1.

(a)

From the generic form

$$V = V_0 J_0\left(\frac{x_{01}\rho}{a}\right)\frac{\sinh\left(\frac{x_{01}z}{a}\right)}{\sinh\left(\frac{x_{01}L}{a}\right)}$$

(b)

$$V = V_0 J_0\left(\frac{\rho}{a}\right)\exp\left(-\frac{z}{a}\right)$$

2.

(a)

The flux through a circle defined by r and θ, for $r < R$

$$\Phi = \pi r^2 \sin^2\theta B$$

For $r > R$

$$\begin{aligned}\Phi =& \int_{\cos\theta}^{1} 2\pi r^2 \mathrm{d}z \frac{\mu_0 m}{4\pi r^3} 2z\\ =& \pi\frac{R^3}{r}\sin^2\theta B\end{aligned}$$

E field

$$\begin{aligned}E_\phi =& \frac{1}{2\pi r\sin\theta}\frac{\mathrm{d}\Phi}{\mathrm{d}t}\\ =& \frac{\mathrm{d}B}{\mathrm{d}t}\sin\theta\begin{cases}\frac{r}{2} & (r < R)\\ \frac{R^3}{2r^2} & (r > R)\end{cases}\end{aligned}$$

(b)

$$\begin{aligned}U_{B_{out}} =& \frac{1}{2\mu_0}\int_R^\infty \mathrm{d}r \int_0^\pi r\mathrm{d}\theta \int_0^{2\pi} r\sin\theta\mathrm{d}\phi \frac{\mu_0^2 m^2}{16\pi^2 r^6}\left(4\cos^2\theta + \sin^2\theta\right)\\ =& \frac{\mu_0 m^2}{16\pi}\int_R^\infty \mathrm{d}r\frac{1}{r^4}\int_{-1}^{1}\mathrm{d}\cos\theta\left(4\cos^2\theta + \sin^2\theta\right)\\ =& \frac{\mu_0 m^2}{4\pi}\frac{1}{3R^3}\\ =& \frac{\mu_0 m^2}{12\pi R^3}\end{aligned}$$

Inside

$$\begin{aligned}U_{B_{in}} =&\frac{4\pi}{3}R^3\frac{B^2}{2\mu_0}\\ =&\frac{4\pi}{3}R^3\frac{1}{2\mu_0}\frac{4\mu_0^2}{9}\frac{9}{16\pi^2R^6}m^2\\ =&\frac{\mu_0m^2}{6\pi R^3}\end{aligned}$$

Total

$$U_B =\frac{\mu_0m^2}{4\pi R^3}$$

(c)

(d)

(e)

(f)

(g)