

11. Log likelihood function (1.54)

$$\ln P(x|\mu, \sigma^2) = -\frac{1}{2\sigma^2} \sum_{n=1}^N (x_n - \mu)^2 - \frac{N}{2} \ln \sigma^2 - \frac{N}{2} \ln(2\pi)$$

$$\frac{\partial \ln P(x|\mu, \sigma^2)}{\partial \mu} = -\frac{1}{2\sigma^2} \cdot 2 \cdot \sum_{n=1}^N (x_n - \mu) = 0$$

$$\Rightarrow \sum_{n=1}^N x_n = N\mu$$

$$\Rightarrow \mu_{ML} = \frac{1}{N} \sum_{n=1}^N x_n$$

$$\frac{\partial \ln P(x|\mu_{ML}, \sigma^2)}{\partial \sigma^2} = -\frac{N}{2} \frac{1}{\sigma^2} + \frac{1}{2(\sigma^2)^2} \sum (x - \mu_{ML})^2 = 0$$

$$\Rightarrow \sigma_{ML}^2 = \frac{\sum (x - \mu_{ML})^2}{N}$$