

$$1.21 \quad \int_{R_1} P(x, C_2) dx \leq \int_{R_1} \sqrt{P(x, C_2) P(x, C_1)} dx$$

$$\int_{R_2} P(x, C_1) dx \leq \int_{R_2} \sqrt{P(x, C_1) P(x, C_2)} dx$$

$$P(\text{mistake}) \leq \int_{R_1} \sqrt{P(x, C_2) P(x, C_1)} dx + \int_{R_2} \sqrt{P(x, C_1) P(x, C_2)} dx$$

$$= \int_{R_1 \cup R_2} \sqrt{P(x, C_2) P(x, C_1)} dx$$

$$= \int \sqrt{P(x, C_2) P(x, C_1)} dx$$