1.

(a)

In x, y, z basis

$$Q_{ij} = \begin{pmatrix} -2d^2q & & \\ & -2d^2q & \\ & & 4d^2q \end{pmatrix}$$

$$= 2d^2q \begin{pmatrix} -1 & & \\ & -1 & \\ & & 2 \end{pmatrix}$$

Potential

$$\Phi = \frac{1}{8\pi\varepsilon_0} \frac{Q_{ij}x_i x_j}{r^5}$$

$$= \frac{d^2q}{4\pi\varepsilon_0} \frac{-x^2 - y^2 + 2z^2}{r^5}$$

$$= \frac{d^2q}{4\pi\varepsilon_0} \frac{3\cos^2\theta - 1}{r^3}$$

Electric field

$$E_r = \frac{3d^2q}{4\pi\varepsilon_0} \frac{3\cos^2\theta - 1}{r^4}$$

$$E_\theta = \frac{d^2q}{4\pi\varepsilon_0} \frac{3\sin 2\theta}{r^4}$$

Field line

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = r\frac{3\cos^2\theta - 1}{\sin 2\theta}$$

$$\frac{\mathrm{d}r}{r} = \frac{3\cos^2\theta - 1}{\sin 2\theta}\mathrm{d}\theta$$

$$r = R_0 \sin\theta\sqrt{\cos\theta}$$

The field line is the furthest when $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = 0$

$$\theta = \arccos\frac{1}{\sqrt{3}}$$

(b)

Magnetic field

$$
\begin{aligned}
\vec{B} &= -\frac{\mu_0}{24\pi c^2 r}\hat{r} \times \left(\hat{r} \cdot \dddot{\overleftrightarrow{Q}}\right) \\
&= -\frac{\mu_0}{24\pi c^2 r}\hat{r} \times \left(\hat{r} \cdot \frac{\mathrm{d}^3 2 d^2 q (3\hat{z}\hat{z}-1)}{\mathrm{d}t^3}\right) \\
&= -\frac{q\mu_0 d_0^2}{12\pi c^2 r}\hat{r} \times (3(\hat{r}\cdot\hat{z})\hat{z}-\hat{r})\frac{\mathrm{d}^3\cos^2\omega t}{\mathrm{d}t^3} \\
&= \frac{q\mu_0 d_0^2\omega^3}{\pi c^2 r}(\hat{r}\cdot\hat{z})(\hat{r}\times\hat{z})\sin 2\omega t \\
&= -\frac{q\mu_0 d_0^2\omega^3}{2\pi c^2 r}\sin 2\theta\sin 2\omega t\hat{e}_\phi
\end{aligned}
$$

Electric field

$$
\begin{aligned}
\vec{E} &= c\vec{B}\times\vec{r} \\
&= -\frac{q\mu_0 d_0^2\omega^3}{2\pi c r}\sin 2\theta\sin 2\omega t\hat{e}_\theta
\end{aligned}
$$

Power

$$
\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2 d_0^4\omega^6}{8\pi^2\varepsilon_0 c^5}\sin^2 2\theta
$$

The radiation is the strongest for $\theta = \dfrac{\pi}{4}$ and $\theta = \dfrac{3\pi}{4}$

$$
\begin{aligned}
P &= \int_0^\pi \mathrm{d}\theta\sin\theta\int_0^{2\pi}\mathrm{d}\phi\frac{q^2 d_0^4\omega^6}{8\pi^2\varepsilon_0 c^5}\sin^2 2\theta \\
&= \frac{4q^2 d_0^4\omega^6}{15\pi\varepsilon_0 c^5}
\end{aligned}
$$

The radiation appears at 2ω

(c)

The radiation will be a dipole radiation appears at ω with total power

$$
P = \frac{q^2 d_0^2\omega^4}{12\pi\varepsilon_0 c^3}
$$

Ratio of power

$$
\begin{aligned}
\frac{P_{quad}}{P} &= \frac{16 d_0^2\omega^2}{5c^2} \\
&\ll 1
\end{aligned}
$$

2.

(a)

$$\vec{B} = \nabla \times A$$

$$= \frac{\mu_0}{4\pi} \int d^3\vec{r}' \nabla \times \left(\frac{1}{|\vec{r} - \vec{r}'|} \vec{j}\left(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c}\right) \right)$$

$$= \frac{\mu_0}{4\pi} \int d^3\vec{r}' \frac{1}{|\vec{r} - \vec{r}'|} \nabla \times \vec{j}\left(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c}\right) + \left(\nabla \frac{1}{|\vec{r} - \vec{r}'|}\right) \times \vec{j}\left(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c}\right)$$

$$= \frac{\mu_0}{4\pi} \int d^3\vec{r}' \vec{j}\left(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c}\right) \times \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} + \frac{\partial}{\partial t}\vec{j}\left(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c}\right) \times \frac{\vec{r} - \vec{r}'}{c|\vec{r} - \vec{r}'|^2}$$

(b)

The ratio of the two term is on the order of (replacing time derivative with T and replace $|\vec{r} - \vec{r}'|$ with d)

$$\frac{d}{cT} \ll 1$$

3.

(a)

$$P = \frac{e^4 E_0^2}{12\pi\varepsilon_0 m_e^2 c^3}$$

$$= \langle S \rangle \frac{e^4}{6\pi\varepsilon_0^2 m_e^2 c^4}$$

$$= \langle S \rangle \frac{8\pi}{3} r_e^2$$

(b)

The radiation (and therefore the stress tensor) has mirror symmetry along each axis.

(c)

$$F = \frac{P}{c}$$

$$= \langle S \rangle \frac{8\pi}{3c} r_e^2$$

4.

(a)

5.

6.

(a)

(b)

(c)

(d)

(e)