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8.422 Problem Set 9
Recitation Section 1

1. A Zeeman Slower

(a)

The maximum deceleration is achieved when the scattering rate is maximized. Since the maximum population when pumping with a laser in such a two level system is 50% the maximum scattering rate is $\dfrac{\gamma}{2}$. Maximum deceleration

$$\begin{aligned}a_{max} =&\frac{\gamma}{2}\frac{p_{rec}}{m}\\=&\frac{h\gamma}{2m\lambda}\\=&9.3\cdot 10^5\mathrm{m}\cdot\mathrm{s}^{-2}\end{aligned}$$

(b)

Assuming the atom flux (and therefore the optical depth of the slower) is small so that the intensity of light is almost constant in the slower.
Length of the slower,

$$\begin{aligned}L =&\frac{v_{max}^2}{2a}\\=&\frac{v_{max}^2}{2fa_{max}}\\=&\frac{1}{f}\frac{k_BT\lambda}{h\gamma}\\=&\frac{0.12}{f}\mathrm{m}\end{aligned}$$

Maximum velocity in the slower

$$v =\sqrt{2fa_{max}(L-x)}$$

Doppler shift

$$\begin{aligned}\delta_{doppler} =&\frac{v}{\lambda}\\=&\frac{\sqrt{2fa_{max}(L-x)}}{\lambda}\end{aligned}$$

This should be canceled by the Zeeman shift

$$B =\frac{\sqrt{2fa_{max}(L-x)}}{g\mu_B\lambda}$$

(c)

Variance of Δx for each emission for three situations

$$\begin{aligned}\left\langle\Delta p^2\right\rangle_0 =&p_{rec}^2\begin{cases}1 & \text{i}\\ \displaystyle\int_{-1}^1 x^2\mathrm{d}x & \text{ii}\\ \displaystyle\int_{-1}^1 x^4\mathrm{d}x & \text{iii}\end{cases}\\ =&p_{rec}^2\begin{cases}1 & \text{i}\\ \frac{1}{3} & \text{ii}\\ \frac{1}{5} & \text{iii}\end{cases}\end{aligned}$$

$$\begin{aligned}\mathcal{D} =&\frac{\mathrm{d}\left\langle\Delta p^2\right\rangle}{\mathrm{d}t}\\ =&\left\langle\Delta p^2\right\rangle_0\Gamma_s\\ =&\Gamma_s p_{rec}^2\begin{cases}1 & \text{i}\\ \frac{1}{3} & \text{ii}\\ \frac{1}{5} & \text{iii}\end{cases}\end{aligned}$$

2. Slowing an atom with off-resonant light

(a)

Scattering rate

$$\begin{aligned}\Gamma_s =&\frac{\gamma}{2}\frac{s}{1+s+\left(\frac{2\delta}{\gamma}\right)^2}\\ =&\frac{\gamma}{2}\frac{s}{1+s+\left(\frac{2v}{\lambda\gamma}\right)^2}\\ \frac{\mathrm{d}v}{\mathrm{d}t} =&-\frac{p_{rec}}{m}\Gamma_s\\ =&-a_{max}\frac{s}{1+s+\left(\frac{2v}{\lambda\gamma}\right)^2}\end{aligned}$$

Total time

$$\begin{aligned}T =&\int_0^{v_{max}}\frac{1}{sa_{max}}\left(1+s+\left(\frac{2v}{\lambda\gamma}\right)^2\right)\mathrm{d}v\\ =&\frac{v_{max}}{sa_{max}}\left(1+s+\frac{4v_{max}^2}{3\lambda^2\gamma^2}\right)\\ =&43.5\frac{v_{max}}{a_{max}}\\ =&21.9\text{ms}\end{aligned}$$

(b)

Let t be the time until the atom stops

$$\begin{aligned}
t =& \frac{v}{sa_{max}}\left(1 + s + \frac{4v^2}{3\lambda^2\gamma^2}\right) \\
\approx& \\
v \approx& \sqrt[3]{\frac{3sa_{max}\lambda^2\gamma^2}{4}} t^{1/3} \\
l \approx& \sqrt[3]{\frac{3sa_{max}\lambda^2\gamma^2}{4}} \int_0^T t^{1/3}\mathrm{d}t \\
=& \sqrt[3]{\frac{3sa_{max}\lambda^2\gamma^2}{4}} \frac{3}{4} T^{4/3} \\
\approx& 7.8\mathrm{m}
\end{aligned}$$

(c)

This is much longer and less efficient than a Zeeman slower.

3. Density Limit in a MOT

(a)

(b)

(c)

(d)

(e)