Yichao Yu

1.

(a)

(b)

(c)

2.

$$
\begin{aligned}
F_c =& -\frac{\hbar c\pi^2}{240a^2}\\
F_e =& \frac{e^2}{16\pi\varepsilon_0 a^2}\\
\frac{\hbar c\pi^2}{240} =& \frac{e^2}{16\pi\varepsilon_0}\\
\alpha =& \frac{e^2}{4\pi\varepsilon_0\hbar c}\\
=& \frac{\pi^2}{60}
\end{aligned}
$$

3.

(a)

$$
\begin{aligned}
\rho(\theta) =& \begin{pmatrix} a & b\mathrm{e}^{\mathrm{i}\theta}\\ c\mathrm{e}^{-\mathrm{i}\theta} & d \end{pmatrix}\\
\langle\rho\rangle =& \int_{-\infty}^{\infty}\mathrm{d}\theta\frac{1}{\sqrt{4\pi\lambda t}}\begin{pmatrix} a & b\mathrm{e}^{\mathrm{i}\theta}\\ c\mathrm{e}^{-\mathrm{i}\theta} & d \end{pmatrix}\exp\left(-\frac{\theta^2}{2\lambda t}\right)\\
=& \begin{pmatrix} a & \frac{b}{\sqrt{4\pi\lambda t}}\int_{-\infty}^{\infty}\mathrm{d}\theta\exp\left(\mathrm{i}\theta-\frac{\theta^2}{2\lambda t}\right)\\ \frac{c}{\sqrt{4\pi\lambda t}}\int_{-\infty}^{\infty}\mathrm{d}\theta\exp\left(-\mathrm{i}\theta-\frac{\theta^2}{2\lambda t}\right) & d \end{pmatrix}\\
=& \begin{pmatrix} a & \frac{b}{\sqrt{4\pi\lambda t}}\int_{-\infty}^{\infty}\mathrm{d}\theta\exp\left(-\frac{(\theta-\mathrm{i}\lambda t)^2+(\lambda t)^2}{2\lambda t}\right)\\ \frac{c}{\sqrt{4\pi\lambda t}}\int_{-\infty}^{\infty}\mathrm{d}\theta\exp\left(-\frac{(\theta+\mathrm{i}\lambda t)^2+(\lambda t)^2}{2\lambda t}\right) & d \end{pmatrix}\\
=& \begin{pmatrix} a & b\exp\left(-\frac{\lambda t}{2}\right)\\ c\exp\left(-\frac{\lambda t}{2}\right) & d \end{pmatrix}
\end{aligned}
$$

(b)

Since the Hamiltonian does nothing to $|g\rangle$, $|g0\rangle$ will remain the same. For $|e*\rangle$, the environment will undergo Rabi flopping. Also assume γ is real since the phase of it can be absorbed in $|1\rangle$. The evolution of state,

$$|\psi(t)\rangle = a|g0\rangle + b(\cos\gamma t|e0\rangle + \sin\gamma t|e1\rangle)$$

Atomic density matrix

$$\rho = \begin{pmatrix} |a|^2 & ab^*\cos\gamma t \\ a^*b\cos\gamma t & |b|^2 \end{pmatrix}$$

(c)

At time t the average density matrix is

$$\begin{aligned}\langle\rho\rangle =& \frac{1+\mathrm{e}^{-\lambda t}}{2}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \frac{1-\mathrm{e}^{-\lambda t}}{2}\begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \\ =& \begin{pmatrix} a & b\mathrm{e}^{-\lambda t} \\ c\mathrm{e}^{-\lambda t} & d \end{pmatrix}\end{aligned}$$