

1.13 given (1.56) $\sigma_{ML}^2 = \frac{1}{N} \sum_n (x_n - \mu_{ML})^2$
replace μ_{ML} with the true value μ of the mean,
we have.

$$\sigma_{ML}^2 = \frac{1}{N} \sum_n (x_n - \mu)^2$$

$$\begin{aligned} E(\sigma_{ML}^2) &= E\left[\frac{1}{N} \sum_n (x_n - \mu)^2\right] \\ &= \frac{1}{N} E\left[\sum_n (x_n - \mu)^2\right] \\ &= \frac{1}{N} E\left[\sum_n x_n^2 - 2\mu x_n + \mu^2\right] \\ &= E[x_n^2] - 2\mu E[x_n] + E[\mu^2] \\ &= \mu^2 + \sigma^2 - 2\mu^2 + \mu^2 \\ &= \sigma^2 \end{aligned}$$