

box r
apple x 3
orange x 4
limes x 3

box b
apple x 1
orange x 1
limes x 0

box g
apple x 3
orange x 3
limes x 4

$$P(r) = 0.2$$

$$P(b) = 0.2$$

$$P(g) = 0.6$$

What is the probability of selecting an apple?

$$\begin{aligned} P(X = \text{apple}) &= P(X = \text{apple} | Y = \text{box r}) P(Y = \text{box r}) \\ &\quad + P(X = \text{apple} | Y = \text{box b}) P(Y = \text{box b}) \\ &\quad + P(X = \text{apple} | Y = \text{box g}) P(Y = \text{box g}) \\ &= 0.3 \times 0.2 \\ &\quad + 0.5 \times 0.2 \\ &\quad + 0.3 \times 0.6 \\ &= 0.34 \end{aligned}$$

if we observe that the selected fruit is in fact an orange, what is the probability that it come from the green box?

$$\begin{aligned} P(Y = \text{box g} | X = \text{orange}) &= \frac{P(X = \text{orange} | Y = \text{box g}) P(Y = \text{box g})}{P(X = \text{orange})} \\ &= \frac{0.5 \times 0.6}{(P(X = \text{orange} | Y = \text{box r}) P(Y = \text{box r}) \\ &\quad + P(X = \text{orange} | Y = \text{box g}) P(Y = \text{box g}) \\ &\quad + P(X = \text{orange} | Y = \text{box b}) P(Y = \text{box b}))} \\ &= \frac{0.3}{0.3 + 0.6 + 0.1} \end{aligned}$$