Confinement induced resonance

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1. INTRODUCTION

2. FESHBACH RESONANCE

2.1. T-Matrix and scatter length

$$\Psi(\vec{r}) = \mathrm{e}^{\mathrm{i}\vec{k}_0\cdot\vec{r}} - \Psi_s \approx \qquad \mathrm{e}^{\mathrm{i}\vec{k}_0\cdot\vec{r}} - \frac{a}{r}$$

$$\frac{1}{m}\left(k^2 - k'^2\right)\Psi_s\left(\vec{k}'\right) = U\left(\vec{k}', \vec{k}\right) + \int \frac{\mathrm{d}^3 k''}{(2\pi)^3} U\left(\vec{k}', \vec{k}''\right)\Psi_s\left(\vec{k}''\right)$$

Define T matrix

$$\begin{aligned}
T\left(\vec{k}', \vec{k}\right) =& U\left(\vec{k}', \vec{k}\right) + \int \frac{\mathrm{d}^3 k''}{(2\pi)^3} \frac{U\left(\vec{k}', \vec{k}''\right)}{k^2/m - k''^2/m + \mathrm{i}0} T\left(\vec{k}'', \vec{k}\right) \\
\Psi_s\left(\vec{k}'\right) =& \frac{m}{k^2 - k'^2 + \mathrm{i}0} T\left(\vec{k}', \vec{k}, \frac{k^2}{m}\right) \\
f\left(\vec{k}', \vec{k}\right) =& -\frac{m}{4\pi\hbar^2} T\left(\vec{k}', \vec{k}, \frac{k^2}{m}\right)
\end{aligned}$$

$$\tilde{U}\left(\vec{k}', \vec{k}\right) = U\left(\vec{k}', \vec{k}\right) + \int_{k''^2/m > \varepsilon_c} \frac{\mathrm{d}^3 k''}{(2\pi)^3} \frac{\tilde{U}\left(\vec{k}', \vec{k}''\right)}{k^2/m - k''^2/m + \mathrm{i}0} \tilde{U}\left(\vec{k}'', \vec{k}\right)$$

$$T\left(\vec{k}', \vec{k}\right) = \tilde{U}\left(\vec{k}', \vec{k}\right) + \int_{k''^2/m < \varepsilon_c} \frac{\mathrm{d}^3 k'}{(2\pi)^3} \frac{\tilde{U}\left(\vec{k}', \vec{k}''\right)}{k^2/m - k''^2/m + \mathrm{i}0} T\left(\vec{k}'', \vec{k}\right)$$

$$\begin{aligned}
\frac{1}{f(k)} =& -\frac{4\pi}{mU_0} + 4\pi \int_{|q|<1/R} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{k^2 - q^2 + \mathrm{i}0} \\
=& \frac{1}{2\pi^2}\left(-\frac{1}{R} - \frac{k}{2}\ln\left(-\frac{R^{-1} - k - \mathrm{i}0}{R^{-1} + k + \mathrm{i}0}\right)\right) \\
\approx& \frac{1}{2\pi^2}\left(-\frac{1}{R} - \frac{\mathrm{i}\pi k}{2} + k^2 R\right)
\end{aligned}$$

$$\begin{aligned}
a =& \frac{\pi}{2}\frac{R}{1 + \frac{2\pi^2 R}{mU_0}} \\
f(k) =& \frac{1}{a^{-1} + r_{eff}k^2/2 - \mathrm{i}k}
\end{aligned}$$

2.2. Multichannel scattering and Feshbach resonance

3. CONFINEMENT INDUCED RESONANCE

4. CONCLUSION

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