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8.311 Problem Set 5
Recitation Section 1

1.

(a)

$$\begin{aligned}\frac{\mathrm{d}W_{rad}}{\mathrm{d}t'} =& \frac{e^2}{4\pi\varepsilon_0}\frac{2}{3c}\gamma^6\left(\Omega^2\beta^2 - \Omega^2\beta^4\right)\\ =& \frac{e^2}{4\pi\varepsilon_0}\frac{2}{3c}\gamma^4\Omega^2\beta^2\\ =& \frac{e^2}{4\pi\varepsilon_0}\frac{2\omega_0^2}{3c}\gamma^2\beta^2\end{aligned}$$

(b)

$$\begin{aligned}\frac{\mathrm{d}\gamma}{\mathrm{d}t}mc^2 =& -\frac{e^2}{4\pi\varepsilon_0}\frac{2\omega_0^2}{3c}\gamma^2\beta^2\\ \frac{\mathrm{d}\gamma}{\mathrm{d}t} =& -\frac{e^2}{4\pi\varepsilon_0}\frac{2\omega_0^2}{3mc^3}\gamma^2\beta^2\\ =& -\frac{2\omega_0^2 r_e}{3c}\gamma^2\beta^2\\ T_0 =& \frac{3c}{2\omega_0^2 r_e}\end{aligned}$$

(c)

For $\gamma \gg 1$, $\beta \approx 1$

$$\begin{aligned}\frac{\mathrm{d}\gamma}{\mathrm{d}t} =& -\frac{\gamma^2}{T_0}\\ \frac{1}{\gamma} =& \frac{1}{\gamma_0} + \frac{t}{T_0}\\ T =& \frac{\gamma_0 - \gamma}{\gamma\gamma_0}T_0\end{aligned}$$

(d)

$$\begin{aligned}\omega_{break} =& 3\gamma_e^2\omega_0\\ \gamma_e =& \sqrt{\frac{\omega_{break}}{3\omega_0}}\end{aligned}$$

(e)

$$\begin{aligned}T =& \frac{T_0}{\gamma}\\ =& T_0\sqrt{\frac{3\omega_0}{\omega_{break}}}\end{aligned}$$

(f)

$$\begin{aligned}\omega_0 =& \frac{eB}{m}\\ =& 1.76 \cdot 10^3\\ T =& \frac{3c}{2\omega_0^2 r_e}\sqrt{\frac{3\omega_0}{\omega_{break}}}\\ =& 5.16 \cdot 10^{10} s\\ =& 1635\text{a}\end{aligned}$$

2.

(a)

$$\begin{aligned}E_r =& \begin{cases} \frac{2p_0 \cos\theta}{4\pi\varepsilon_0 r^3} & (r > R)\\ E_0 \cos\theta & (r < R)\end{cases}\\ E_\theta =& \begin{cases} \frac{p_0 \sin\theta}{4\pi\varepsilon_0 r^3} & (r > R)\\ -E_0 \sin\theta & (r < R)\end{cases}\\ E_0 =& -\frac{p_0}{4\pi\varepsilon_0 R^3}\\ \frac{\sigma_0}{\varepsilon_0} =& \frac{2p_0}{4\pi\varepsilon_0 R^3} - E_0\\ p_0 =& \frac{4\pi R^3 \sigma_0}{3}\\ E_0 =& \frac{\sigma_0}{3\varepsilon_0}\end{aligned}$$

(b)

$$\begin{aligned}B_r =& \begin{cases} \frac{2\mu_0 m_0 \cos\theta}{4\pi r^3} & (r > R)\\ B_0 \cos\theta & (r < R)\end{cases}\\ B_\theta =& \begin{cases} \frac{\mu_0 m_0 \sin\theta}{4\pi r^3} & (r > R)\\ -B_0 \sin\theta & (r < R)\end{cases}\\ B_0 =& \frac{\mu_0 m_0}{2\pi R^3}\\ \mu_0\kappa_0 =& \frac{\mu_0 m_0}{4\pi R^3} + B_0\\ m_0 =& \frac{4\pi R^3 \kappa_0}{3}\\ B_0 =& \frac{2\mu_0\kappa_0}{3}\end{aligned}$$

3.

(a)

(b)

(c)

(d)

(e)

4.

(a)

(b)

(c)