1.

(a)

$$
\begin{aligned}
H =&\hbar\omega b^\dagger b + \mathrm{i}\hbar\Lambda\left({b^\dagger}^2 + b^2\right)\\
\frac{\mathrm{d}b}{\mathrm{d}t} =&\frac{\mathrm{i}}{\hbar}[H, b] + \frac{\partial b}{\partial t}\\
=&\left[\mathrm{i}\omega b^\dagger b - \Lambda {b^\dagger}^2, b\right] + \mathrm{i}\omega b\\
=&2\Lambda b^\dagger\\
\frac{\mathrm{d}b^\dagger}{\mathrm{d}t} =&2\Lambda b
\end{aligned}
$$

(b)

$$
\begin{aligned}
\vec{E} =&\mathrm{i}\mathcal{E}_\omega\vec{\varepsilon}\left(b\mathrm{e}^{\mathrm{i}\left(\vec{k}\cdot\vec{r}-\omega t\right)} - b^\dagger\mathrm{e}^{-\mathrm{i}\left(\vec{k}\cdot\vec{r}-\omega t\right)}\right)\\
=&\mathrm{i}\mathcal{E}_\omega\vec{\varepsilon}\left(b\cos\left(\vec{k}\cdot\vec{r}-\omega t\right) + \mathrm{i}b\sin\left(\vec{k}\cdot\vec{r}-\omega t\right) - b^\dagger\cos\left(\vec{k}\cdot\vec{r}-\omega t\right) + \mathrm{i}b^\dagger\sin\left(\vec{k}\cdot\vec{r}-\omega t\right)\right)\\
=&-2\mathcal{E}_\omega\vec{\varepsilon}\left(\frac{b-b^\dagger}{2\mathrm{i}}\cos\left(\vec{k}\cdot\vec{r}-\omega t\right) + \frac{b+b^\dagger}{2}\sin\left(\vec{k}\cdot\vec{r}-\omega t\right)\right)\\
=&-2\mathcal{E}_\omega\vec{\varepsilon}\left(b_Q\cos\left(\vec{k}\cdot\vec{r}-\omega t\right) + b_P\sin\left(\vec{k}\cdot\vec{r}-\omega t\right)\right)\\
\frac{\mathrm{d}b_\pm}{\mathrm{d}t} \equiv&\frac{\mathrm{d}b\pm b^\dagger}{\mathrm{d}t}\\
=&2\Lambda b^\dagger \pm 2\Lambda b\\
=&\pm 2\Lambda b_\pm\\
\frac{\mathrm{d}b_P}{\mathrm{d}t} =&2\Lambda b_P\\
\frac{\mathrm{d}b_Q}{\mathrm{d}t} =&-2\Lambda b_Q
\end{aligned}
$$

Therefore

$$
\begin{aligned}
b_P =&\mathrm{e}^{2\Lambda t}b_{P0}\\
b_Q =&\mathrm{e}^{-2\Lambda t}b_{Q0}\\
b =&b_P + \mathrm{i}b_Q\\
=&\mathrm{e}^{2\Lambda t}b_{P0} + \mathrm{i}\mathrm{e}^{-2\Lambda t}b_{Q0}\\
=&b_0\cosh 2\Lambda t + b_0^\dagger\sinh 2\Lambda t\\
b^\dagger =&b_0^\dagger\cosh 2\Lambda t + b_0\sinh 2\Lambda t
\end{aligned}
$$

(c)

$$
\begin{aligned}
\langle N\rangle =&\langle 0|b^\dagger b|0\rangle\\
=&\langle 0|\left(b_0^\dagger\cosh 2\Lambda t+b_0\sinh 2\Lambda t\right)\left(b_0\cosh 2\Lambda t+b_0^\dagger\sinh 2\Lambda t\right)|0\rangle\\
=&\sinh^2 2\Lambda t\\
\Delta b_P =&\mathrm{e}^{2\Lambda t}\Delta b_{P0}\\
=&\frac{1}{2}\mathrm{e}^{2\Lambda t}\\
\Delta b_Q =&\mathrm{e}^{-2\Lambda t}\Delta b_{Q0}\\
=&\frac{1}{2}\mathrm{e}^{-2\Lambda t}
\end{aligned}
$$

The state is squeezed in Q direction while the product of the uncertainty in P and Q remains the same.

(d)

Under the transformation $U=\mathrm{e}^{\mathrm{i}\omega t a^\dagger a}$

$$
\begin{aligned}
UaU^\dagger =&\mathrm{e}^{\mathrm{i}\omega t a^\dagger a}a\mathrm{e}^{-\mathrm{i}\omega t a^\dagger a}\\
=&\sum_N\frac{(\mathrm{i}\omega t)^N}{N!}\left[a^\dagger a,a\right]_N\\
=&\mathrm{e}^{-\mathrm{i}\omega t}a\\
Ua^\dagger U^\dagger =&\mathrm{e}^{\mathrm{i}\omega t}a^\dagger\\
\frac{\mathrm{d}}{\mathrm{d}t}|\psi'\rangle =&\frac{\mathrm{d}}{\mathrm{d}t}U|\psi\rangle\\
=&\frac{\mathrm{d}U}{\mathrm{d}t}|\psi\rangle+U\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle\\
=&\frac{\mathrm{d}\mathrm{e}^{\mathrm{i}\omega t a^\dagger a}}{\mathrm{d}t}|\psi\rangle+\frac{U}{\mathrm{i}\hbar}H|\psi\rangle\\
=&\mathrm{i}\omega a^\dagger a|\psi'\rangle+\frac{1}{\mathrm{i}\hbar}UHU^\dagger|\psi'\rangle\\
=&\Lambda U\left({a^\dagger}^2\mathrm{e}^{-2\mathrm{i}\omega t}-a^2\mathrm{e}^{2\mathrm{i}\omega t}\right)U^\dagger|\psi'\rangle\\
=&\Lambda\left({a^\dagger}^2-a^2\right)|\psi'\rangle
\end{aligned}
$$

Therefore, the state $|\psi'\rangle$ is transforming as

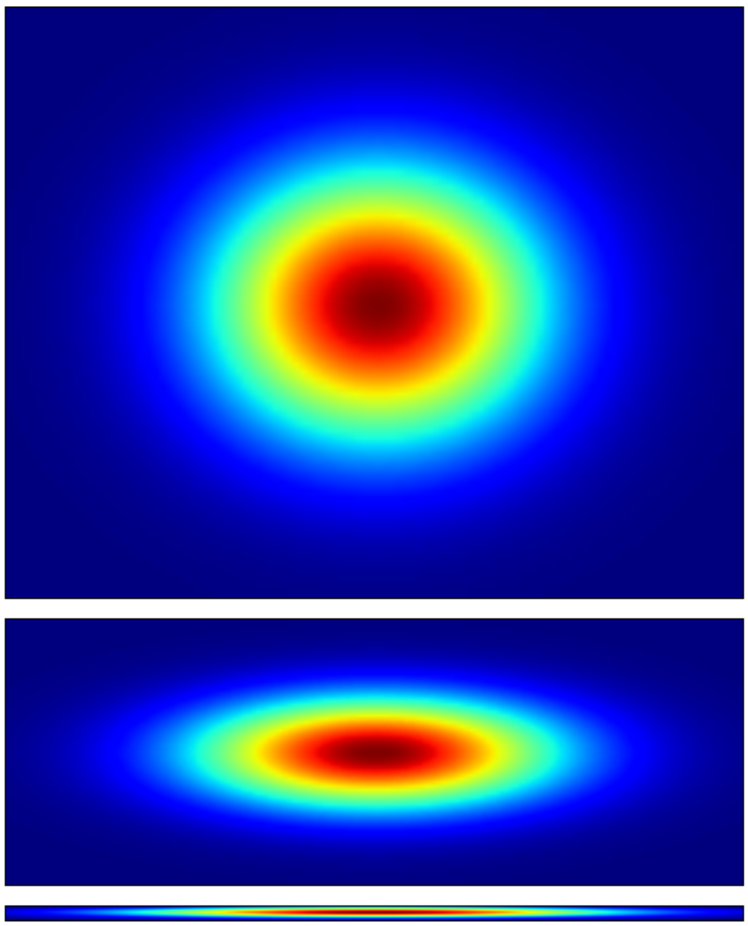
$$
\mathrm{e}^{\Lambda t\left({a^\dagger}^2-a^2\right)}
$$

where

$$
\varepsilon=-2\Lambda t
$$

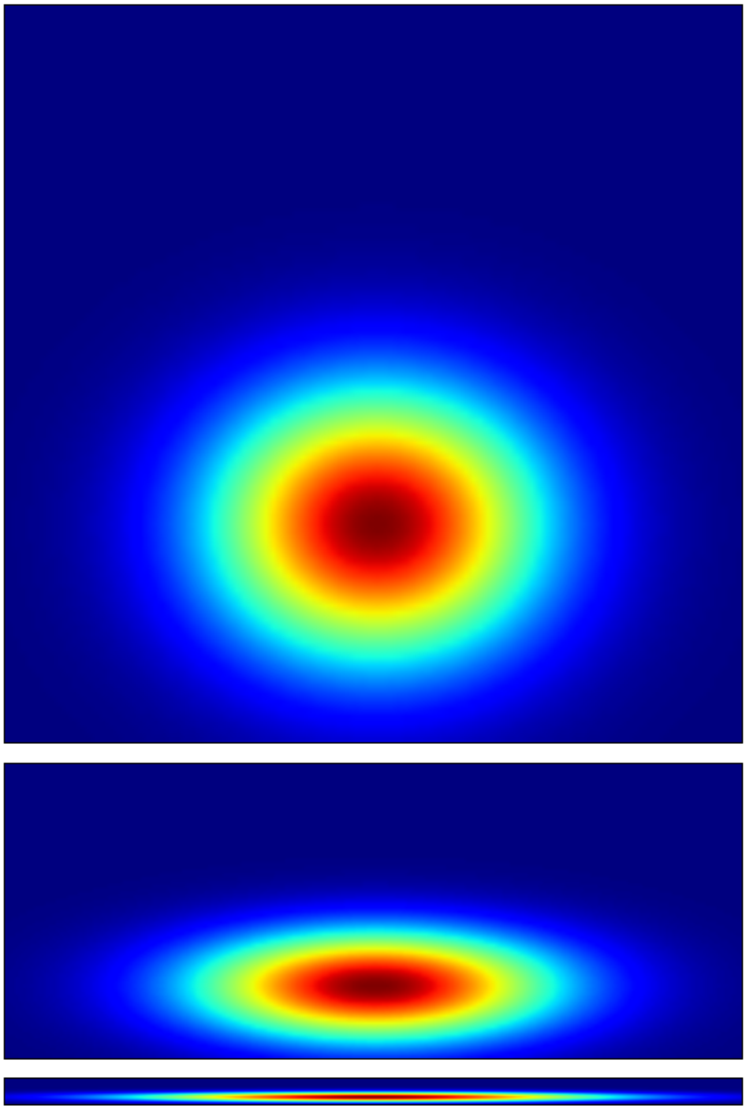
(e)

i.



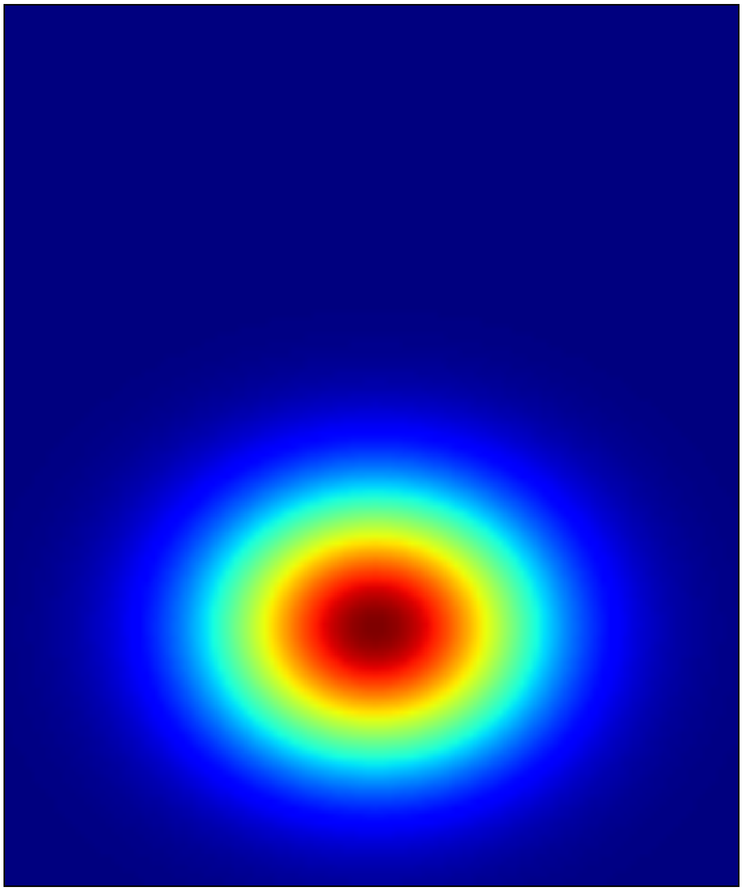
ii.

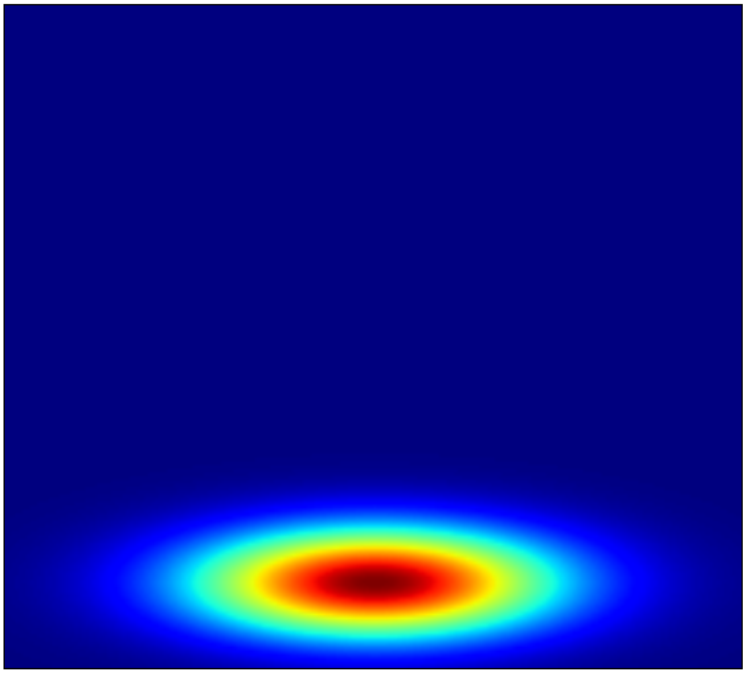
$\beta = \mathrm{i}$

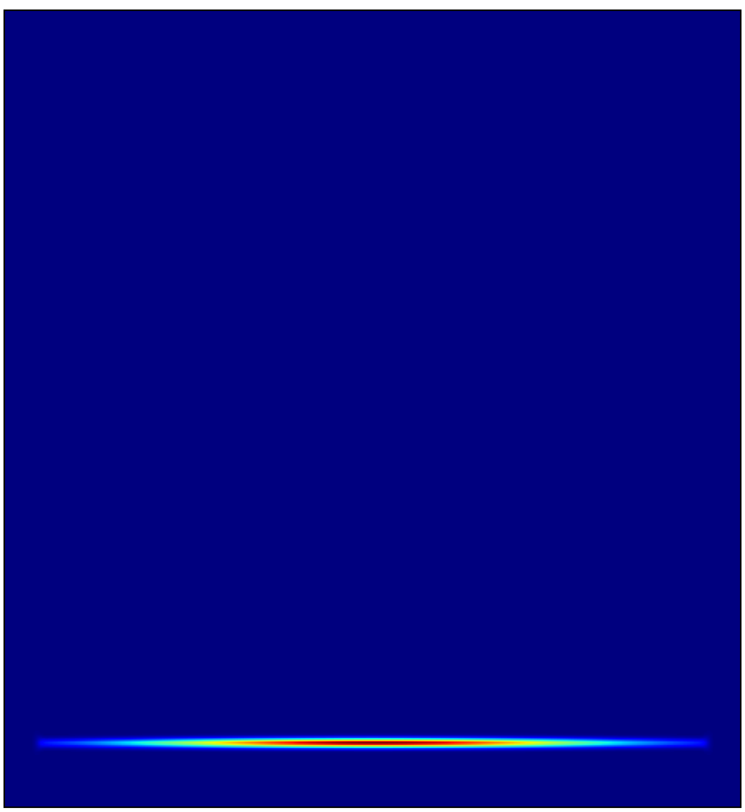


iii.

$\beta = \mathrm{i}$







Weither a phase or number squeezed state is created depends on the initial state.

2.

(a)

(b)

(c)

i.

ii.

iii.

iv.

3.

(a)

(b)

(c)

(d)