Confinement induced resonance

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We present a formal derivation of the confinement induced resonance for particles in a lower dimension. The generic T-matrix method is used to calculate the S-wave collisional property between two ultra-code atoms. We show the breakdown of a naive approach to describe such low dimensional system near a Feshbach resonance and preform a real three dimensional calculation to include additional effect from the confinement. The normalization of the high energy states in such cases are done by comparing to the result in free space.

1. INTRODUCTION

In the field of ultra-cold atoms and molecules, the collisional properties between the particles play an important role both for cooling into quantum degeneracy and as a source of correlation and entanglement in a many-body system.

2. FESHBACH RESONANCE

2.1. T-Matrix and scatter length

$$\Psi(\vec{r}) = \mathrm{e}^{\mathrm{i}\vec{k}_0\cdot\vec{r}} - \Psi_s \approx \qquad \mathrm{e}^{\mathrm{i}\vec{k}_0\cdot\vec{r}} - \frac{a}{r}$$

$$\frac{1}{m}\left(k^2 - k'^2\right)\Psi_s\left(\vec{k}'\right) = U\left(\vec{k}',\vec{k}\right) + \int\frac{\mathrm{d}^3k''}{(2\pi)^3}U\left(\vec{k}',\vec{k}''\right)\Psi_s\left(\vec{k}''\right)$$

Define T matrix

$$\begin{aligned}
T\left(\vec{k}',\vec{k}\right) =& U\left(\vec{k}',\vec{k}\right) + \int\frac{\mathrm{d}^3k''}{(2\pi)^3}\frac{U\left(\vec{k}',\vec{k}''\right)}{k^2/m - k''^2/m + \mathrm{i}0}T\left(\vec{k}'',\vec{k}\right)\\
\Psi_s\left(\vec{k}'\right) =& \frac{m}{k^2 - k'^2 + \mathrm{i}0}T\left(\vec{k}',\vec{k},\frac{k^2}{m}\right)\\
f\left(\vec{k}',\vec{k}\right) =& -\frac{m}{4\pi\hbar^2}T\left(\vec{k}',\vec{k},\frac{k^2}{m}\right)
\end{aligned}$$

$$\tilde{U}\left(\vec{k}',\vec{k}\right) = U\left(\vec{k}',\vec{k}\right) + \int_{k''^2/m>\varepsilon_c}\frac{\mathrm{d}^3k''}{(2\pi)^3}\frac{\tilde{U}\left(\vec{k}',\vec{k}''\right)}{k^2/m - k''^2/m + \mathrm{i}0}\tilde{U}\left(\vec{k}'',\vec{k}\right)$$

$$T\left(\vec{k}',\vec{k}\right) = \tilde{U}\left(\vec{k}',\vec{k}\right) + \int_{k''^2/m<\varepsilon_c}\frac{\mathrm{d}^3k'}{(2\pi)^3}\frac{\tilde{U}\left(\vec{k}',\vec{k}''\right)}{k^2/m - k''^2/m + \mathrm{i}0}T\left(\vec{k}'',\vec{k}\right)$$

$$\begin{aligned}
\frac{1}{f(k)} =& -\frac{4\pi}{mU_0} + 4\pi\int_{|q|<1/R}\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{1}{k^2 - q^2 + \mathrm{i}0}\\
=& \frac{1}{2\pi^2}\left(-\frac{1}{R} - \frac{k}{2}\ln\left(-\frac{R^{-1} - k - \mathrm{i}0}{R^{-1} + k + \mathrm{i}0}\right)\right)\\
\approx& \frac{1}{2\pi^2}\left(-\frac{1}{R} - \frac{\mathrm{i}\pi k}{2} + k^2R\right)
\end{aligned}$$

$$\begin{aligned}
a =& \frac{\pi}{2}\frac{R}{1 + \frac{2\pi^2R}{mU_0}}\\
f(k) =& \frac{1}{a^{-1} + r_{eff}k^2/2 - \mathrm{i}k}
\end{aligned}$$

2.2. Multichannel scattering and Feshbach resonance

$$\hat{H}_0|\vec{k}\rangle =$$

3. CONFINEMENT INDUCED RESONANCE

4. CONCLUSION

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