Ko 2 |
| 4 | Erq | Ei 1,3 |

Cqr, KCrqs/Ko, Ei

| KCsqq, Cqs \rightarrow KEsqKsq | | |
|----------------------------------|---------|--------|
| 1 | KCsqq | s |
| 2 | Cqs | s |
| 3 | Csq | Ko 1 |
| 4 | Esq | Ei 2,3 |
| 5 | q | Ko 1 |
| 6 | s | Co 2,5 |
| 7 | Ksq | Ki 5,6 |
| 8 | KEsqKsq | Ki 4,7 |

KCsqq, Cqs/Ko, Ki, Co, Ei

| KCEsqrCsq, p, CpCqs \rightarrow ANrr | | |
|--|-----------|------------|
| 1 | KCEsqrCsq | s |
| 2 | p | s |
| 3 | CpCqs | s |
| 4 | Cqs | Co 2 and 3 |
| 5 | Csq | Ko 1 |
| 6 | Esq | Ei 4 and 5 |
| 7 | CEsqr | Ko 1 |
| 8 | r | Co 6 and 7 |
| 9 | ANrr | Ai 8 |

KCEsqrCsq, p, CpCqs/
Co, Ko, Ki, Ai

Practice Proofs:

1. KCrCr \rightarrow Err
2. Cps, KpCsp \rightarrow Esp
3. Cps, KpCsp \rightarrow KsEps
4. Csr, Crs \rightarrow Esr
5. KEsrCqp, Cpq \rightarrow KEqpEsr

Eo: Eo says if you have an E WFF, then you can get a C WFF with the two little WFFs of the E WFF reversed. This rule can be stated as $Erp \rightarrow Cpr$ or Crp .

Example proofs and solutions:

| <u>Eps, $s \rightarrow p$</u> | | |
|--|-----|---------|
| 1 | Eps | s |
| 2 | s | s |
| 3 | Csp | Eo 1 |
| 4 | p | Co 2, 3 |

Eps, s/Eo, Co

| <u>Erp \rightarrow KCrpCpr</u> | | |
|---|---------|--------|
| 1 | Erp | s |
| 2 | Crp | Eo 1 |
| 3 | Cpr | Eo 1 |
| 4 | KCrpCpr | Ki 2,3 |

Erp/Eo, Ki

| <u>KEspp \rightarrow ANKrps</u> | | |
|--|--------|--------|
| 1 | KEspp | s |
| 2 | Esp | Ko 1 |
| 3 | Cps | Eo 2 |
| 4 | p | Ko 1 |
| 5 | s | Co 3,4 |
| 6 | ANKrps | Ai 5 |

KEspp/Ko, Eo, Co, Ai

| <u>KEsqApr \rightarrow AKEqsAprKrp</u> | | |
|---|-------------|---------|
| 1 | KEsqApr | s |
| 2 | Esq | Ko 1 |
| 3 | Csq | Eo 2 |
| 4 | Cqs | Eo 2 |
| 5 | Eqs | Ei 3, 4 |
| 6 | Apr | Ko 1 |
| 7 | KEqsApr | Ki 5,6 |
| 8 | AKEqsAprKrp | Ai 7 |

KEsqApr/Ko, Eo, Ei, Ko, Ki, Ai

Practice Proofs:

1. $Erp \rightarrow KCrpCpr$
2. $Esq \rightarrow Eqs$
3. $EqKrs, q \rightarrow s$
4. $Erp \rightarrow KCrpAEprAss$
5. $KCprEpr \rightarrow Erp$

How To Present A Solution

A solution consists of your proof, and the cubes you used (including the rules). In addition to a solution, you must also write a proof:

| | Given \rightarrow Goal | | |
|---|--------------------------|-----------|--------------------|
| 1 | Given | s | Given/RuleA, RuleB |
| 2 | ~~~~ | RuleA 1 | |
| 3 | ---~ | RuleB 1,2 | |

When writing a solution you should

- 1) Number your lines (this will make it easier for the checker to check)
 - 2) Include the line numbers with which you used the rule (Ko uses one line, Ki uses two lines, Ai uses one line, Co uses two lines, Ei uses two lines, and Eo uses one line)
 - 3) Put a line after your given in your proof
 - 4) Put arrows under the cubes in essentials in your solution.
- Remember, each proof must have at least one rule, or it's wrong, and if you use a rule twice, you only need to list it once in your solution.

Setting the Goal: You have two minutes to set the goal. The maximum number of cubes in the goal is seven. A goal must be a WFF, or it is a Never.

The Playing Mats:

The Equations playing mat, with Required, Permitted and Forbidden is used to play WFF N Proof. The permitted and forbidden section operate exactly as they do in Equations. The required section, however, is slightly different. When a cube is placed in the required section in WFF, it must be an essential part of any acceptable solution. If you can take a cube that is in required out of a solution and the solution still works, that cube is non-essential, and therefore the solution is incorrect. The cube(s) you take out must be a WFF or a rule, not a part of a WFF or part of a rule. You are allowed to take out as many rules and WFFs out of a solution as you want, but you can not add any rules. To prove non-essentiality, you must write a proof showing that the parts you took out are non-essential. If you can prove a cube that is in essentials is non-essential, then the solution is wrong. Here is an example of this:

Someone calls Now on you and presents a solution like this

| <u>Krp \rightarrow KKrpKrp</u> | | |
|---|---------|----------------------------|
| 1 | Krp | s |
| 2 | r | Ko 1 Krp/Ko, Ki, Rp |
| 3 | p | Ko 1 \uparrow \uparrow |
| 4 | Krp | Ki 2,3 |
| 5 | Krp | Rp 1 |
| 6 | KKrpKrp | Ki 1,5 |

You say that that the K in essentials is non-essential. Here is your proof:

| <u>Krp \rightarrow KKrpKrp</u> | | |
|---|--------------|-------------------|
| 1 | Krp | s Krp/Rp, Ki |
| 2 | \bar{K} rp | Rp 1 |
| 3 | KKrpKrp | Ki 1,2 |

You have proven your opponent is wrong, and they lose the shake.

Regular WFF

In addition to all the basic WFF Rules, regular WFF uses five new rules, No, Ni, Ao, Ci, and R(reiterate). All regular rules require the use of sub proofs, or proofs within a proof. You can start a sub proof with any given that you want to, but you cannot take something that is in a sub proof back out to the regular proof, unless you are using one of the five regular rules.

R(Reiterate)

Reiterate is similar to Repeat in Basic Wff, except that Reiterate allows you to repeat a Wff from the regular proof into a sub proof within that proof.

No(N Out)

N Out states that if you start a sub proof with an N WFF, and within that sub proof you prove a contradiction(p, Np for example), then you can take the N out of the WFF. $(Nr \rightarrow s, Ns) \rightarrow r$

| r, Nr \rightarrow KCrqApr | | |
|-----------------------------|--------------|------|
| 1 | r | s |
| 2 | Nr | s |
| 3 | a NKCrqApr | s |
| | b r | R, 1 |
| | c Nr | R, 2 |
| 4 | KCrqApr | No 3 |

r, Nr/R, No

| Crs, r, Ns \rightarrow Kpq | | |
|------------------------------|----------|--------|
| 1 | Crs | s |
| 2 | r | s |
| 3 | Ns | s |
| 4 | s | Co 1,2 |
| 5 | a NKpq | s |
| | b s | R,4 |
| | c Ns | R,3 |
| 6 | Kpq | No 5 |

Crs, r, Ns/Co, R, No

Practice Problems:

1. Eps, Ns, $p \rightarrow Aqr$
2. CApsNr, $Kpr \rightarrow NKqq$
3. KKEqrCrNqr \rightarrow Ess
4. $NNp \rightarrow p$
5. $KsNs \rightarrow \uparrow$ (show how that given can lead to any goal)

Using Reiterate and No instead of using Rp

In regular WFF it is not necessary to use the Rp rule if you are also using No in your proof. The following proof accomplishes the same thing as Rp, without using that rule:

| | | | |
|---|--|-------------------|--------|
| | | $p \rightarrow p$ | |
| 1 | | p | s |
| 2 | | a Np | s |
| | | I NNp | s |
| | | II p | R,R, 1 |
| | | III Np | R, a |
| | | b Np | No, a |
| | | c p | R, 1 |
| 3 | | p | No 2 |

This proof is especially useful when an opponent uses Rp in their proof and the R or the p cube is in essentials. You can then prove them wrong by using the proof above to show that the p cube is non-essential.

For example, suppose an opponent presents the following solution:

| | | | |
|---|--|---------------------|-------|
| | | $NNq \rightarrow q$ | |
| 1 | | NNq | s |
| 2 | | a Nq | s |
| | | b Nq | Rp, a |
| | | c NNq | R, 1 |
| 3 | | q | No 2 |

NNq/R, Rp, No
↑ ↑↑

You could prove the solution wrong by stating the R and p cubes were not essential and providing the following proof:

| | | | |
|---|--|---------------------|--------|
| | | $NNq \rightarrow q$ | |
| 1 | | NNq | s |
| 2 | | a Nq | s |
| | | b I NNq | s |
| | | II NNq | R,R, 1 |
| | | III Nq | R, a |
| | | c Nq | No, b |
| | | d NNq | R, 1 |
| 3 | | q | No 2 |

NNq/R, No

Ni(N in)

N In states that if you start a sub proof with any WFF, and within that sub proof you prove a contradiction(r, Nr for example), then you can put an N in the front of the WFF. $(r \rightarrow s, Ns) \rightarrow Nr$

| <u>$r, Nr \rightarrow NKCrqApr$</u> | | | |
|--|----------|-------------------------------|-------------|
| 1 | r | s | r, Nr/R, Ni |
| 2 | Nr | s | |
| 3 | a | <u>$KCrqApr s$</u> | |
| | b | r | R, 1 |
| | c | Nr | R, 2 |
| 4 | NKCrqApr | | Ni 3 |

| <u>$Krq \rightarrow NCrNq$</u> | | | |
|---|-------|--------------------------|-------------------|
| 1 | Krq | s | Krq/Ko, Co, Ni, R |
| 2 | r | Ko 1 | |
| 3 | q | Ko 1 | |
| 4 | a | <u>$CrNq$</u> | s |
| | b | r | R 2 |
| | c | Nq | Co a,b |
| | d | q | R 3 |
| 5 | NCrNq | | Ni 4 |

| <u>$Epr, r, Np \rightarrow NKpq$</u> | | | |
|---|------|-------------------------|--------------------------|
| 1 | Epr | s | Epr, r, Np/Eo, Co, R, Ni |
| 2 | r | s | |
| 3 | Np | s | |
| 4 | Crp | Eo 1 | |
| 5 | p | Co 2,4 | |
| 6 | a | <u>Kpq</u> | s |
| | b | p | R 4 |
| | c | Np | R 3 |
| 7 | NKpq | | Ni 6 |

Practice Problems:

- $p \rightarrow N N p$
- $C p q, N q \rightarrow N p$
- $C r s \rightarrow N K r N s$
- $C q K s N s \rightarrow N q$
- $K q N q \rightarrow N \uparrow$ (show how that given can lead to any N WFF)

Ci(C in)

C In states that if you start a sub proof with WFF 1, and within that sub proof you prove WFF 2, then you can get C WFF1 WFF2. $(r \rightarrow s) \rightarrow Crs$

| | | | | |
|---|---|----------|------|---------------|
| | <u>Krq \rightarrow Crq</u> | | | |
| 1 | Krq | | s | Krq/R, Ci, Ko |
| 2 | a | <u>r</u> | s | |
| | b | Krq | R 1 | |
| | c | q | Ko b | |
| 3 | Crq | | | Ci 2 |

| | | | | |
|---|---------------------------------------|------------|------|---------|
| | <u>\rightarrow CKsrs</u> | | | |
| 1 | a | <u>Ksr</u> | s | /Ci, Ko |
| | b | s | Ko a | |
| 2 | CKsrs | | | Ci 1 |

| | | | | |
|---|--|--------------|--------|------------------|
| | <u>Crs, r \rightarrow CNNErqs</u> | | | |
| 1 | Crs | | s | Crs, r/R, Ci, Co |
| 2 | r | | s | |
| 3 | a | <u>NNErq</u> | s | |
| | b | Crs | R 1 | |
| | c | r | R 2 | |
| | d | s | Co b,c | |
| 4 | CNNErqs | | | Ci 3 |

Practice Problems:

1. $Ksq \rightarrow Eqs$
2. $\rightarrow CqCKrpp$
3. $Crs \rightarrow CNsNr$
4. $NKsr \rightarrow CsNr$
5. $r \rightarrow C_r$

(show how you can put any WFF as the first WFF in the C WFF)

Ao(A out)

A out states that if you have an A WFF, and you can start one sub proof with the first WFF of the A WFF and prove another WFF, and you can start a second sub proof with the second WFF of the A WFF and prove the same WFF as the first sub proof, then you have proven that WFF.

Arq, (r→s), (q→s)→ s

| Asp → Aps | | | |
|-----------|---------|------|------------|
| 1 | Asp | s | Asp/Ai, Ao |
| 2 | a s | s | |
| | b Aps | Ai a | |
| 3 | a p | s | |
| | b Aps | Ai a | |
| 4 | Aps | | Ao 1,2,3 |

| AKqrKpr → r | | | |
|-------------|---------|------|----------------|
| 1 | AKqrKpr | s | AKqrKpr/Ko, Ao |
| 2 | a Kqr | s | |
| | b r | Ko a | |
| 3 | a Kpr | s | |
| | b r | Ko a | |
| 4 | r | | Ao 1,2,3 |

| AKpCpqKr q → q | | | |
|----------------|------------|--------|-----------------------|
| 1 | AKpCpqKr q | s | AKpCpqKr q/Ko, Co, Ao |
| 2 | a KpCpq | s | |
| | b p | Ko a | |
| | c Cp q | Ko a | |
| | d q | Co b,c | |
| 3 | a Kr q | s | |
| | b q | Ko a | |
| 4 | q | | Ao 1,2,3 |

Practice Problems:

1. Ars, Ns → r
2. AErpEpr → KCprCrp
3. AKsqKqs → Esq
4. Apr → CNpr
5. Asp → NKNsNp

Additional Practice Problems

1. $Cqs \rightarrow ANqs$
2. $Krq \rightarrow NCrNq$
3. $\rightarrow ArNr$
4. $Ksp \rightarrow NANsNp$
5. $NKpq \rightarrow ANpNq$
6. $NCpr \rightarrow KpNr$
7. $NAqs \rightarrow KNqNs$
8. $Eps \rightarrow ENpNs$
9. $Np, Nq \rightarrow Epq$
10. $Epq, Eqr \rightarrow Epr$

Answer Key

No

| | | | | |
|----|------------------------------|------|--------|---------------------------|
| 1) | Eps, Ns, p \rightarrow Aqr | | | |
| 1 | Eps | | s | Eps, Ns, p /R, Eo, Co, No |
| 2 | Ns | | s | |
| 3 | p | | s | |
| 4 | a | NAqr | s | |
| | b | Eps | R 1 | |
| | c | Ns | R 2 | |
| | d | p | R 3 | |
| | e | Cps | Eo b | |
| | f | s | Co d,e | |
| 4 | Aqr | | No 4 | |

| | | | |
|----|--------------------|-------|--------|
| 2) | CApsNr, Kpr → NKqq | | |
| 1 | CApsNr | | s |
| 2 | Kpr | | s |
| 3 | p | | Ko 2 |
| 4 | r | | Ko 2 |
| 5 | Aps | | Ai 3 |
| 6 | Nr | | Co 1,4 |
| 7 | a | NNKqq | s |
| | b | r | R, 4 |
| | c | Nr | R, 6 |
| 8 | NKqq | | No 7 |

| | | | | | | | | | | | | | |
|----|--|--------|--|-----------------------------------|------|---|---|---|------|---|----|------|--|
| 3) | KKEqrCrNqr → Ess | | | | | | | | | | | | |
| 1 | KKEqrCrNqr | s | | KKEqrCrNqr / R, No, Co, Eo, Ko | | | | | | | | | |
| 2 | r | Ko 1 | | | | | | | | | | | |
| 3 | KEqCrNq | Ko 1 | | | | | | | | | | | |
| 4 | Eqr | Ko 3 | | | | | | | | | | | |
| 5 | CrNq | Ko 3 | | | | | | | | | | | |
| 6 | Nq | Co 2,5 | | | | | | | | | | | |
| 7 | Crq | Eo 4 | | | | | | | | | | | |
| 8 | q | Co 2,7 | | | | | | | | | | | |
| 9 | <table><tr><td>a</td><td>NEss</td><td>s</td></tr><tr><td>b</td><td>q</td><td>R, 8</td></tr><tr><td>c</td><td>Nq</td><td>R, 6</td></tr></table> | | | a | NEss | s | b | q | R, 8 | c | Nq | R, 6 | |
| a | NEss | s | | | | | | | | | | | |
| b | q | R, 8 | | | | | | | | | | | |
| c | Nq | R, 6 | | | | | | | | | | | |
| 10 | Ess | No 9 | | | | | | | | | | | |

Answer Key

No

4)

| | | | |
|---|---|---------|---------|
| | | NNq → q | |
| 1 | | NNq | s |
| 2 | a | Nq | s |
| | b | I NNq | s |
| | | II NNq | R, R, 1 |
| | | III Nq | R, a |
| | c | Nq | No, b |
| | d | NNq | R, 1 |
| 3 | q | | No 2 |

NNq/R, No

5)

| | | | |
|---|---|----------------|------|
| | | KsNs → | |
| 1 | | KsNs | s |
| 2 | | s | Ko 1 |
| 3 | | Ns | Ko 1 |
| 4 | a | N ¹ | s |
| | b | s | R, 2 |
| | c | Ns | R, 3 |
| 5 | | | No 4 |

KsNs / Ko, R, No

Ni

1)

| | | | |
|---|-----|---------|--------|
| | | p → NNp | |
| 1 | | p | s |
| 2 | a | Np | s |
| | b | I p | s |
| | | II p | R, R 1 |
| | | III Np | R a |
| | c | Np | Ni b |
| | d | p | R 1 |
| 3 | NNp | | Ni 2 |

p/R, Ni

2)

| | | | |
|---|----|--------------|---------|
| | | Cpq, Nq → Np | |
| 1 | | Nq | s |
| 2 | | Cpq | s |
| 3 | a | p | s |
| | b | Cpq | R 2 |
| | c | q | Co a, b |
| | d | Nq | R 1 |
| 4 | Np | | Ni 3 |

Cpq, Nq /R, Ni, Co

3)

| | | | |
|---|-------|-------------|---------|
| | | Crs → NKrNs | |
| 1 | | Crs | s |
| 2 | a | KrNs | s |
| | b | r | Ko a |
| | c | Crs | R 1 |
| | d | s | Co b, c |
| | e | Ns | Ko a |
| 4 | NKrNs | | Ni 2 |

Crs/ R, Ni, Co, Ko

Answer Key

Ni

4)

| | CqKsNs → Nq | | | | | | | | | | | | | | | | |
|---|---|--------|---|---|---|--------|-----|---|------|--------|---|---|------|---|----|------|--|
| 1 | CqKsNs | s | | | | | | | | | | | | | | | |
| 2 | <table> <tr> <td>a</td><td>q</td><td>s</td></tr> <tr> <td>b</td><td>CqKsNs</td><td>R 1</td></tr> <tr> <td>c</td><td>KsNs</td><td>Co a,b</td></tr> <tr> <td>d</td><td>s</td><td>Ko c</td></tr> <tr> <td>e</td><td>Ns</td><td>Ko c</td></tr> </table> | a | q | s | b | CqKsNs | R 1 | c | KsNs | Co a,b | d | s | Ko c | e | Ns | Ko c | |
| a | q | s | | | | | | | | | | | | | | | |
| b | CqKsNs | R 1 | | | | | | | | | | | | | | | |
| c | KsNs | Co a,b | | | | | | | | | | | | | | | |
| d | s | Ko c | | | | | | | | | | | | | | | |
| e | Ns | Ko c | | | | | | | | | | | | | | | |
| 3 | Nq | Ni 2 | | | | | | | | | | | | | | | |

CqKsNs/
R, Ni, Ko, Co

5)

| | KqNq → N_ | | | | | | | | | | |
|---|--|------|---|---|---|---|------|---|----|------|--|
| 1 | KsNs | s | | | | | | | | | |
| 2 | s | Ko 1 | | | | | | | | | |
| 3 | Ns | Ko 1 | | | | | | | | | |
| 4 | <table> <tr> <td>a</td><td>s</td><td>s</td></tr> <tr> <td>b</td><td>s</td><td>R, 2</td></tr> <tr> <td>c</td><td>Ns</td><td>R, 3</td></tr> </table> | a | s | s | b | s | R, 2 | c | Ns | R, 3 | |
| a | s | s | | | | | | | | | |
| b | s | R, 2 | | | | | | | | | |
| c | Ns | R, 3 | | | | | | | | | |
| 5 | N_ | Ni 4 | | | | | | | | | |

KqNq / Ko, R, Ni

Ci

1)

| | Ksq → Eqs | | | | | | | |
|---|--|--------|---|---|---|---|-----|--|
| 1 | Ksq | s | | | | | | |
| 2 | s | Ko 1 | | | | | | |
| 3 | q | Ko 1 | | | | | | |
| 4 | <table> <tr> <td>a</td><td>s</td><td>s</td></tr> <tr> <td>b</td><td>q</td><td>R 3</td></tr> </table> | a | s | s | b | q | R 3 | |
| a | s | s | | | | | | |
| b | q | R 3 | | | | | | |
| 5 | Csq | Ci 4 | | | | | | |
| 6 | <table> <tr> <td>a</td><td>q</td><td>s</td></tr> <tr> <td>b</td><td>s</td><td>R 2</td></tr> </table> | a | q | s | b | s | R 2 | |
| a | q | s | | | | | | |
| b | s | R 2 | | | | | | |
| 7 | Cqs | Ci 6 | | | | | | |
| 8 | Eqs | Ei 5,7 | | | | | | |

Ksq/R, Ci, Ei, Ko

2)

| | → CqCKrpp | | | | | | | | | | | | | | | | |
|----|--|------|-----|---|----|--|------|-----|---|----|---|------|--|---|-------|------|--|
| 1 | <table> <tr> <td>a</td><td>q</td><td>s</td></tr> <tr> <td>b</td><td> <table> <tr> <td>I</td><td>Krp</td><td>s</td></tr> <tr> <td>II</td><td>p</td><td>Ko I</td></tr> </table> </td><td></td></tr> <tr> <td>c</td><td>CKrpp</td><td>Ci b</td></tr> </table> | a | q | s | b | <table> <tr> <td>I</td><td>Krp</td><td>s</td></tr> <tr> <td>II</td><td>p</td><td>Ko I</td></tr> </table> | I | Krp | s | II | p | Ko I | | c | CKrpp | Ci b | |
| a | q | s | | | | | | | | | | | | | | | |
| b | <table> <tr> <td>I</td><td>Krp</td><td>s</td></tr> <tr> <td>II</td><td>p</td><td>Ko I</td></tr> </table> | I | Krp | s | II | p | Ko I | | | | | | | | | | |
| I | Krp | s | | | | | | | | | | | | | | | |
| II | p | Ko I | | | | | | | | | | | | | | | |
| c | CKrpp | Ci b | | | | | | | | | | | | | | | |
| 2 | CqCKrpp | Ci 1 | | | | | | | | | | | | | | | |

/Ko, Ci

3)

| | Crs → CNsNr | | | | | | | | | | | | | | | | | | | | | | |
|-----|--|---------|----|---|-----|--|-----|----|---|---------|-----|-----|-----|---|---------|--|----|-----|--|---|----|----|--|
| 1 | Crs | s | | | | | | | | | | | | | | | | | | | | | |
| 2 | <table> <tr> <td>a</td><td>Ns</td><td>s</td></tr> <tr> <td>b</td><td> <table> <tr> <td>II</td><td>r</td><td>s</td></tr> <tr> <td>III</td><td>Crs</td><td>R 1</td></tr> <tr> <td>IV</td><td>s</td><td>Co I,II</td></tr> <tr> <td></td><td>Ns</td><td>R a</td></tr> </table> </td><td></td></tr> <tr> <td>c</td><td>Nr</td><td>Ni</td></tr> </table> | a | Ns | s | b | <table> <tr> <td>II</td><td>r</td><td>s</td></tr> <tr> <td>III</td><td>Crs</td><td>R 1</td></tr> <tr> <td>IV</td><td>s</td><td>Co I,II</td></tr> <tr> <td></td><td>Ns</td><td>R a</td></tr> </table> | II | r | s | III | Crs | R 1 | IV | s | Co I,II | | Ns | R a | | c | Nr | Ni | |
| a | Ns | s | | | | | | | | | | | | | | | | | | | | | |
| b | <table> <tr> <td>II</td><td>r</td><td>s</td></tr> <tr> <td>III</td><td>Crs</td><td>R 1</td></tr> <tr> <td>IV</td><td>s</td><td>Co I,II</td></tr> <tr> <td></td><td>Ns</td><td>R a</td></tr> </table> | II | r | s | III | Crs | R 1 | IV | s | Co I,II | | Ns | R a | | | | | | | | | | |
| II | r | s | | | | | | | | | | | | | | | | | | | | | |
| III | Crs | R 1 | | | | | | | | | | | | | | | | | | | | | |
| IV | s | Co I,II | | | | | | | | | | | | | | | | | | | | | |
| | Ns | R a | | | | | | | | | | | | | | | | | | | | | |
| c | Nr | Ni | | | | | | | | | | | | | | | | | | | | | |
| 3 | CNsNr | Ci 2 | | | | | | | | | | | | | | | | | | | | | |

Crs/ R, Ni, Ci, Co

Answer Key

Ci

4)

| NKsr \rightarrow CsNr | | |
|-------------------------|-----------------|------|
| 1 | NKsr | s |
| 2 | a s | s |
| | b I r s | |
| | II s R a | |
| | III Ksr Ki I,II | |
| | IV NKsr R,R 1 | |
| 3 | c Nr | Ni a |
| | CsNr | Ci 2 |

NKsr/ R, Ni, Ki, Ci

5)

| r \rightarrow C'r | | |
|---------------------|------------|------|
| 1 | r | s |
| 2 | a I s | |
| | b r R, 1 | |
| 3 | C'r | Ci 2 |

r / R, Ci

Ao

1)

| Ars, Ns \rightarrow r | | |
|-------------------------|--------------|--------|
| 1 | Ars | s |
| 2 | Ns | s |
| 3 | a r | s |
| | b I Nr s | |
| | II A NNr s | |
| | B Nr R I | |
| | C r R,R a | |
| | III Nr No I | |
| | IV r R a | |
| 4 | a s | s |
| | b I Nr s | |
| | II s R a | |
| | III NS R,R 2 | |
| 5 | c r | No, b |
| | r | Ao 3,4 |

Ars, Ns/
R, No, Ao

2)

| AErpEpr \rightarrow KCprCrp | | |
|-------------------------------|-------------|--------|
| 1 | AErpEpr | s |
| 2 | a Erp | s |
| | b Crp | Eo a |
| | c Cpr | Eo a |
| | d KCprCrp | Ki b,c |
| 3 | a Epr | s |
| | b Crp | Eo a |
| | c Cpr | Eo a |
| | d KCprCrp | Ki b,c |
| 4 | KCprCrp | Ao 2,4 |

AErpEpr / Ao, Eo, Ki

Answer Key

Ao

3)

| | | | |
|---|---|---------------|--------|
| | | AKsqKqs → Esq | |
| 1 | | AKsqKqs | s |
| 2 | a | Ksq | s |
| | b | s | Ko a |
| | c | q | Ko a |
| | d | I s | s |
| | | II q | R c |
| | e | Csq | Ci d |
| | f | I q | s |
| | | II s | R b |
| | g | Cqs | Ci f |
| | h | Esq | Ei e,g |
| 3 | a | Kqs | s |
| | b | s | Ko a |
| | c | q | Ko a |
| | d | I s | s |
| | | II q | R c |
| | e | Csq | Ci d |
| | f | I q | s |
| | | II s | R b |
| | g | Cqs | Ci f |
| | h | Esq | Ei e,g |
| 4 | | Esq | Ao 2,3 |

AKsqKqs /
Ao, Ei, Ci, Ko

4)

| | | | |
|---|---|-------------|-----------------|
| | | Apr → CNpr | |
| 1 | | Apr | s |
| 2 | a | Np | s |
| | b | Aps | R 1 |
| | c | I r | s |
| | | II A Nr | s |
| | | | B I NNr s |
| | | | 2 Nr R A |
| | | | 3 r R, R I |
| | | C Nr | No, B |
| | | D r | R I |
| | | III r | No, II |
| | d | I p | s |
| | | II A Nr | s |
| | | | B Np R, R a |
| | | | C p R, I |
| | | III r | No II |
| | E | r | Ao b,c,d |
| 3 | | CNpr | Ci 2 |

Apr / R, No, Ci, Ao

Answer Key

Ao

5)

| | | | |
|---|--------|--------------|----------|
| | | Asp → NKNsNp | |
| 1 | Asp | | s |
| 2 | a | KNsNp | s |
| | b | Np | Ko a |
| | c | Ns | Ko a |
| | d | Asp | R 1 |
| | e | I p | s |
| | | II A NNp | s |
| | | B Np | R,R b |
| | | C p | R I |
| | | III p | No II |
| | f | I s | s |
| | | II A Np | s |
| | | B Ns | R,R c |
| | | C s | R I |
| | | III p | No II |
| 3 | g | p | Ao d,e,f |
| | NKNsNp | | No 2 |

Asp /
Ko, R, No, Ao

Additional Practice Problems

1)

| | | | |
|---|------|-------------|--------|
| | | Cqs → ANqs | |
| 1 | Cqs | | s |
| 2 | a | NANqs | s |
| | b | I Nq | s |
| | | II ANqs | Ai I |
| | | III NANqs | R a |
| | c | q | No b |
| | d | Cqs | R 1 |
| | e | s | Co c,d |
| | f | ANqs | Ai e |
| | g | I NNANqs | s |
| | | II NANqs | R a |
| | | III ANqs | R f |
| | h | NANqs | No g |
| 3 | ANqs | | No, 2 |

Cqs /
Ai, R, No, Co

2)

| | | | |
|---|-------|-------------|--------|
| | | Krq → NCrNq | |
| 1 | Krq | | s |
| 2 | r | | Ko 1 |
| 3 | q | | Ko 1 |
| 4 | a | CrNq | s |
| | b | r | R 2 |
| | c | Nq | Co a,b |
| | d | q | R 3 |
| 5 | NCrNq | | Ni 4 |

Krq/
Ko, R, Co, Ni

Answer Key

Additional Practice Problems

3)

| → ArNr | |
|--------|-------------------|
| 1 | a NArNr s |
| | b I Nr s |
| | II ArNr Ai I |
| | III NArNr R a |
| | c r No b |
| | d ArNr Ai c |
| | e I NArNr s |
| | II ArNr R d |
| | III NArNr R a |
| | f NArNr No e |
| 2 | ArNr No 1 |

/Ai, No, R

4)

| Ksp → NAnsNp | |
|--------------|-------------------|
| 1 | Ksp s |
| 2 | a ANsNp s |
| | b Ksp R 1 |
| | c s Ko b |
| | d p Ko b |
| | e I Ns s |
| | II A s s |
| | B s R, R c |
| | C Ns R I |
| | III Ns Ni II |
| | f I Np s |
| | II A s s |
| | B p R, R d |
| | C Np R I |
| | III Ns Ni II |
| | g Ns Ao a,e,f |
| 3 | NAnsNp Ni 2 |

Ksp/R, Ko, Ni, Ao

5)

| NKpq → ANpNq | |
|--------------|--------------------|
| 1 | NKpq s |
| 2 | a NANpNq s |
| | b I Np s |
| | II ANpNq Ai I |
| | III NANpNq R a |
| | c p No b |
| | d I Nq s |
| | II ANpNq Ai I |
| | III NANpNq R a |
| | e q No d |
| | f Kpq Ko c,e |
| | g NKpq R 1 |
| 3 | ANpNq No 2 |

NKpq/R, Ko, No, Ai

Answer Key

Additional Practice Problems

6)

| | | | | |
|---|------|-------------|---|-------------|
| | | NCpr → KpNr | | |
| 1 | NCpr | | | s |
| 2 | a | NKpNr | | s |
| | b | I | p | s |
| | | II | A | Nr s |
| | | | B | p R I |
| | | | C | KpNr Ki A,B |
| | | | D | NKpNr R,R a |
| | | III | r | No II |
| | c | Cpr | | Ci b |
| | d | NCpr | | R 1 |
| 3 | KpNr | | | No 2 |

NCpr/R, Ki, No, Ci

7)

| | | | | |
|---|-------|--------------|--|--------|
| | | NAqs → KNqNs | | |
| 1 | NAqs | | | s |
| 2 | a | q | | s |
| | b | Aqs | | Ai a |
| | c | NAqs | | R 1 |
| 3 | Nq | | | Ni 2 |
| 4 | a | s | | s |
| | b | Aqs | | Ai a |
| | c | NAqs | | R 1 |
| 5 | Ns | | | Ni 4 |
| 6 | KNqNs | | | Ki 3,5 |

NAqs/R, Ki, Ni, Ai

8)

| | | | | |
|---|-------|-------------|-----|--------|
| | | Eps → ENpNs | | |
| 1 | Eps | | | s |
| 2 | a | Np | | s |
| | b | I | s | s |
| | | II | Eps | R,R 1 |
| | | III | Csp | Eo II |
| | | IV | p | Co III |
| | | V | Np | R a |
| | c | Ns | | Ni b |
| 3 | CNpNs | | | Ci 2 |
| 4 | a | Ns | | s |
| | b | I | p | s |
| | | II | Eps | R,R 1 |
| | | III | Cps | Eo II |
| | | IV | s | Co III |
| | | V | Ns | R a |
| | c | Np | | Ni b |
| 5 | CNsNp | | | Ci 4 |
| 6 | ENpNs | | | Ei 3,5 |

Eps/
R, Eo, Co, Ci, Ei, Ni

Answer Key

Additional Practice Problems

9)

| | Np, Nq → Epq | | | | | | | | | | |
|-----|--|--------|----|---|----|----|--------|-----|---|-----|--|
| 1 | Np | s | | | | | | | | | |
| 2 | Nq | s | | | | | | | | | |
| 3 | a p | s | | | | | | | | | |
| | b <table border="1"> <tr> <td>I</td><td>Nq</td><td>s</td></tr> <tr> <td>II</td><td>Np</td><td>R, R 1</td></tr> <tr> <td>III</td><td>p</td><td>R a</td></tr> </table> | I | Nq | s | II | Np | R, R 1 | III | p | R a | |
| I | Nq | s | | | | | | | | | |
| II | Np | R, R 1 | | | | | | | | | |
| III | p | R a | | | | | | | | | |
| | c q | No b | | | | | | | | | |
| 4 | Cpq | Ci 3 | | | | | | | | | |
| 5 | a q | s | | | | | | | | | |
| | b <table border="1"> <tr> <td>I</td><td>Np</td><td>s</td></tr> <tr> <td>II</td><td>Nq</td><td>R, R 2</td></tr> <tr> <td>III</td><td>q</td><td>R a</td></tr> </table> | I | Np | s | II | Nq | R, R 2 | III | q | R a | |
| I | Np | s | | | | | | | | | |
| II | Nq | R, R 2 | | | | | | | | | |
| III | q | R a | | | | | | | | | |
| | c p | No b | | | | | | | | | |
| 6 | Cqp | Ci 5 | | | | | | | | | |
| 7 | Epq | Ei 4,6 | | | | | | | | | |

Np, Nq/ R, No, Ci, Ei

10)

| | Epq, Eqr → Epr | |
|---|----------------|--------|
| 1 | Epq | s |
| 2 | Eqr | s |
| 3 | a p | s |
| | b Epq | R 1 |
| | c Cpq | Eo b |
| | d q | Co a,c |
| | e Eqr | R 2 |
| | f Cqr | Eo e |
| | g r | Co d,f |
| 4 | Cpr | Ci 3 |
| 5 | a r | s |
| | b Eqr | R 2 |
| | c Crq | Eo b |
| | d q | Co a,c |
| | e Epq | R 1 |
| | f Cqp | Eo e |
| | g p | Co d,f |
| 6 | Crp | Ci 3 |
| 7 | Epr | Ei 4,6 |

Epq, Eqr/
R, Eo, Co, Ci, Ei