1.38 (1.114) $f(\lambda a + (1-\lambda)b) \le \lambda f(a) + (1-\lambda) f(b)$

(1.115) $f(\sum_{i=1}^{M} \lambda_i x_i) \le \sum_{i=1}^{M} \lambda_i f(x_i)$

show that (1.114) $\Rightarrow$ (1.115)

for $M=1$, let $b=0$, (1.114) becomes $f(\lambda a) \le \lambda f(a)$, we know that (1.115) holds.

suppose that (1.115) holds for M, we need to show that it also holds for $M+1$

$$f(\sum_{i=1}^{M+1} \lambda_i x_i) = f(\lambda_{M+1} x_{M+1} + \sum_{i=1}^{M} \lambda_i x_i)$$

$$\le f(\lambda_{M+1} x_{M+1} + (1-\lambda_{M+1}) \frac{1}{1-\lambda_{M+1}} \sum_{i=1}^{M} \lambda_i x_i)$$

using (1.114) $= \lambda_{M+1} f(x_{M+1}) + (1-\lambda_{M+1}) f(\sum_{i=1}^{M} \frac{\lambda_i}{1-\lambda_{M+1}} x_i)$

by definition $\sum_{i=1}^{M+1} \lambda_i = 1 \Rightarrow \sum_{i=1}^{M} \lambda_i = 1-\lambda_{M+1} \Rightarrow \sum_{i=1}^{M} \frac{\lambda_i}{1-\lambda_{M+1}} = 1$

Hence, we can use (1.115) for $f(\sum_{i=1}^{M} \frac{\lambda_i}{1-\lambda_{M+1}} x_i)$

so $f(\sum_{i=1}^{M+1} \lambda_i x_i)$ ~~becomes~~ $\le$

$\lambda_{M+1} f(x_{M+1}) + (1-\lambda_{M+1}) f(\sum_{i=1}^{M} \frac{\lambda_i}{1-\lambda_{M+1}} x_i) \le$

$\lambda_{M+1} f(x_{M+1}) + (1-\lambda_{M+1}) (\sum_{i=1}^{M} \frac{\lambda_i}{1-\lambda_{M+1}} f(x_i)) =$

$\lambda_{M+1} f(x_{M+1}) + \sum_{i=1}^{M} \lambda_i f(x_i) =$

$\sum_{i=1}^{M+1} \lambda_i f(x_i)$