1.

(a)

(Shouldn't the condition be $\Omega \ll \delta$ instead of $\Omega \ll \Gamma$?)
Let $\delta = (\omega_0 - \omega_L)$

$$\begin{aligned}
\rho_{ee} =&\rho'_{ee}\mathrm{e}^{-\Gamma t}\\
\rho_{ge} =&\rho'_{ge}\mathrm{e}^{-\Gamma t/2+\mathrm{i}\delta t}\\
\dot{\rho}'_{ee} =&\mathrm{i}\frac{\Omega\mathrm{e}^{\Gamma t/2}}{2}\left({\rho'_{ge}}^*\mathrm{e}^{-\mathrm{i}\delta t} - \rho'_{ge}\mathrm{e}^{\mathrm{i}\delta t}\right)\\
\dot{\rho}'_{ge} =&-\mathrm{i}\frac{\Omega\mathrm{e}^{\Gamma t/2}}{2}\left(2\rho'_{ee}\mathrm{e}^{-\Gamma t} - 1\right)\mathrm{e}^{-\mathrm{i}\delta t}
\end{aligned}$$

0th order in Ω

$$\begin{aligned}
\rho'_{ee0} =&0\\
\rho'_{ge0} =&0
\end{aligned}$$

1st order in Ω

$$\begin{aligned}
\rho'_{ee1} =&0\\
\dot{\rho}'_{ge1} =&\mathrm{i}\frac{\Omega}{2}\mathrm{e}^{\Gamma t/2-\mathrm{i}\delta t}\\
\rho'_{ge1} =&\mathrm{i}\frac{\Omega}{2(\Gamma/2-\mathrm{i}\delta)}\left(\mathrm{e}^{\Gamma t/2-\mathrm{i}\delta t} - 1\right)\\
{\rho'_{ge1}}^* =&-\mathrm{i}\frac{\Omega}{2(\Gamma/2+\mathrm{i}\delta)}\left(\mathrm{e}^{\Gamma t/2+\mathrm{i}\delta t} - 1\right)
\end{aligned}$$

2nd order in Ω

$$\begin{aligned}
\dot{\rho}'_{ee2} =&\mathrm{i}\frac{\Omega\mathrm{e}^{\Gamma t/2}}{2}\left({\rho'_{ge1}}^*\mathrm{e}^{-\mathrm{i}\delta t} - \rho'_{ge1}\mathrm{e}^{\mathrm{i}\delta t}\right)\\
=&\mathrm{i}\frac{\Omega\mathrm{e}^{\Gamma t/2}}{2}\left(-\mathrm{i}\frac{\Omega}{2(\Gamma/2+\mathrm{i}\delta)}\left(\mathrm{e}^{\Gamma t/2+\mathrm{i}\delta t} - 1\right)\mathrm{e}^{-\mathrm{i}\delta t} - \mathrm{i}\frac{\Omega}{2(\Gamma/2-\mathrm{i}\delta)}\left(\mathrm{e}^{\Gamma t/2-\mathrm{i}\delta t} - 1\right)\mathrm{e}^{\mathrm{i}\delta t}\right)\\
=&\frac{\Omega^2}{4}\left(\frac{1}{\Gamma/2+\mathrm{i}\delta}\left(\mathrm{e}^{\Gamma t} - \mathrm{e}^{\Gamma t/2-\mathrm{i}\delta t}\right) + \frac{1}{\Gamma/2-\mathrm{i}\delta}\left(\mathrm{e}^{\Gamma t} - \mathrm{e}^{\Gamma t/2+\mathrm{i}\delta t}\right)\right)\\
\rho'_{ee2} =&C_0 + \frac{\Omega^2}{4}\left(\frac{1}{\Gamma/2+\mathrm{i}\delta}\left(\frac{1}{\Gamma}\mathrm{e}^{\Gamma t} - \frac{1}{\Gamma/2-\mathrm{i}\delta}\mathrm{e}^{\Gamma t/2-\mathrm{i}\delta t}\right) + \frac{1}{\Gamma/2-\mathrm{i}\delta}\left(\frac{1}{\Gamma}\mathrm{e}^{\Gamma t} - \frac{1}{\Gamma/2+\mathrm{i}\delta}\mathrm{e}^{\Gamma t/2+\mathrm{i}\delta t}\right)\right)\\
=&C_0 + \frac{\Omega^2}{\Gamma^2+4\delta^2}\left(\frac{\Gamma/2-\mathrm{i}\delta}{\Gamma}\mathrm{e}^{\Gamma t} - \mathrm{e}^{\Gamma t/2-\mathrm{i}\delta t} + \frac{\Gamma/2+\mathrm{i}\delta}{\Gamma}\mathrm{e}^{\Gamma t} - \mathrm{e}^{\Gamma t/2+\mathrm{i}\delta t}\right)\\
=&C_0 + \frac{\Omega^2}{\Gamma^2+4\delta^2}\left(\mathrm{e}^{\Gamma t} - 2\cos{(\delta t)}\mathrm{e}^{\Gamma t/2}\right)
\end{aligned}$$

Since $\rho'(0) = 0$

$$\rho'_{ee2} =\frac{\Omega^2}{\Gamma^2+4\delta^2}\left(1 + \mathrm{e}^{\Gamma t} - 2\cos{(\delta t)}\mathrm{e}^{\Gamma t/2}\right)$$

Therefore, to the second order in Ω

$$\rho_{ee} = \frac{\Omega^2}{\Gamma^2 + 4\delta^2}\left(1 + \mathrm{e}^{-\Gamma t} - 2\cos(\delta t)\mathrm{e}^{-\Gamma t/2}\right)$$

In the limit of $\Gamma \to 0$, this turns into a Rabi flopping at the frequency of the detuning.

(b)

Expand the solution to second order in t

$$\begin{aligned}
\rho_{ee} =& \frac{\Omega^2}{\Gamma^2 + 4\delta^2}\left(1 + 1 - \Gamma t + \frac{\Gamma^2 t^2}{2} - 2\left(1 - \frac{\delta^2 t^2}{2}\right)\left(1 - \frac{\Gamma t}{2} + \frac{\Gamma^2 t^2}{8}\right)\right)\\
=& \frac{\Omega^2}{\Gamma^2 + 4\delta^2}\left(1 + 1 - \Gamma t + \frac{\Gamma^2 t^2}{2} - 2 + \delta^2 t^2 + \Gamma t - \frac{\Gamma^2 t^2}{4}\right)\\
=& \frac{\Omega^2}{\Gamma^2 + 4\delta^2}\left(\frac{\Gamma^2 t^2}{4} + \delta^2 t^2\right)\\
=& \frac{\Omega^2 t^2}{4}
\end{aligned}$$

When the pulse is very short, the decay haven't started yet and the atom also doesn't have enough time to figure out that the frequency is wrong.

2.

(a)

In the $|0g\rangle$ subspace, the Hamiltonian is $\frac{\hbar}{2}(\delta - 2\omega_0)$ and in the $|1g\rangle$, $|0e\rangle$ subspace, the Hamiltonian is $\frac{\hbar}{2}(\Omega_1\sigma_x - \delta\sigma_z)$

$$\begin{aligned}
\mathrm{e}^{-\mathrm{i}Ht/\hbar} =& \mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t} \oplus \mathrm{e}^{-\mathrm{i}(\Omega_1\sigma_x - \delta\sigma_z)t/2}\\
=& \mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)t} \oplus \left(\cos\frac{\Omega t}{2} - \mathrm{i}\left(\frac{\Omega_1}{\Omega}\sigma_x - \frac{\delta}{\Omega}\sigma_z\right)\sin\frac{\Omega t}{2}\right)\\
=& \mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t}|0g\rangle\langle 0g| + \left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right)|0e\rangle\langle 0e| + \left(\cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right)|1g\rangle\langle 1g|\\
&- \mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2}(|0e\rangle\langle 1g| + |1g\rangle\langle 0e|)
\end{aligned}$$

(b)

Since the system is either in $|0e\rangle$ or $|1g\rangle$ the atom is in state e if there's no photon in the cavity and g if there's one photon in the cavity.

(c)

In the same subspace witht the first problem,

$$
\begin{aligned}
\rho_0 =& \begin{pmatrix} \rho_{gg} & \rho_{ge} & \\ \rho_{eg} & \rho_{ee} & \\ & & 0 \end{pmatrix}\\
U =& \begin{pmatrix} \mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t} & & \\ & \cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} & -\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} \\ & -\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} \end{pmatrix}\\
\rho_t =& U^\dagger\rho_0 U\\
=& \begin{pmatrix} \mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t} & & \\ & \cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} & -\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} \\ & -\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} \end{pmatrix} \begin{pmatrix} \rho_{gg} & \rho_{ge} & \\ \rho_{eg} & \rho_{ee} & \\ & & 0 \end{pmatrix}\\
& \begin{pmatrix} \mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)t} & & \\ & \cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} & +\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} \\ & \mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} \end{pmatrix}\\
=& \begin{pmatrix} \rho_{gg}\mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t} & \rho_{ge}\mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t} & \\ \rho_{eg}\left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) & \rho_{ee}\left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) & \\ -\rho_{eg}\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & -\rho_{ee}\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & 0 \end{pmatrix}\\
& \begin{pmatrix} \mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)t} & & \\ & \cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} & +\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} \\ & \mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & \cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2} \end{pmatrix}
\end{aligned}
$$

Let $\rho'_{ge} = \rho_{ge}\mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t}$ and $\rho'_{eg} = \rho_{eg}\mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)t}$

$$
\rho = \begin{pmatrix} \rho_{gg} & \rho'_{ge}\left(\cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) & \rho'_{ge}\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} \\ \rho'_{eg}\left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) & \rho_{ee}\left(\cos^2\frac{\Omega t}{2} + \frac{\delta^2}{\Omega^2}\sin^2\frac{\Omega t}{2}\right) & \rho_{ee}\left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right)\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} \\ -\rho'_{eg}\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2} & -\rho_{ee}\mathrm{i}\frac{\Omega_1}{\Omega}\sin\frac{\Omega t}{2}\left(\cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) & \rho_{ee}\frac{\Omega_1^2}{\Omega^2}\sin^2\frac{\Omega t}{2} \end{pmatrix}
$$

The density matrix of the atom is

$$
\rho_{atom} = \begin{pmatrix} \rho_{gg} + \rho_{ee}\frac{\Omega_1^2}{\Omega^2}\sin^2\frac{\Omega t}{2} & \rho_{ge}\mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t}\left(\cos\frac{\Omega t}{2} - \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) \\ \rho_{eg}\mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)t}\left(\cos\frac{\Omega t}{2} + \mathrm{i}\frac{\delta}{\Omega}\sin\frac{\Omega t}{2}\right) & \rho_{ee}\left(\cos^2\frac{\Omega t}{2} + \frac{\delta^2}{\Omega^2}\sin^2\frac{\Omega t}{2}\right) \end{pmatrix}
$$

(d)

$$\begin{aligned}
\rho_{atom}(\Delta t) \approx& \begin{pmatrix} \rho_{gg} + \rho_{ee}\dfrac{\Omega_1^2\Delta t^2}{4} & \rho_{ge}\mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)\Delta t}\left(1 - \dfrac{\Omega^2\Delta t^2}{8} - \mathrm{i}\dfrac{\delta\Delta t}{2}\right) \\ \rho_{eg}\mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)\Delta t}\left(1 - \dfrac{\Omega^2\Delta t^2}{8} + \mathrm{i}\dfrac{\delta\Delta t}{2}\right) & \rho_{ee}\left(1 - \dfrac{\Omega_1^2\Delta t^2}{4}\right) \end{pmatrix}\\
\rho'_{atom}(t) \approx& \begin{pmatrix} 1 - \rho'_{ee}(t) & \rho_{ge}\mathrm{e}^{\mathrm{i}(\delta/2+\omega_0)t}\left(1 - \dfrac{\Omega^2\Delta t^2}{8} - \mathrm{i}\dfrac{\delta\Delta t}{2}\right)^{t/\Delta t} \\ \rho_{eg}\mathrm{e}^{-\mathrm{i}(\delta/2+\omega_0)t}\left(1 - \dfrac{\Omega^2\Delta t^2}{8} + \mathrm{i}\dfrac{\delta\Delta t}{2}\right)^{t/\Delta t} & \rho_{ee}\left(1 - \dfrac{\Omega_1^2\Delta t^2}{4}\right)^{t/\Delta t} \end{pmatrix}\\
\approx& \begin{pmatrix} 1 - \rho_{ee}\exp\left(-\dfrac{\Omega_1^2\Delta tt}{4}\right) & \rho_{ge}\mathrm{e}^{\mathrm{i}\omega_0 t}\exp\left(-\dfrac{\Omega_1^2\Delta tt}{8}\right) \\ \rho_{eg}\mathrm{e}^{-\mathrm{i}\omega_0 t}\exp\left(-\dfrac{\Omega_1^2\Delta tt}{8}\right) & \rho_{ee}\exp\left(-\dfrac{\Omega_1^2\Delta tt}{4}\right) \end{pmatrix}\\
=& \begin{pmatrix} 1 - \rho_{ee}\mathrm{e}^{-\Gamma t} & \rho_{ge}\mathrm{e}^{\mathrm{i}\omega_0 t}\exp\left(-\dfrac{\Gamma t}{2}\right) \\ \rho_{eg}\mathrm{e}^{-\mathrm{i}\omega_0 t}\exp\left(-\dfrac{\Gamma t}{2}\right) & \rho_{ee}\mathrm{e}^{-\Gamma t} \end{pmatrix}
\end{aligned}$$

where $\Gamma = \dfrac{\Omega_1^2\Delta t}{4}$

(e)

On the Bloch sphere

$$\vec{r} = \begin{pmatrix} 2\rho_{ge}\cos\omega_0 t\exp\left(-\dfrac{\Gamma t}{2}\right) \\ -2\rho_{ge}\sin\omega_0 t\exp\left(-\dfrac{\Gamma t}{2}\right) \\ 2\rho_{ee}\mathrm{e}^{-\Gamma t} - 1 \end{pmatrix}$$

For $(|e\rangle + |g\rangle)/\sqrt{2}$

$$\begin{aligned}
\rho_{ee} =& \frac{1}{2}\\
\rho_{ge} =& \frac{1}{2}\\
\vec{r} =& \begin{pmatrix} \cos\omega_0 t\exp\left(-\dfrac{\Gamma t}{2}\right) \\ -\sin\omega_0 t\exp\left(-\dfrac{\Gamma t}{2}\right) \\ \mathrm{e}^{-\Gamma t} - 1 \end{pmatrix}
\end{aligned}$$

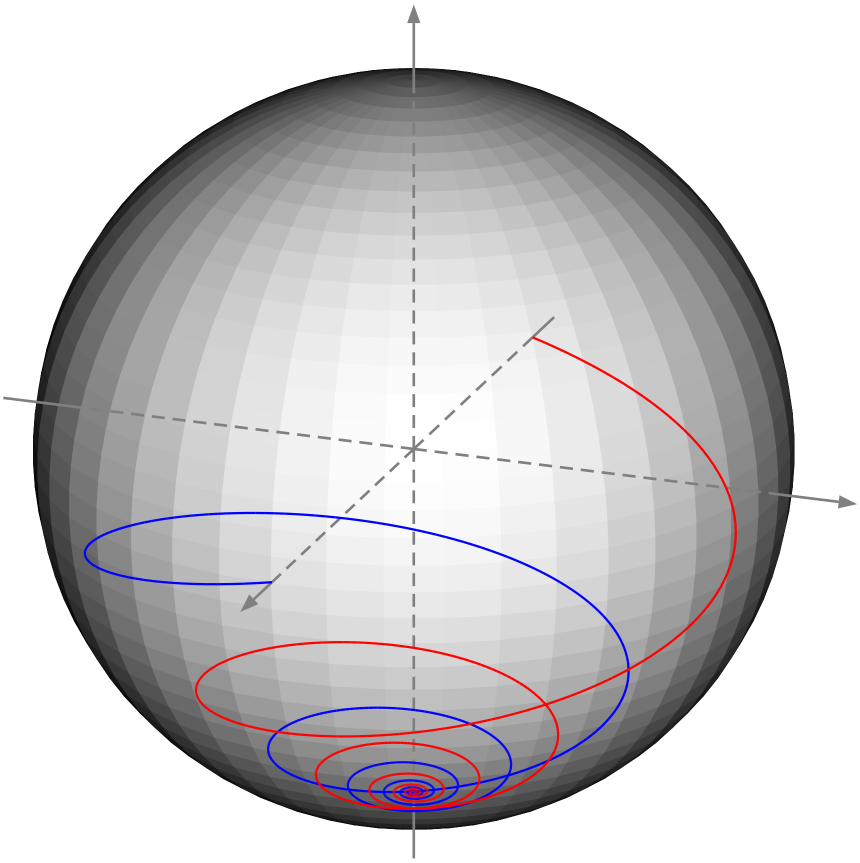
For $(|e\rangle - |g\rangle)/\sqrt{2}$

$$\rho_{ee} = \frac{1}{2}$$
$$\rho_{ge} = -\frac{1}{2}$$
$$\vec{r} = \begin{pmatrix} -\cos\omega_0 t\exp\left(-\frac{\Gamma t}{2}\right) \\ \sin\omega_0 t\exp\left(-\frac{\Gamma t}{2}\right) \\ \mathrm{e}^{-\Gamma t} - 1 \end{pmatrix}$$

For $|e\rangle$

$$\rho_{ee} = 1$$
$$\rho_{ge} = 0$$
$$\vec{r} = \begin{pmatrix} 0 \\ 0 \\ 2\mathrm{e}^{-\Gamma t} - 1 \end{pmatrix}$$

Plotting the first and the second one, (the third one is just a line connecting the north and south pole)



(f)

In the limit of $\Delta t \to 0$ the atom haven't got enough time (δ^{-1}) to figure out that it is actually detuned from the cavity resonance yet.

3.

(a)

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle =&(\dot{a} + \mathrm{i}\omega_1 a)\mathrm{e}^{\mathrm{i}\omega_1 t}|g\rangle + \left(\dot{b} + \mathrm{i}\omega_2 b\right)\mathrm{e}^{\mathrm{i}\omega_2 t}|e\rangle\\
\frac{H}{\hbar}|\psi\rangle =& -a\frac{\omega_0}{2}\mathrm{e}^{\mathrm{i}\omega_1 t}|g\rangle + b\frac{\omega_0}{2}\mathrm{e}^{\mathrm{i}\omega_2 t}|e\rangle + \frac{\Omega_1}{2}\left(\mathrm{e}^{\mathrm{i}\omega_L t} b\mathrm{e}^{\mathrm{i}\omega_2 t}|g\rangle + \mathrm{e}^{-\mathrm{i}\omega_L t} a\mathrm{e}^{\mathrm{i}\omega_1 t}|e\rangle\right)
\end{aligned}$$

If $\mathrm{e}^{-\mathrm{i}\omega_L t}\mathrm{e}^{\mathrm{i}\omega_1 t} = \mathrm{e}^{\mathrm{i}\omega_2 t}$ (and for convinienice having $\omega_1 + \omega_2 = 0$)

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle =&\left(\dot{a} + \mathrm{i}\frac{\omega_L}{2}a\right)\mathrm{e}^{\mathrm{i}\omega_L t/2}|g\rangle + \left(\dot{b} - \mathrm{i}\frac{\omega_L}{2}b\right)\mathrm{e}^{-\mathrm{i}\omega_L t/2}|e\rangle\\
\frac{H}{\hbar}|\psi\rangle =& -a\frac{\omega_0}{2}\mathrm{e}^{\mathrm{i}\omega_L t/2}|g\rangle + b\frac{\omega_0}{2}\mathrm{e}^{-\mathrm{i}\omega_L t/2}|e\rangle + \frac{\Omega_1}{2}\left(b\mathrm{e}^{\mathrm{i}\omega_L t/2}|g\rangle + a\mathrm{e}^{-\mathrm{i}\omega_L t/2}|e\rangle\right)\\
=&\frac{b\Omega_1 - a\omega_0}{2}\mathrm{e}^{\mathrm{i}\omega_L t/2}|g\rangle + \frac{a\Omega_1 + b\omega_0}{2}\mathrm{e}^{-\mathrm{i}\omega_L t/2}|e\rangle
\end{aligned}$$

Since $\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \frac{H}{\hbar}|\psi\rangle$

$$\begin{aligned}
\mathrm{i}\dot{a} =&\frac{b\Omega_1 + a\delta}{2}\\
\mathrm{i}\dot{b} =&\frac{a\Omega_1 - b\delta}{2}
\end{aligned}$$

(b)

$$H' = \frac{\hbar}{2}(\delta\sigma_z + \Omega_1\sigma_x)$$

(c)

$$H' = \frac{\hbar\Omega}{2}(\cos 2\theta\sigma_z + \sin 2\theta\sigma_x)$$

where $\sin 2\theta = \frac{\Omega_1}{\Omega}$ and $\cos 2\theta = \frac{\delta}{\Omega}$

(d)

Since the Hamiltonian is proportional to $\vec{r}\cdot\vec{\sigma}$ where $\vec{r} = \cos 2\theta\hat{z} + \sin 2\theta\hat{x}$, the eigenvalues are $\pm\dfrac{\hbar\Omega}{2}$ with eigenvectors

$$\begin{aligned}
|+\rangle =& \cos\theta|e'\rangle + \sin\theta|g'\rangle\\
|-\rangle =& -\sin\theta|e'\rangle + \cos\theta|g'\rangle\\
H|+\rangle =& \frac{\hbar\Omega}{2}(\cos 2\theta\cos\theta|e'\rangle - \cos 2\theta\sin\theta|g'\rangle + \sin 2\theta\cos\theta|g'\rangle + \sin 2\theta\sin\theta|e'\rangle)\\
=& \frac{\hbar\Omega}{2}|+\rangle\\
H|-\rangle =& -\frac{\hbar\Omega}{2}|-\rangle
\end{aligned}$$

(e)

The generic solution is

$$|\psi\rangle = \left(a\mathrm{e}^{-\mathrm{i}\Omega t/2}\cos\theta - b\mathrm{e}^{\mathrm{i}\Omega t/2}\sin\theta\right)\mathrm{e}^{-\mathrm{i}\omega_L t/2}|e\rangle + \left(a\mathrm{e}^{-\mathrm{i}\Omega t/2}\sin\theta + b\mathrm{e}^{\mathrm{i}\Omega t/2}\cos\theta\right)\mathrm{e}^{\mathrm{i}\omega_L t/2}|g\rangle$$