

$$1.1 \quad E(w) = \frac{1}{2} \sum_{n=1}^N \{ y(x_n, w) - t_n \}^2$$

$$\text{where } y(x_n, w) = w_0 + w_1 x_n + \dots + w_M x_n^M = \sum_{j=0}^M w_j x_n^j$$

$E(w)$  is convex quadratic function

To minimize the  $E(w)$ , apply partial derivative with respect to  $w_j$

$$\frac{\partial E(w)}{\partial w_i} = \frac{\partial \left( \frac{1}{2} \sum_{n=1}^N \left\{ \sum_{j=0}^M w_j x_n^j - t_n \right\}^2 \right)}{\partial w_i}$$

$$= \sum_{n=1}^N \left( \sum_{j=0}^M (w_j x_n^j - t_n) \right) \cdot x_n^i$$

$$= \sum_{n=1}^N \sum_{j=0}^M w_j x_n^{i+j} - \sum_{n=1}^N t_n x_n^i$$

$$= 0$$

$$\sum_{n=1}^N \sum_{j=0}^M w_j x_n^{i+j} = \sum_{n=1}^N t_n x_n^i$$

$$\Rightarrow \sum_{j=0}^M \sum_{n=1}^N x_n^{i+j} w_j = \sum_{n=1}^N (x_n)^i t_n$$

$$\text{Let } A_{ij} = \sum_{n=1}^N (x_n)^{i+j}, \quad T_i = \sum_{n=1}^N (x_n)^i t_n$$

$$\text{we have } \sum_{j=0}^M A_{ij} w_j = T_i$$