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8.311 Problem Set 2
Recitation Section 1

1.

(a)

$$\begin{aligned} x'_i x'_i =& a_{ij} a_{ik} x_j x_k \\ x_i x_i =& \delta_{jk} x_j x_k \end{aligned}$$

Since $x'_i x'_i = x_i x_i$ for all x_i

$$\delta_{jk} = a_{ij} a_{ik}$$

(b)

$$\begin{aligned} x_i =& \delta_{ik} x_k \\ =& a_{ji} a_{jk} x_k \\ =& a_{ji} x'_j \end{aligned}$$

(c)

$$\begin{aligned} \frac{\partial f}{\partial x'_i} =& \frac{\partial f}{\partial x_j} \frac{\partial x_j}{\partial x'_i} \\ =& \frac{\partial f}{\partial x_j} \frac{\partial a_{kj} x'_k}{\partial x'_i} \\ =& a_{ij} \frac{\partial f}{\partial x_j} \end{aligned}$$