

1.39 Table 1.3

		y	
		0	1
x	0	$\frac{1}{3}$	$\frac{1}{3}$
	1	0	$\frac{1}{3}$
		$\frac{1}{3}$	$\frac{2}{3}$

$$\begin{aligned} (a) \quad H[X] &= -\int P(x) \ln(P(x)) dx \\ &= -\frac{2}{3} \ln\left(\frac{2}{3}\right) - \frac{1}{3} \ln\left(\frac{1}{3}\right) \end{aligned}$$

$$\begin{aligned} H[Y] &= -\int P(y) \ln(P(y)) dy \\ &= -\frac{1}{3} \ln\left(\frac{1}{3}\right) - \frac{2}{3} \ln\left(\frac{2}{3}\right) \end{aligned}$$

$$\begin{aligned} H[Y|X] &= -\iint P(y, x) \ln P(y|x) dy dx \\ &= -\left( P(0,0) \ln P(0|0) + P(0,1) \ln P(0|1) \right. \\ &\quad \left. + P(1,0) \ln P(1|0) + P(1,1) \ln P(1|1) \right) \\ &= -\left( \frac{1}{3} \ln \frac{1}{2} + 0 \cdot 0 + \frac{1}{3} \ln \frac{1}{2} + \frac{1}{3} \ln 1 \right) \end{aligned}$$

$$\begin{aligned} H[X|Y] &= -\iint P(x, y) \ln P(x|y) dx dy \\ &= -\left( P(0,0) \ln P(0|0) + P(0,1) \ln P(0|1) \right. \\ &\quad \left. + P(1,0) \ln P(1|0) + P(1,1) \ln P(1|1) \right) \\ &= -\left( \frac{1}{3} \cdot \ln(1) + \frac{1}{3} \ln\left(\frac{1}{2}\right) + 0 \cdot \ln\left(\frac{1}{2}\right) + \frac{1}{3} \ln\left(\frac{1}{2}\right) \right) \end{aligned}$$

$$\begin{aligned} H[X, Y] &= -\iint P(x, y) \ln P(x, y) dx dy \\ &= -\left( \frac{1}{3} \ln \frac{1}{3} + \frac{1}{3} \ln\left(\frac{1}{3}\right) + 0 + \frac{1}{3} \ln\left(\frac{1}{3}\right) \right) \end{aligned}$$

$$I[X, Y] = H[X] - H[X|Y] = H[Y] - H[Y|X]$$