Assignment #6

Due: Wednesday, April 8, 2015
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Office Hours: Friday, April 3, 9:30-11:00 am
and Monday April 6, 3:00-4:00 pm, or by appointment

Problem 1. Long-range interaction between an excited atom and a ground-state atom

Consider the case where one atom is excited and the other atom is in its ground state. For simplicity model each atom as a two level system with one ground state and one excited state.

a) Assume you have two atoms a and b with almost (but not quite) degenerate ground $\leftrightarrow$ excited state transition energies $(E_i^{(a)} - E_g^{(a)}) \approx (E_i^{(b)} - E_g^{(b)})$. How does the energy of the state $|i_a g_b\rangle$ change as a function of the separation R for large distances? What about state $|g_a i_b\rangle$? For what separation does perturbation theory become invalid?

b) Now assume you have two identical (i.e. same transition energy) atoms. Calculate the long-range interaction potential curves for the case of one excited atom and one ground state atom.

c) For case (b) what is the relation between the spontaneous decay rate of the atom and its long-range interaction coefficient?

Problem 2. Casimir model of the electron

Model the electron as two parallel plates of area a^2, separated by distance a and carrying charge $q = \frac{e}{2}$. Balance the Casimir and electrostatic forces and from this determine a value for the fine-structure constant $\alpha \equiv \frac{e^2}{\hbar c}$ (cgs units).

Problem 3. Physical origin of T_2

We will soon discuss a general derivation of optical Bloch equations which include damping of elements of the atomic density matrix. In this problem, in preparation for the general discussion, you will look at different processes which lead to such damping.

What is the physical meaning of the decay of the off-diagonal elements of the atomic density matrix, ρ_{eg} and ρ_{ge}, due to spontaneous emission and other processes? Three very simple models for this kind of quantum noise, which leads to loss of *phase coherence*, and gives rise to decoherence times characterized by T_2, are the following.

a) One physical origin for the decay of the off-diagonal elements of the atomic density matrix (this decay is known as "phase damping") is random phase noise. Suppose that we have an atom in the arbitrary state

$$\rho = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \tag{1}$$

excited by far off-resonance light of random intensity. The effect of this light on the state is an AC stark shift $|e\rangle \to e^{i\theta}|e\rangle$, which we may model as a rotation $R_z(\theta)$, where the angle of rotation θ is random, distributed as a Gaussian with mean 0 and variance $2\lambda t$. Give $\rho(\theta) = R_z(\theta)\rho R_z^\dagger(\theta)$, and the expected density matrix $\bar{\rho}(t) = \langle \rho(\theta) \rangle$, averaged over this Gaussian distribution.

b) Another physical origin for phase damping is elastic collisions. Assume the two-level atom bounces along a waveguide, interacting with the walls without loosing kinetic energy, but changing its trajectory slightly at each bounce, in a manner depending on the state of the atom. This can be modeled by a Hamiltonian interacting the atom with a single mode environment,

$$H_{SE} = |e\rangle\langle e| \otimes \Big[\gamma |0\rangle\langle 1| + \gamma^* |1\rangle\langle 0| \Big] \,, \tag{2}$$

with coupling constant γ. This Hamiltonian flips the state of the environment between $|0\rangle$ and $|1\rangle$ with a rate gamma, when the atom is in the excited state. This is a simple model for a state-dependent interaction with the environment. Compute the evolution of an initial atomic state $a|g\rangle + b|e\rangle$ coupled to an environment $|0\rangle$, evolved for a small differential timestep dt, and give the density matrix of the atom $\rho(t)$ for small t.

c) A third physical model for phase damping is the following scenario (which you may think a bit unphysical). Suppose the two-level atom is subject to a force that randomly flips the phase of the atom by -1, changing $|e\rangle \to -|e\rangle$, with probability $(1 - e^{-\lambda t})/2$ as a function of time. Assuming the density matrix is initially arbitrary, give $\rho(t)$ averaged over this random phase flip process.

An amazing fact about these three models is that if phase damping is known to be happening to an atom, say during traversal through some black box, without additional knowledge about what is inside the black box, there is *in principle, no way to distinguish* between which of the above physical processes is causing the phase damping! They are perfectly equivalent and equally legitimate. This is fundamental to why quantum error correction can work: phase noise can be modeled as either no error happens to the state, or a phase flip "error" occurs (analogous to bit errors in a communication channel), even if the actual physical process is a different one.