

1.18 given (1.126) $\int_{-\infty}^{\infty} \exp\left(-\frac{1}{2\sigma^2} x^2\right) dx = (2\pi\sigma^2)^{\frac{1}{2}}$

Let $u = r^2$, the right hand side can be rewritten as

$$\begin{aligned} S_D \int_0^{\infty} e^{-r^2} r^{D-1} dr &= S_D \int_0^{\infty} e^{-u} u^{\frac{1}{2}(D-1)} \frac{1}{2} u^{-\frac{1}{2}} du \\ &= \frac{1}{2} S_D \int_0^{\infty} e^{-u} u^{\frac{D}{2}-1} du \\ &= \frac{1}{2} S_D \Gamma(D/2) \end{aligned}$$

using (1.126) the left hand side can be written as

$$\prod_{i=1}^D \int_{-\infty}^{\infty} e^{-x_i^2} dx_i = \pi^{D/2}$$

Therefore $S_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$