Yichao Yu

1.

(a)

Since $\mathrm{i}\big(\lambda a^\dagger - \lambda^* a\big)$ is Hermitian, $S_\lambda \equiv \exp\big(\lambda a^\dagger - \lambda^* a\big)$ is unitary and $|\lambda\rangle \equiv S_\lambda|0\rangle$ is normalized.
Since $[a, a^\dagger] = 1$ commutes with both a and $a^\dagger$

$$\begin{aligned}
|\lambda\rangle =& \exp\big(\lambda a^\dagger - \lambda^* a\big)|0\rangle\\
=& \exp\big(\lambda a^\dagger\big)\exp\left(-\lambda^* a\right)\exp\left(-\frac{1}{2}\big[\lambda a^\dagger, -\lambda^* a\big]\right)|0\rangle\\
=& \exp\big(\lambda a^\dagger\big)\exp\left(-\lambda^* a\right)\exp\left(-\frac{|\lambda|^2}{2}\right)|0\rangle\\
=& \exp\left(-\frac{|\lambda|^2}{2}\right)\exp\big(\lambda a^\dagger\big)|0\rangle
\end{aligned}$$

$$\begin{aligned}
a|\lambda\rangle =& \exp\left(-\frac{|\lambda|^2}{2}\right)a\exp\big(\lambda a^\dagger\big)|0\rangle\\
=& \exp\left(-\frac{|\lambda|^2}{2}\right)\big(\exp\big(\lambda a^\dagger\big)a + \big[a, \exp\big(\lambda a^\dagger\big)\big]\big)|0\rangle\\
=& \exp\left(-\frac{|\lambda|^2}{2}\right)\big[a, a^\dagger\big]\lambda\exp\big(\lambda a^\dagger\big)|0\rangle\\
=& \lambda|\lambda\rangle
\end{aligned}$$

(b)

(c)

(d)

2.

(a)

(b)

(c)

(d)

3.

4.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

5.

(a)

(b)