

1.

(a)

$$\begin{aligned}\langle \alpha | \beta \rangle &= \exp\left(-\frac{|\alpha|^2 + |\beta|^2}{2}\right) \sum_n \frac{(\alpha^*)^n \beta^n}{n!} \\ &= \exp\left(-\frac{|\alpha|^2 + |\beta|^2}{2} + \alpha^* \beta\right)\end{aligned}$$

(b)

$$\begin{aligned}\langle n | \int |\alpha\rangle \langle \alpha| \frac{d^2\alpha}{\pi} |m\rangle &= e^{-|\alpha|^2} \int \frac{|\alpha|^{2n} \alpha^{m-n}}{\sqrt{n!m!}} |\alpha| \frac{d|\alpha| d\theta}{\pi} \\ &= \delta_{mn} e^{-|\alpha|^2} \int \frac{|\alpha|^{2n}}{n!} d|\alpha|^2 \\ &= \delta_{mn}\end{aligned}$$

(c)

$$\begin{aligned}D(\alpha)|0\rangle &= \exp(\alpha a^\dagger - \alpha^* a)|0\rangle \\ &= e^{\alpha a^\dagger} e^{-\alpha^* a} e^{\alpha \alpha^* [a^\dagger, a]/2} |0\rangle \\ &= \exp\left(-\frac{|\alpha|^2}{2}\right) e^{\alpha a^\dagger} |0\rangle \\ &= |\alpha\rangle\end{aligned}$$

(d)

$$\begin{aligned}\langle E_x \rangle &= E_0 \sin(kz) \langle a + a^\dagger \rangle \\ &= E_0 \sin(kz) (\alpha + \alpha^*) \\ \langle E_x^2 \rangle &= E_0^2 \sin^2(kz) \langle (a + a^\dagger)^2 \rangle \\ &= E_0^2 \sin^2(kz) \langle a^2 + 1 + 2a^\dagger a + a^{\dagger 2} \rangle \\ &= E_0^2 \sin^2(kz) ((\alpha + \alpha^*)^2 + 1) \\ \sqrt{\langle \Delta E_x^2 \rangle} &= E_0 \sin(kz)\end{aligned}$$

The deviation of the field does not change (even for vacuum) because the coherent state is just a displacement of the vacuum state.

(e)

For number state

$$P(\phi) = \frac{1}{2\pi}$$

so the number state has equal probability of having any phase.

$$\begin{aligned} P(\phi, \theta) &= \frac{1}{2\pi} \left| \sum_n \langle n | e^{-in\phi} (|\alpha| e^{i\theta}) \rangle \right|^2 \\ &= \frac{1}{2\pi} \left| \sum_n e^{-in(\phi-\theta)} \langle n | (|\alpha|) \rangle \right|^2 \\ &= P(\phi - \theta, 0) \end{aligned}$$

Therefore P rotates in the same way as the phase of α changes

$$\begin{aligned} P(\phi, 0) &= \frac{1}{2\pi} \left| \sum_n \langle n | e^{-in\phi} (|\alpha|) \rangle \right|^2 \\ &= \frac{e^{-|\alpha|^2}}{2\pi} \left| \sum_n e^{-in\phi} \frac{|\alpha|^n}{\sqrt{n!}} \right|^2 \end{aligned}$$

So $P(\phi, 0)$ is symmetrically distributed and maximized at $\phi = 0$ (therefore $P(\phi, \theta)$ is maximized when $\phi = \theta$) if $|\alpha| \neq 0$. Therefore the expected value of ϕ (when $\alpha \neq 0$) is θ .

2.

(a)

$$\begin{aligned} \frac{V_\psi}{V_0} &= \frac{1}{4} \langle \psi, 0 | (a^\dagger + b^\dagger)(a + b)(a^\dagger - b^\dagger)(a - b) | \psi, 0 \rangle \\ &= \frac{1}{4} \langle \psi, 0 | a^\dagger (aa^\dagger - ab^\dagger + ba^\dagger - bb^\dagger) a | \psi, 0 \rangle \\ &= \frac{1}{4} \langle a^\dagger a^\dagger a a \rangle \end{aligned}$$

(b)

$$\begin{aligned} g_{cl}^{(2)}(0) &= \frac{\langle I^2 \rangle}{\langle I \rangle^2} \\ &= 1 + \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2} \\ &= 1 + \frac{\langle (I - \langle I \rangle)^2 \rangle}{\langle I \rangle^2} \\ &\geq 1 \end{aligned}$$

(c)

$$\begin{aligned}
 g_{\alpha}^{(2)} &= \frac{|a|^4}{|a|^4} \\
 &= 1 \\
 g_{n=2}^{(2)} &= \frac{2 \cdot 3}{2^2} \\
 &= \frac{3}{2}
 \end{aligned}$$

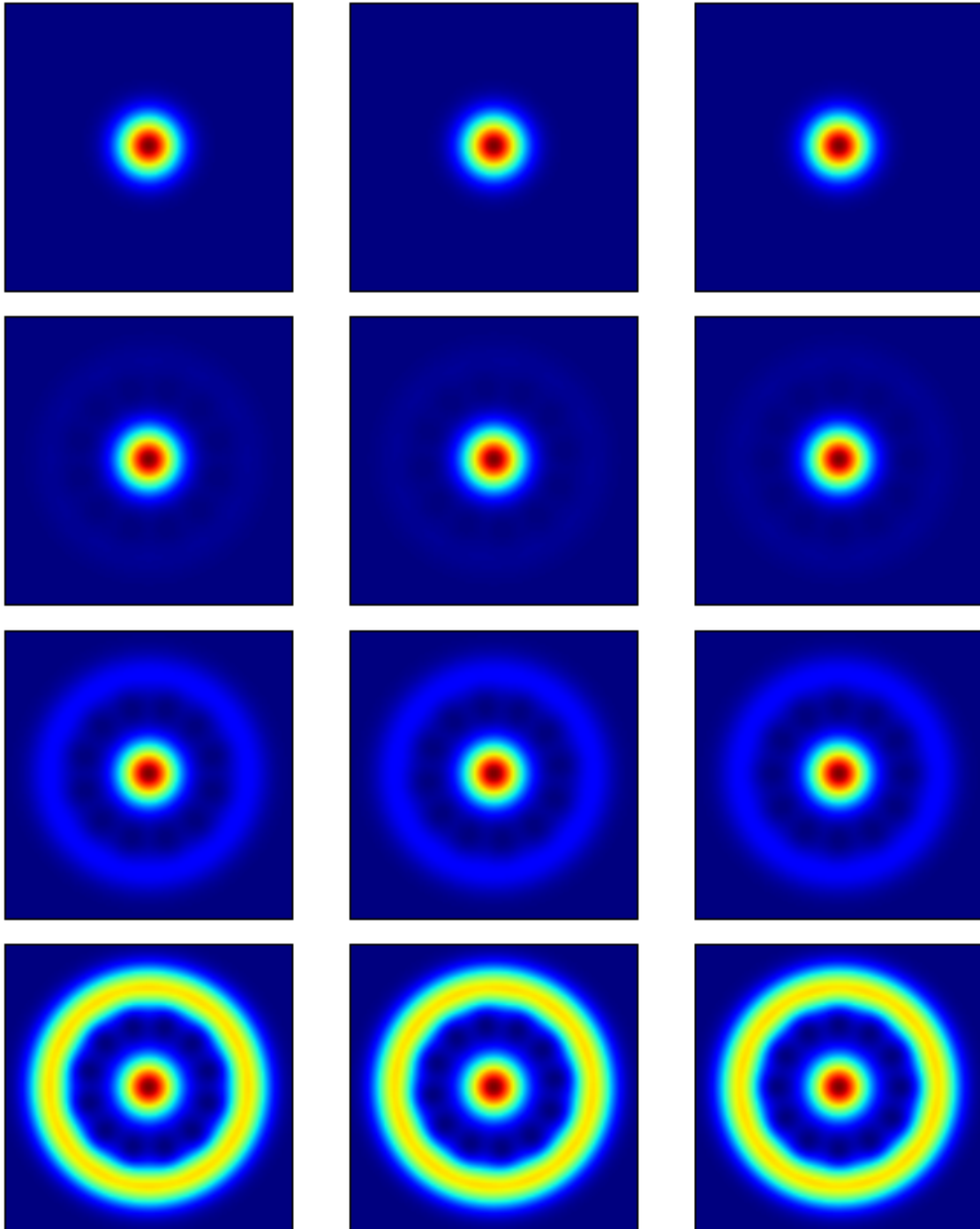
(d)

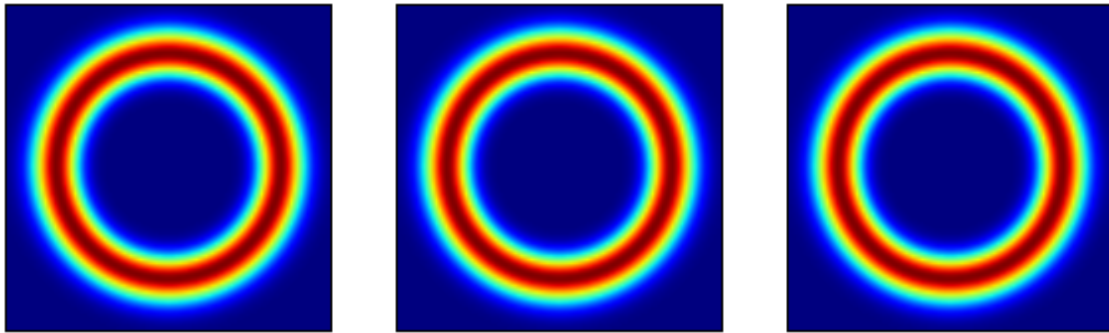
$$\begin{aligned}
 \langle \psi_3 | \psi_3 \rangle &= (1 + e^{2|\alpha|^2}) \\
 \langle \psi_4 | \psi_4 \rangle &= (1 - e^{2|\alpha|^2}) \\
 g_{3,4}^{(2)} &= \frac{\langle \psi_{3,4} | a^\dagger a^\dagger a a | \psi_{3,4} \rangle \langle \psi_{3,4} | \psi_{3,4} \rangle}{\langle \psi_{3,4} | a^\dagger a | \psi_{3,4} \rangle^2} \\
 &= \frac{(\langle \alpha | \pm \langle -\alpha |) a^\dagger a^\dagger a a (| \alpha \rangle \pm | -\alpha \rangle) (1 \pm e^{2|\alpha|^2})}{(\langle \alpha | \pm \langle -\alpha |) a^\dagger a (| \alpha \rangle \pm | -\alpha \rangle)^2} \\
 &= \frac{|\alpha|^4 (\langle \alpha | \pm \langle -\alpha |) (| \alpha \rangle \pm | -\alpha \rangle) (1 \pm e^{2|\alpha|^2})}{|\alpha|^4 (\langle \alpha | \mp \langle -\alpha |) (| \alpha \rangle \mp | -\alpha \rangle)^2} \\
 &= \left(\frac{1 \pm e^{2|\alpha|^2}}{1 \mp e^{2|\alpha|^2}} \right)^2
 \end{aligned}$$

Therefore $g^{(2)}$ can be smaller than 1 for ψ_4 .

3.

(a)

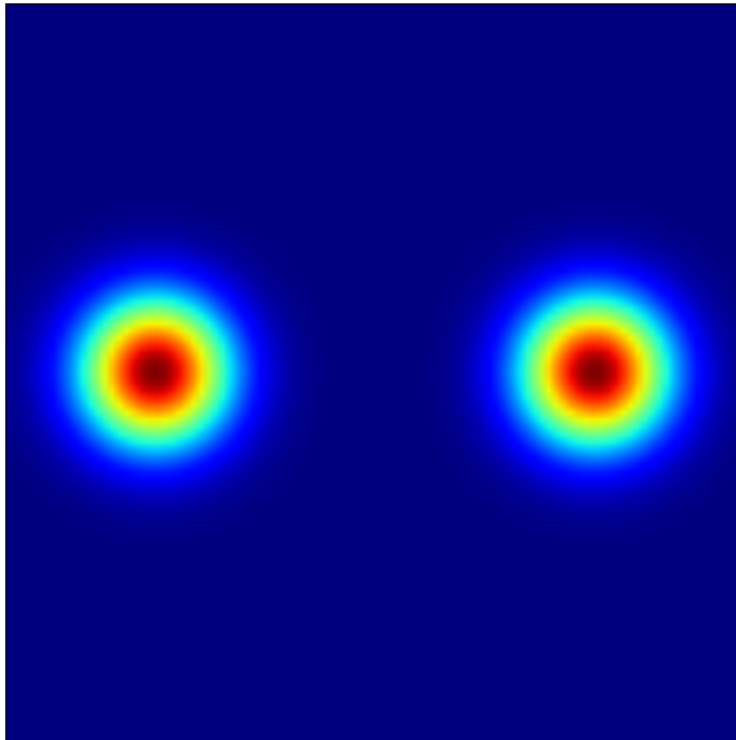




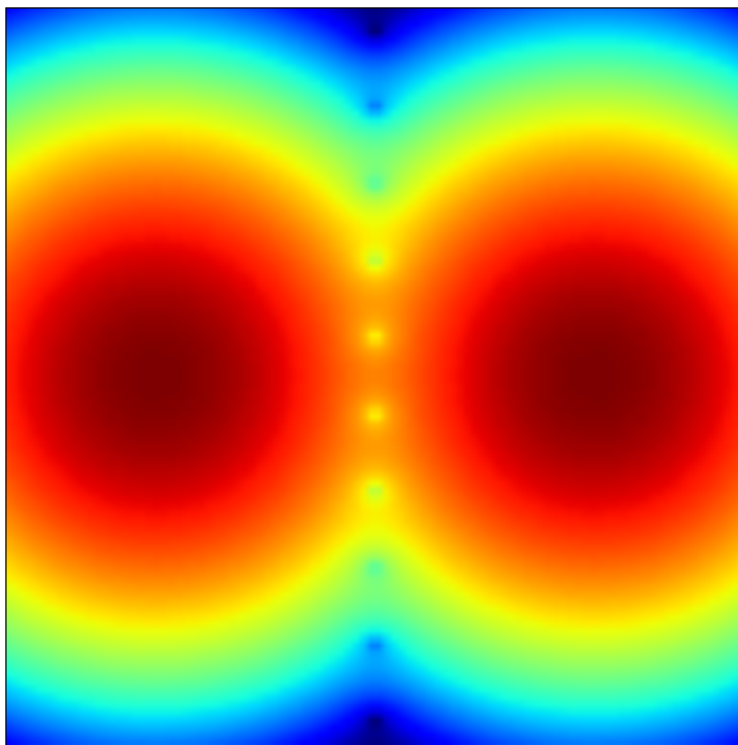
These are not minimum uncertainty states (except when it's $|0\rangle$) and not squeezed states.

(b)

$\alpha = 3i$ Linear scale



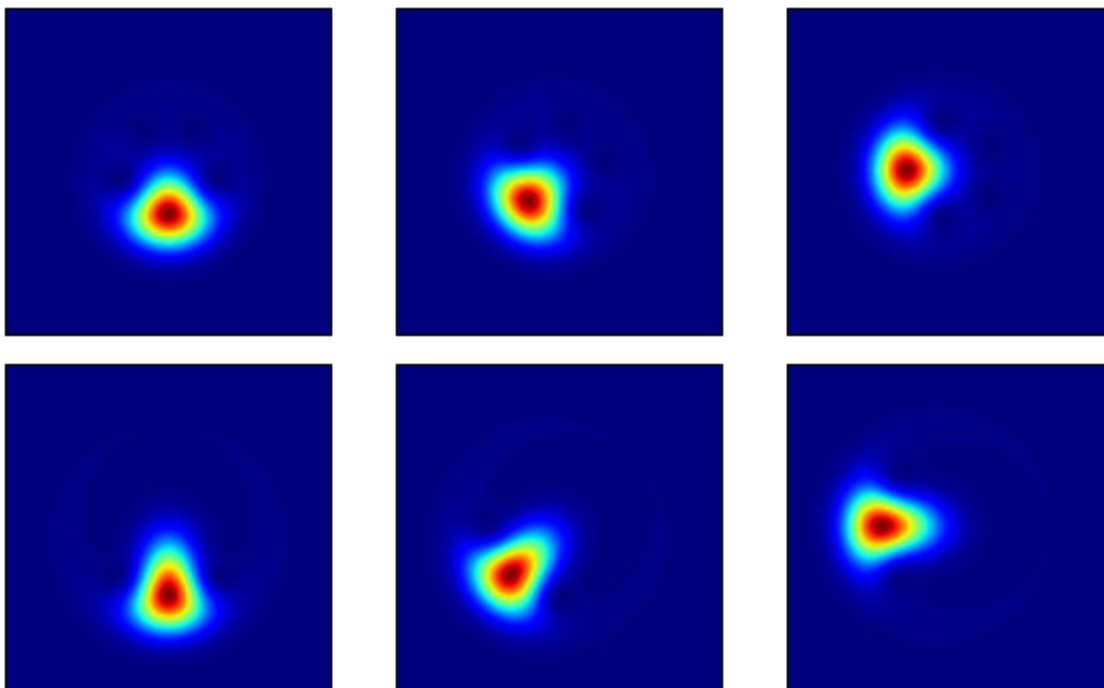
Log scale

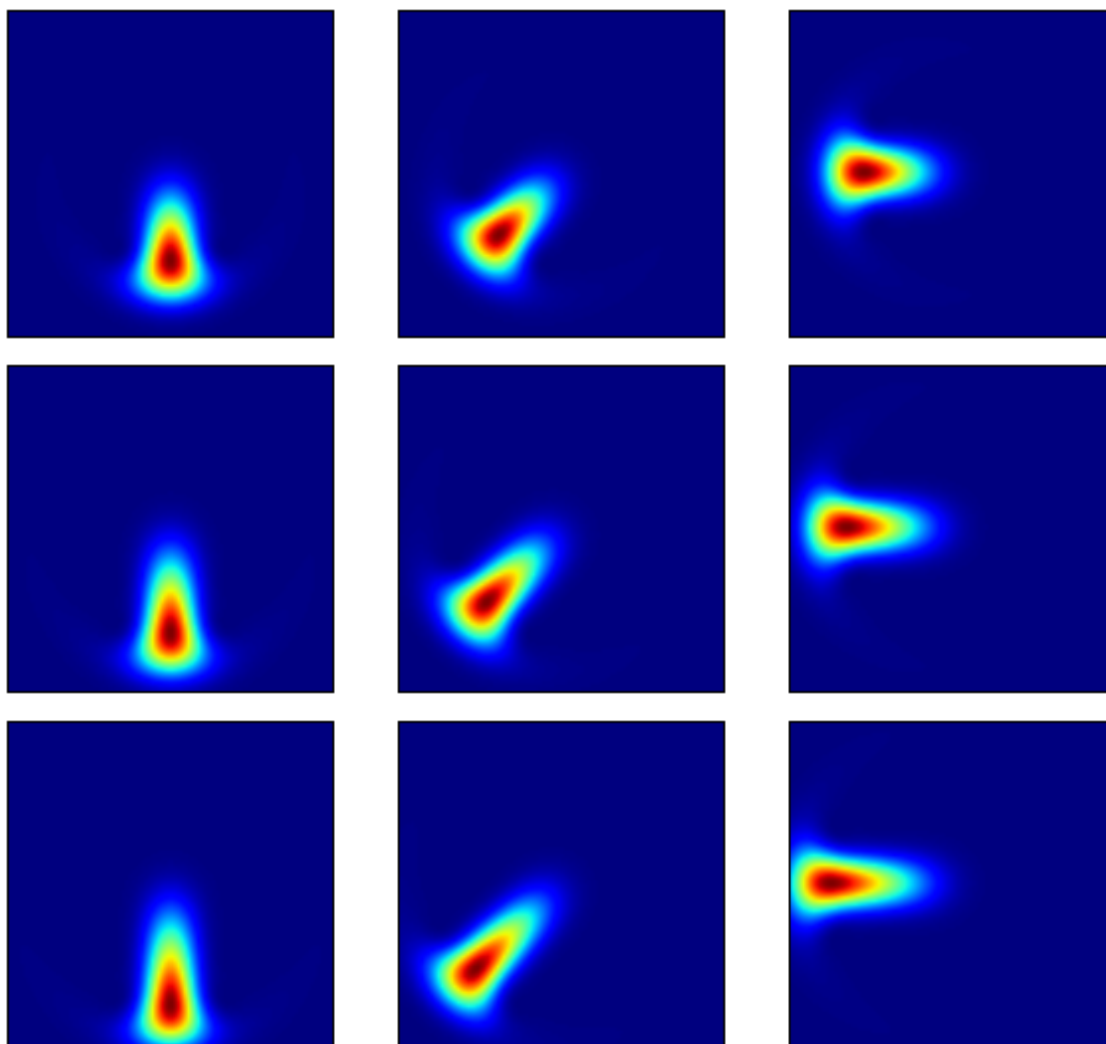


Interference fringes appears because the two wavefunction overlaps.

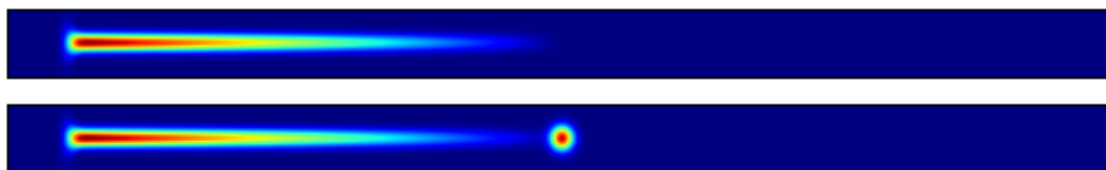
(c)

States with $\phi = 0, \frac{\pi}{4}, \frac{\pi}{2}$ and $N = 5, 10, 15, 20, 15$



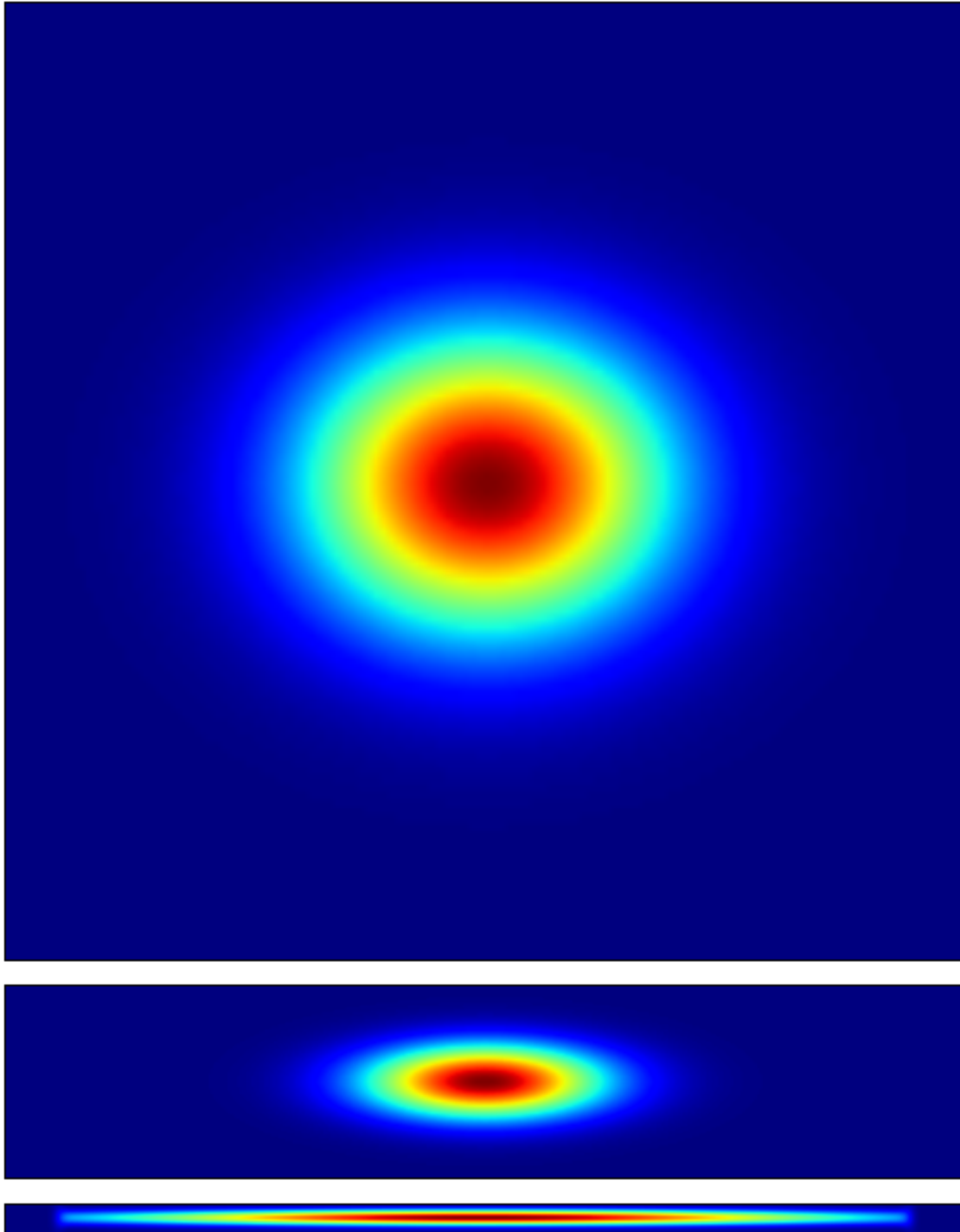


With $N = 2000$ and compare to $N = 1$



(d)

(Transposed and not at the same scale)



(e)