

1. Gauge invariance and the Lorentz force

(a)

Schroedinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right)^2 \psi + q\phi\psi$$

Complex conjugate

$$\begin{aligned} -i\hbar \frac{\partial \psi^*}{\partial t} &= -\frac{\hbar^2}{2m} \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right)^2 \psi^* + q\phi\psi^* \\ i\hbar \psi^* \frac{\partial \psi}{\partial t} &= -\psi^* \frac{\hbar^2}{2m} \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right)^2 \psi + q\phi\psi^*\psi \\ -i\hbar \psi \frac{\partial \psi^*}{\partial t} &= -\frac{\hbar^2}{2m} \psi \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right)^2 \psi^* + q\phi\psi^*\psi \\ i\hbar \frac{\partial \psi^*\psi}{\partial t} &= -\psi^* \frac{\hbar^2}{2m} \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right)^2 \psi + \frac{\hbar^2}{2m} \psi \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right)^2 \psi^* \\ &= -\frac{\hbar^2}{2m} \left(\nabla \left(\psi^* \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right) \psi \right) - \left| \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right) \psi \right|^2 \right) \\ &\quad + \frac{\hbar^2}{2m} \left(\nabla \left(\psi \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right) \psi^* \right) - \left| \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right) \psi^* \right|^2 \right) \\ &= \frac{\hbar^2}{2m} \nabla \left(\psi \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right) \psi^* - \psi^* \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right) \psi \right) \\ \frac{\partial \rho}{\partial t} &= \frac{\hbar}{2mi} \nabla \left(\psi \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right) \psi^* - \psi^* \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right) \psi \right) \\ &= -\nabla \cdot \vec{j} \\ 0 &= \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} \end{aligned}$$

(b)

Under time reversal transformation

$$\begin{aligned} &-i\hbar \frac{\partial \psi^*}{\partial t} + \frac{\hbar^2}{2m} \left(\nabla - \frac{iq}{\hbar c} \vec{A} \right)^2 \psi^* - q\phi\psi^* \\ &= \frac{\hbar^2}{2m} \left(\left(\nabla - \frac{iq}{\hbar c} \vec{A} \right)^2 - \left(\nabla + \frac{iq}{\hbar c} \vec{A} \right)^2 \right) \psi^* \\ &\neq 0 \end{aligned}$$

(c)

2.

(a)

(b)

(c)

3.

(a)

(b)

(c)

4.

(a)

(b)

(c)

(d)