

## Assignment #5

Due: Monday, March 30, 2015

TA: Colin Kennedy / 26-269 / colink@mit.edu / x2-2501

Office Hours: 1:30pm - 3:30pm, March 27th (Fri) & 1pm - 3pm, March 29th (Sun).

### 1. Rayleigh and Thomson scattering using two different interaction Hamiltonians.

Elastic light scattering takes the system from state  $|i\rangle = |a; \mathbf{k}\rangle$  to state  $|f\rangle = |a; \mathbf{k}'\epsilon'\rangle$  where  $a$  is the (unchanged) state of the atom, and  $\mathbf{k}(\mathbf{k}')$  and  $\epsilon(\epsilon')$  the wavevector and polarization of the incident (scattered) photon.

- (a) Calculate the transition matrix element  $\mathcal{T}_{fi}$  (i.e. matrix element of the  $T$  matrix) in leading (non-zero) order for both Rayleigh and Thomson scattering ( $\hbar\omega$  smaller/larger than excitation energies), for both the electric-dipole Hamiltonian:

$$H_I' = -\mathbf{d} \cdot \mathbf{E} \quad (1)$$

and the Coulomb-gauge interaction Hamiltonian:

$$H_I = -\frac{q}{m} \mathbf{p} \cdot \mathbf{A}(\mathbf{0}) + \frac{q^2}{2m} \mathbf{A}^2(\mathbf{0}) \quad (2)$$

You will realize that Rayleigh scattering is more easily calculated with the dipole Hamiltonian, whereas the reverse is true for Thomson scattering. Of course, both Hamiltonians lead to the same answer.

- (b) Show that the total cross section for Thomson scattering is given by  $\sigma_0 = \frac{8\pi}{3} r_0^2$  where  $r_0$  is the classical electron radius.

Try to advance with this problem as far as you can by yourself. If you get stuck, you will find a discussion of this problem in Exercises 3 and 4 in the book Atom-Photon Interactions.

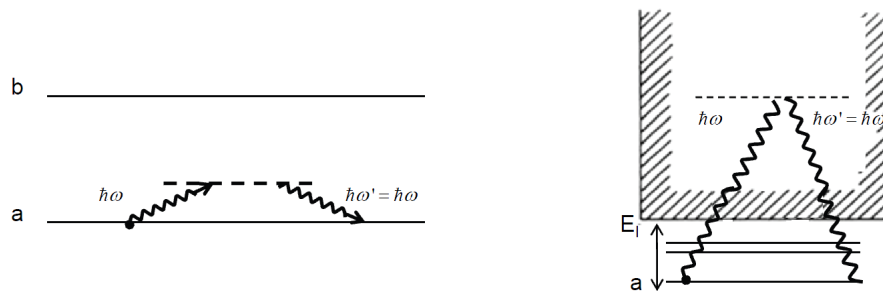


FIG. 1: Rayleigh and Thomson scattering

## 2. Long-range (Van der Waals) interaction between ground-state atoms.

The electrostatic interaction between atoms  $a$  and  $b$  is described to first order by the dipole-dipole term:

$$H_{el}(R) = \frac{\mathbf{d}_a \cdot \mathbf{d}_b - 3(\mathbf{d}_a \cdot \hat{\mathbf{R}})(\mathbf{d}_b \cdot \hat{\mathbf{R}})}{R^3} \quad (3)$$

where  $\mathbf{d}_a = e\mathbf{r}_a$  is the electric dipole operator of atom  $a$ ,  $\mathbf{d}_b = e\mathbf{r}_b$  is the electric dipole operator of atom  $b$ , and  $\mathbf{R} = \mathbf{R}_{nb} - \mathbf{R}_{na}$  is a position vector pointing from the nuclei of  $a$  to the nuclei of  $b$ .

We will use time-independent perturbation theory to calculate the effect of  $H_{el}$ . This is simpler than the time-dependent perturbation expansion discussed in class and does not lead to a virtual-photon picture of the Van der Waals interaction.

Notation: Let  $|g_a g_b\rangle$  denote atom  $a$  and atom  $b$  in the ground state.

Let  $|i_a g_b\rangle$  denote atom  $a$  in an excited state  $i$  and atom  $b$  in the ground state.

.

.

.

- (a) What is the first non-vanishing term in the series for the perturbed ground state energy of the system?
- (b) Dipole matrix elements in atomic physics are often discussed in terms of “oscillator strength,”  $f_{ig} = \frac{2m\omega_{ig}}{\hbar} |\langle i|x|g\rangle|^2$ . Note:  $\omega_{ig} = \frac{E_i - E_g}{\hbar}$ , so  $f_{ig}$  is positive for absorption and negative for emission. Also,  $\sum_i f_{ig} = 1$ , the Thomas-Reiche-Kuhn sum rule. Express your result from (a) in terms of oscillator strengths. You will have to make some arguments (non-mathematical if you prefer) about the symmetry of photon emission to get rid of annoying cross terms.
- (c) We can estimate  $C_6$  using the approximation that the oscillator strength  $f_{ig}$  is large for only one transition,  $|g\rangle \rightarrow |i\rangle$ . The  $|nS\rangle \rightarrow |(n+1)P\rangle$  transitions in alkali atoms are the classic examples, with  $f \approx 0.98$ . Use this in combination with the sum rule above and the definition of the static polarizability of the ground state:

$$\alpha_g = 2e^2 \sum_i \frac{|\langle i|z|g\rangle|^2}{E_i - E_g} \quad (4)$$

and express your result for  $C_6$  from (b) in terms of polarizabilities  $\alpha_g^{(a)}$  and  $\alpha_g^{(b)}$ . (You should not have any summation signs in your final answer.)