

1.

(a)

$$\begin{aligned}
 & \left(\vec{A} \times (\vec{B} \times \vec{C}) \right)_i \\
 &= \varepsilon_{ijk} A_j (\vec{B} \times \vec{C})_k \\
 &= \varepsilon_{kij} \varepsilon_{klm} A_j B_l C_m \\
 &= (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) A_j B_l C_m \\
 &= B_i A_j C_j - C_i A_j B_j \\
 &= \left(\vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B}) \right)_i
 \end{aligned}$$

(b)

$$\begin{aligned}
 & \vec{A} \times (\nabla \times \vec{A}) \\
 &= (\nabla \otimes \vec{A}) \cdot \vec{A} - (\vec{A} \cdot \nabla) \vec{A} \\
 &= \frac{1}{2} \nabla (A^2) - (\vec{A} \cdot \nabla) \vec{A}
 \end{aligned}$$

(c)

Use X_c to represent treating X as constant during the derivative.

$$\begin{aligned}
 & \nabla \times (\vec{A} \times \vec{B}) \\
 &= \nabla \times (\vec{A}_c \times \vec{B}) + \nabla \times (\vec{A} \times \vec{B}_c) \\
 &= \vec{A}_c (\nabla \cdot \vec{B}) - (\vec{A}_c \cdot \nabla) \vec{B} - \vec{B}_c (\nabla \cdot \vec{A}) + (\vec{B}_c \cdot \nabla) \vec{A} \\
 &= \vec{A} (\nabla \cdot \vec{B}) - (\vec{A} \cdot \nabla) \vec{B} - \vec{B} (\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla) \vec{A}
 \end{aligned}$$

2.

(a)

$$\begin{aligned}
 & \int_{min}^{max} f(x) \Theta'(x-a) dx \\
 &= \int_{min}^{max} f(x) d\Theta(x-a) \\
 &= f(x) \Theta(x-a) \Big|_{min}^{max} - \int_{min}^{max} \Theta(x-a) df(x) \\
 &= f(max) - \int_a^{max} df(x) \\
 &= f(a)
 \end{aligned}$$

(b)

$$\begin{aligned}\frac{d\operatorname{sgn}(t)}{dt} &= \frac{d2\Theta(t) - 1}{dt} \\ &= 2\delta(t)\end{aligned}$$

(c)

$$\rho(r, \theta, \phi) = \frac{Q}{4\pi R^2} \delta(r - R)$$

(d)

$$\rho(\rho, \theta, z) = \frac{\lambda}{2\pi b} \delta(\rho - b)$$

(e)

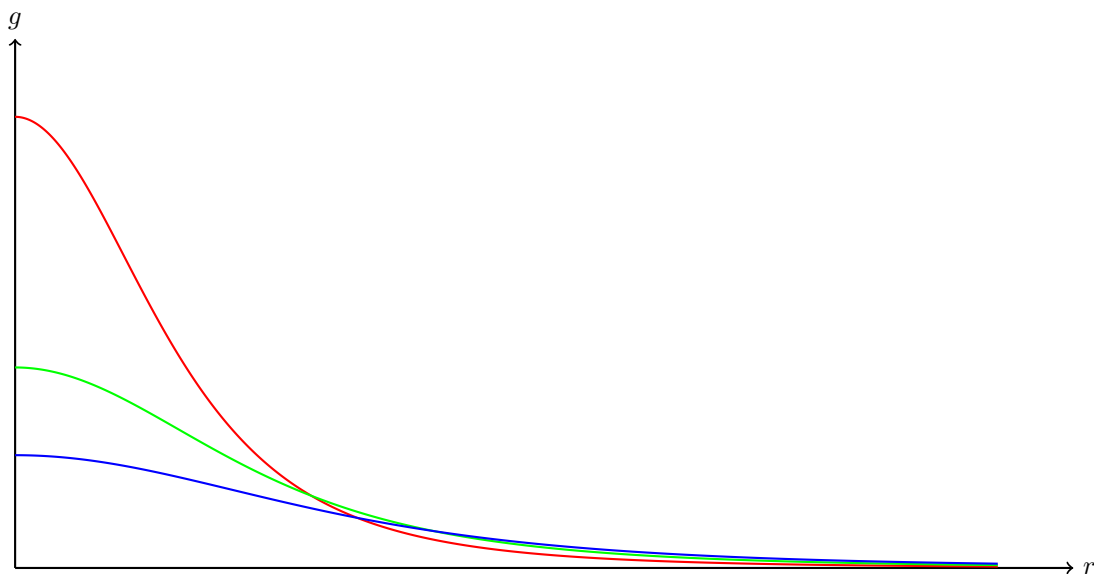
Assuming the disk is parallel to the $x - y$ plane at z_0 .

$$\rho(\rho, \theta, z) = \frac{Q}{\pi b^2} \delta(z - z_0) \Theta(b - r)$$

3.

(a)

$$\begin{aligned}g_a &= \nabla^2 f_a \\ &= -\frac{1}{4\pi r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \left(\frac{1}{\sqrt{r^2 + a^2}} \right) \right) \\ &= \frac{1}{4\pi r^2} \frac{\partial}{\partial r} \left(\frac{r^3}{\sqrt{r^2 + a^2}^3} \right) \\ &= \frac{3}{4\pi(r^2 + a^2)} \frac{\partial}{\partial r} \left(\frac{r}{\sqrt{r^2 + a^2}} \right) \\ &= \frac{3a^2}{4\pi(r^2 + a^2)^{5/2}}\end{aligned}$$



(b)

$$\begin{aligned}
 \int dr 4\pi r^2 g_a &= \int dr 4\pi r^2 \frac{3a^2}{4\pi(r^2 + a^2)^{5/2}} \\
 &= \int \frac{3\rho^2 d\rho}{(1 + \rho^2)^{5/2}} \\
 &= \int_0^{\pi/2} 3 \sin^2 \theta d \sin \theta \\
 &= 1
 \end{aligned}$$

(c)

$$\begin{aligned}
 \lim_{a \rightarrow 0, r \neq 0} g_a(r) &= \lim_{a \rightarrow 0, r \neq 0} \frac{3a^2}{4\pi(r^2 + a^2)^{5/2}} \\
 &= \lim_{a \rightarrow 0, r \neq 0} \frac{3a^2}{4\pi r^5} \\
 &= 0
 \end{aligned}$$

4.

Area of the spherical cap

$$S = 2\pi \sqrt{z^2 + R^2} \left(\sqrt{z^2 + R^2} - |z| \right)$$

Magnitude of solid angle

$$|\Omega| = 2\pi \left(1 - \frac{|z|}{\sqrt{z^2 + R^2}} \right)$$

Solid angle

$$\begin{aligned}\Omega &= 2\pi \left(\text{sgn}(z) - \frac{\text{sgn}(z)|z|}{\sqrt{z^2 + R^2}} \right) \\ &= 2\pi \left(\text{sgn}(z) - \frac{z}{\sqrt{z^2 + R^2}} \right)\end{aligned}$$

For $z = \pm 0$, $\Omega = \pm 2\pi$.

5.

$$\begin{aligned}\Phi(z) &= \frac{1}{4\pi\epsilon_0} D\Omega \\ &= \frac{D}{2\epsilon_0} \left(\text{sgn}(z) - \frac{z}{\sqrt{z^2 + R^2}} \right)\end{aligned}$$

$$\Phi(z = \pm 0) = \pm \frac{D}{2\epsilon_0}, \quad \Delta\Phi = \frac{D}{\epsilon_0}$$

6.

(a)

$$\begin{aligned}\Phi &= \frac{1}{4\pi\epsilon_0} \frac{p_o \cos \theta}{R^2} \\ \frac{d\Phi}{d\vec{n}} &= \frac{1}{2\pi\epsilon_0} \frac{p \cos \theta}{R^3}\end{aligned}$$

(b)

$$\begin{aligned}\sigma &= -\frac{1}{2\pi} \frac{p \cos \theta}{R^3} \\ D &= -\frac{1}{4\pi} \frac{p_o \cos \theta}{R^2}\end{aligned}$$