Assignment 11: Due Friday May 8, 2015 at 2:30 pm

Problems

This problem set has only one problem worth 60 points.

Problem 11.1 : The spinning shell of charge in general

A spherical shell of radius R, carries a uniform surface charge σ. Its total charge Q is $4\pi R^2 \sigma$, and its Coulomb electric field is as given on Problem Set 10. The sphere spins at an angular frequency $\Omega(t) = \Omega_o \cos(\omega t)$, where $\Omega_o R << c$. The motion of the charge glued onto the surface of the spinning sphere results in a surface current

$$\vec{\mathbf{J}}(\mathbf{X},t) = \mathrm{Re}\left\{\sigma\Omega_o R\delta(r-R)\sin\theta e^{-i\omega t}\,\hat{\boldsymbol{\phi}}\right\}$$

In addition to the radial coulomb electric field, there will be a time varying electric field in the azimuthal direction $E_\phi(\mathbf{X},t)\hat{\boldsymbol{\phi}}$. From arguments in lecture the azimuthal electric field must be of the form

$$E_\phi(r,\theta,t) = \mathrm{Re}\left\{A j_1(kr_<)h_1^1(kr_>)\sin\theta e^{-i\omega t}\right\}$$

where $r_< = \min(r,R)$ and $r_> = \max(r,R)$ and A is a constant to be determined.

(a) Given the definitions on page 426 of Jackson, and applying the proper boundary conditions at $r = R$, show that

$$|A| = \mu_o c\sigma \mathrm{R}\,\Omega_o (kR)^2 = \frac{Q}{4\pi R^2\varepsilon_o}\frac{\Omega_o \mathrm{R}}{c}(kR)^2$$

(b) Calculate the time average over one period of $\int_{\text{all space}} -\mathbf{J}\cdot\mathbf{E}\, d^3x$, and multiply this quantity by the period T. Divide this energy by the magnetostatic energy stored in a sphere spinning at a constant rate Ω_o . Plot this quantity versus $kR = 2\pi R/\lambda$ from $kR = 0$ to $kR = 10$. This is the energy radiated away in one period normalized to the magnetostatic energy.

(c) Does the small argument behavior of your expression in (b) make sense to you? Explain *quantitatively*.

(d) Does the behavior of this energy radiated over one period between $kR = 1$ and $kR = 10$ make sense to you? Explain *qualitatively*. Is there a frequency at which there is a maximum in the energy radiated during one period?