Q1: For $H_0: \beta_1 = 0$ vs. $H_1: \beta_1 < 0$

test statistic is $t = \dfrac{\hat{\beta}_1}{se(\hat{\beta}_1)}$

why do we reject H_0 when t is very small?

Intuitively, this makes sense. But how to get this from strict mathematical proof?

A1: (Credit goes to Ganghua.)

The family of normal densities has monotone likelihood ratio property. By Thm 12.9 on Keener's book ("Theoretical Topics for a Core Course"),

$$\varphi^*(x) = \begin{cases} 1, & T(x) < c \\ 0, & T(x) > c \end{cases}$$ will be the UMP test,

which means uniformly most powerful.

And by checking the density of normal dist, we can show that

$$\varphi(x) = \begin{cases} 1, & t = \dfrac{\hat{\beta}_1}{se(\hat{\beta})} < c \\ 0, & t > c \end{cases}$$

is equivalent to $\varphi^*(x)$.