

1.12 if $m=n$, then $E[X_m X_n] = E[X^2] = \mu^2 + \sigma^2$
 if $m \neq n$, then X_m and X_n are independent. $E[X_m X_n] = E[X_m] E[X_n] = \mu^2$

Next $E[\mu_{ML}] = E\left[\frac{1}{N} \sum X_n\right] = \frac{1}{N} \sum E[X_n] = \frac{1}{N} \cdot N \mu = \mu$.

Finally, $E[\sigma_{ML}^2] = E\left[\frac{1}{N} \sum_n (X_n - \mu_{ML})^2\right]$ using (1.56)

$$= E\left[\frac{1}{N} \sum_n \left(X_n - \frac{1}{N} \sum_m X_m\right)^2\right]$$

$$= E\left[\frac{1}{N} \left(\sum_n X_n^2 - 2X_n \left(\frac{1}{N} \sum_m X_m\right) + \left(\frac{1}{N} \sum_m X_m\right)^2\right)\right]$$

$$= \frac{1}{N} E\left[\sum_n X_n^2 - \sum_n 2X_n \frac{1}{N} \sum_m X_m + \sum_n \frac{1}{N^2} \left(\sum_m X_m\right)^2\right]$$

$$= \frac{1}{N} \sum_n E\left[X_n^2 - \frac{2X_n}{N} \sum_m X_m + \frac{1}{N^2} \sum_{\ell} \sum_k X_\ell X_k\right]$$

$$= \mu^2 + \sigma^2 - 2\left(\mu^2 + \frac{1}{N} \sigma^2\right) + \mu^2 + \frac{1}{N} \sigma^2$$

$$= \frac{(N-1)}{N} \sigma^2$$

using $\left(\sum_m X_m\right)^2 = \sum_{\ell} \sum_k X_\ell X_k$