

1.

(a)

$$\begin{aligned}\nabla \times \vec{E} &= i\vec{k} \times E \\ \frac{\partial \vec{B}}{\partial t} &= -i\omega \vec{B} \\ \omega \vec{B} &= \vec{k} \times E \\ \vec{B} &= \frac{\vec{k}}{\omega} \times E\end{aligned}$$

when $\vec{k} \parallel \hat{z}$

$$\begin{aligned}\vec{B} &= \frac{k}{\omega} \hat{z} \times E \\ &= \frac{n}{c} \hat{z} \times E\end{aligned}$$

Since the material is non-magnetic, both \vec{E} and \vec{B} are continuous across the surface

$$\begin{aligned}E_{0i} + E_{0r} &= E_{0t} \\ E_{0i} - E_{0r} &= nE_{0t} \\ E_{0t} &= \frac{2}{n+1} E_{0i} \\ E_{0r} &= -\frac{n-1}{n+1} E_{0i}\end{aligned}$$

(b)

Define

$$\begin{aligned}\tilde{a} &= a_0(z) e^{i\phi_a(z)} \\ \tilde{b} &= b_0(z) e^{i\phi_b(z)} \\ a(z, t) b^*(z, t) &= \tilde{a} \tilde{b}^* \\ \langle a(z, t) b^*(z, t) \rangle &= \tilde{a} \tilde{b}^* \\ \langle a^*(z, t) b(z, t) \rangle &= \tilde{a}^* \tilde{b} \\ \langle a(z, t) b(z, t) \rangle &= \tilde{a} \tilde{b} \langle e^{-2i\omega t} \rangle = 0 \\ \langle a^*(z, t) b^*(z, t) \rangle &= \tilde{a}^* \tilde{b}^* \langle e^{-2i\omega t} \rangle = 0 \\ \langle \Re(a(z, t)) \Re(b(z, t)) \rangle &= \frac{1}{4} (\langle a(z, t) b^*(z, t) \rangle + \langle a^*(z, t) b(z, t) \rangle + \langle a(z, t) b(z, t) \rangle + \langle a^*(z, t) b^*(z, t) \rangle) \\ &= \frac{1}{4} (\tilde{a}^* \tilde{b} + \tilde{a} \tilde{b}^*) \\ &= \frac{1}{2} \Re(\tilde{a}^* \tilde{b}) \\ &= \frac{1}{2} \Re(a(z, t) b^*(z, t))\end{aligned}$$

2.

- (a)
- (b)
- (c)
- (d)
- (e)
- (f)
- (g)

3.

- (a)
- (b)
- (c)
- (d)
- (e)
- (f)

4.