1.

$$\begin{aligned}
S_{1,0} =&\frac{1}{2}\frac{\omega k a^2\mu}{\pi^2}H_0^2\sin^2\frac{\pi x}{a}\\
P_{1,0} =&\frac{1}{4}\frac{\omega k a^3 b\mu}{\pi^2}H_0^2\\
-\frac{\mathrm{d}P}{\mathrm{d}z} =&\frac{1}{2\sigma\delta}\int\left|\vec{n}\times\vec{H}\right|^2\mathrm{d}l\\
=&\frac{H_0^2}{4\sigma\delta}\left(2b+a+\frac{k^2a^3}{\pi^2}\right)\\
\beta_{1,0} =&\frac{\pi^2}{2\mu_0 k a^3 b\sqrt{2\mu_c\sigma\omega}}\left(2b+a+\frac{k^2a^3}{\pi^2}\right)\\
=&\frac{\pi^2}{2\mu_0 a^3 b\sqrt{2\mu_c\mu\varepsilon\sigma\omega\left(\omega^2-\omega_{1,0}^2\right)}}\left(2b+a+\frac{\mu\varepsilon\left(\omega^2-\omega_{1,0}^2\right)a^3}{\pi^2}\right)
\end{aligned}$$

2.

$$\begin{aligned}
B_z =&B_0\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}\mathrm{e}^{-\mathrm{i}\omega t}\\
B_x =&-\frac{a}{d}B_0\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\mathrm{e}^{-\mathrm{i}\omega t}\\
E_y =&\mathrm{i}\frac{\omega a}{\pi}B_0\sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\mathrm{e}^{-\mathrm{i}\omega t}\\
U =&\frac{a^2d}{8\mu}\left(B_0^2+B_0^2\frac{a^2}{d^2}+\mu\varepsilon\frac{\omega^2a^2}{\pi^2}B_0^2\right)\\
=&\frac{a^2dB_0^2}{4\mu}\left(1+\frac{a^2}{d^2}\right)\\
P =&\frac{B_0^2a^2}{2\mu^2\sigma\delta}\left(\frac{d}{a}+\frac{a^2}{d^2}+\frac{a}{2d}\right)\\
Q =&\omega\frac{U}{P}\\
=&\frac{\mu\sigma\delta\omega d}{2}\left(1+\frac{a^2}{d^2}\right)\left(\frac{d}{a}+\frac{a^2}{d^2}+\frac{a}{2d}\right)^{-1}
\end{aligned}$$

3.

For each solution to the triangular waveguide, mirroring the solution along the diagonal should always create a valid solution for the square waveguide. Therefore, we should be able to construct all solutions to the triangular waveguide from the solutions of the square ones.
For TM modes,

$$E_z^{mn} =E_0\sin\frac{m\pi x}{a}\sin\frac{n\pi x}{a}$$

Since $E_z=0$ on the diagonal

$$E_z^{mn,tri} =E_z^{mn}-E_z^{nm}$$

Since E_z^{mn} is symmetric for m, n, we should have $m > n$ for non-vanishing solution (and to avoid double counting). The cutoff frequencies are the same with the ones for square waveguide

$$\omega_{mn} = \frac{c\pi}{a}\sqrt{m^2 + n^2}$$

For TE modes,

$$B_z^{mn} = E_0 \cos\frac{m\pi x}{a}\cos\frac{n\pi x}{a}$$

Since $\partial_n B_z = 0$ on the diagonal

$$B_z^{mn,tri} = B_z^{mn} + B_z^{nm}$$

Since B_z^{mn} is symmetric for m, n, we should have $m \geqslant n$ to avoid double counting. The cutoff frequencies are the same with the ones for square waveguide

$$\omega_{mn} = \frac{c\pi}{a}\sqrt{m^2 + n^2}$$