

1.

(a)

$$\begin{aligned}\frac{dW_{rad}}{dt'} &= \frac{e^2}{4\pi\epsilon_0} \frac{2}{3c} \gamma^6 (\Omega^2 \beta^2 - \Omega^2 \beta^4) \\ &= \frac{e^2}{4\pi\epsilon_0} \frac{2}{3c} \gamma^4 \Omega^2 \beta^2 \\ &= \frac{e^2}{4\pi\epsilon_0} \frac{2\omega_0^2}{3c} \gamma^2 \beta^2\end{aligned}$$

(b)

$$\begin{aligned}\frac{d\gamma}{dt} mc^2 &= - \frac{e^2}{4\pi\epsilon_0} \frac{2\omega_0^2}{3c} \gamma^2 \beta^2 \\ \frac{d\gamma}{dt} &= - \frac{e^2}{4\pi\epsilon_0} \frac{2\omega_0^2}{3mc^3} \gamma^2 \beta^2 \\ &= - \frac{2\omega_0^2 r_e}{3c} \gamma^2 \beta^2 \\ T_0 &= \frac{3c}{2\omega_0^2 r_e}\end{aligned}$$

(c)

For $\gamma \gg 1$, $\beta \approx 1$

$$\begin{aligned}\frac{d\gamma}{dt} &= - \frac{\gamma^2}{T_0} \\ \frac{1}{\gamma} &= \frac{1}{\gamma_0} + \frac{t}{T_0} \\ T &= \frac{\gamma_0 - \gamma}{\gamma\gamma_0} T_0\end{aligned}$$

(d)

$$\begin{aligned}\omega_{break} &= 3\gamma_e^2 \omega_0 \\ \gamma_e &= \sqrt{\frac{\omega_{break}}{3\omega_0}}\end{aligned}$$

(e)

$$\begin{aligned}T &= \frac{T_0}{\gamma} \\ &= T_0 \sqrt{\frac{3\omega_0}{\omega_{break}}}\end{aligned}$$

(f)

$$\begin{aligned}\omega_0 &= \frac{eB}{m} \\ &= 1.76 \cdot 10^3 \\ T &= \frac{3c}{2\omega_0^2 r_e} \sqrt{\frac{3\omega_0}{\omega_{break}}} \\ &= 5.16 \cdot 10^{10} s \\ &= 1635 \text{a}\end{aligned}$$

2.

(a)

$$\begin{aligned}E_r &= \begin{cases} \frac{2p_0 \cos \theta}{4\pi\epsilon_0 r^3} & (r > R) \\ E_0 \cos \theta & (r < R) \end{cases} \\ E_\theta &= \begin{cases} \frac{p_0 \sin \theta}{4\pi\epsilon_0 r^3} & (r > R) \\ -E_0 \sin \theta & (r < R) \end{cases} \\ E_0 &= -\frac{p_0}{4\pi\epsilon_0 R^3} \\ \frac{\sigma_0}{\epsilon_0} &= \frac{2p_0}{4\pi\epsilon_0 R^3} - E_0 \\ p_0 &= \frac{4\pi R^3 \sigma_0}{3} \\ E_0 &= \frac{\sigma_0}{3\epsilon_0}\end{aligned}$$

(b)

$$\begin{aligned}B_r &= \begin{cases} \frac{2\mu_0 m_0 \cos \theta}{4\pi r^3} & (r > R) \\ B_0 \cos \theta & (r < R) \end{cases} \\ B_\theta &= \begin{cases} \frac{\mu_0 m_0 \sin \theta}{4\pi r^3} & (r > R) \\ -B_0 \sin \theta & (r < R) \end{cases} \\ B_0 &= \frac{\mu_0 m_0}{2\pi R^3} \\ \mu_0 \kappa_0 &= \frac{\mu_0 m_0}{4\pi R^3} + B_0 \\ m_0 &= \frac{4\pi R^3 \kappa_0}{3} \\ B_0 &= \frac{2\mu_0 \kappa_0}{3}\end{aligned}$$

3.

(a)

$$\begin{aligned} E_i &= -\frac{p_j}{4\pi\epsilon_0} \partial_i \frac{r_j}{r^3} \\ &= \frac{p_j}{4\pi\epsilon_0} \partial_i \partial_j \frac{1}{r} \end{aligned}$$

Since $\nabla^2 \frac{1}{r} = -4\pi\delta(\vec{r})$

$$\begin{aligned} \partial_i \frac{r_j}{r^3} &= \left(\frac{\delta_{ij}}{r^3} - 3 \frac{x_i x_j}{r^5} \right) + \frac{4\pi}{3} \delta_{ij} \delta(\vec{r}) \\ E_i &= \frac{1}{4\pi\epsilon_0} \left(3 \frac{x_i x_j p_j}{r^5} - \frac{p_i}{r^3} \right) - \frac{1}{3\epsilon_0} p_i \delta(\vec{r}) \end{aligned}$$

(b)

$$\begin{aligned} \nabla \times \left(\frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3} \right) &= \frac{\mu_0}{4\pi} \left(\vec{m} \left(\nabla \cdot \frac{\vec{r}}{r^3} \right) - \vec{m} \cdot \left(\nabla \frac{\vec{r}}{r^3} \right) \right) \\ &= \frac{\mu_0}{4\pi} \left(\vec{m} 4\pi\delta(\vec{r}) + 3 \frac{(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} - \frac{4\pi\vec{m}}{3} \delta(\vec{r}) \right) \\ &= \frac{\mu_0}{4\pi} \left(3 \frac{(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right) + \frac{2\mu_0\vec{m}}{3} \delta(\vec{r}) \end{aligned}$$

(c)

(d)

(e)

4.

(a)

$$\vec{E}(x, t) = -\hat{y} \frac{c\mu_0}{2} \int_{-\infty}^{\infty} dx' J_0 \exp \left(i \left(kx' - \omega \left(t - \frac{|x - x'|}{c} \right) \right) \right)$$

(b)

$$\begin{aligned}\vec{E}(x, t) &= -\hat{y} \frac{c\mu_0 J_0 e^{i(kx - \omega t)}}{2} \left(\int_{-\infty}^0 dx'' \exp\left(i\left(k - \frac{\omega}{c}\right)x''\right) + \int_0^{\infty} dx'' \exp\left(i\left(k + \frac{\omega}{c}\right)x''\right) \right) \\ &= -\hat{y} \frac{c\mu_0 J_0 e^{i(kx - \omega t)}}{2i} \left(\frac{1}{k - \omega/c} - \frac{1}{k + \omega/c} \right) \\ &= i\hat{y} c\mu_0 J_0 e^{i(kx - \omega t)} \frac{\omega/c}{k^2 - \omega^2/c^2}\end{aligned}$$

(c)

$$\begin{aligned}J_{pol} &= \varepsilon_0 (k_e - 1) \omega \hat{y} c\mu_0 J_0 e^{i(kx - \omega t)} \frac{\omega/c}{k^2 - \omega^2/c^2} \\ &= J_0 \hat{y} e^{i(kx - \omega t)} \\ 1 &= \varepsilon_0 (k_e - 1) \omega c\mu_0 \frac{\omega/c}{k^2 - \omega^2/c^2}\end{aligned}$$

Let $v \equiv \frac{\omega}{k}$

$$\begin{aligned}1 &= (k_e - 1) \frac{v^2}{c^2 - v^2} \\ \frac{c^2}{v^2} &= k_e \\ v &= \frac{c}{\sqrt{k_e}}\end{aligned}$$