1.

(a)

$$
\begin{aligned}
\langle\alpha|\beta\rangle =& \exp\left(-\frac{|\alpha|^2+|\beta|^2}{2}\right)\sum_n\frac{(\alpha^*)^n\beta^n}{n!}\\
=& \exp\left(-\frac{|\alpha|^2+|\beta|^2}{2}+\alpha^*\beta\right)
\end{aligned}
$$

(b)

$$
\begin{aligned}
\langle n\int|\alpha\rangle\langle\alpha|\frac{\mathrm{d}^2\alpha}{\pi}|m\rangle =& \mathrm{e}^{-|\alpha|^2}\int\frac{|\alpha|^{2n}\alpha^{m-n}}{\sqrt{n!m!}}|\alpha|\frac{\mathrm{d}|\alpha|\mathrm{d}\theta}{\pi}\\
=& \delta_{mn}\mathrm{e}^{-|\alpha|^2}\int\frac{|\alpha|^{2n}}{n!}\mathrm{d}|\alpha|^2\\
=& \delta_{mn}
\end{aligned}
$$

(c)

$$
\begin{aligned}
D(\alpha)|0\rangle =& \exp\left(\alpha a^\dagger-\alpha^*a\right)|0\rangle\\
=& \mathrm{e}^{\alpha a^\dagger}\mathrm{e}^{-\alpha^*a}\mathrm{e}^{\alpha\alpha^*[a^\dagger,a]/2}|0\rangle\\
=& \exp\left(-\frac{|\alpha|^2}{2}\right)\mathrm{e}^{\alpha a^\dagger}|0\rangle\\
=& |\alpha\rangle
\end{aligned}
$$

(d)

$$
\begin{aligned}
\langle E_x\rangle =& E_0\sin(kz)\langle a+a^\dagger\rangle\\
=& E_0\sin(kz)(\alpha+\alpha^*)\\
\left\langle E_x^2\right\rangle =& E_0^2\sin^2(kz)\left\langle\left(a+a^\dagger\right)^2\right\rangle\\
=& E_0^2\sin^2(kz)\left\langle a^2+1+2a^\dagger a+{a^\dagger}^2\right\rangle\\
=& E_0^2\sin^2(kz)\left((\alpha+\alpha^*)^2+1\right)\\
\sqrt{\langle\Delta E_x^2\rangle} =& E_0\sin(kz)
\end{aligned}
$$

The deviation of the field does not change (even for vacuum) because the coherent state is just a displacement of the vacuum state.

(e)

For number state

$$P(\phi) = \frac{1}{2\pi}$$

so the number state has equal probability of having any phase.

$$\begin{aligned}P(\phi,\theta) =& \frac{1}{2\pi}\left|\sum_n \left\langle n|\mathrm{e}^{-\mathrm{i}n\phi}|\left(|\alpha|\mathrm{e}^{\mathrm{i}\theta}\right)\right\rangle\right|^2\\ =& \frac{1}{2\pi}\left|\sum_n \mathrm{e}^{-\mathrm{i}n(\phi-\theta)}\langle n|(|\alpha|)\rangle\right|^2\\ =& P(\phi-\theta,0)\end{aligned}$$

Therefore P rotates in the same way as the phase of α changes

$$\begin{aligned}P(\phi,0) =& \frac{1}{2\pi}\left|\sum_n \left\langle n|\mathrm{e}^{-\mathrm{i}n\phi}|(|\alpha|)\right\rangle\right|^2\\ =& \frac{\mathrm{e}^{-|\alpha|^2}}{2\pi}\left|\sum_n \mathrm{e}^{-\mathrm{i}n\phi}\frac{|\alpha|^n}{\sqrt{n!}}\right|^2\end{aligned}$$

So $P(\phi,0)$ is symmetrically distributed and maximized at $\phi = 0$ (therefore $P(\phi,\theta)$ is maximized when $\phi = \theta$) if $|\alpha| \neq 0$. Therefore the expected value of ϕ (when $\alpha \neq 0$) is θ.

2.

(a)

$$\begin{aligned}\frac{V_\psi}{V_0} =& \frac{1}{4}\langle\psi,0|\left(a^\dagger + b^\dagger\right)(a+b)\left(a^\dagger - b^\dagger\right)(a-b)|\psi,0\rangle\\ =& \frac{1}{4}\langle\psi,0|a^\dagger\left(aa^\dagger - ab^\dagger + ba^\dagger - bb^\dagger\right)a|\psi,0\rangle\\ =& \frac{1}{4}\langle a^\dagger a^\dagger aa\rangle\end{aligned}$$

(b)

$$\begin{aligned}g_{cl}^{(2)}(0) =& \frac{\left\langle I^2\right\rangle}{\langle I\rangle^2}\\ =& 1 + \frac{\left\langle I^2\right\rangle - \langle I\rangle^2}{\langle I\rangle^2}\\ =& 1 + \frac{\left\langle (I - \langle I\rangle)^2\right\rangle}{\langle I\rangle^2}\\ \geqslant& 1\end{aligned}$$

(c)

$$
\begin{aligned}
g_\alpha^{(2)} =& \frac{|a|^4}{|a|^4}\\
=&1\\
g_{n=2}^{(2)} =& \frac{2\cdot 3}{2^2}\\
=&\frac{3}{2}
\end{aligned}
$$

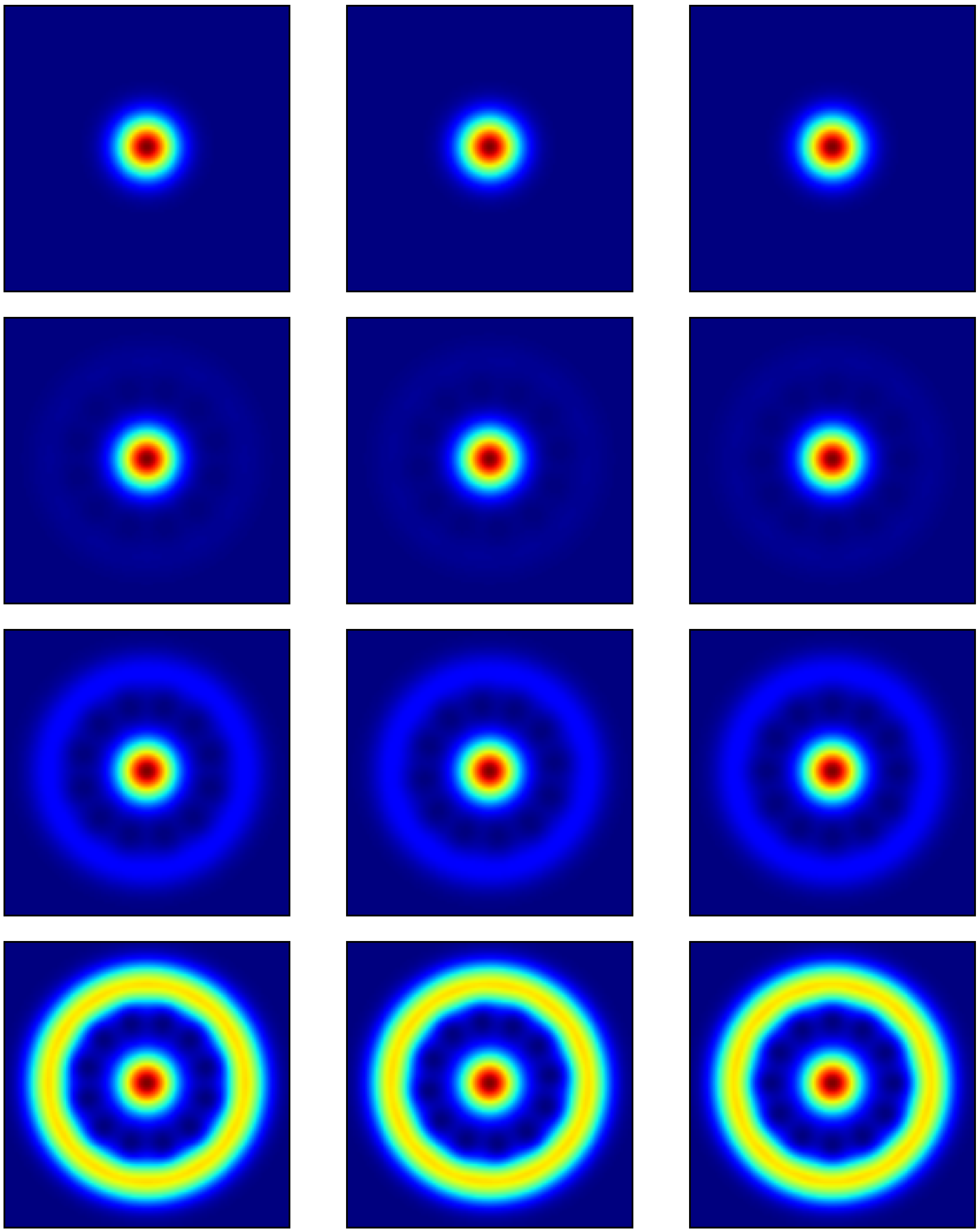
(d)

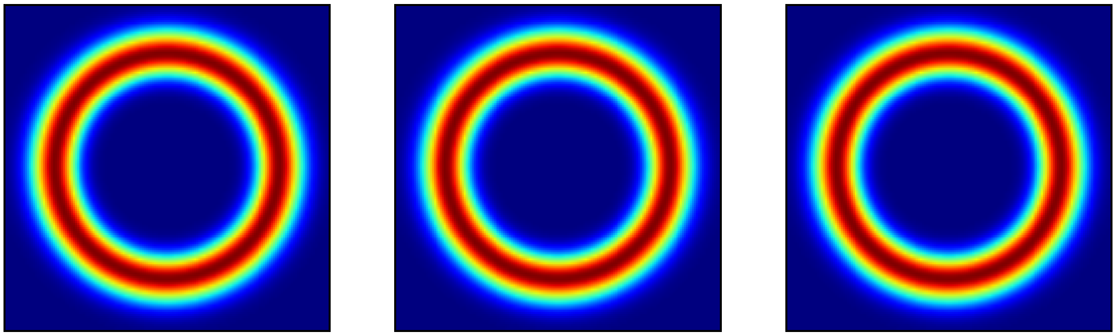
$$
\begin{aligned}
\langle\psi_3|\psi_3\rangle =&\left(1+\mathrm{e}^{2|\alpha|^2}\right)\\
\langle\psi_4|\psi_4\rangle =&\left(1-\mathrm{e}^{2|\alpha|^2}\right)\\
g_{3,4}^{(2)} =& \frac{\langle\psi_{3,4}|a^\dagger a^\dagger aa|\psi_{3,4}\rangle\langle\psi_{3,4}|\psi_{3,4}\rangle}{\langle\psi_{3,4}|a^\dagger a|\psi_{3,4}\rangle^2}\\
=&\frac{(\langle\alpha|\pm\langle-\alpha|)a^\dagger a^\dagger aa(|\alpha\rangle\pm|-\alpha\rangle)\left(1\pm\mathrm{e}^{2|\alpha|^2}\right)}{\left((\langle\alpha|\pm\langle-\alpha|)a^\dagger a(|\alpha\rangle\pm|-\alpha\rangle)\right)^2}\\
=&\frac{|\alpha|^4(\langle\alpha|\pm\langle-\alpha|)(|\alpha\rangle\pm|-\alpha\rangle)\left(1\pm\mathrm{e}^{2|\alpha|^2}\right)}{|\alpha|^4\left((\langle\alpha|\mp\langle-\alpha|)(|\alpha\rangle\mp|-\alpha\rangle)\right)^2}\\
=&\left(\frac{1\pm\mathrm{e}^{2|\alpha|^2}}{1\mp\mathrm{e}^{2|\alpha|^2}}\right)^2
\end{aligned}
$$

Therefore $g^{(2)}$ can be smaller than 1 for ψ_4.

3.

(a)

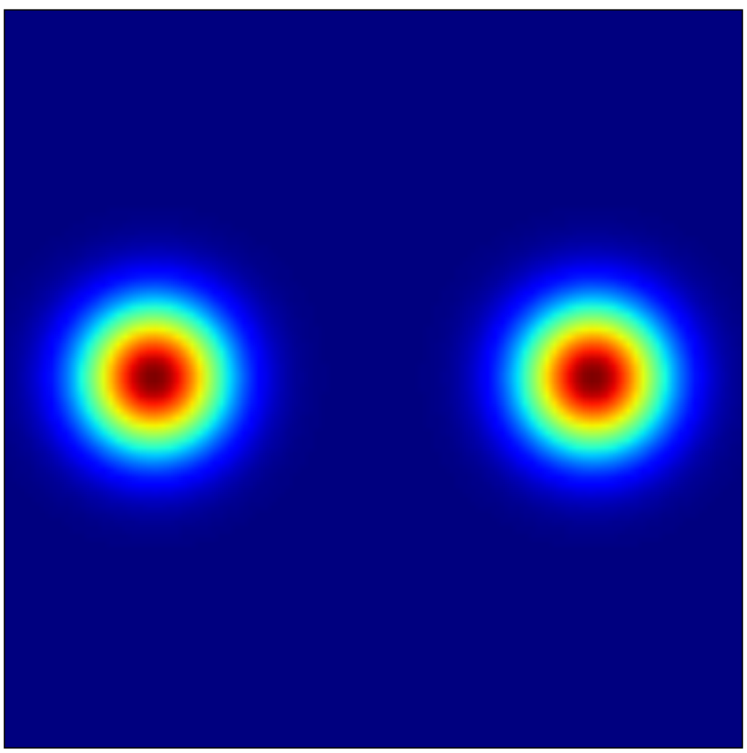




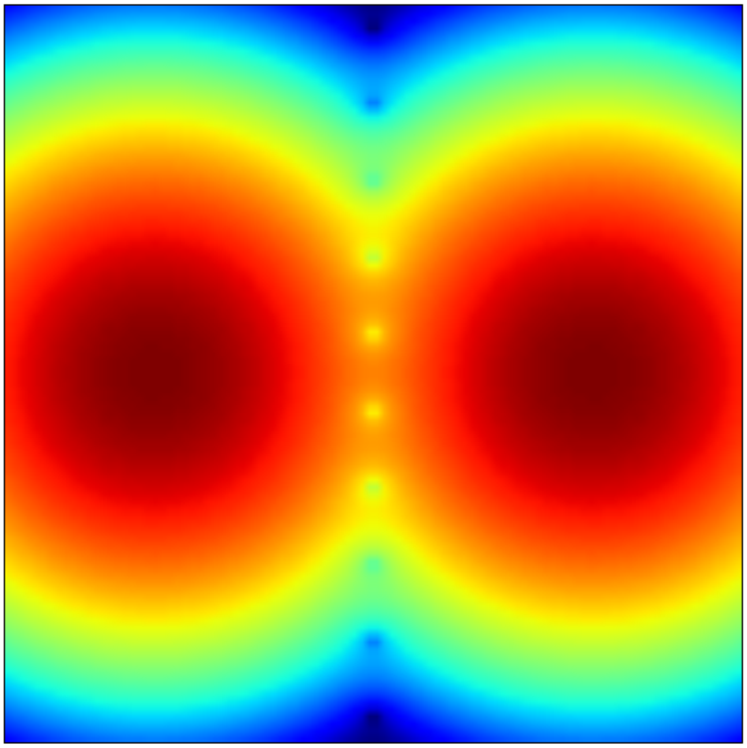
These are not minimum uncertainty states (except when it's $|0\rangle$) and not squeezed states.

(b)

$\alpha = 3\mathrm{i}$ Linear scale



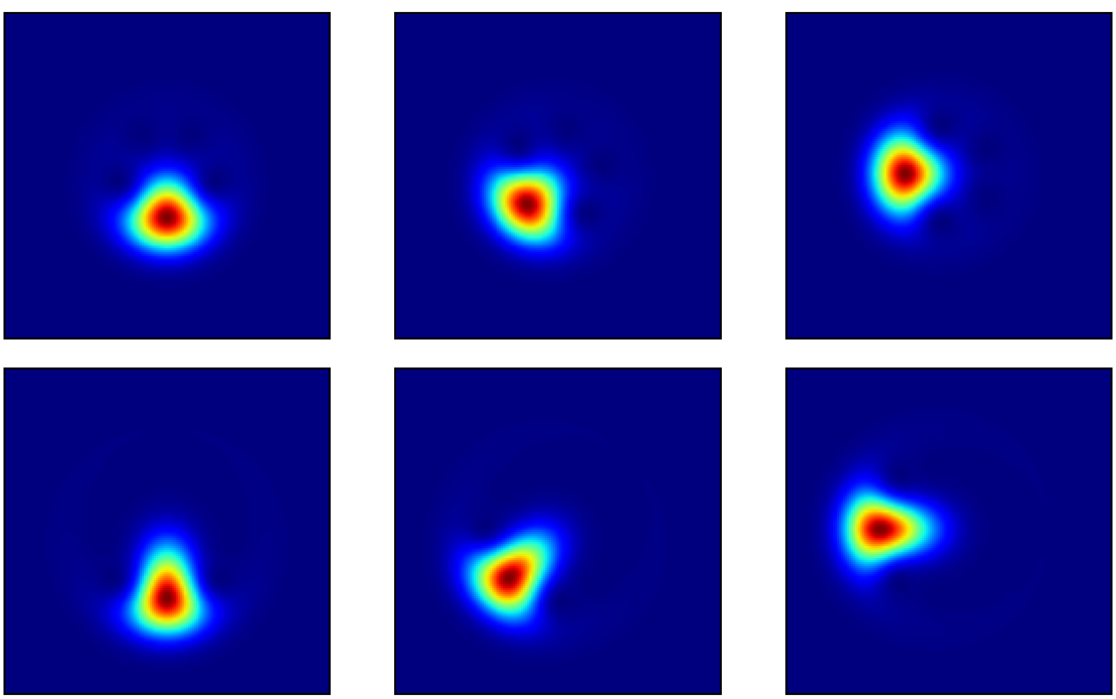
Log scale

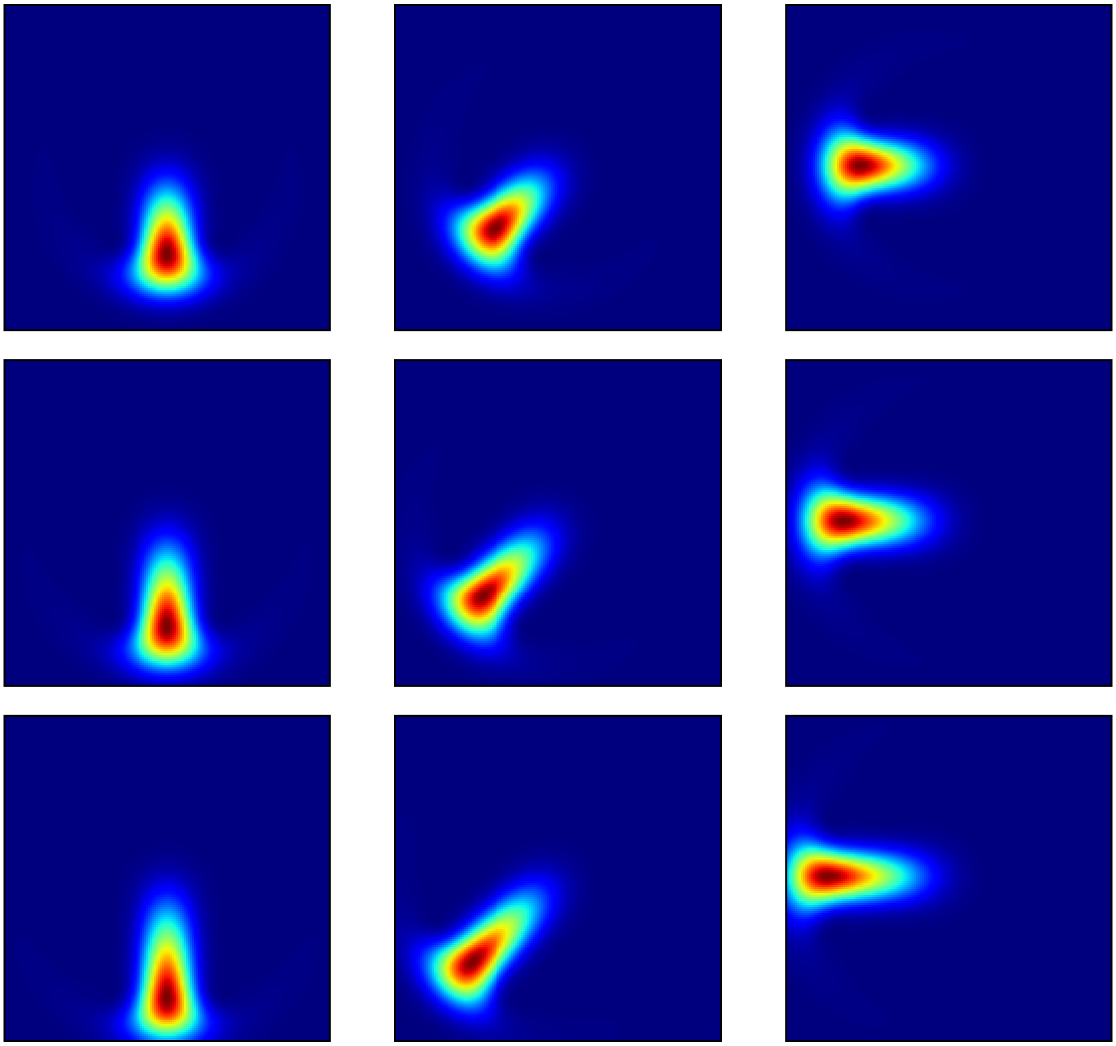


Interference fringes appears because the two wavefunction overlaps.

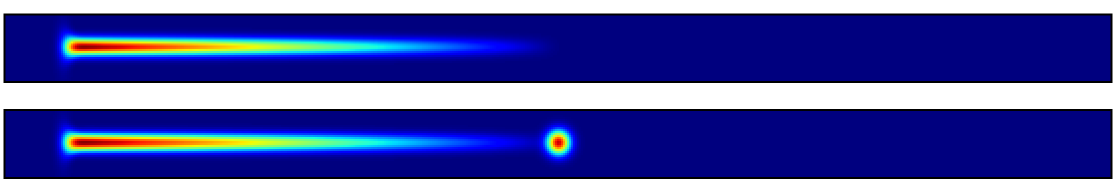
(c)

States with $\phi = 0, \frac{\pi}{4}, \frac{\pi}{2}$ and $N = 5, 10, 15, 20, 15$





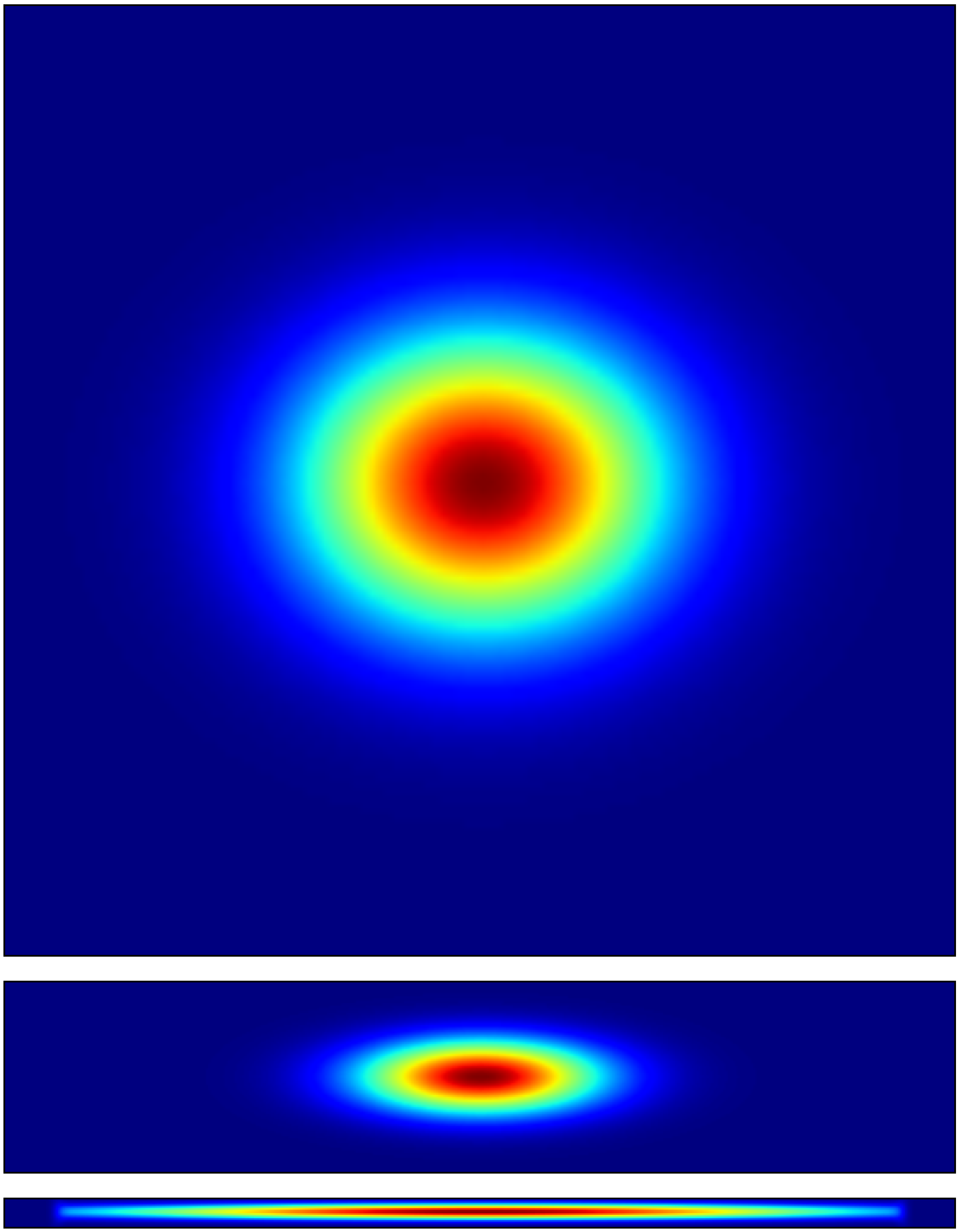
With $N = 2000$ and compare to $N = 1$



These are not squeezed states since they do not decrease the uncertainty in one direction.

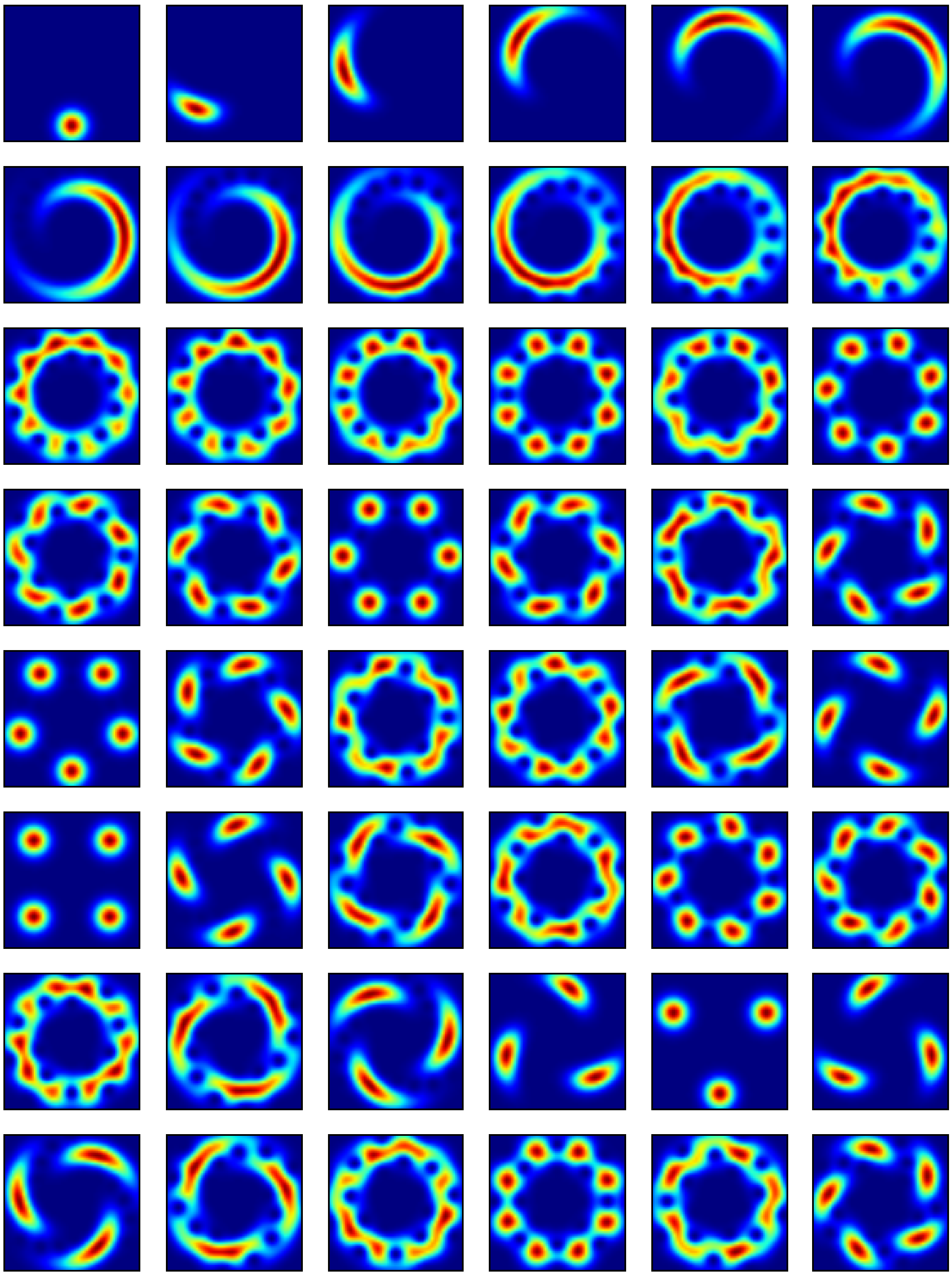
(d)

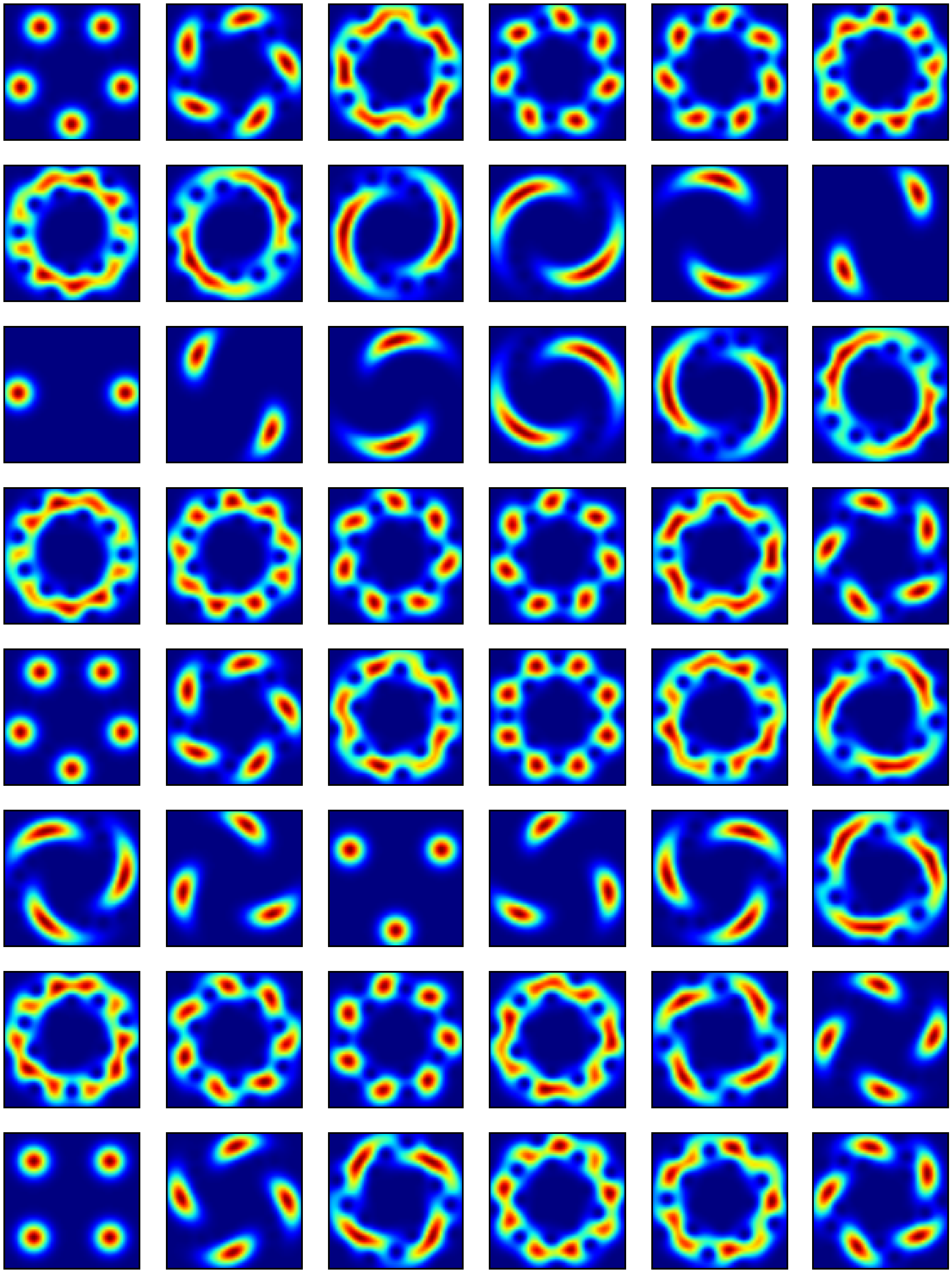
(Transposed and not at the same scale)

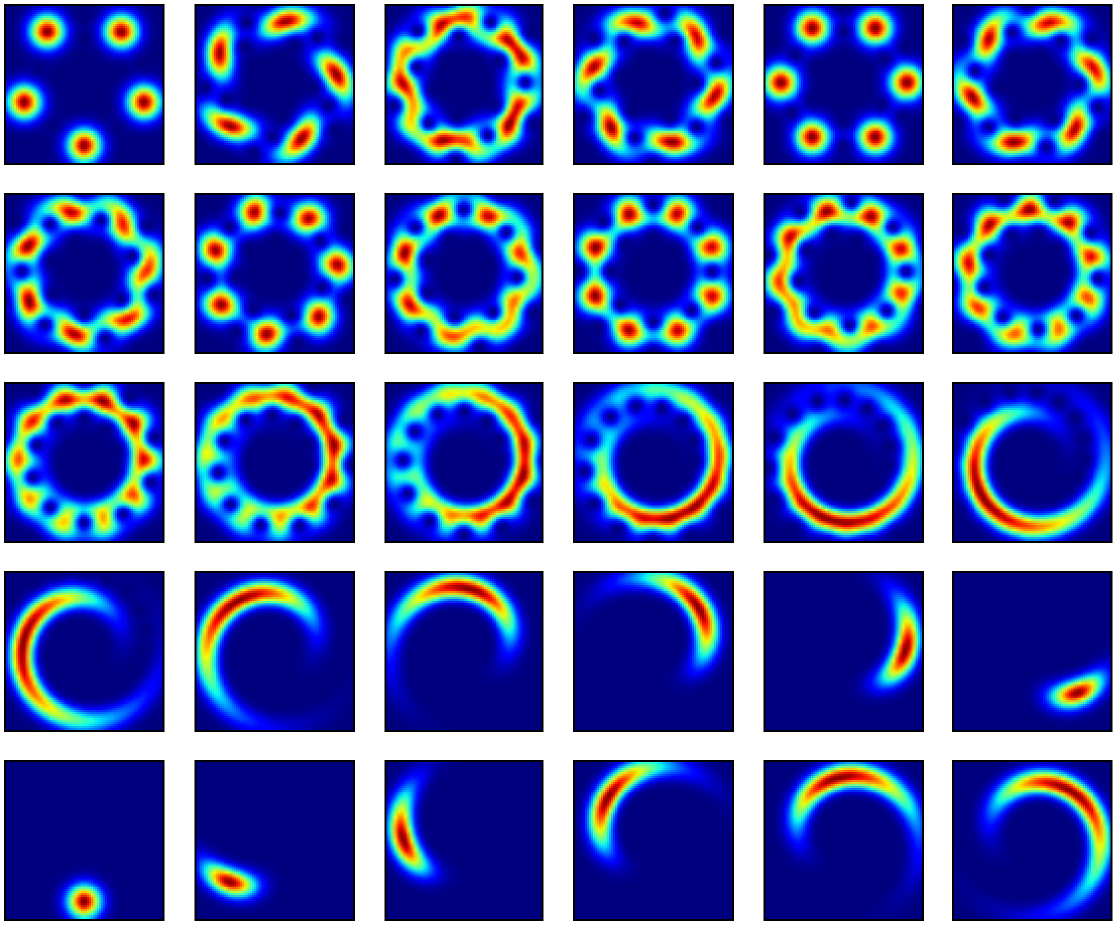


(e)

ξt from 0 to $\dfrac{25}{24}$ every $\dfrac{1}{120}$







It become two superposed coherent state when $\xi t = \frac{1}{2}$ and come back to its original state when $\xi t = 1$. For a video of the evolution, see "http://git.yuyichao.com/yuyichao/8-422/raw/master/pset/pset1/3-5.webm".