# 1.

# 2.

## (a)

Since $J_z = L_z + S_z$ and $[L_z, S_z] = 0$, the $J_z$ eigenstates are sum of $L_z$ and $S_z$ eigenstates that has the same sum of $m_l$ and $m_s$. (The only variable on the RHS are $m_l$ and $m_s$ among which $m_s$ can only be $\pm\dfrac{1}{2}$ so the constraint above limit the decomposition to the given form.)

## (b)

Hamiltonian

$$\begin{aligned} H =& \frac{L^2}{2ma^2} + V_0 + \frac{e^2 \vec{L}\cdot\vec{S}}{2mc^2a^3} \\ =& \frac{L^2}{2ma^2} + V_0 + \frac{e^2}{4mc^2a^3}\left(J^2 - L^2 - S^2\right) \\ =& \frac{l(l+1)}{2ma^2} + V_0 + \frac{e^2}{4mc^2a^3}\left(j(j+1) - l(l+1) - \frac{3}{4}\right) \end{aligned}$$

When $l = 0$ there's only one manifold instead of two.

# 3.

# 4.

# 5.

## (a)

Eigenvalue $\lambda$

$$\begin{aligned} 0 =& (h-\lambda)^2 - |g|^2 \\ h - \lambda =& \pm|g| \\ \lambda =& h \pm |g| \end{aligned}$$

Corresponding eigen vectors are $\dfrac{1}{\sqrt{2}}\left(1, \pm\dfrac{g}{|g|}\right)$

## (b)

Initial state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

At time $t$,

$$\begin{aligned}
|\psi_t\rangle =&\frac{1}{\sqrt{2}}\left(\exp\left(-\mathrm{i}\frac{h+|g|}{\hbar}t\right)|+\rangle+\exp\left(-\mathrm{i}\frac{h-|g|}{\hbar}t\right)|-\rangle\right)\\
=&\frac{\mathrm{e}^{-\mathrm{i}ht/\hbar}}{\sqrt{2}}\left(\mathrm{e}^{-\mathrm{i}|g|t/\hbar}|+\rangle+\mathrm{e}^{\mathrm{i}|g|t/\hbar}|-\rangle\right)\\
=&\frac{\mathrm{e}^{-\mathrm{i}ht/\hbar}}{2}\left(\mathrm{e}^{-\mathrm{i}|g|t/\hbar}\left(|1\rangle+\frac{g}{|g|}|2\rangle\right)+\mathrm{e}^{\mathrm{i}|g|t/\hbar}\left(|1\rangle-\frac{g}{|g|}|2\rangle\right)\right)\\
=&\mathrm{e}^{-\mathrm{i}ht/\hbar}\left(\cos\left(\frac{|g|t}{\hbar}\right)|1\rangle-\mathrm{i}\frac{g}{|g|}\sin\left(\frac{|g|t}{\hbar}\right)|2\rangle\right)
\end{aligned}$$

# 6.

## (a)

## (b)