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1.

(a)

$$\begin{aligned}
&\left(\vec{A}\times\left(\vec{B}\times\vec{C}\right)\right)_i\\
=&\varepsilon_{ijk}A_j\left(\vec{B}\times\vec{C}\right)_k\\
=&\varepsilon_{kij}\varepsilon_{klm}A_jB_lC_m\\
=&(\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl})A_jB_lC_m\\
=&B_iA_jC_j-C_iA_jB_j\\
=&\left(\vec{B}\left(\vec{A}\cdot\vec{C}\right)-\vec{C}\left(\vec{A}\cdot\vec{B}\right)\right)_i
\end{aligned}$$

(b)

$$\begin{aligned}
&\vec{A}\times\left(\nabla\times\vec{A}\right)\\
=&\left(\nabla\otimes\vec{A}\right)\cdot\vec{A}-\left(\vec{A}\cdot\nabla\right)\vec{A}\\
=&\frac{1}{2}\nabla(A^2)-\left(\vec{A}\cdot\nabla\right)\vec{A}
\end{aligned}$$

(c)

Use X_c to respresent treating X as constant during the derivative.

$$\begin{aligned}
&\nabla\times(\vec{A}\times\vec{B})\\
=&\nabla\times(\vec{A}_c\times\vec{B})+\nabla\times(\vec{A}\times\vec{B}_c)\\
=&\vec{A}_c(\nabla\cdot\vec{B})-(\vec{A}_c\cdot\nabla)\vec{B}-\vec{B}_c(\nabla\cdot\vec{A})+(\vec{B}_c\cdot\nabla)\vec{A}\\
=&\vec{A}(\nabla\cdot\vec{B})-(\vec{A}\cdot\nabla)\vec{B}-\vec{B}(\nabla\cdot\vec{A})+(\vec{B}\cdot\nabla)\vec{A}
\end{aligned}$$

2.

(a)

$$\begin{aligned}
&\int_{min}^{max}f(x)\Theta'(x-a)\mathrm{d}x\\
=&\int_{min}^{max}f(x)\mathrm{d}\Theta(x-a)\\
=&f(x)\Theta(x-a)|_{min}^{max}-\int_{min}^{max}\Theta(x-a)\mathrm{d}f(x)\\
=&f(max)-\int_a^{max}\mathrm{d}f(x)\\
=&f(a)
\end{aligned}$$

(b)

$$\frac{\mathrm{d}\mathbf{sgn}(t)}{\mathrm{d}t} = \frac{\mathrm{d}2\Theta(t)-1}{\mathrm{d}t}$$
$$=2\delta(t)$$

(c)

$$\rho(r,\theta,\phi) = \frac{Q}{4\pi R^2}\delta(r-R)$$

(d)

$$\rho(\rho,\theta,z) = \frac{\lambda}{2\pi b}\delta(\rho-b)$$

(e)

Assuming the disk is parallel to the $x-y$ plane at z_0.

$$\rho(\rho,\theta,z) = \frac{Q}{\pi b^2}\delta(z-z_0)\Theta(b-r)$$