1.

(a)

$$\mu_0\frac{\partial}{\partial t}\vec{J} =\nabla^2\vec{E}_\perp - \frac{1}{c^2}\frac{\partial^2\vec{E}_\perp}{\partial t^2}$$

On the shell

$$\begin{aligned}
\mu_0\frac{\partial}{\partial t}\tilde{J}_\phi =&\frac{\partial^2}{\partial r^2}\tilde{E}_\phi\\
\mu_0\frac{\partial}{\partial t}\tilde{J}_\phi =&-\mathrm{i}\omega\mu_0\sigma\Omega_0R\delta(r-R)\sin\theta\mathrm{e}^{-\mathrm{i}\omega t}\\
\frac{\partial^2}{\partial r^2}\tilde{E}_\phi =&kA\sin\theta\mathrm{e}^{-\mathrm{i}\omega t}\delta(r-R)\\
&\left(-\frac{\mathrm{e}^{\mathrm{i}x}}{x^5}\left(\mathrm{i}x^2-2x-2\mathrm{i}\right)\left(\sin x-x\cos x\right)+\frac{\mathrm{e}^{\mathrm{i}x}}{x^5}\left(x+\mathrm{i}\right)\left(2x\cos x-2\sin x+x^2\sin x\right)\right)
\end{aligned}$$

where $x=kr$

$$\begin{aligned}
=&Ak\sin\theta\mathrm{e}^{-\mathrm{i}\omega t}\delta(r-R)\frac{\mathrm{i}}{x^2}\\
A=&-c\mu_0\sigma\Omega_0R\left(kR\right)^2
\end{aligned}$$

(b)

With $X=kR$

$$\begin{aligned}
\tilde{E}_\phi(R) =&c\mu_0\sigma\Omega_0RX^2\mathrm{e}^{\mathrm{i}X}\frac{\sin X-X\cos X}{X^2}\frac{X+\mathrm{i}}{X^2}\sin\theta\mathrm{e}^{-\mathrm{i}\omega t}\\
=&c\mu_0\sigma\Omega_0R\mathrm{e}^{\mathrm{i}X}\frac{\sin X-X\cos X}{X^2}(X+\mathrm{i})\sin\theta\mathrm{e}^{-\mathrm{i}\omega t}
\end{aligned}$$

Average power

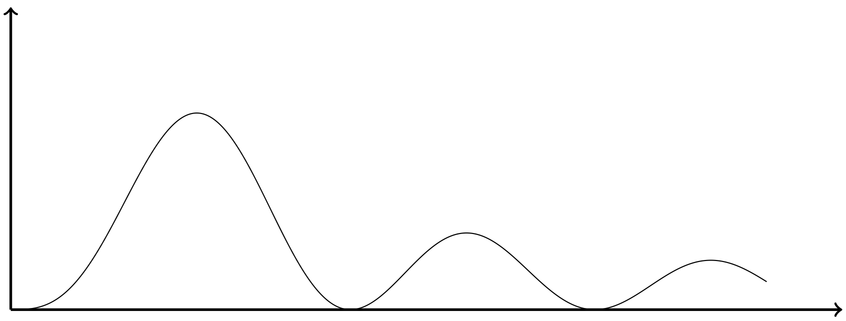
$$\begin{aligned}
P=&-R^2\int\mathrm{d}\Omega\Re\left(c\mu_0\sigma^2\Omega_0^2R^2\mathrm{e}^{\mathrm{i}X}\frac{\sin X-X\cos X}{X^2}(X+\mathrm{i})\sin^2\theta\right)\\
=&\frac{8\pi}{3}c\mu_0\sigma^2\Omega_0^2R^4\frac{\left(\sin X-X\cos X\right)^2}{X^2}\\
=&\frac{8\pi}{3}c\mu_0\sigma^2\Omega_0^2R^2\frac{\left(\sin kR-kR\cos kR\right)^2}{k^2}
\end{aligned}$$

Magnetostatic energy

$$U_B =\mu_0\frac{4\pi R^3}{9}\sigma^2\Omega_0^2R^2$$

Normalized radiation

$$p =6\pi\frac{\left(\sin kR-kR\cos kR\right)^2}{\left(kR\right)^3}$$



(c)

For small kR, $p \propto (kR)^3$. This makes sense since it is the same scaling with a dipole radiation with constant change rate (i.e. one less factor of ω)

(d)

The pattern for larger kR appears because of the interference of radiation from different part of the sphere. The maximum radiation appears when $kR \approx 2.46$.