1.

2.

(a)

Since $J_z = L_z + S_z$ and $[L_z, S_z] = 0$, the J_z eigenstates are sum of L_z and S_z eigenstates that has the same sum of m_l and m_s. (The only variable on the RHS are m_l and m_s among which m_s can only be $\pm\frac{1}{2}$ so the constraint above limit the decomposition to the given form.)

(b)

3.

4.

5.

(a)

Eigenvalue λ

$$\begin{aligned}
0 =&(h-\lambda)^2 - |g|^2\\
h-\lambda =&\pm|g|\\
\lambda =&h\pm|g|
\end{aligned}$$

Corresponding eigen vectors are $\frac{1}{\sqrt{2}}\left(1, \pm\frac{g}{|g|}\right)$

(b)

Initial state

$$|\psi_0\rangle =\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

At time t,

$$\begin{aligned}
|\psi_t\rangle =&\frac{1}{\sqrt{2}}\left(\exp\left(-\mathrm{i}\frac{h+|g|}{\hbar}t\right)|+\rangle + \exp\left(-\mathrm{i}\frac{h-|g|}{\hbar}t\right)|-\rangle\right)\\
=&\frac{\mathrm{e}^{-\mathrm{i}ht/\hbar}}{\sqrt{2}}\left(\mathrm{e}^{-\mathrm{i}|g|t/\hbar}|+\rangle + \mathrm{e}^{\mathrm{i}|g|t/\hbar}|-\rangle\right)\\
=&\frac{\mathrm{e}^{-\mathrm{i}ht/\hbar}}{2}\left(\mathrm{e}^{-\mathrm{i}|g|t/\hbar}\left(|1\rangle + \frac{g}{|g|}|2\rangle\right) + \mathrm{e}^{\mathrm{i}|g|t/\hbar}\left(|1\rangle - \frac{g}{|g|}|2\rangle\right)\right)\\
=&\mathrm{e}^{-\mathrm{i}ht/\hbar}\left(\cos\left(\frac{|g|t}{\hbar}\right)|1\rangle - \mathrm{i}\frac{g}{|g|}\sin\left(\frac{|g|t}{\hbar}\right)|2\rangle\right)
\end{aligned}$$

6.

(a)

(b)