Yichao Yu
8.311 Problem Set 9
Recitation Section 1

1.

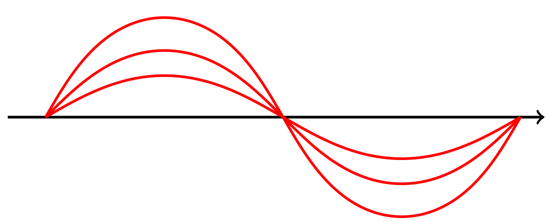
(a)

$$\begin{aligned}
z =& \bar{n}\int_0^x \frac{\mathrm{d}x}{\sqrt{n^2-\bar{n}^2}}\\
=& \bar{n}\int_0^x \frac{\mathrm{d}x}{\sqrt{n_0^2\mathrm{sech}^2(\alpha x)-\bar{n}^2}}\\
=& \frac{\cos\theta_0}{\alpha}\int_0^{\alpha x}\frac{\cosh(\alpha x)\mathrm{d}\alpha x}{\sqrt{1-\cos^2\theta_0\cosh^2(\alpha x)}}\\
=& \frac{\cos\theta_0}{\alpha}\int_0^{\sinh(\alpha x)}\frac{\mathrm{d}\sinh(\alpha x)}{\sqrt{\sin^2\theta_0-\cos^2\theta_0\sinh^2(\alpha x)}}\\
=& \frac{1}{\alpha}\int_0^{\cot\theta_0\sinh(\alpha x)}\frac{\mathrm{d}y}{\sqrt{1-y^2}}\\
=& \frac{1}{\alpha}\arcsin(\cot\theta_0\sinh(\alpha x))\\
\sin(\alpha z) =& \cot\theta_0\sinh(\alpha x)
\end{aligned}$$

Since $\max(\sin(\alpha z)) = 1$

$$\begin{aligned}
\sin(\alpha z) =& \frac{\sinh(\alpha x)}{\sinh(\alpha x_{max})}\\
\alpha x =& \mathrm{arcsinh}(\sinh(\alpha x_{max})\sin(\alpha z))
\end{aligned}$$

Rays for $\theta_0 = \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3}$



(b)

$$Z = \frac{\pi}{\alpha}$$

Independent from $\bar{n}$

(c)

2.

(a)

(b)

3.

(a)

(b)

(c)