Chapter 2: Exploring Data

Yu Yang

School of Statistics
University of Minnesota

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Types of Data

A *variable* is any characteristic of a subject in a population.
ex: height, IQ, income, # of hot dogs eaten last year, gender, eye color

1. Categorical (Qualitative) Variable:
   Classifies subjects as belonging to a certain group/category.
   ex: gender, eye color, car make, race, major, area code
2. Quantitative Variable:
   Takes on numerical values that represent different magnitudes.
   2.1 Discrete : The possible values of a discrete quantitative variable form a set of separate numbers (i.e. can be listed).
   ex: # of hot dogs eaten, # of t.v.'s, # of accidents/day
   2.2 Continuous : The possible values of a continuous quantitative variable form an interval. That is, there is an infinite continuum of possible values.
   ex: height, blood pressure, amount of rainfall

Numerical Summaries of Categorical Data

Frequency Table

A *frequency table* is a listing of possible values for a variable, together with the number of observations for each value. (Note that we can also construct frequency tables for quantitative variables.)

Proportion

A *proportion* of observations that fall in a certain category is the count of observations in that category divided by the total number of observations. (NOTE: percent $= 100 \times$ proportion)

Frequency Table

Social Media	Frequency	Proportion	Percent
Facebook	18	.050	5%
Instagram	172	.480	48%
Twitter	45	.126	12.6%
YouTube	82	.229	22.9%
others	41	.115	11.5%
Total	358	1	100%

Figure 1: Example Frequency Table

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Numerical Summaries of Quantitative Data

Notations

1. $n =$ the number of observations in a sample
2. $x_i =$ the ith observation of a sample (so the list of observations is $x_1, x_2, \cdots, x_n$)
3. $\sum =$ summation

$$\sum x_i = x_1 + x_2 + \cdots + x_n$$

Measures of Center

1. **mean** ($\overline{x}$) = the average of all observations

$$\overline{x} = \frac{\sum x_i}{n}$$

2. **median** (M) = the middle number when measurements are ordered from smallest to largest
   When n is odd, $M =$ **the middle value.**
   When n is even, $M =$ **the average of the middle two values.**
3. mean vs. median

Measures of Spread I

Range

The *range* is the difference between the largest and smallest observations. That is,

$$\textbf{range} = \textbf{maximum} - \textbf{minimum}$$

Measures of Spread II

Percentile

The pth *percentile* of a distribution is the value below which p% of the observations fall.

Interquartile Range (IQR)

The *interquartile range* is the difference between the first and third quartiles. That is,

$$\textbf{IQR} = \textbf{Q3} - \textbf{Q1}$$

1. **First Quartile (Q1)** = 25th percentile
   The lowest 25% of the data lies below Q1.
2. **Second Quartile (Q2)** = 50th percentile = median
   50% of the data are below and 50% are above M
3. **Third Quartile (Q3)** = 75th percentile
   The highest 25% of the data lies above Q3.

Measures of Spread III

Sample Variance (s^2)

The *sample variance* of a set of observations is the "average" of the squared deviations from the mean.

$$s^2 = \frac{\sum(x_i - \overline{x})^2}{n-1} = \frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}$$

Sample Standard Deviation (s)

The *standard deviation* is the square root of the sample variance:

$$s = \sqrt{s^2} = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n-1}}$$

Measure of Spread III (cont.)

Properties of s

1. Interpretation: distance that a "typical" observation falls from the mean
2. s is measured in the same units as the original observations
3. Use s in conjunction with mean
4. $s \geq 0$
5. The larger s is, the greater the spread of the data.
6. $s = 0 \Rightarrow$ there is no variation in the data.
7. s is also not robust to outliers and skewness.

Measure of Spread III (cont.)

Interpreting the Magnitude of s

Unless the data set is extremely skewed or has extreme outliers, nearly all of the observations will fall within $3s$ of the mean $\overline{x}$.

$$[\overline{x} - 3s, \overline{x} + 3s] \tag{1}$$

5-number Summary

The *5-number summary* is a brief numerical description of the center *and* spread of a distribution:

minimum **Q1** M **Q3** **maximum**

Choosing the Proper Numerical Summaries

- mean vs. median
- range vs. IQR vs. s
- (Read Handout p31)

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Graphical Summaries of Categorical Data

Pie Chart

A circle is drawn with a "slice of pie" representing each category's % of observations.

Bar Graph

A bar is drawn for each category with the bar's height representing the % or count of observations.

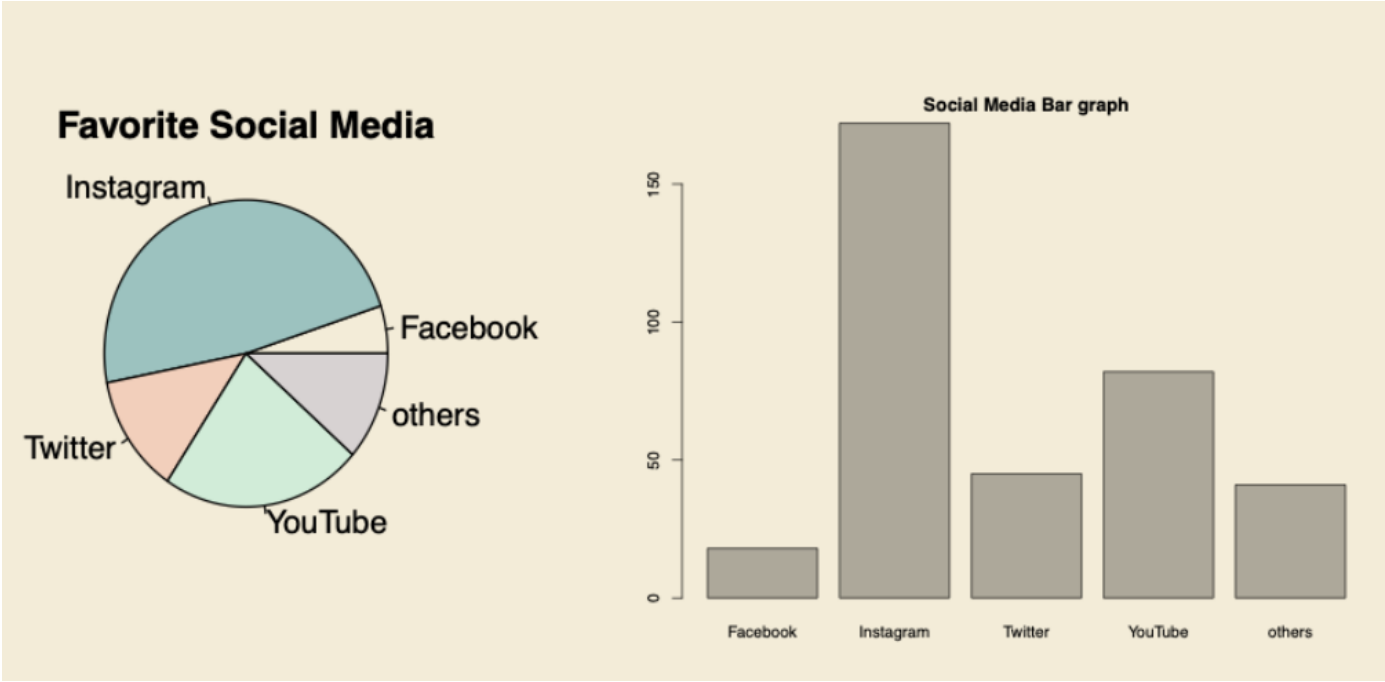

Figure 2: Pie Chart and Barplot

Graphical Summaries of Categorical Data (cont.)

Pie Charts vs. Bar Graphs

1. Pie charts emphasize a category's relation to the whole, but make it difficult to compare categories to each other.
2. Bar graphs compare the sizes of each group of a categorical variable (not in relation to the whole).
3. Bar graphs are easier to read and more flexible than pie charts.

Graphical Summaries of Quantitative Data

Distribution

A distribution of data shows the values a variable takes and how often they occur.

Major Focuses

1. Generating plots
   - 1.1 Stem-and-Leaf Plot
   - 1.2 Histogram
   - 1.3 Boxplot
2. Interpreting plots (understand distribution)
   - 2.1 overall shape
   - 2.2 center and spread
   - 2.3 outliers
3. Comparing plots

Stem-and-Leaf Plot

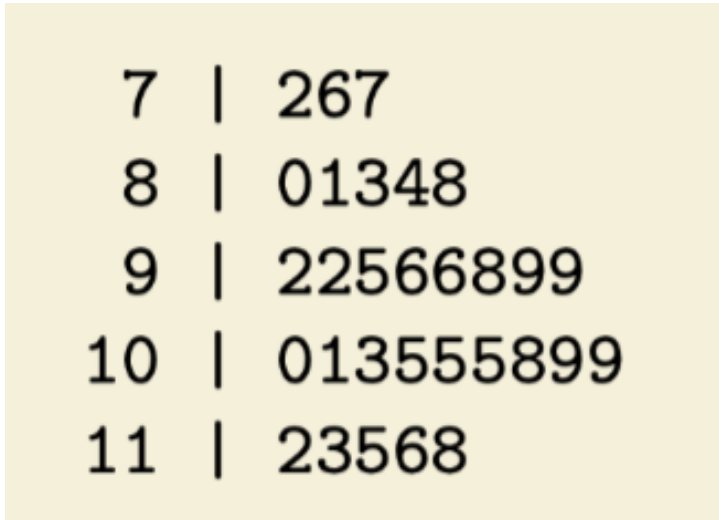

Figure 3: Example 2.2 Stem-and-Leaf Plot

1. Shape: The lower "tail" extends further than the upper "tail".
2. Center: 99 million.
3. Spread: The salaries range from 72 to 118 million.
4. Outlier: No outliers. No team appears to have a salary much smaller or larger than the other teams.

Histogram

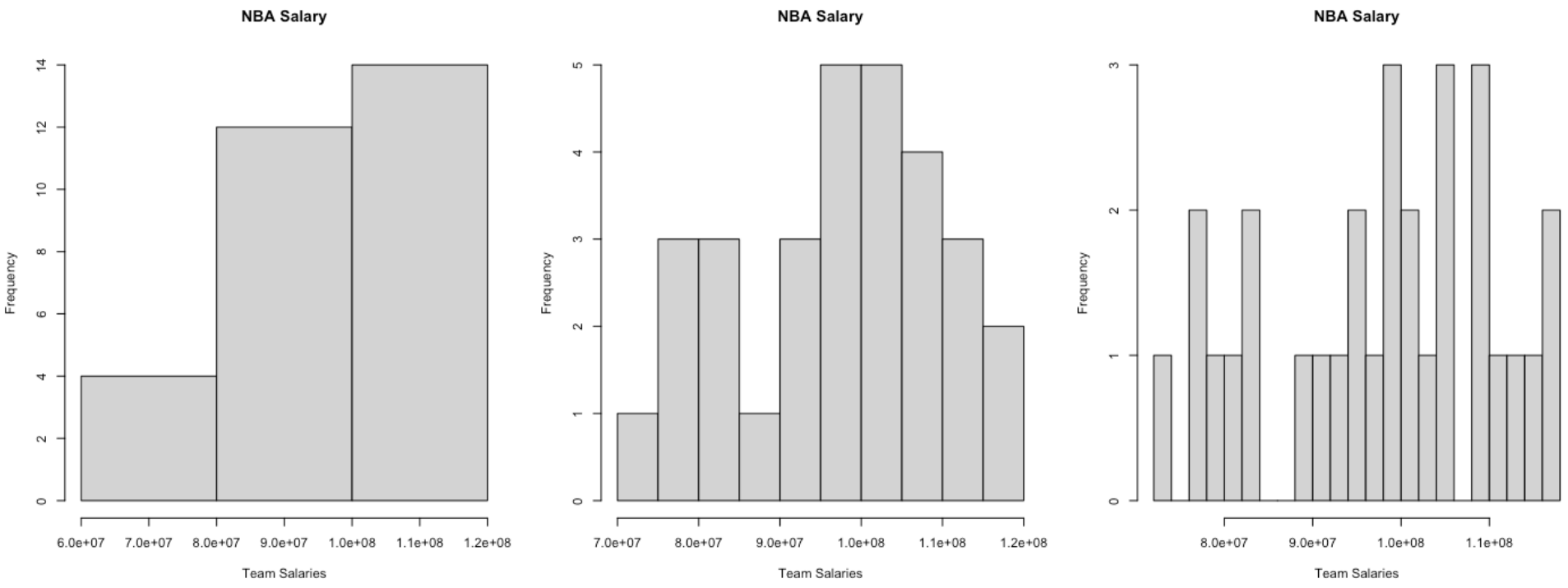

Figure 4: Example 2.2 Histogram

Common Distribution Shapes

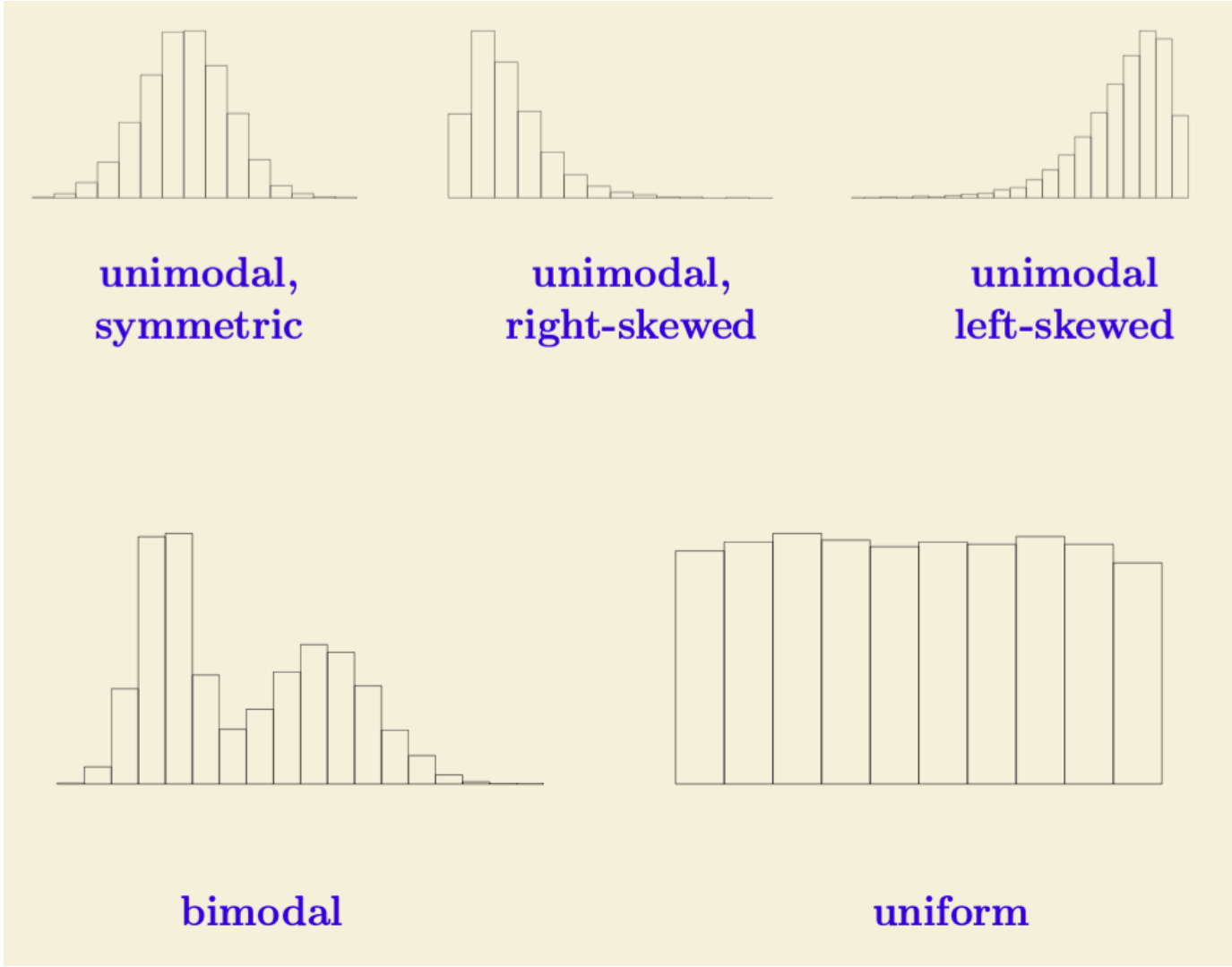

Figure 5: Common Distribution Shapes

Use Mean and Median to tell Skewness

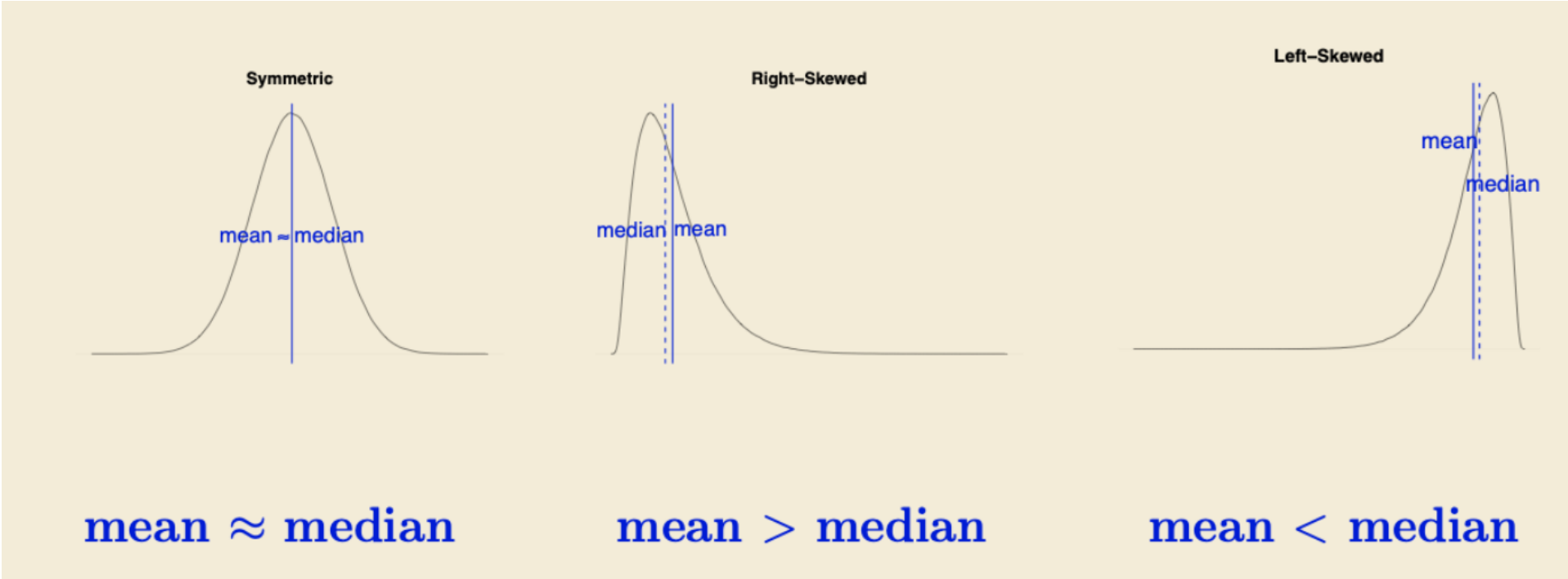

Figure 6: Mean vs. Median in different shapes

Comparing Mean and Median in Histograms

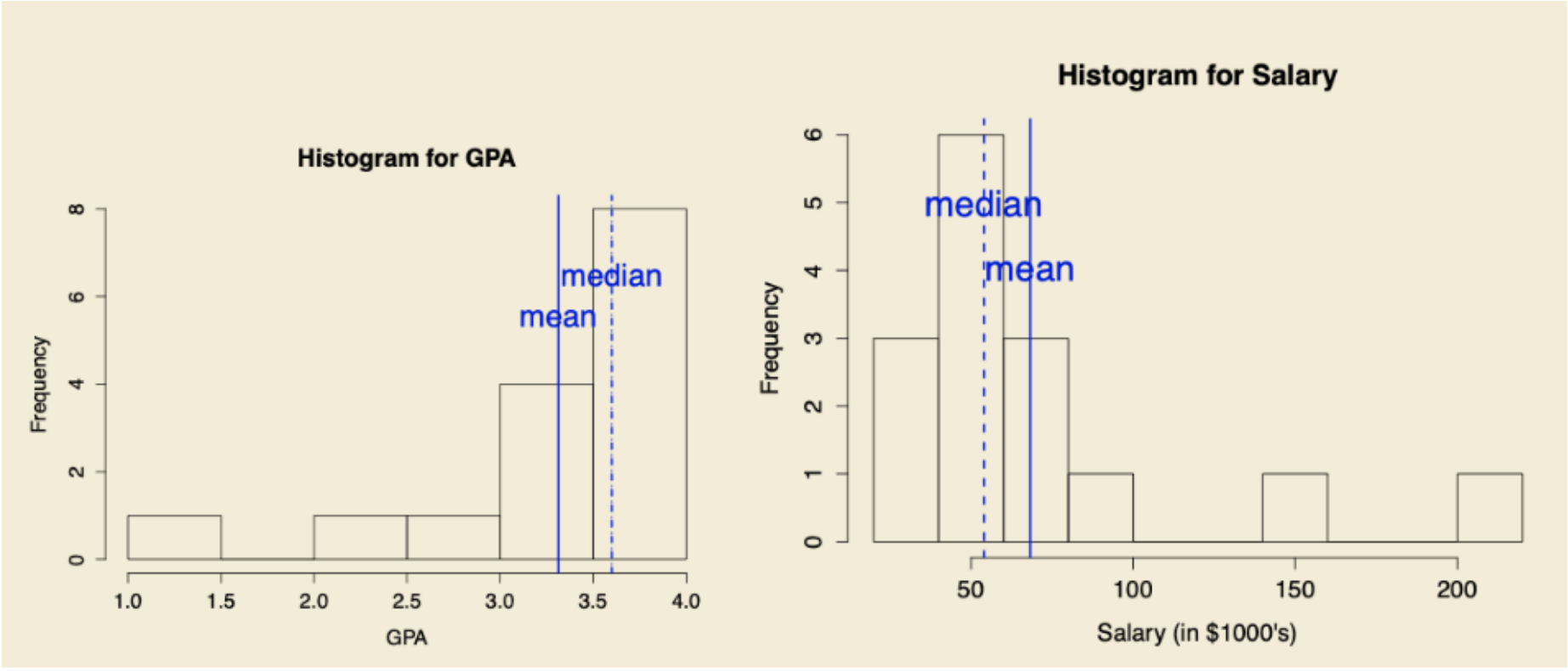

Figure 7: Example 2.3

Comparing Standard Deviations in Histograms

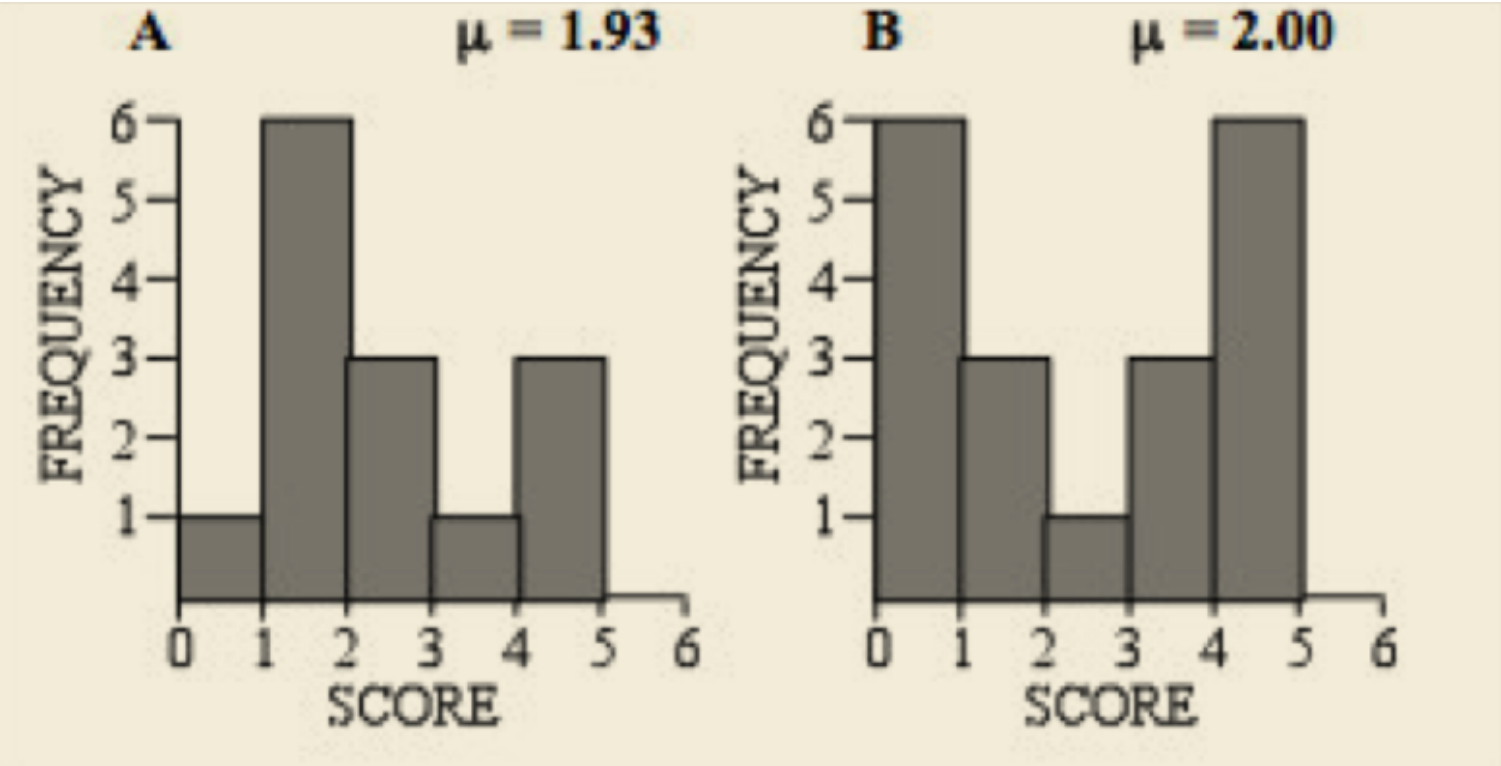

Figure 8: Comparing Standard Deviations Example 1

Comparing Standard Deviations in Histograms

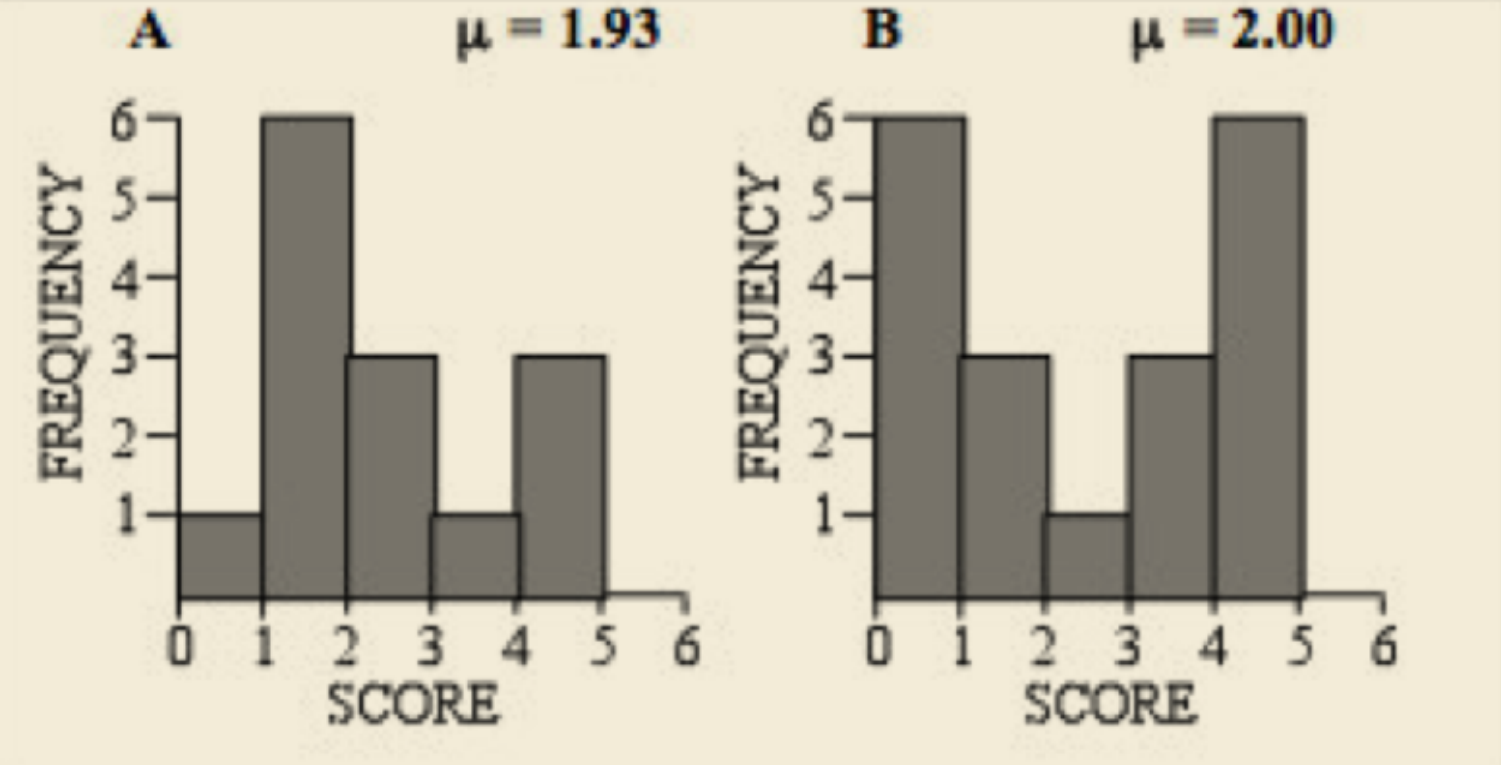

Figure 8: Comparing Standard Deviations Example 1

B has a larger standard deviation, data is more spread out from the mean

Comparing Standard Deviations in Histograms (cont.)

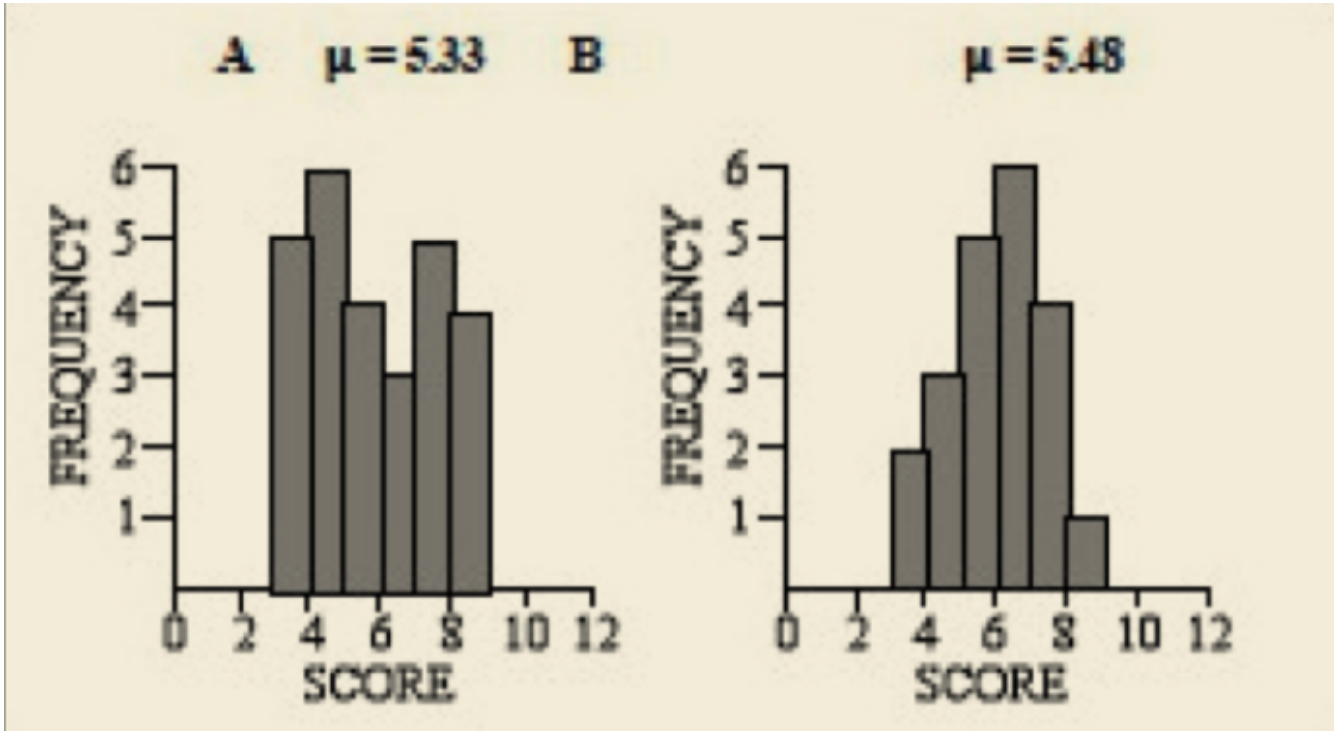

Figure 9: Comparing Standard Deviations Example 2

Comparing Standard Deviations in Histograms (cont.)

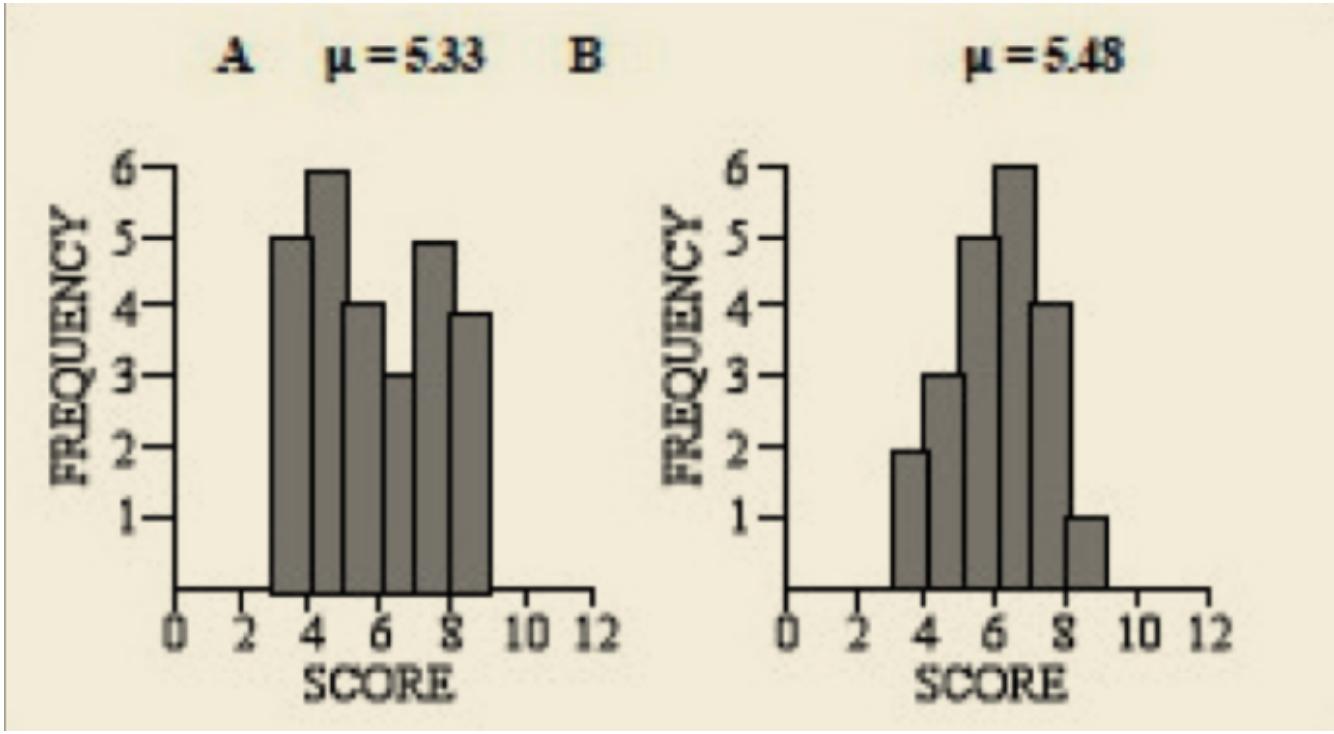

Figure 9: Comparing Standard Deviations Example 2

A has a larger standard deviation, data is more spread out from the mean

Comparing Standard Deviations in Histograms (cont.)

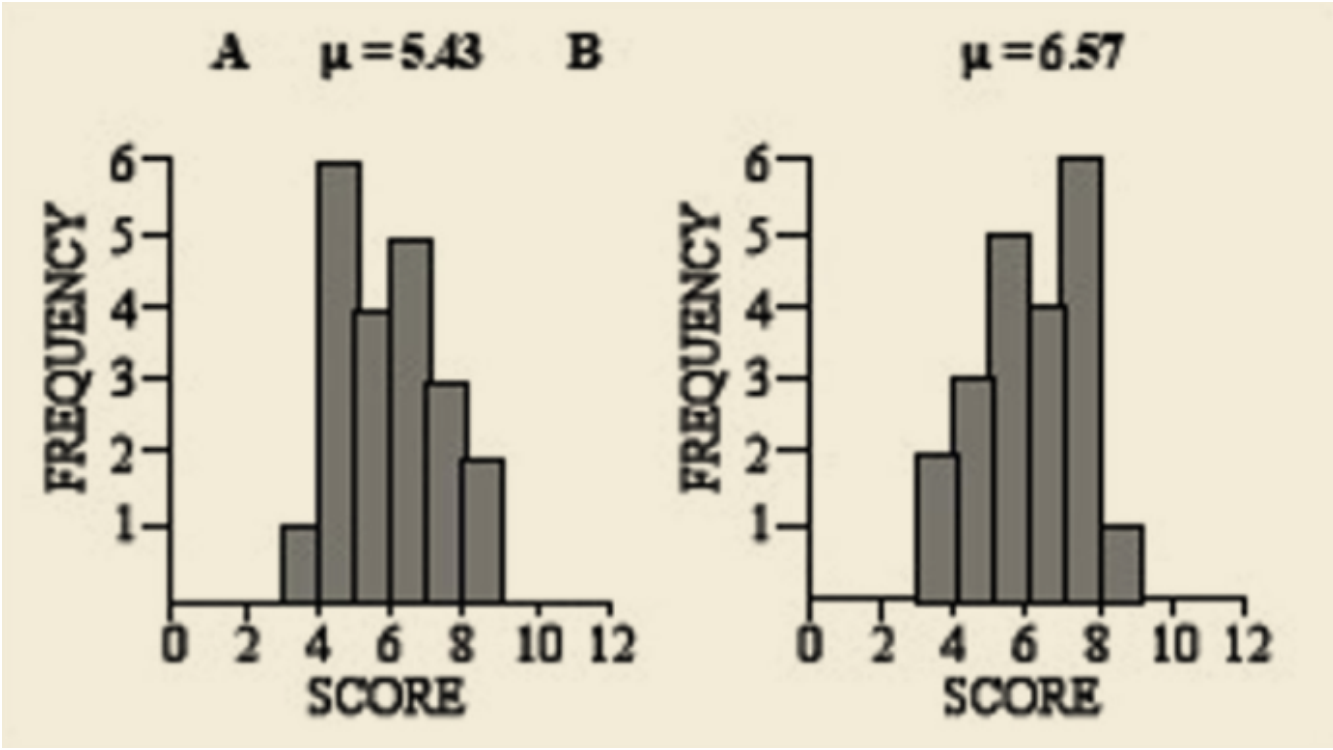

Figure 10: Comparing Standard Deviations Example 3

Comparing Standard Deviations in Histograms (cont.)

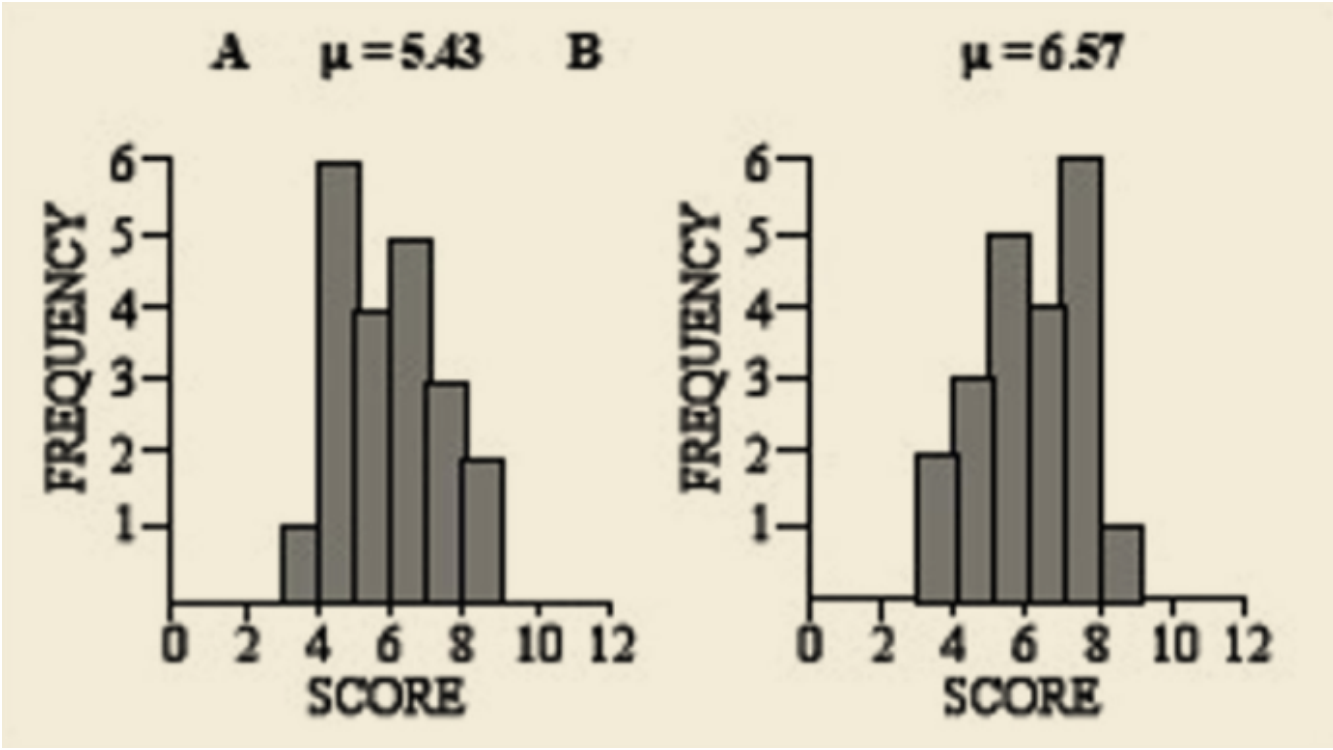

Figure 10: Comparing Standard Deviations Example 3

Both graphs have the same standard deviation, graphs are mirror image of each other

Comparing Standard Deviations in Histograms (cont.)

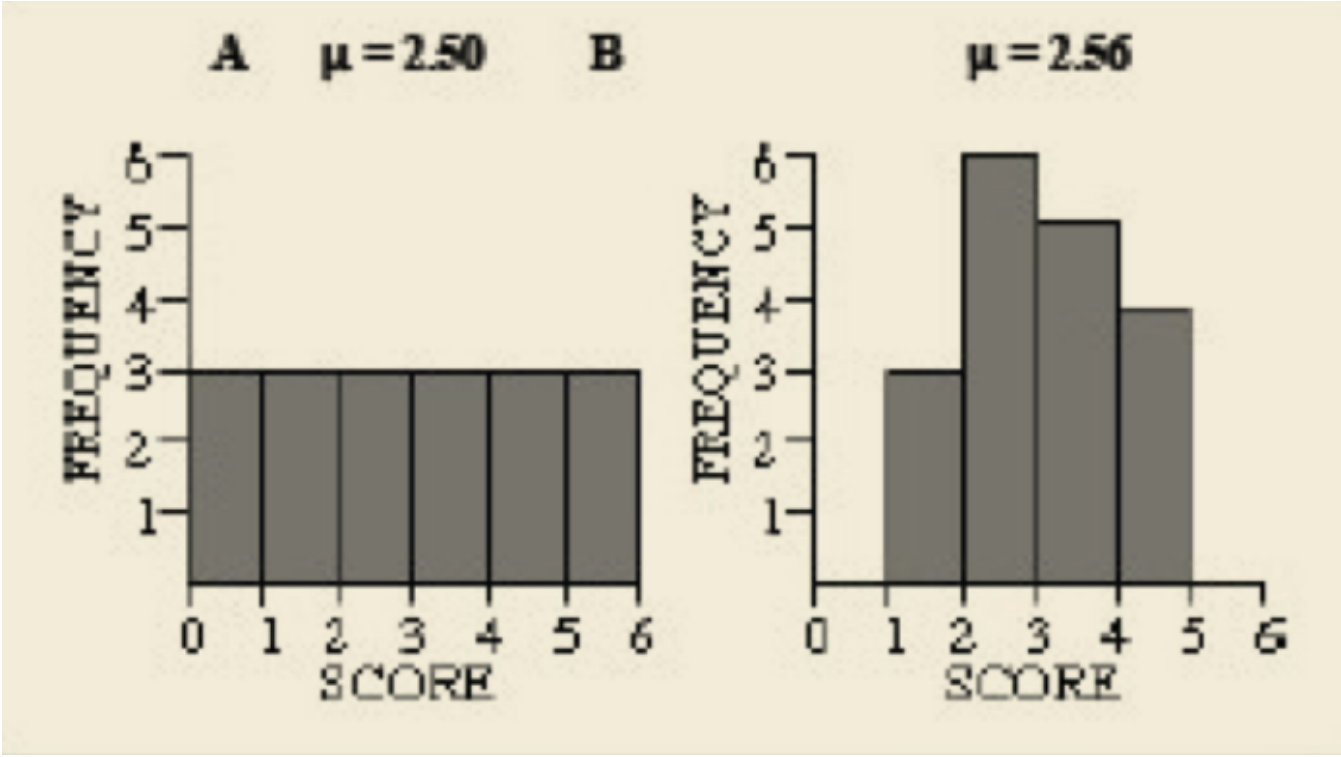

Figure 11: Comparing Standard Deviations Example 4

Comparing Standard Deviations in Histograms (cont.)

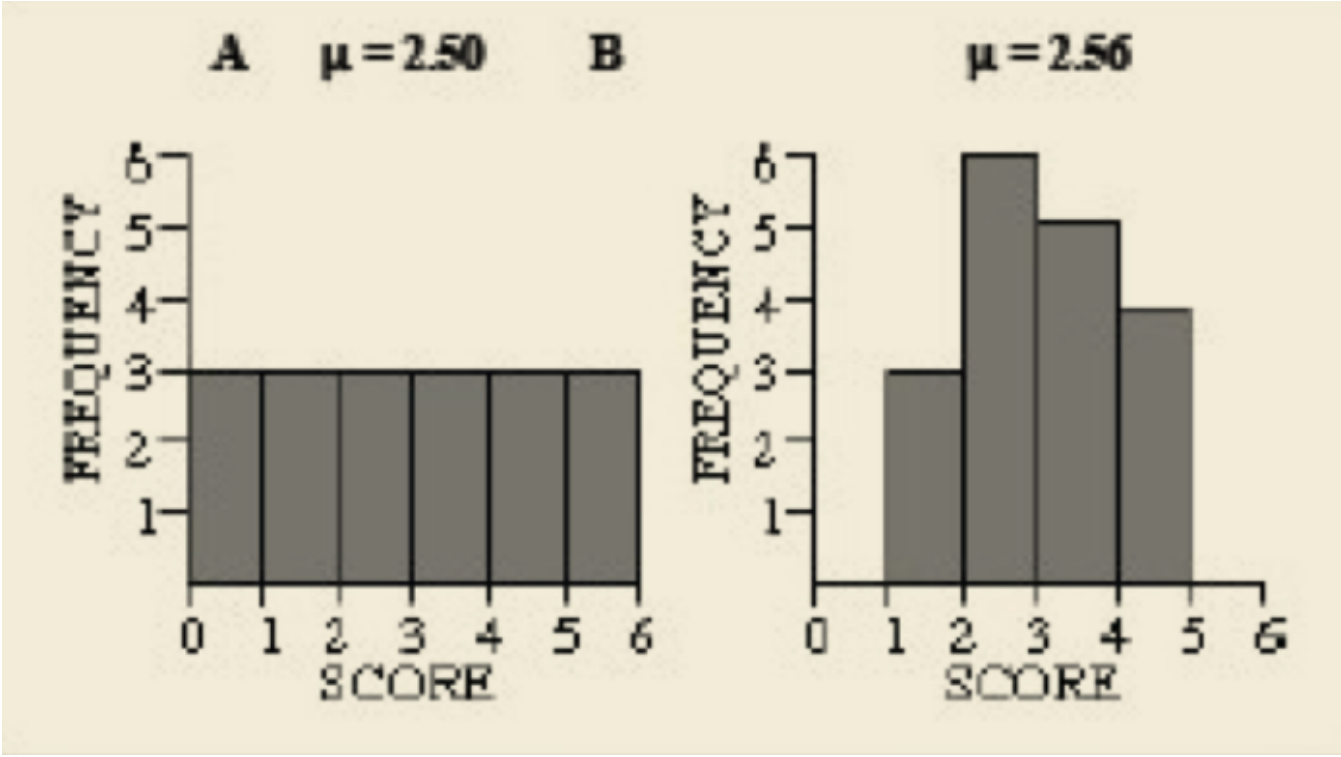

Figure 11: Comparing Standard Deviations Example 4

A has a larger standard deviation, data is more spread out from the mean

Comparing Standard Deviations in Histograms (cont.)

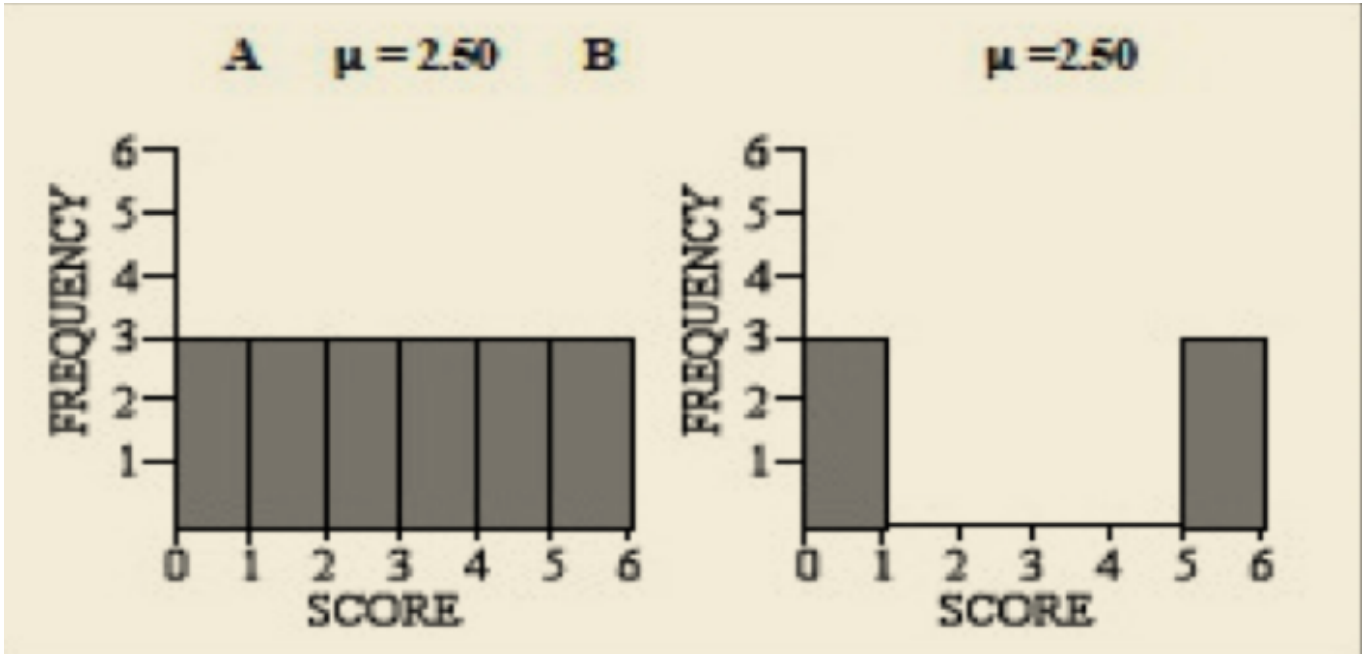

Figure 12: Comparing Standard Deviations Example 5

Comparing Standard Deviations in Histograms (cont.)

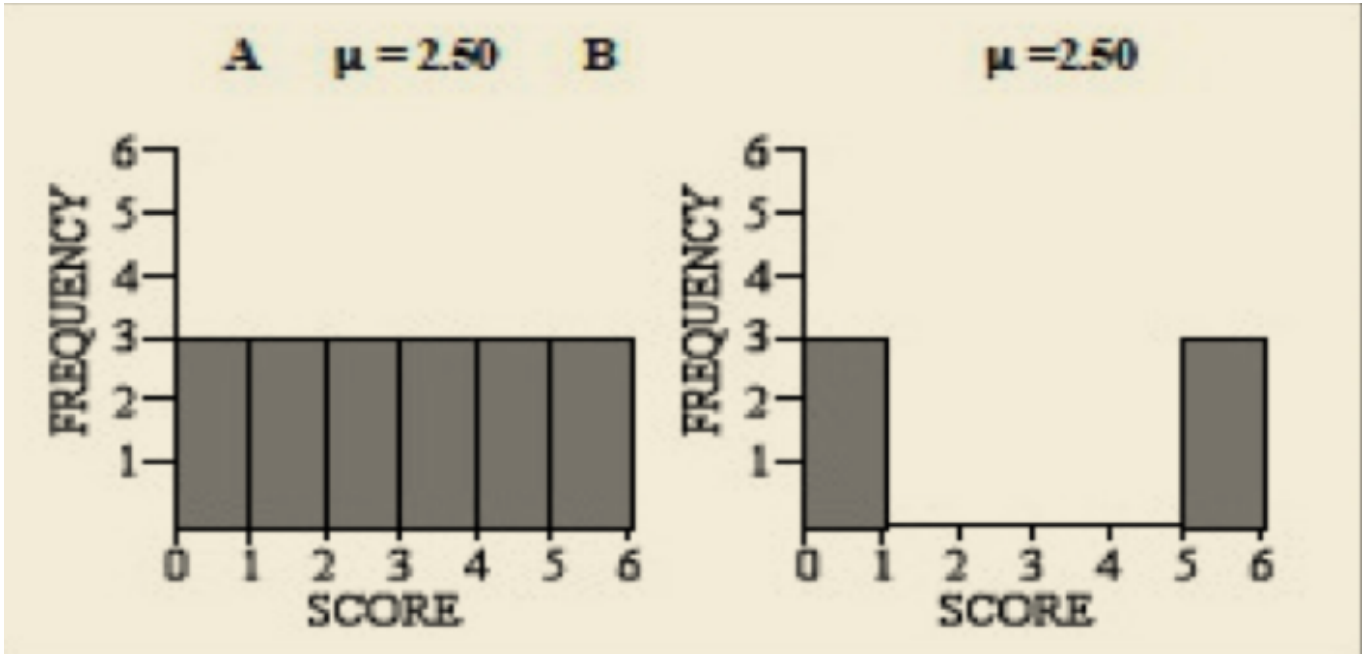

Figure 12: Comparing Standard Deviations Example 5

B has a larger standard deviation, data is more spread out from the mean the data close to the mean in graph A balance out while in B the data is further away from the mean resulting in more variabilit

Box Plot

Box Plot

The box plot is a plot of the five number summary.

Side-by-side Box Plot

A side-by-side boxplot includes boxplots for more than one distribution.

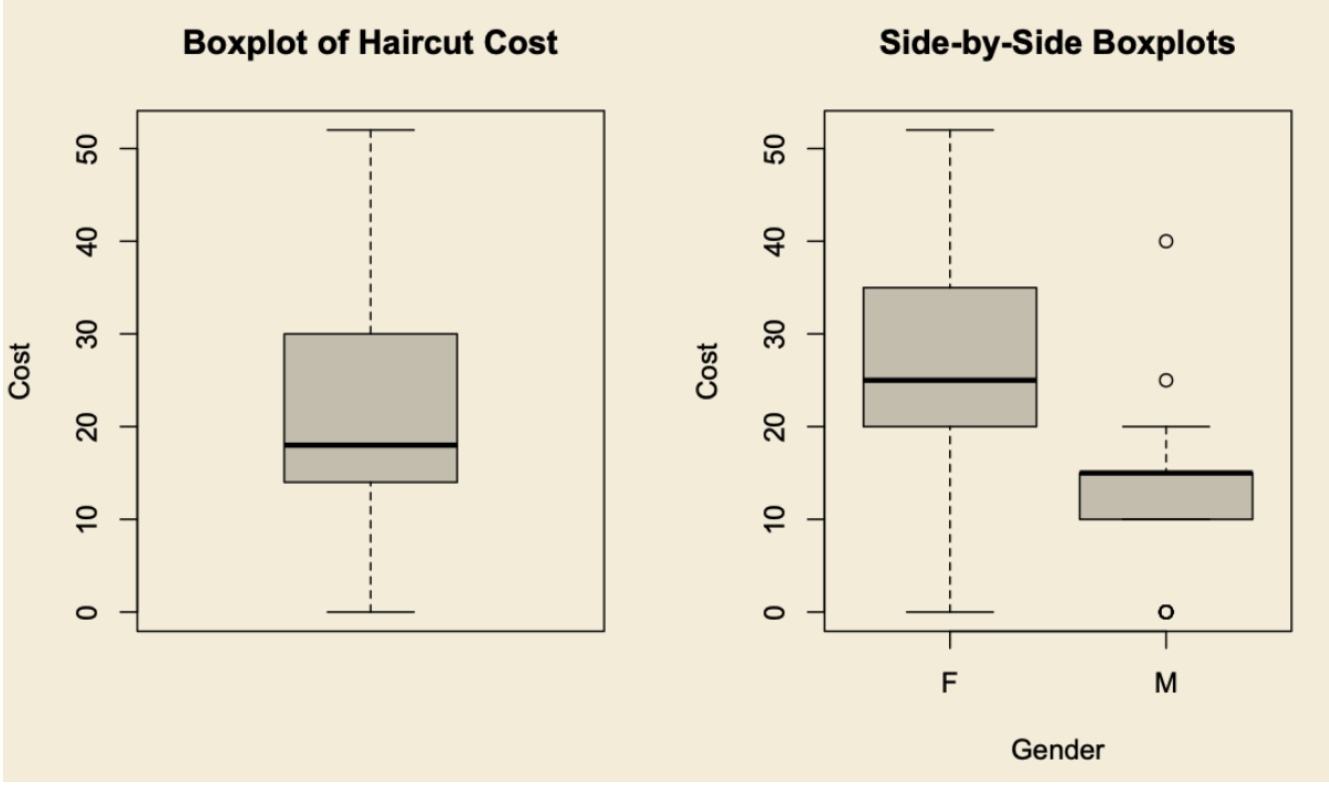

Figure 13: Box Plot and Side-by-side Boxplot (Example 2.4)

Read Skewness from Boxplots

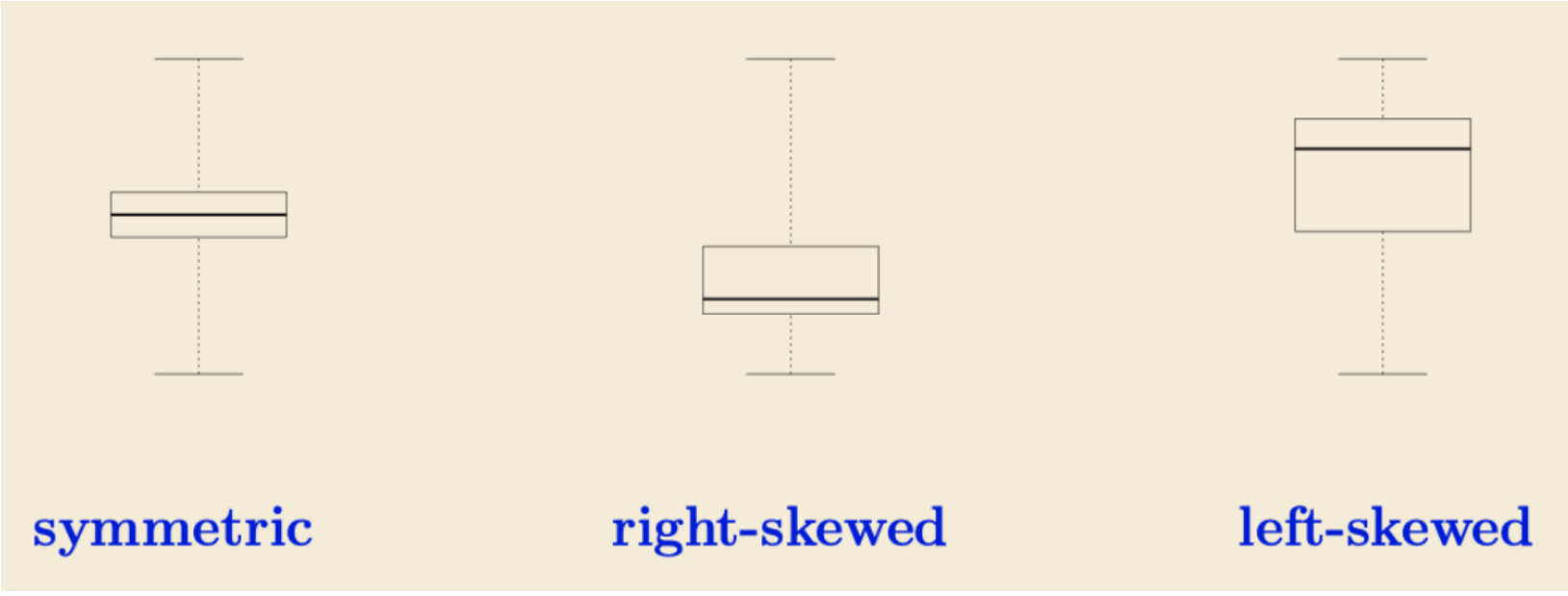

Figure 14: Boxplots with different shapes

Read Spread and Outliers from Boxplots

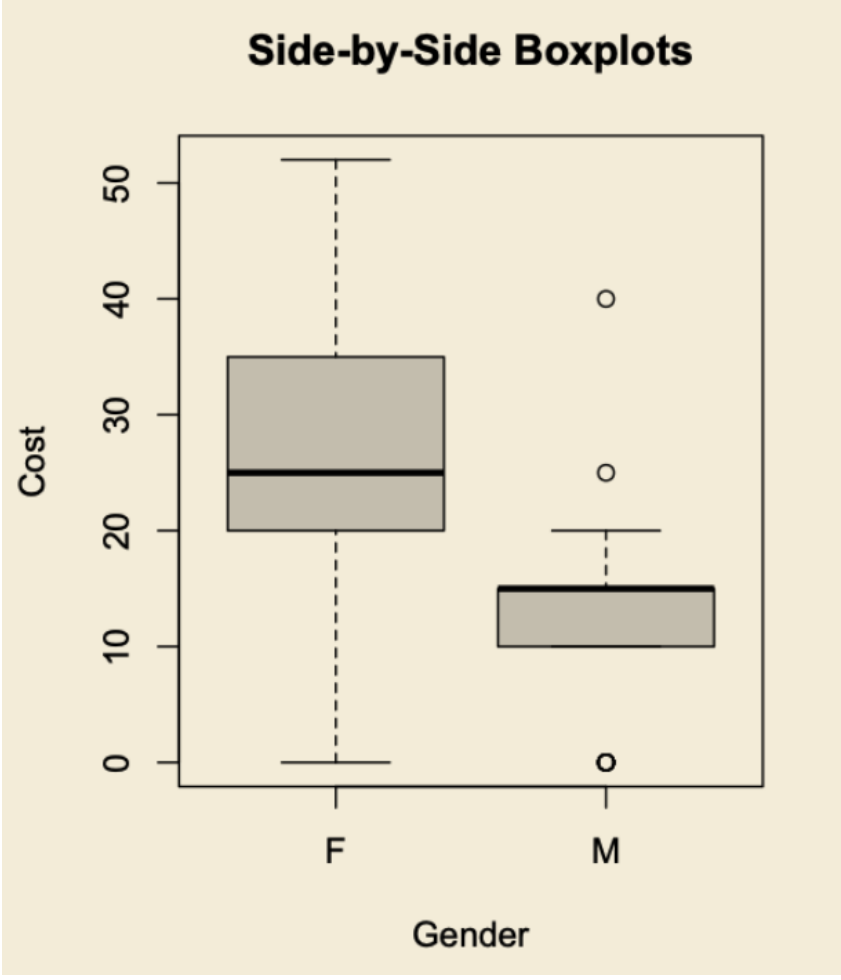

Figure 15: Spread and Outliers from different boxplots