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8.422 Problem Set 9
Recitation Section 1

1. A Zeeman Slower

(a)

The maximum deceleration is achieved when the scattering rate is maximized. Since the maximum population when pumping with a laser in such a two level system is 50% the maximum scattering rate is $\frac{\gamma}{2}$. Maximum deceleration

$$\begin{aligned}a_{max} =&\frac{\gamma}{2}\frac{p_{rec}}{m}\\ =&\frac{h\gamma}{2m\lambda}\\ =&9.3\cdot 10^5\mathrm{m}\cdot\mathrm{s}^{-2}\end{aligned}$$

(b)

Assuming the atom flux (and therefore the optical depth of the slower) is small so that the intensity of light is almost constant in the slower.
Length of the slower,

$$\begin{aligned}L =&\frac{v_{max}^2}{2a}\\ =&\frac{v_{max}^2}{2fa_{max}}\\ =&\frac{1}{f}\frac{k_BT\lambda}{h\gamma}\\ =&\frac{0.12}{f}\mathrm{m}\end{aligned}$$

Maximum velocity in the slower

$$v =\sqrt{2fa_{max}(L-x)}$$

Doppler shift

$$\begin{aligned}\delta_{doppler} =&\frac{v}{\lambda}\\ =&\frac{\sqrt{2fa_{max}(L-x)}}{\lambda}\end{aligned}$$

This should be canceled by the Zeeman shift

$$B =\frac{\sqrt{2fa_{max}(L-x)}}{g\mu_B\lambda}$$

(c)

Variance of Δx for each emission for three situations

$$\begin{aligned}\left\langle\Delta p^2\right\rangle_0 =& p_{rec}^2\begin{cases}1 & \text{i}\\ \displaystyle\int_{-1}^1 x^2\mathrm{d}x & \text{ii}\\ \displaystyle\int_{-1}^1 x^4\mathrm{d}x & \text{iii}\end{cases}\\ =& p_{rec}^2\begin{cases}1 & \text{i}\\ \frac{1}{3} & \text{ii}\\ \frac{1}{5} & \text{iii}\end{cases}\end{aligned}$$

$$\begin{aligned}\mathcal{D} =& \frac{\mathrm{d}\left\langle\Delta p^2\right\rangle}{\mathrm{d}t}\\ =& \left\langle\Delta p^2\right\rangle_0\Gamma_s\\ =& \Gamma_s p_{rec}^2\begin{cases}1 & \text{i}\\ \frac{1}{3} & \text{ii}\\ \frac{1}{5} & \text{iii}\end{cases}\end{aligned}$$

2. Slowing an atom with off-resonant light

(a)

Scattering rate

$$\begin{aligned}\Gamma_s =& \frac{\gamma}{2}\frac{s}{1+s+\left(\frac{2\delta}{\gamma}\right)^2}\\ =& \frac{\gamma}{2}\frac{s}{1+s+\left(\frac{2v}{\lambda\gamma}\right)^2}\\ \frac{\mathrm{d}v}{\mathrm{d}t} =& -\frac{p_{rec}}{m}\Gamma_s\\ =& -a_{max}\frac{s}{1+s+\left(\frac{2v}{\lambda\gamma}\right)^2}\end{aligned}$$

Total time

$$\begin{aligned}T =& \int_0^{v_{max}}\frac{1}{sa_{max}}\left(1+s+\left(\frac{2v}{\lambda\gamma}\right)^2\right)\mathrm{d}v\\ =& \frac{v_{max}}{sa_{max}}\left(1+s+\frac{4v_{max}^2}{3\lambda^2\gamma^2}\right)\\ =& 43.5\frac{v_{max}}{a_{max}}\\ =& 21.9\mathrm{ms}\end{aligned}$$

(b)

Let t be the time until the atom stops

$$\begin{aligned}
t =& \frac{v}{sa_{max}}\left(1 + s + \frac{4v^2}{3\lambda^2\gamma^2}\right)\\
\approx&\\
v \approx& \sqrt[3]{\frac{3sa_{max}\lambda^2\gamma^2}{4}}t^{1/3}\\
l \approx& \sqrt[3]{\frac{3sa_{max}\lambda^2\gamma^2}{4}}\int_0^T t^{1/3}\mathrm{d}t\\
=& \sqrt[3]{\frac{3sa_{max}\lambda^2\gamma^2}{4}}\frac{3}{4}T^{4/3}\\
\approx& 7.8\mathrm{m}
\end{aligned}$$

(c)

This is much longer and less efficient than a Zeeman slower.

3. Density Limit in a MOT

(a)

The radiation from a single atom is

$$S_{single} = \frac{6\sigma_L I}{4\pi r^2}\hat{r}$$

Force

$$F_{single} = \frac{6\sigma_L\sigma_R I}{4\pi r^2 c}\hat{r}$$

From Gauss' law

$$\nabla\cdot F_R = \frac{6\sigma_L\sigma_R I n}{c}$$

(b)

For a isotropic light field, the light force is proportional to the gradient of light intensity. For the case of scattering force, each atom is acting as a source for the gradient while for attenuation force, each atom is acting as a drain for the gradient (thus the minus sign). Furthermore, the frequency of the light is the trap light so the absorption cross section should be replaced by σ_L.

(c)

$$\begin{aligned}
\nabla\cdot\vec{F} =& \frac{6\sigma_L(\sigma_R - \sigma_L)In}{c} - 3\kappa\\
n_{max} =& \frac{3\kappa c}{6\sigma_L(\sigma_R - \sigma_L)I}
\end{aligned}$$

(d)

On average $\sigma_R > \sigma_L$ due to the resonance component of the scattered light (Mollow triplet).

(e)

This effectively replaces n by $fn < n$ in F_R and F_A so the both F_R and F_A decreases (with same n) while F_T remains the same (and might increase a little due to the lack of attenuation from the atoms). n_{max} will increase by f^{-1}.