

1.

(a)

From the generic form

$$V = V_0 J_0 \left(\frac{x_{01} \rho}{a} \right) \frac{\sinh \left(\frac{x_{01} z}{a} \right)}{\sinh \left(\frac{x_{01} L}{a} \right)}$$

(b)

$$V = V_0 J_0 \left(\frac{\rho}{a} \right) \exp \left(-\frac{z}{a} \right)$$

2.

(a)

The flux through a circle defined by r and θ , for $r < R$

$$\Phi = \pi r^2 \sin^2 \theta B$$

For $r > R$

$$\begin{aligned} \Phi &= \int_{\cos \theta}^1 2\pi r^2 dz \frac{\mu_0 m}{4\pi r^3} 2z \\ &= \pi \frac{R^3}{r} \sin^2 \theta B \end{aligned}$$

E field

$$\begin{aligned} E_\phi &= \frac{1}{2\pi r \sin \theta} \frac{d\Phi}{dt} \\ &= \frac{dB}{dt} \sin \theta \begin{cases} \frac{r}{2} & (r < R) \\ \frac{R^3}{2r^2} & (r > R) \end{cases} \end{aligned}$$

(b)

$$\begin{aligned} U_{B_{out}} &= \frac{1}{2\mu_0} \int_R^\infty dr \int_0^\pi r d\theta \int_0^{2\pi} r \sin \theta d\phi \frac{\mu_0^2 m^2}{16\pi^2 r^6} (4 \cos^2 \theta + \sin^2 \theta) \\ &= \frac{\mu_0 m^2}{16\pi} \int_R^\infty dr \frac{1}{r^4} \int_{-1}^1 d \cos \theta (4 \cos^2 \theta + \sin^2 \theta) \\ &= \frac{\mu_0 m^2}{4\pi} \frac{1}{3R^3} \\ &= \frac{\mu_0 m^2}{12\pi R^3} \end{aligned}$$

Inside

$$\begin{aligned} U_{B_{in}} &= \frac{4\pi}{3} R^3 \frac{B^2}{2\mu_0} \\ &= \frac{4\pi}{3} R^3 \frac{1}{2\mu_0} \frac{4\mu_0^2}{9} \frac{9}{16\pi^2 R^6} m^2 \\ &= \frac{\mu_0 m^2}{6\pi R^3} \end{aligned}$$

Total

$$U_B = \frac{\mu_0 m^2}{4\pi R^3}$$

(c)

$$\begin{aligned} W &= \int_0^\pi R d\theta \int_0^{2\pi} R \sin \theta d\phi \kappa \sin \theta \frac{dB}{dt} \sin \theta \frac{R}{2} \\ &= \pi \frac{\mu_0}{3} R^3 \frac{d\kappa^2}{dt} \int_{-1}^1 dz (1 - z^2) \\ &= \frac{\mu_0}{4\pi R^3} \frac{dm^2}{dt} \\ &= \frac{dU_B}{dt} \end{aligned}$$

(d)

For $r = R + 0^+$

$$\begin{aligned} W_+ &= \int_0^\pi R d\theta \int_0^{2\pi} R \sin \theta d\phi \frac{\mu_0 m}{4\pi R^3} \sin \theta \frac{1}{\mu_0} \frac{dB}{dt} \sin \theta \frac{R}{2} \\ &= \int_0^\pi R d\theta \int_0^{2\pi} R \sin \theta d\phi \frac{\kappa}{3} \sin \theta \frac{dB}{dt} \sin \theta \frac{R}{2} \\ &= \frac{W}{3} \end{aligned}$$

For $r = R - 0^+$

$$\begin{aligned} W_- &= \int_0^\pi R d\theta \int_0^{2\pi} R \sin \theta d\phi B \sin \theta \frac{1}{\mu_0} \frac{dB}{dt} \sin \theta \frac{R}{2} \\ &= \int_0^\pi R d\theta \int_0^{2\pi} R \sin \theta d\phi \frac{2}{3} \kappa \sin \theta \frac{dB}{dt} \sin \theta \frac{R}{2} \\ &= \frac{2W}{3} \end{aligned}$$

This is the same as what one would expect from (b) and (c) with determines the ratio and the sum of W_- and W_+ respectively.

(e)

(f)

(g)