2017 Applied Exam Q2

November

(i)

```
d <- read.csv("2017QualDataQ2.csv", header = T)
code <- function(x) {
  y <- x
  y[x == min(x)] <- -1
  y[x == max(x)] <- 1
  y[x > min(x) & x < max(x)] <- 0
  y
}
d <- within(d, {
  Design <- as.factor(Design)
  Machine <- as.factor(Machine)
  block <- as.factor(block)
  cTime <- code(Time)
  cP <- code(P)
  cTemp <- code(Temp)
  y.penal <- ifelse(Effort <= 4, Strength, 0)
})
```

Test whether `Design` interacts with the three settings. It shows that we can include `Design` as additive in the model.

```
m1.1 <- lm(Strength ~ block + Machine + Design * (cTime + cP + cTemp)^2 +
             Design * (I(cTime^2) + I(cP^2) + I(cTemp^2)), data = d)
m1.2 <- lm(Strength ~ block + Machine + Design + (cTime + cP + cTemp)^2 +
             (I(cTime^2) + I(cP^2) + I(cTemp^2)), data = d)
anova(m1.1, m1.2)
```

```
...
##   Res.Df     RSS Df Sum of Sq      F Pr(>F)
## 1     42 0.43117
## 2     51 0.50125 -9 -0.070076 0.7584 0.6543
...
```

Now, use `{rsm}` to fit the response surface model.

```
m1 <- rsm(Strength ~ block + Machine + Design + FO(cTime, cP, cTemp),
          data = d)
summary(m1)
```

```
...
## Analysis of Variance Table
##
## Response: Strength
##                       Df  Sum Sq Mean Sq  F value    Pr(>F)
## block                  1  0.5175  0.5175  16.2675 0.0001654
## Machine                1  0.0113  0.0113   0.3548 0.5537451
## Design                 1  0.4016  0.4016  12.6243 0.0007727
## FO(cTime, cP, cTemp)   3 16.3636  5.4545 171.4472 < 2.2e-16
## Residuals             57  1.8134  0.0318
## Lack of fit           10  1.3313  0.1331  12.9793 1.762e-10
## Pure error            47  0.4821  0.0103
...
```

```
m2 <- rsm(Strength ~ block + Machine + Design + SO(cTime, cP, cTemp),
          data = d)
summary(m2)
```

```
...
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept)  1.3709375  0.0391878 34.9838 < 2.2e-16 ***
## block2      -0.0219792  0.0314589 -0.6987   0.48794
## Machine2     0.0265625  0.0247845  1.0717   0.28888
## Design2     -0.1584375  0.0247845 -6.3926 4.953e-08 ***
## cTime        0.5935000  0.0156751 37.8626 < 2.2e-16 ***
## cP           0.2050000  0.0156751 13.0781 < 2.2e-16 ***
## cTemp       -0.1217500  0.0156751 -7.7671 3.366e-10 ***
## cTime:cP    -0.0303125  0.0175253 -1.7296   0.08974 .
## cTime:cTemp -0.0096875  0.0175253 -0.5528   0.58283
## cP:cTemp     0.0153125  0.0175253  0.8737   0.38636
## cTime^2      0.3082292  0.0314589  9.7978 2.576e-13 ***
## cP^2        -0.0317708  0.0314589 -1.0099   0.31730
## cTemp^2      0.0744792  0.0314589  2.3675   0.02174 *
## Analysis of Variance Table
##
## Response: Strength
##                        Df  Sum Sq Mean Sq  F value    Pr(>F)
## block                   1  0.5175  0.5175  52.6584 2.141e-09
## Machine                 1  0.0113  0.0113   1.1486    0.2889
## Design                  1  0.4016  0.4016  40.8653 4.953e-08
## FO(cTime, cP, cTemp)    3 16.3636  5.4545 554.9788 < 2.2e-16
## TWI(cTime, cP, cTemp)   3  0.0399  0.0133   1.3535    0.2675
## PQ(cTime, cP, cTemp)    3  1.2723  0.4241  43.1499 5.001e-14
## Residuals              51  0.5012  0.0098
## Lack of fit             4  0.0191  0.0048   0.4667    0.7598
## Pure error             47  0.4821  0.0103
...
```

```
m3 <- rsm(Strength ~ block + Machine + Design + FO(cTime, cP, cTemp) +
            PQ(cTime, cTemp), data = d)
summary(m3)
```

```
##
## Call:
## rsm(formula = Strength ~ block + Machine + Design + FO(cTime,
##     cP, cTemp) + PQ(cTime, cTemp), data = d)
##
##               Estimate Std. Error t value  Pr(>|t|)
## (Intercept)  1.357791   0.037324 36.3786 < 2.2e-16 ***
## block2      -0.014310   0.030827 -0.4642   0.64433
## Machine2     0.026563   0.025027  1.0614   0.29316
## Design2     -0.158437   0.025027 -6.3307 4.680e-08 ***
## cTime        0.593500   0.015828 37.4960 < 2.2e-16 ***
## cP           0.205000   0.015828 12.9514 < 2.2e-16 ***
## cTemp       -0.121750   0.015828 -7.6919 2.776e-10 ***
## cTime^2      0.300560   0.030827  9.7499 1.372e-13 ***
## cTemp^2      0.066810   0.030827  2.1673   0.03456 *
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Multiple R-squared:  0.9712, Adjusted R-squared:  0.967
## F-statistic: 231.5 on 8 and 55 DF,  p-value: < 2.2e-16
##
## Analysis of Variance Table
##
## Response: Strength
##                       Df  Sum Sq Mean Sq  F value    Pr(>F)
## block                  1  0.5175  0.5175  51.6437 1.867e-09
## Machine                1  0.0113  0.0113   1.1265    0.2932
## Design                 1  0.4016  0.4016  40.0779 4.680e-08
## FO(cTime, cP, cTemp)   3 16.3636  5.4545 544.2855 < 2.2e-16
## PQ(cTime, cTemp)       2  1.2623  0.6311  62.9776 5.981e-15
## Residuals             55  0.5512  0.0100
## Lack of fit            8  0.0691  0.0086   0.8419    0.5710
## Pure error            47  0.4821  0.0103
##
## Stationary point of response surface:
##      cTime          cP      cTemp
## -0.9873225  0.0000000  0.9111613
##
## Eigenanalysis:
## eigen() decomposition
## $values
## [1] 0.30056034 0.06681034 0.00000000
##
## $vectors
##       [,1] [,2] [,3]
## cTime    1    0    0
## cP       0    0    1
## cTemp    0    1    0
```

The estimated coefficient for `Design2` is negative, suggesting that at the same level of the covariates, Design2 has smaller strength. The contour plots suggest border would give larger strength. Use grid search to find the optimal setting, and it gives the same result. `cTemp=-1, cTime=1, cP=1, Design=1` would provide the strongest seal. From the contour plot, we can see no big difference between the machines.

`cTime` and `cTemp` affects the strength quadratically, while `cP` has linear influence.

```
# grid search
newdata1 <- expand.grid(cTemp = seq(-1, 1, by = 0.1),
                        cTime = seq(-1, 1, by = 0.1),
                        cP = seq(-1, 1, by = 0.1))
n <- dim(newdata1)[1]
newdata2 <- data.frame(Design = rep("1", n),
                       block = rep("1", n),
                       Machine = rep("1", n))
newdata <- cbind(newdata1, newdata2)
pred <- predict(m3, newdata)
ind <- which.max(pred)
newdata[ind, ]
```

```
##      cTemp cTime cP Design block Machine
## 9241    -1     1  1      1     1       1
```

```
pred[ind]
```

```
##     9241
## 2.645412
```

(ii)

As shown in (i), the first-order model doesn't fit well, and in order to fit a second-order model, we need more points, both axial and center points.

(iii)

As shown in Figure 5 in Appendix, for Design 2, Effort is acceptable for whatever `Strength`. So I would recommend the same setting as in (i) to achieve strong seals with effort ≤ 4.

(iv)

```
m4 <- rsm(y.penal ~ block + Machine + Design + SO(cTime, cP, cTemp),
          data = d)
```

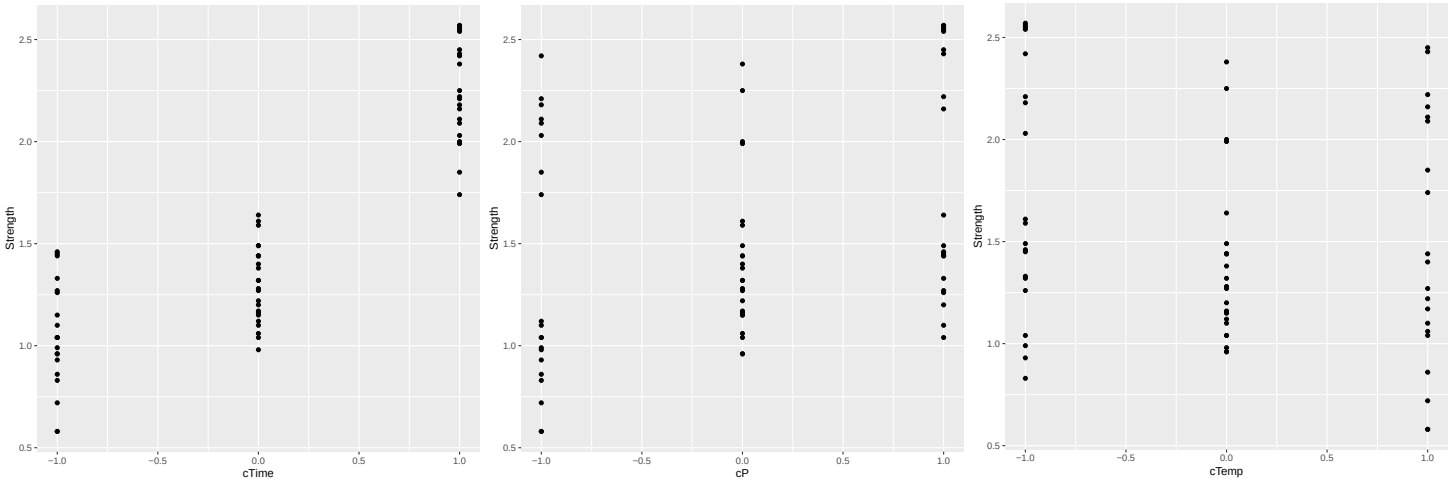
Packages

All R packages used in this problem are listed below.

```
library(gmodels)
library(MASS)
library(car)
library(dplyr)
```

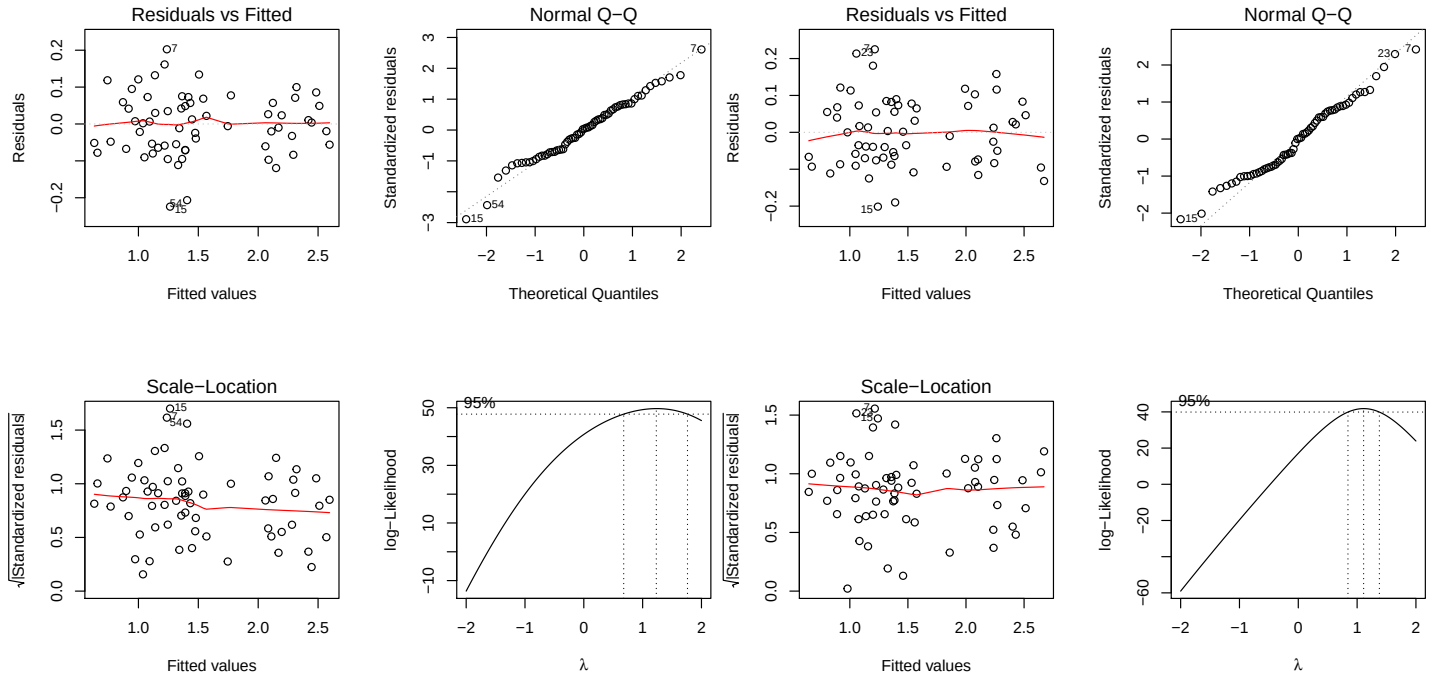
Appendix

Figures



(a) cTime vs Strength (b) cP vs Strength (c) cTemp vs Strength

Figure 1: Scatter plots



(a) Diagnostics for m1.1

(b) Diagnostics for m3

Figure 2: Model Diagnostics

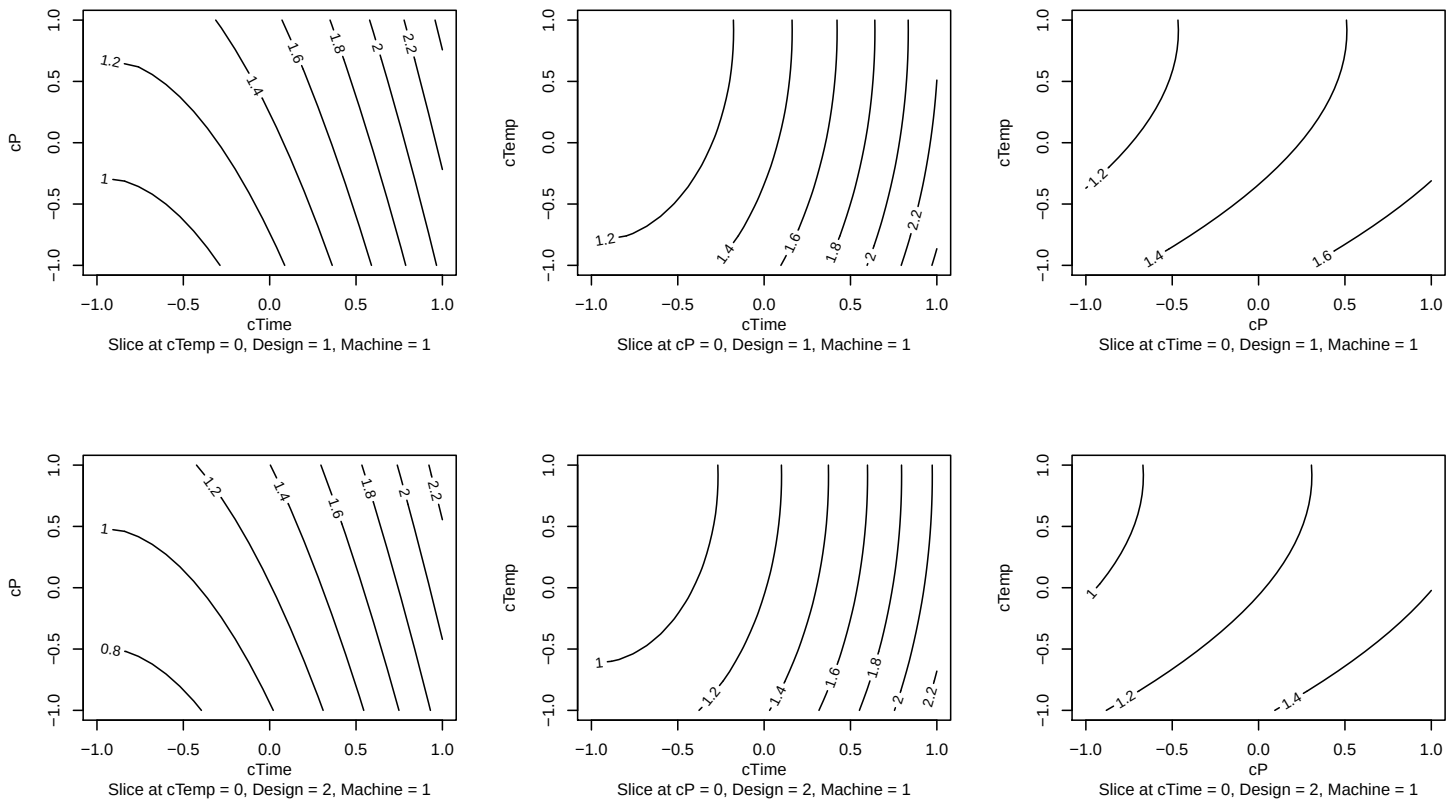


Figure 3: Contour Plots: Different Designs

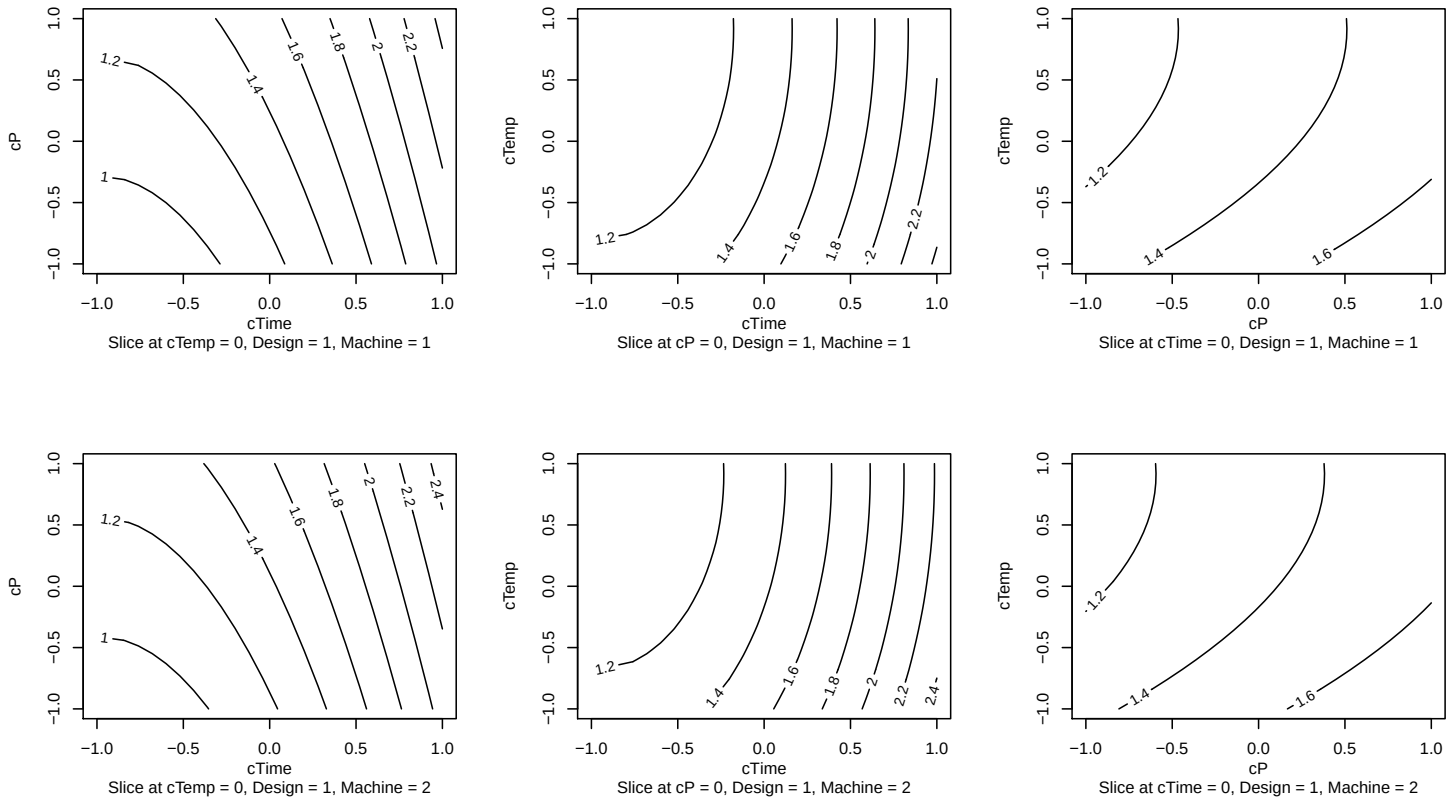


Figure 4: Contour Plots: Different Machines

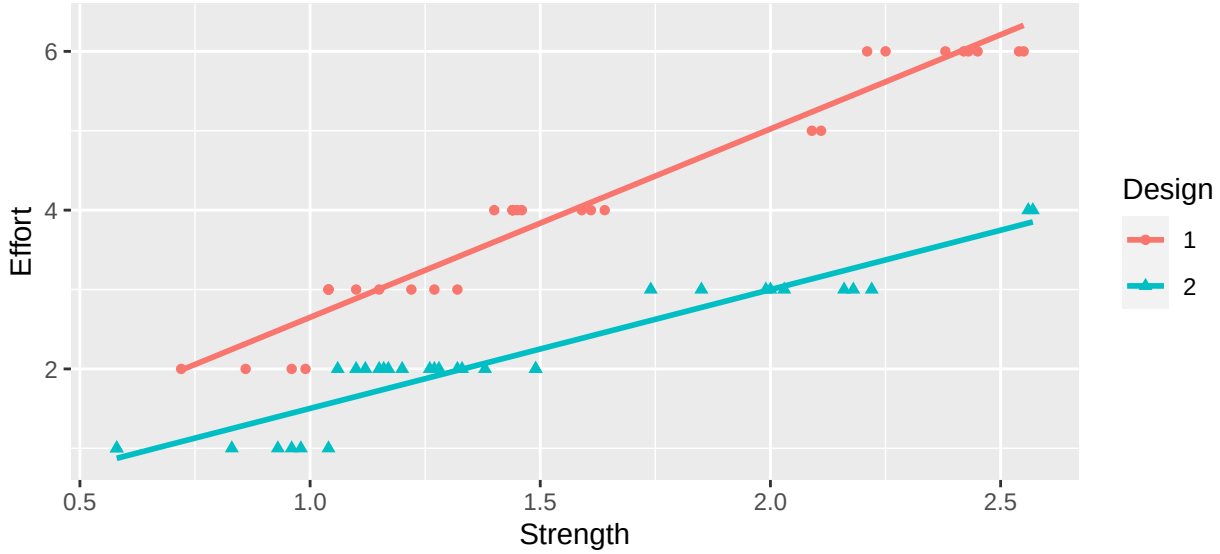


Figure 5: Strength vs Effort

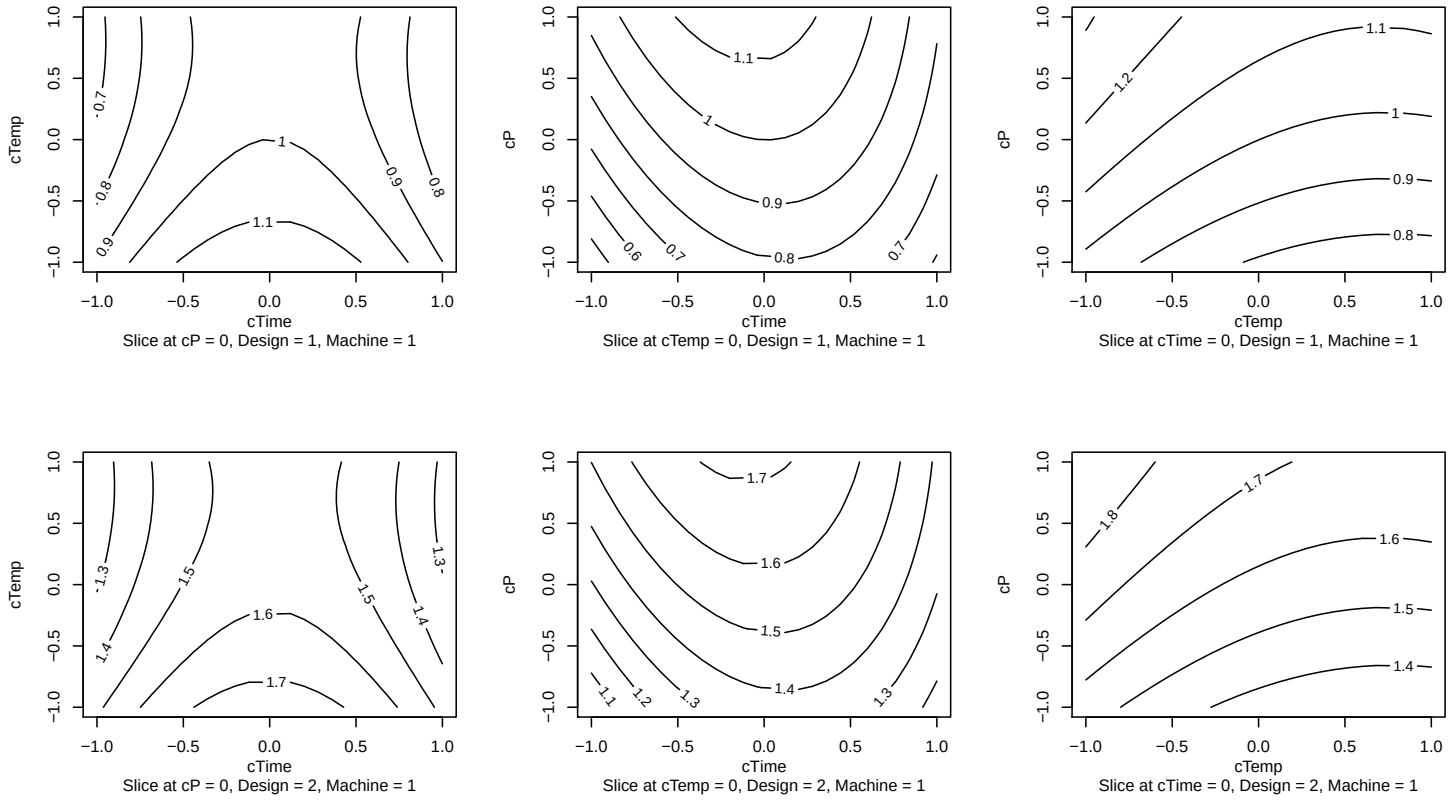


Figure 6: Contour Plots for m4