1.

(a)

Communator of each component

$$
\begin{aligned}
[L_i + g_0 S_i, J_j] =& [L_i + g_0 S_i, L_j + S_j] \\
=& [L_i, L_j] + g_0 [S_i, S_j] \\
=& \mathrm{i}\hbar\varepsilon_{ijk}(L_k + g_0 S_k) \\
\left[L_i + g_0 S_i, \hat{n}\cdot\vec{J}\right] =& \mathrm{i}\hbar\varepsilon_{ijk} n_j (L_k + g_0 S_k) \\
=& \mathrm{i}\hbar\Big(\hat{n}\times\left(\vec{L} + g_0\vec{S}\right)\Big)_i \\
\left[\vec{L} + g_0\vec{S}, \hat{n}\cdot\vec{J}\right] =& \mathrm{i}\hbar\varepsilon_{ijk} n_j (L_k + g_0 S_k) \\
=& \mathrm{i}\hbar\hat{n}\times\left(\vec{L} + g_0\vec{S}\right)
\end{aligned}
$$

Therefore for any $\vec{n}$

$$
\begin{aligned}
& \mathrm{i}\hbar\hat{n}\times\langle 0|\vec{L} + g_0\vec{S}|0\rangle \\
=& \langle 0|\left[L_i + g_0 S_i, \hat{n}\cdot\vec{J}\right]|0\rangle \\
=& \langle 0|[L_i + g_0 S_i, 0]|0\rangle \\
=& 0 \\
& \langle 0|\vec{L} + g_0\vec{S}|0\rangle \\
=& 0
\end{aligned}
$$

This is a special case of the Wigner-Eckart Theorem because the $|0\rangle$ state is spherical symmetric. The physical origin of the factor g_0 is the low energy limit of the Dirac equation of electron (and QED corrections on top of it).

(b)

2.

3.

(a)

(b)

(c)

(d)

4.

(a)

(b)

5.

(a)

(b)

(c)