1. Gauge invariance and the Lorentz force

(a)

Schroedinger equation

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)^2\psi + q\phi\psi$$

Complex conjugate

$$\begin{aligned}
-\mathrm{i}\hbar\frac{\partial\psi^*}{\partial t} =& -\frac{\hbar^2}{2m}\left(\nabla + \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)^2\psi^* + q\phi\psi^*\\
\mathrm{i}\hbar\psi^*\frac{\partial\psi}{\partial t} =& -\psi^*\frac{\hbar^2}{2m}\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)^2\psi + q\phi\psi^*\psi\\
-\mathrm{i}\hbar\psi\frac{\partial\psi^*}{\partial t} =& -\frac{\hbar^2}{2m}\psi\left(\nabla + \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)^2\psi^* + q\phi\psi^*\psi\\
\mathrm{i}\hbar\frac{\partial\psi^*\psi}{\partial t} =& -\psi^*\frac{\hbar^2}{2m}\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)^2\psi + \frac{\hbar^2}{2m}\psi\left(\nabla + \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)^2\psi^*\\
=& -\frac{\hbar^2}{2m}\left(\nabla\left(\psi^*\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi\right) - \left|\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi\right|^2\right)\\
&+\frac{\hbar^2}{2m}\left(\nabla\left(\psi\left(\nabla + \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi^*\right) - \left|\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi\right|^2\right)\\
=&\frac{\hbar^2}{2m}\nabla\left(\psi\left(\nabla + \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi^* - \psi^*\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi\right)\\
\frac{\partial\rho}{\partial t} =&\frac{\hbar}{2m\mathrm{i}}\nabla\left(\psi\left(\nabla + \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi^* - \psi^*\left(\nabla - \frac{\mathrm{i}q}{\hbar c}\vec{A}\right)\psi\right)\\
=& -\nabla\cdot\vec{j}\\
0 =&\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j}
\end{aligned}$$

(b)

(c)

2.

(a)

(b)

(c)

3.

(a)

(b)

(c)

4.

(a)

(b)

(c)

(d)