HW8

1. Given the data as listed below

x	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3
y	102.6	113.2	130.1	142.1	167.5	195.1	224.9	256.8

a. Construct the least squares approximation of degree two and compute the error.

b. Construct the least squares approximation of the form be^{ax} and compute the error.

c. Construct the least squares approximation of the form bx^n and compute the error.

2. Find the least squares polynomial approximation of degree two on the interval $[-1,1]$ for the function $f(x)=\frac{1}{2}\cos x+\frac{1}{4}\sin 2x$

3. Determine the discrete least squares trigonometric polynomial S_4 using $m=16$ for $f(x)=x^2\sin x$ on the interval $[0,1]$.

b. Compute $\int_0^1 S_4(x)\,dx$

c. Compare the integral in part (b) to $\int_0^1 x^2\sin x\,dx$

d. Compute the error $E(S_4)$

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==== 第一題：資料擬合 ====
1.a 二次多項式係數 (c2, c1, c0): [ 6.69118439 -1.88374644  3.0863933 ]
1.a 擬合誤差: 0.0052456855825467295

1.b 指數函數擬合參數 (a, b): [ 0.38666401 22.82449552]
1.b 擬合誤差: 74.36078759754199

1.c 冪次函數擬合參數 (n, b): [2.02016212 6.23352099]
1.c 擬合誤差: 0.010270381747465208

==== 第二題：區間 [-1,1] 上的二次多項式擬合 ====
二次多項式擬合係數 a0, a1, a2: [ 0.49827931  0.32654833 -0.23263145]

==== 第三題：離散最小平方三角多項式 S4（改寫版）====
(a) S₄(x) =
    0.209865 +0.097202 * cos(2π*1*x) -0.239780 * sin(2π*1*x) +0.002081 * cos(2π*2*x) -0.128971 * sin(2π*2*x) -0.014104 * cos(2π*3*x) -0.085607 * sin
(2π*3*x) -0.019664 * cos(2π*4*x) -0.062873 * sin(2π*4*x)

(b) ∫₀¹ S₄(x) dx ≈ 0.209865
(c) ∫₀¹ x² sin(x) dx ≈ 0.223244
    差異 ≈ 1.337931e-02
(d) E(S₄) ≈ 0.098750
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