1.

(a)

$$\begin{aligned}
\vec{E}_{dipole} =& -\nabla\left(\frac{1}{4\pi\varepsilon_0}\frac{\vec{r}\cdot\vec{p}}{r^3}\right)\\
=& -\frac{1}{4\pi\varepsilon_0}\nabla\left(\frac{\vec{r}\cdot\vec{p}}{r^3}\right)\\
=& -\frac{\vec{r}\cdot\vec{p}}{4\pi\varepsilon_0}\nabla\left(\frac{1}{r^3}\right)-\frac{1}{4\pi\varepsilon_0 r^3}\nabla(\vec{r}\cdot\vec{p})\\
=&\frac{\vec{r}\cdot\vec{p}}{4\pi\varepsilon_0}\frac{3\hat{n}}{r^4}-\frac{\vec{p}}{4\pi\varepsilon_0 r^3}\\
=&\frac{3\hat{n}(\hat{n}\cdot\vec{p})-\vec{p}}{4\pi\varepsilon_0 r^3}
\end{aligned}$$

(b)

$$\begin{aligned}
&\int\vec{J}(\vec{r}')(\vec{r}'\cdot\hat{n})\mathrm{d}^3x\\
=&\frac{1}{2}\int\vec{J}(\vec{r}')(\vec{r}'\cdot\hat{n})\mathrm{d}^3x+\frac{1}{2}\int\left(\hat{n}\times\left(\vec{J}(\vec{r}')\times\vec{r}'\right)-\vec{r}'\left(\hat{n}\cdot\vec{J}(\vec{r}')\right)\right)\mathrm{d}^3x\\
=&\hat{n}\times\frac{1}{2}\int\vec{J}(\vec{r}')\times\vec{r}'\mathrm{d}^3x+\frac{1}{2}\int\left(\vec{J}(\vec{r}')(\vec{r}'\cdot\hat{n})-\vec{r}'\left(\hat{n}\cdot\vec{J}(\vec{r}')\right)\right)\mathrm{d}^3x
\end{aligned}$$

For arbitrary vector $\vec{l}$

$$\begin{aligned}
&\nabla'\cdot\left(\left(\vec{l}\cdot\vec{r}'\right)(\hat{n}\cdot\vec{r}')\vec{J}(r')\right)\\
=&(\hat{n}\cdot\vec{r}')\vec{J}(r')\cdot\nabla'\left(\vec{l}\cdot\vec{r}'\right)+\left(\vec{l}\cdot\vec{r}'\right)\vec{J}(r')\cdot\nabla'(\hat{n}\cdot\vec{r}')+\left(\vec{l}\cdot\vec{r}'\right)(\hat{n}\cdot\vec{r}')\nabla'\cdot\vec{J}(r')\\
=&(\hat{n}\cdot\vec{r}')\vec{J}(r')\cdot\vec{l}+\left(\vec{l}\cdot\vec{r}'\right)\vec{J}(r')\cdot\hat{n}\\
=&\vec{l}\cdot\left((\hat{n}\cdot\vec{r}')\vec{J}(r')+\vec{r}'\vec{J}(r')\cdot\hat{n}\right)
\end{aligned}$$

Integrate both sides

$$\begin{aligned}
0 =&\vec{l}\cdot\int\mathrm{d}^3x'\left((\hat{n}\cdot\vec{r}')\vec{J}(r')+\vec{r}'\vec{J}(r')\cdot\hat{n}\right)\\
0 =&\int\mathrm{d}^3x'\left((\hat{n}\cdot\vec{r}')\vec{J}(r')+\vec{r}'\vec{J}(r')\cdot\hat{n}\right)\\
&\int\vec{J}(\vec{r}')(\vec{r}'\cdot\hat{n})\mathrm{d}^3x\\
=&\vec{m}\times\vec{n}
\end{aligned}$$

(c)

$$
\begin{aligned}
\frac{4\pi}{\mu_0}\vec{B}_{dipole} =&\nabla\times\frac{\vec{m}\times\vec{r}}{r^3}\\
=&\vec{m}\left(\nabla\cdot\frac{\vec{r}}{r^3}\right)-(\vec{m}\cdot\nabla)\frac{\vec{r}}{r^3}\\
=&-\vec{r}\left(\vec{m}\cdot\nabla\frac{1}{r^3}\right)-\frac{1}{r^3}(\vec{m}\cdot\nabla)\vec{r}\\
=&\vec{r}\frac{3\vec{m}\cdot\hat{n}}{r^4}-\frac{\vec{m}}{r^3}\\
=&\frac{3\hat{n}(\vec{m}\cdot\hat{n})-\vec{m}}{r^3}
\end{aligned}
$$

2.

(a)

(b)

3.

(a)

(b)

(c)

(d)

(e)

(f)

4.

(a)

(b)

(c)

(d)

5.

(a)

(b)

(c)

(d)

(e)

(f)

6.

(a)

(b)

(c)