Yewno-Quantitative Analyst Question 2

Code

Leo Guoyuan Liu

May 21, 2018

Question 2 Implement one Smart Beta strategy and discuss pros and cons compared to a chosen benchmark.

Traditional market-cap weighted portfolios have been criticized that they are poorly diversified. The risk-based smart beta approach corresponds to the criticism, and adopts risk-based weighing in portfolio construction, thus manages the risk more effectively and achieve a better performance. The well-known risk-based approaches are MV (minimum variance), equally-weighted, and ERC (equally-weighted risk contributions) In this exercise, I implement the ERC smart beta strategy in R and compare its performance to other two as benchmarks.

Definition of ERC problem

Given a portfolio $x = (x_1, x_2, \ldots, x_n)$ of n risky assets, x_i is the weight of assets i. The corresponding return is $r = (r_1, r_2, \ldots, r_n)$. The goal of the ERC strategy is to find a risk-balanced portfolio such that the risk contribution is the same for all assets without short selling.

Let $\Sigma = [\sigma_{ij}]$ be the covariance matrix of the return. The portfolio risk is given by $x^T \Sigma x$. The contribution of $i^t h$ asset to the portfolio risk can be calculated by the weight x_i multiply marginal risk contribution.

$$x_i \times \partial_{x_i}(x^T \Sigma x) = x_i(\Sigma x)_i$$

The problem can be rewritten as finding $\hat{x}$ such that

$$\hat{x} = \{x \in [0,1]^n : 1^T x = 1, x_i(\Sigma x)_i = x_j(\Sigma x)_j, \\ \text{for all } i, j\}$$

We can use SQP (Sequential Quadratic programming) to solve it

$$\text{minimize } f(x) = \sum_{i,j}(x_i(\Sigma x)_i - x_j(\Sigma x)_j)^2 \\ \text{subject to: } 1^T x = 1\,, 0 \leq x \leq 1$$

The function that calculates the optimized weights is *erc*.

Comparison with 1/n and minimum-variance portfolios

Equally weighted portfolio(1/n) and minimum-variance (MV) portfolios are widely used in practice. ERC portfolios are naturally located between both and thus appear as good potential substitutes for these traditional approaches Literature shown that

- The volatilities of these three portfolios as follows, with the ERC located between.

$$V_{mv} \leq V_{erc} \leq V_{1/n}$$

- ERC portfolio is optimal if we assume a constant correlation matrix and supposing that the assets have all the same Sharpe ratio.

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```r
# calulate the center quardratic moment
M2<-function (r)
{
    L <- nrow(r)
    rc <- apply(r, 2, scale, scale = FALSE)
    ans <- 1/(L - 1) * crossprod(rc)
    ans
}
# Portfoio performance
performance<-function(r){
  c(
  Return=Return.cumulative(r),
  Sharpe=SharpeRatio(r,FUN="StdDev"),
  VaR=VaR(r,method="historical")
)
}
## Function calculates ERC distrituion
## with respect to higher moments
##
#'
#' @param start \code{numeric}, a \eqn{(N x 1)} vector of starting values.
#' @param returns \code{matrix}, a \eqn{(T x N)} matrix of asset returns.
#' @param sigma \code{numeric}, a \eqn{(N x N)} Covariance matrix'
#'
#' @return  code{numeric}, a \eqn{(N x 1)} vector of weights
#'
erc<- function(returns=NULL,sigma=NULL){
  #covariance matrix
  if(is.null(sigma))
    sigma<-M2(returns)
  #objective function
  f<-function(x){
    pctb<- x * sigma %*% x / c(crossprod(x, sigma) %*% x)
    var(pctb)
  }

  #optimization
  m<-ncol(sigma)
  opt <-nlminb(start = runif(m), objective = f, lower = rep(0, m))
  #output weight
  w <- opt$par / sum(opt$par)
}
#calculate risk contribution
risk_contribution<-function(r,w){
  dm2<- (M2(r) %*% w)
  mp2<-c(crossprod(w, M2(r)) %*% w)
  w*dm2/mp2
}
mv<-function(returns=NULL,sigma=NULL){
  if(is.null(sigma))
    sigma<-M2(returns)
  require("NMOF")
  res<-minvar(sigma)
  as.vector(res)

}
#calculate risk contribution
risk_contribution<-function(r,w){
  dm2<- (M2(r) %*% w)
  mp2<-c(crossprod(w, M2(r)) %*% w)
  w*dm2/mp2
}
get_return<-function(x) x/lag.xts(x)-1
```

A Numerical example

A numerical example is illustrated as follows. The portfolio has four assets with constant volatility $(0.1, 0.2, 0.3, 0.4)$ and correlation matrix rho. Then 1/n portfolio give equal contribution of $(0.25, 0.25, 0.25, 0.25)$, while the mv portfolio is concentrated in the first portfolio. The ERC portfolio is invested in all assets. Therefore ERC portfolio is more balanced in terms of risk contribution. The variance of the ERC portfolio is also between the other two.

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```
rho<-matrix(
  c(1,0.8,0,0, 0.8,1,0,0,0,0,1,-0.5,0,0,-0.5,1),
  c(4,4)
)
s<-c(0.1,0.2,0.3,0.4)
d<-diag(s)
print("The correlation matrix portfolio")
```

```
[1] "The correlation matrix portfolio"
```

Hide

```
print(rho)
```

```
     [,1] [,2] [,3] [,4]
[1,]  1.0  0.8  0.0  0.0
[2,]  0.8  1.0  0.0  0.0
[3,]  0.0  0.0  1.0 -0.5
[4,]  0.0  0.0 -0.5  1.0
```

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```
print("The stdv of the portfolio")
```

```
[1] "The stdv of the portfolio"
```

Hide

```
print(s)
```

```
[1] 0.1 0.2 0.3 0.4
```

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```
Sigma<-d%*%rho%*%d
Vr<-function(w,sigma=Sigma){
 as.numeric( t(w)%*%sigma%*%w)
}
print("The solution for the 1/n portfolio")
```

```
[1] "The solution for the 1/n portfolio"
```

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```
w=rep(1/4,4)
print(w )
```

```
[1] 0.25 0.25 0.25 0.25
```

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```
print("The variance")
```

```
[1] "The variance"
```

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```
print(Vr(w))
```

```
[1] 0.01325
```

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```
print("The solution for the MV portfolio")
```

```
[1] "The solution for the MV portfolio"
```

Hide

```
w=mv(sigma=sigma)
print(w)
```

```
[1] 0.7448276 0.0000000 0.1517241 0.1034483
```

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```
print("The variance")
```

```
[1] "The variance"
```

Hide

```
print(Vr(w))
```

```
[1] 0.007448276
```

Hide

```
print("The solution for the ERC portfolio")
```

```
[1] "The solution for the ERC portfolio"
```

Hide

```
w=erc(sigma=sigma)
print(w )
```

```
[1] 0.3836125 0.1918063 0.2426179 0.1819634
```

Hide

```
print("The variance")
```

```
[1] "The variance"
```

Hide

```
print(Vr(w))
```

```
[1] 0.01059542
```

Implement the ERC strategy.

I write an Run_Strategy function to implement the three strategies. The idea behind the strategy is that we rebalance the portfolio every 30 days based on the weights calculated by the strategy. We using the rolling window of 360 days to calculate the variance matrix. The performance function measure the performance of the strategies, including the cumulative return, the Sharpe ratio and the VaR. The performance of the all three metrics are between 1/n and MV.

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```
data <- new.env()
getSymbols(c('AAPL','WMT','GE','IBM'), src = 'yahoo',from = "2000-01-01", adjust =  TRUE,env=dat
a )
```

```
[1] "AAPL" "WMT"  "GE"   "IBM"
```

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```r
x<-merge(data$AAPL[,6], data$WMT[,6], data$GE[,6],data$IBM[,6])
```

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```
## Function runs the rebalance strategy E
## with respect to higher moments
##
#'
#' @param x market price data.
#' @param stragegy name of the strategy namely 1/n, mv, and erc#'
#'
#' @return  return of the portfolio
#'
Run_Strategy<-function(x,strategy){
  n=nrow(x)
  m =ncol(x)
  r=get_return(x)



  res<-xts(data.frame(Value=rep(1,n)),order.by=index(x))
  #starting from equal weights
  shares=rep(1/m, m)/as.numeric(x[360])
  for(i in 361:n){
     res[i]$Value=sum(x[i]*shares)
     # re-balance the portfolio every 30 days
    if((i-361)%%30==0){
      # switch strategy
      sigma=M2(r[(i-360+1):i])
      if(strategy=="1/n"){
        w=rep(1/m,m)
      }else if(strategy=="mv"){
        w=mv(sigma=sigma)
      }else if(strategy=="erc"){
         w=erc(sigma=sigma)
      }else{
        stop("not implented")
      }

      shares=as.numeric(res[i]$Value*w)/as.numeric(x[i])
    }

  }
  # get return
  get_return(res)["2002/"]
}
r1<-Run_Strategy(x,"1/n")
```

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r2<-Run_Strategy(x,"mv")
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Found more than one class "xts" in cache; using the first, from namespace 'FRAPO'
Also defined by □quantmod□
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Hide

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r3<-Run_Strategy(x,"erc")
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  Use c() or as.vector() instead.
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  Use c() or as.vector() instead.
Found more than one class "xts" in cache; using the first, from namespace 'FRAPO'
Also defined by ‘quantmod’
Found more than one class "xts" in cache; using the first, from namespace 'FRAPO'
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Also defined by ‘quantmod’
Found more than one class "xts" in cache; using the first, from namespace 'FRAPO'
Also defined by ‘quantmod’
```

Hide

```
performance(r1)
```

```
     Return      Sharpe         VaR 
 5.06867194  0.04111114 -0.01960185 
```

Hide

```
performance(r2)
```

```
     Return      Sharpe         VaR 
 1.18994752  0.02273673 -0.01709234 
```

Hide

```
performance(r3)
```

```
     Return      Sharpe         VaR 
 3.04278114  0.03463334 -0.01852141 
```

Conclusion

Minimum variance (MV) and equality-weighted (1/n) portfolio are popular risk-based smart beta strategy. They both have limitations. 1/n lacks risk monitoring while mv suffers dramatic asset concentration. The ERC strategy offer a middle ground between both the distribution and the risk. From the implementation of the simple rebalance strategy, the performance of the ERC is also between the other two, thus give a meaningful option for portfolio construciton.

Reference On the properties of equally-weighted risk contributions portfolios Sébastien Maillard, Thierry Roncalli and Jérôme Teiletche May 2009 https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1271972 (https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1271972)