Assignment #7: Due Friday April 10, 2015 at 2:30 pm

Problems

There are only two problems in this problem set. Each one is worth 30 points.

Problem 7-1: E&M Wave Packet: Where is the Angular Momentum?

(a) A circularly polarized plane wave moving in the *z*-direction has a finite extent in the *x*- and *y*-directions. Assuming that the amplitude modulation is slowly varying (the wave is many wavelengths broad), show that the electric and magnetic fields are given approximately by

$$\mathbf{E}(x,y,z,t) \simeq \left[E_o(x,y)(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) + \frac{i}{k}\left(\frac{\partial E_o}{\partial x} \pm i\frac{\partial E_o}{\partial y} \right)\hat{\mathbf{z}} \right] e^{ikz - i\omega t}$$

$$\mathbf{B} \simeq \mp\sqrt{\mu\varepsilon}\,\mathbf{E}$$

(b) For this circularly polarized wave, calculate the time-averaged component of the angular momentum parallel to the direction of propagation. Show that the ratio of this component of angular momentum to the energy of the wave is

$$\frac{L_z}{U} = \pm\frac{1}{\omega}$$

Interpret this result in terms of quanta of radiation (photons). Show that for a cylindrically symmetric finite plane wave the transverse components of angular momentum vanish.

Problem 7-2: Jackson 10.11 page 510 parts (a) and (b). Do not do part (c).

See http://en.wikipedia.org/wiki/Fresnel_integral for definitions of the Fresnel Integrals.