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Physics 251b Problem Set 1
Recitation Section 1

1.

(a)

Represent the operation using 4×4 matrices that shows the mapping between the nodes.

$$T_1 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \qquad T_2 = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & & & 1 \\ & 1 & & \end{pmatrix} \qquad T_3 = \begin{pmatrix} 1 & & & \\ & & & 1 \\ & 1 & & \\ & & 1 & \end{pmatrix}$$

$$T_4 = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix} \qquad T_5 = \begin{pmatrix} & 1 & & \\ & & 1 & \\ 1 & & & \\ & & & 1 \end{pmatrix} \qquad T_6 = \begin{pmatrix} & 1 & & \\ & & & 1 \\ & & 1 & \\ 1 & & & \end{pmatrix}$$

$$T_7 = \begin{pmatrix} & & 1 & \\ 1 & & & \\ & 1 & & \\ & & & 1 \end{pmatrix} \qquad T_8 = \begin{pmatrix} & & 1 & \\ & 1 & & \\ & & & 1 \\ 1 & & & \end{pmatrix} \qquad T_9 = \begin{pmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{pmatrix}$$

$$T_{10} = \begin{pmatrix} & & & 1 \\ 1 & & & \\ & & 1 & \\ & 1 & & \end{pmatrix} \qquad T_{11} = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} \qquad T_{12} = \begin{pmatrix} & & & 1 \\ & 1 & & \\ 1 & & & \\ & & 1 & \end{pmatrix}$$

(b)

$$g_{123} = \begin{pmatrix} & & 1 & \\ 1 & & & \\ & 1 & & \\ & & & 1 \end{pmatrix}$$

$$g_{234} = \begin{pmatrix} 1 & & & \\ & & & 1 \\ & 1 & & \\ & & 1 & \end{pmatrix}$$

$$g_{234}g_{123} = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix} = T_4$$

(180° rotation around the the axis connecting the middle of 1-2 and 3-4)

$$g_{123}g_{234} = \begin{pmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{pmatrix} = T_9 \neq T_4$$

(c)

See (a)

(d)

$$H = \begin{pmatrix} \varepsilon_0 & -t & -t & -t \\ -t & \varepsilon_0 & -t & -t \\ -t & -t & \varepsilon_0 & -t \\ -t & -t & -t & \varepsilon_0 \end{pmatrix}$$

Eigenvalues are $\varepsilon_0 - 3t$ for eigenvector $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ and $\varepsilon_0 + t$ for eigenvectors, $\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$, $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$, $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$.

2.

(a)

$$H = \begin{pmatrix} \varepsilon_0 & -t & & & & -t & & -t \\ -t & \varepsilon_0 & -t & & -t & & & \\ & -t & \varepsilon_0 & -t & & & & -t \\ & & -t & \varepsilon_0 & -t & & -t & \\ & -t & & -t & \varepsilon_0 & -t & & \\ -t & & & & -t & \varepsilon_0 & -t & \\ & & & -t & & -t & \varepsilon_0 & -t \\ -t & & -t & & & & -t & \varepsilon_0 \end{pmatrix}$$

(b)

(c)

3.

The constrains is that the angular momentum cannot perfectly point in a certain direction and there will always be some fluctuations. This uncertain comes from,

$$\begin{aligned} \langle \Delta L_x, \Delta L_y \rangle \geqslant & \frac{1}{2\mathrm{i}} \langle [L_x, L_y] \rangle \\ = & \frac{\hbar}{2} \langle L_z \rangle \\ = & \frac{\hbar^2 m}{2} \end{aligned}$$

Which can only be 0 when $m = 0$.

4.

(a)

(b)

(c)

(d)

(e)

5.

(a)

(b)

(c)