1.

(a)

$$
\begin{aligned}
\nabla \times \vec{E} =& \mathrm{i}\vec{k} \times E \\
\frac{\partial \vec{B}}{\partial t} =& -\mathrm{i}\omega\vec{B} \\
\omega\vec{B} =& \vec{k} \times E \\
\vec{B} =& \frac{\vec{k}}{\omega} \times E
\end{aligned}
$$

when $\vec{k} \parallel \hat{z}$

$$
\begin{aligned}
\vec{B} =& \frac{k}{\omega}\hat{z} \times E \\
=& \frac{n}{c}\hat{z} \times E
\end{aligned}
$$

Since the material is non-magnetic, both $\vec{E}$ and $\vec{B}$ are continious across the surface

$$
\begin{aligned}
E_{0i} + E_{0r} =& E_{0t} \\
E_{0i} - E_{0r} =& nE_{0t} \\
E_{0t} =& \frac{2}{n+1}E_{0i} \\
E_{0r} =& -\frac{n-1}{n+1}E_{0i}
\end{aligned}
$$

(b)

Define

$$
\begin{aligned}
\tilde{a} =& a_0(z)\mathrm{e}^{\mathrm{i}\phi_a(z)} \\
\tilde{b} =& b_0(z)\mathrm{e}^{\mathrm{i}\phi_b(z)} \\
a(z,t)b^*(z,t) =& \tilde{a}\tilde{b}^* \\
\langle a(z,t)b^*(z,t)\rangle =& \tilde{a}\tilde{b}^* \\
\langle a^*(z,t)b(z,t)\rangle =& \tilde{a}^*\tilde{b} \\
\langle a(z,t)b(z,t)\rangle =& \tilde{a}\tilde{b}\left\langle \mathrm{e}^{-2\mathrm{i}\omega t}\right\rangle = 0 \\
\langle a^*(z,t)b^*(z,t)\rangle =& \tilde{a}^*\tilde{b}^*\left\langle \mathrm{e}^{-2\mathrm{i}\omega t}\right\rangle = 0 \\
\langle \Re(a(z,t))\Re(b(z,t))\rangle =& \frac{1}{4}(\langle a(z,t)b^*(z,t)\rangle + \langle a^*(z,t)b(z,t)\rangle + \langle a(z,t)b(z,t)\rangle + \langle a^*(z,t)b^*(z,t)\rangle) \\
=& \frac{1}{4}\left(\tilde{a}^*\tilde{b} + \tilde{a}\tilde{b}^*\right) \\
=& \frac{1}{2}\Re\left(\tilde{a}^*\tilde{b}\right) \\
=& \frac{1}{2}\Re(a(z,t)b^*(z,t))
\end{aligned}
$$

2.

(a)

$$\begin{aligned}
|n|^2 =&\left|\frac{kc}{\omega}\right|^2\\
=&\left|\frac{k^2}{\omega^2}\right|c^2\\
\approx&\frac{\mu_0\sigma}{\omega}c^2\\
=&\frac{\sigma}{\varepsilon_0\omega}\\
\gg&1\\
\frac{E_{0t}}{E_{0i}} =&\frac{2}{n+1}\\
\approx&\frac{2}{n}\\
\frac{E_{0r}}{E_{0i}} =&-\frac{n-1}{n+1}\\
\approx&-1+\frac{2}{n}
\end{aligned}$$

(b)

Incident

$$\begin{aligned}
\vec{E}_i =&\hat{x}E_{0i}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}k_0z}\\
\vec{B}_i =&\hat{y}\frac{E_{0i}}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}k_0z}
\end{aligned}$$

Transmitted

$$\begin{aligned}
\vec{E}_t =&\hat{x}E_{0t}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}kz}\\
=&\hat{x}E_{0i}\frac{\sqrt{2}\delta\omega}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}(1+\mathrm{i})z/\delta-\mathrm{i}\pi/4}\\
\vec{B}_t =&\hat{y}\frac{nE_{0t}}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}kz}\\
=&\hat{y}\frac{2E_{0i}}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}(1+\mathrm{i})z/\delta}
\end{aligned}$$

Reflected

$$\begin{aligned}
\vec{E}_r =&\hat{x}E_{0r}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0z}\\
=&\hat{x}\left(1-\frac{2\omega\delta}{(1+\mathrm{i})c}\right)E_{0i}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0z}\\
\vec{B}_r =&\hat{y}\frac{E_{0r}}{c}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0z}\\
=&\hat{y}\left(1-\frac{2\omega\delta}{(1+\mathrm{i})c}\right)\frac{E_{0i}}{c}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0z}
\end{aligned}$$

Current density

$$\begin{aligned}\vec{j} =&\sigma\vec{E}_t\\ =&\hat{x}\sigma E_{0i}\frac{\sqrt{2}\delta\omega}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}(1+\mathrm{i})z/\delta-\mathrm{i}\pi/4}\\ =&\hat{x}\frac{2E_{0i}}{\mu_0\omega\delta^2}\frac{\sqrt{2}\delta\omega}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}(1+\mathrm{i})z/\delta-\mathrm{i}\pi/4}\\ =&\hat{x}\frac{2E_{0i}}{\mu_0 c}\frac{\sqrt{2}}{\delta}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}(1+\mathrm{i})z/\delta-\mathrm{i}\pi/4}\end{aligned}$$

(c)

Incident

$$\langle S_i\rangle =\vec{z}\frac{E_{0i}^2}{2c\mu_0}$$

Reflected

$$\begin{aligned}\langle S_r\rangle =&-\vec{z}\frac{\Re(E_{0r})^2}{2c\mu_0}\\ =&-\vec{z}\Re\left(1-\frac{4\omega}{ck}\right)\frac{E_{0i}^2}{2c\mu_0}\\ =&-\langle S_i\rangle\left(1-\frac{2\omega\delta}{c}\right)\end{aligned}$$

Transmitted

$$\begin{aligned}\langle S_t\rangle =&\vec{z}\frac{\left|nE_{0t}^2\right|}{2c\mu_0}\\ =&\vec{z}\Re\left(\frac{4}{n}\right)\frac{E_{0i}^2}{2c\mu_0}\\ =&\langle S_i\rangle\frac{2\omega\delta}{c}\end{aligned}$$

(d)

$$\begin{aligned}R =&\left(1-\frac{2\omega\delta}{c}\right)\\ T =&\frac{2\omega\delta}{c}\end{aligned}$$

(e)

$$\begin{aligned}
P =& \int_0^\infty \left\langle \frac{j^2}{\sigma} \right\rangle \mathrm{d}z \\
=& \frac{1}{2\sigma} \int_0^\infty \mathrm{e}^{-2z/\delta} \mathrm{d}z \left(\frac{2E_{0i}}{\mu_0 c} \frac{\sqrt{2}}{\delta} \right)^2 \\
=& \omega\delta \frac{E_{0i}^2}{\mu_0 c^2} \\
=& \langle S_i \rangle \frac{2\omega\delta}{c}
\end{aligned}$$

(f)

$$\begin{aligned}
\vec{p} =& \int_0^\infty \left\langle \vec{j} \times \vec{B} \right\rangle \mathrm{d}z \\
=& \frac{\hat{z}}{2} \int_0^\infty \Re \left(\frac{2E_{0i}}{\mu_0 c} \frac{\sqrt{2}}{\delta} \mathrm{e}^{-2z/\delta - \mathrm{i}\pi/4} \frac{2E_{0i}}{c} \right) \mathrm{d}z \\
=& \hat{z} \frac{E_{0i}^2}{\mu_0 c^2}
\end{aligned}$$

(g)

$$\begin{aligned}
\lim_{\delta \to 0} \vec{j} =& \vec{x} \frac{2B_{0i}}{\mu_0} \frac{\mathrm{e}^{-z/\delta}}{\delta} \cos\left(\omega t - \frac{z}{\delta} \right) \\
=& \vec{x} \frac{2B_{0i}}{\mu_0} \delta(z) \cos\left(\omega t - \frac{z}{\delta} \right) \\
=& \vec{x} \frac{2B_{0i}}{\mu_0} \delta(z) \cos\omega t
\end{aligned}$$

(h)

The field generated near the surface is $-\vec{y}B_{0i}\cos\omega t$, which is just enough to cancel the incident field.

3.

(a)

$$\begin{aligned}
\vec{a} =& -\frac{e}{m}\vec{E}\\
\vec{v} =& -\frac{e}{-\mathrm{i}m\omega}\vec{E}\\
=& -\mathrm{i}\frac{e}{m\omega}\vec{E}\\
\vec{j} =& \mathrm{i}\frac{n_e e^2}{m\omega}\vec{E}\\
-\vec{k}\times\left(\vec{k}\times\vec{E}\right) =& \mathrm{i}\omega\mu_0\left(\mathrm{i}\frac{n_e e^2}{m\omega}\vec{E} - \mathrm{i}\omega\epsilon_0\vec{E}\right)\\
k^2 =& \frac{1}{c^2}\left(\omega^2 - \frac{n_e e^2}{m\epsilon_0}\right)
\end{aligned}$$

(b)

$n = \frac{kc}{\omega} = \frac{\mathrm{i}c}{\omega\delta}$

Incident

$$\begin{aligned}
\vec{E}_i =& \hat{x}E_{0i}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}k_0 z}\\
\vec{B}_i =& \hat{y}\frac{E_{0i}}{c}\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}k_0 z}
\end{aligned}$$

Transmitted

$$\begin{aligned}
\vec{E}_t =& \hat{x}E_{0t}\mathrm{e}^{-\mathrm{i}\omega t-z/\delta}\\
=& \hat{x}\frac{2}{n+1}E_{0i}\mathrm{e}^{-\mathrm{i}\omega t-z/\delta}\\
\vec{B}_t =& \hat{y}\frac{nE_{0t}}{c}\mathrm{e}^{-\mathrm{i}\omega t-z/\delta}\\
=& \hat{y}\frac{2n}{(n+1)}\frac{E_{0i}}{c}\mathrm{e}^{-\mathrm{i}\omega t-z/\delta}
\end{aligned}$$

Reflected

$$\begin{aligned}
\vec{E}_r =& \hat{x}E_{0r}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0 z}\\
=& -\hat{x}\frac{n-1}{n+1}E_{0i}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0 z}\\
\vec{B}_r =& -\hat{y}\frac{E_{0r}}{c}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0 z}\\
=& \hat{y}\frac{n-1}{n+1}\frac{E_{0i}}{c}\mathrm{e}^{-\mathrm{i}\omega t-\mathrm{i}k_0 z}
\end{aligned}$$

Current density

$$\begin{aligned}
\vec{j} =& \mathrm{i}\frac{n_e e^2}{m\omega}\vec{E}_t\\
=& \mathrm{i}\hat{x}\frac{n_e e^2}{m\omega}\frac{2}{n+1}E_{0i}\mathrm{e}^{-\mathrm{i}\omega t-z/\delta}
\end{aligned}$$

(c)

Incident

$$\langle S_i\rangle = \vec{z}\frac{E_{0i}^2}{2c\mu_0}$$

Reflected

$$\begin{aligned}\langle S_r\rangle =& -\vec{z}\frac{|E_{0r}|^2}{2c\mu_0}\\ =& -\vec{z}\left|\frac{n-1}{n+1}\right|^2\frac{E_{0i}^2}{2c\mu_0}\\ =& -\langle S_i\rangle\end{aligned}$$

Transmitted

$$\langle S_t\rangle = 0$$

(d)

$$\begin{aligned}R =& 1\\ T =& 0\end{aligned}$$

(e)

There's no ohmic heating since nothing is heated up.

(f)

$$\begin{aligned}\left\langle\vec{j}\times\vec{B}\right\rangle =& \frac{1}{2}\Re\left(\mathrm{i}\hat{x}\frac{n_e e^2}{m\omega}\frac{2}{n+1}E_{0i}\mathrm{e}^{-\mathrm{i}\omega t-z/\delta}\times\hat{y}\frac{-2n}{(-n+1)}\frac{E_{0i}}{c}\mathrm{e}^{\mathrm{i}\omega t-z/\delta}\right)\\ =& \hat{z}\frac{n_e e^2}{m\omega c}\frac{2|n|}{1-n^2}E_{0i}^2\mathrm{e}^{-2z/\delta}\\ \vec{p} =& \hat{z}\frac{n_e e^2}{mc}\frac{1}{\omega_p^2}E_{0i}^2\\ =& \hat{z}\frac{\varepsilon_0 E_{0i}^2}{c}\end{aligned}$$

4.

Assuming there is another sheet of stationary opposite charge on top of the moving one so the static electric field is 0. Start from the vector potential which include all the field generated by the moving charge.

$$\begin{aligned}
\vec{A} =&\frac{\mu_0}{4\pi}\int\frac{\vec{j}(t-r/c)}{r}\mathrm{d}\sigma\\
=&\frac{\sigma}{4\pi c^2\varepsilon_0}\int_0^\infty\mathrm{d}\rho\int_0^{2\pi}\mathrm{d}\phi\rho\frac{\vec{v}\Big(t-\sqrt{\rho^2+z^2}/c\Big)}{\sqrt{\rho^2+z^2}}\\
=&\frac{\sigma}{2c^2\varepsilon_0}\int_0^\infty\mathrm{d}\rho\rho\frac{\vec{v}\Big(t-\sqrt{\rho^2+z^2}/c\Big)}{\sqrt{\rho^2+z^2}}\\
\vec{E} =&-\frac{\partial}{\partial t}\frac{\sigma}{2c^2\varepsilon_0}\int_0^\infty\mathrm{d}\rho\rho\frac{\vec{v}\Big(t-\sqrt{\rho^2+z^2}/c\Big)}{\sqrt{\rho^2+z^2}}\\
=&-\frac{\sigma}{4c^2\varepsilon_0}\int_0^\infty\mathrm{d}\rho^2\frac{\vec{a}\Big(t-\sqrt{\rho^2+z^2}/c\Big)}{\sqrt{\rho^2+z^2}}\\
=&-\frac{\sigma}{2c^2\varepsilon_0}\int_z^\infty\mathrm{d}r\vec{a}\Big(t-\frac{r}{c}\Big)\\
=&-\frac{\sigma}{2c\varepsilon_0}\int_{-\infty}^{t-z/c}\mathrm{d}t'\vec{a}(t')\\
=&-\frac{\sigma}{2c\varepsilon_0}\vec{v}(t')\Big|_{-\infty}^{t-z/c}
\end{aligned}$$

Assuming $\vec{v}(-\infty)=0$

$$\vec{E}=-\frac{\sigma}{2c\varepsilon_0}\vec{v}\Big(t-\frac{z}{c}\Big)$$