Yichao Yu

1.

2.

(a)

(b)

3.

4.

5.

(a)

Eigenvalue λ

$$\begin{aligned} 0 =& (h-\lambda)^2 - |g|^2 \\ h-\lambda =& \pm |g| \\ \lambda =& h \pm |g| \end{aligned}$$

Corresponding eigen vectors are $\frac{1}{\sqrt{2}}\left(1, \pm\frac{g}{|g|}\right)$

(b)

Initial state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

At time t,

$$\begin{aligned} |\psi_t\rangle =& \frac{1}{\sqrt{2}}\left(\exp\left(-\mathrm{i}\frac{h+|g|}{\hbar}t\right)|+\rangle + \exp\left(-\mathrm{i}\frac{h-|g|}{\hbar}t\right)|-\rangle\right) \\ =& \frac{\mathrm{e}^{-\mathrm{i}ht/\hbar}}{\sqrt{2}}\left(\mathrm{e}^{-\mathrm{i}|g|t/\hbar}|+\rangle + \mathrm{e}^{\mathrm{i}|g|t/\hbar}|-\rangle\right) \\ =& \frac{\mathrm{e}^{-\mathrm{i}ht/\hbar}}{2}\left(\mathrm{e}^{-\mathrm{i}|g|t/\hbar}\left(|1\rangle + \frac{g}{|g|}|2\rangle\right) + \mathrm{e}^{\mathrm{i}|g|t/\hbar}\left(|1\rangle - \frac{g}{|g|}|2\rangle\right)\right) \\ =& \mathrm{e}^{-\mathrm{i}ht/\hbar}\left(\cos\left(\frac{|g|t}{\hbar}\right)|1\rangle - \mathrm{i}\frac{g}{|g|}\sin\left(\frac{|g|t}{\hbar}\right)|2\rangle\right) \end{aligned}$$

6.

(a)

(b)