Determining Optimal Reorder Quantity “Q” and Optimal Reorder Point “R” for Inventories on a Monthly Basis

SUTD | ESD

40.004 Statistics

40.012 Manufacturing and Service Operations

Team 06

Team Pandas

Name	Student ID	Contributions
Chester Lim Zi Hao	1004343	Report Writing, Data Analyst, Liaising
Hour Youlinsereydevid	1004327	Report Writing, Data Analyst, Liaising
Chia Yu Ying	1004609	Report Writing, Data Analyst, R Coding
Ong Lok Hen	1004647	Video Creation, Slides, Executive Summary
Tan Yun Yi	1004645	Video Creation, Slides, Executive Summary

Executive Summary

Given that SIAEC is Asia’s leading Maintenance, Repair and Overhaul (MRO) provider, effective inventory management is an important aspect of their operations. As such, we were tasked to develop models that auto-generates the optimal stock and reorder level monthly for inventory items such as aircraft spare parts and consumables. Ultimately, our proposed models should minimize unnecessary costs while meeting demand.

During data wrangling, we filtered and sorted our data based on inventory movements, level of importance and the number of data points to ensure that there was a sufficient sample size to determine the demand distribution for each inventory item. Also, we chose to focus on pre-covid data since the aviation industry is gradually recovering from the effects of COVID-19 and will operate as before.

First, we had to determine the demand distribution for the inventory parts to determine a suitable inventory model. By visualizing our data using a Q-Q plot and performing the Wilk Shapiro test, we were able to conclude with 95% confidence that demand is normally distributed for the items we chose to focus on.

Next, we decided to use the (Q,r) to model the inventory after evaluating the key parameters that we had for the dataset. We assumed our lead time to be 30 days and fixed set up cost to be $500 to further improve the fit of our data to the (Q,r) model.

We needed to calculate the optimal stock and reorder quantity that would minimize the fixed setup cost, stockout cost and holding cost. To ensure that inventory levels will meet demand as much as possible, we calculated the optimal stock level and equated fill rate to be close to 100% to determine the optimal reorder level.

We also used the t-distribution to find the confidence interval of the demand and determined that the true monthly demand lies within the 95% confidence interval and our reorder points for the inventory items can meet demand within 1 month of supply lead time.

In conclusion, using the (Q,r) model, we were able to achieve fill rates close to 100% with our optimal stock and reorder level. However, to improve the reliability of our model, we would require more data regarding the holding costs, stockout costs and monthly average demands.

1. Introduction

Our team has obtained the datasets from Singapore Airline Engineering Company (SIAEC), a company that provides aircraft maintenance, repair, and overhaul (MRO) services to more than 80 international and aerospace equipment manufacturers worldwide. The dataset contains an inventory data that comprises aircraft spare parts and consumables. Different items in the inventory have different inventory movement behaviour.

1.1. Problem Statement and Objective

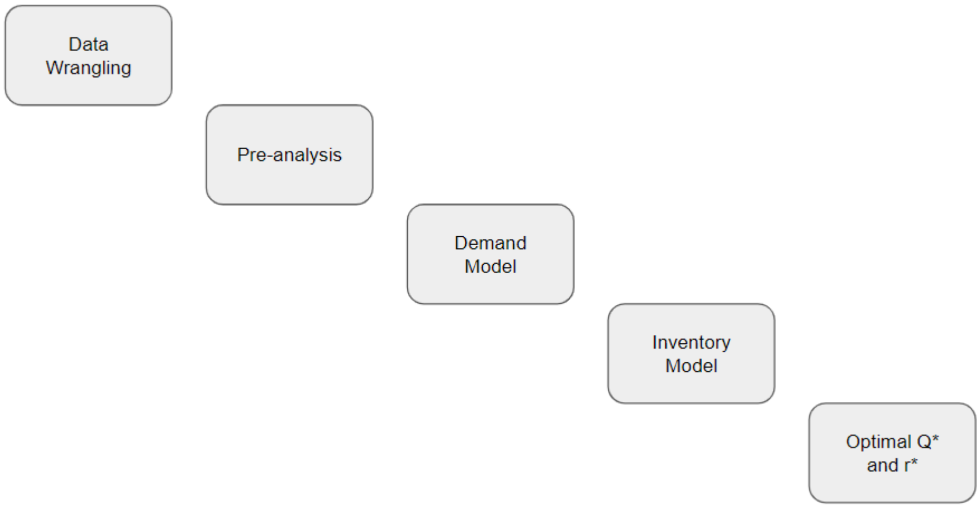
The task given to us is to develop a model (for them to use for their own comparison with their existing model) that automatically assigns these two numbers for every single inventory item:

- Optimal Stock Level (Q*)
- Optimal Reorder Level (r*)

<u>Objective: To propose an inventory model which minimizes unnecessary costs while meeting demands</u>

2. Methodology

The methodology will be illustrated in IDEF0 diagram as followed:



2.1. Data Wrangling

We are given 2 years (2019, 2020) worth of dataset of approx. 1.1 million rows of data with 24 columns. (total of 63,807 distinct part numbers & 11,613 distinct FFF-classes). The important columns that we feel are necessary for our analysis are shown in a table below:

No.	Title	Remarks
1	**Masked part number**	**Each PN represents different product**
2	**Inventory movement description**	**Either return, purchase, purchase & return, usage, reverse purchase**
3	**Masked FFF-class**	**Family of interchangeable parts, parts are interchangeable if they belong to the same FFF class. For example, FFF-38WSAA71475 may contain several masked part numbers but the masked part number "38WSAA71475" is the most preferred part to have in inventory.**
4	**Item category**	**Consists of Aircraft, Consumables and Tools**
5	**Importance**	**High, Medium, or low importance**
6	Quantity	For calculating monthly demand
7	Entry date	
8	Amount (SGD)	For getting the original/premium price

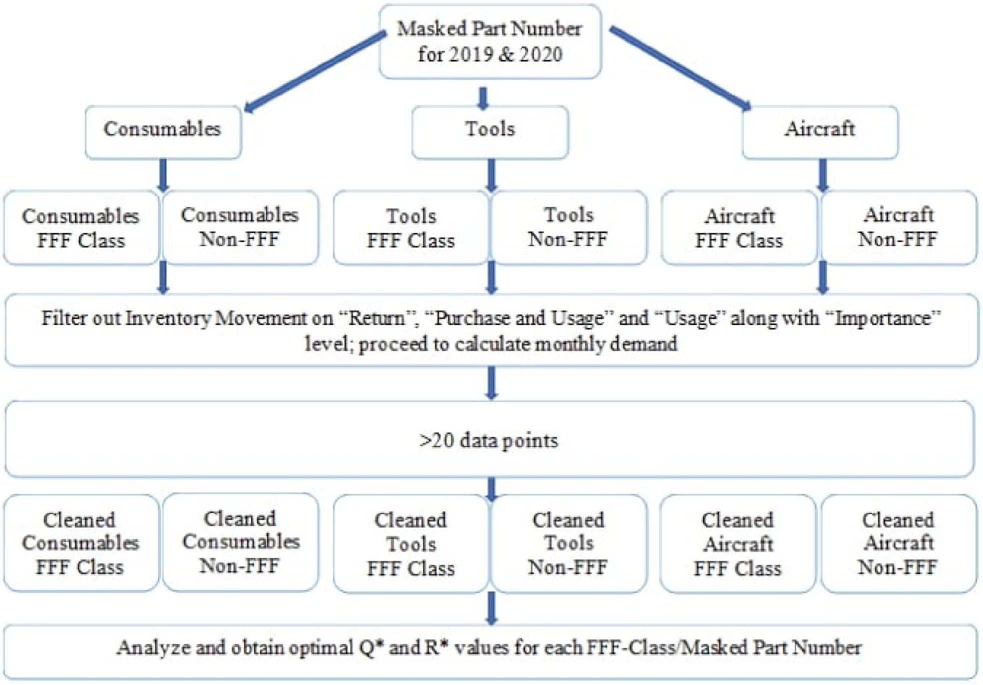
Those marked bold are categories which SIAEC are taking more importance on and they wish that we will take into consideration when doing our calculations.

Steps taken for data wrangling: (By using excel and dplyr package in R)

1. Firstly, using excel, we filter out the columns that are important for our analysis, leaving us with 8 columns of data.
2. Using the dplyr package in R, we grouped the data up based on their item categories, namely: Aircraft parts, Consumables, and Tools.
3. For each item category, we further group them into 2 subcategories – Masked Part Numbers which belongs into a FFF-class and Masked Part Numbers which do not belong in an FFF-class.
4. We filter out all data with the inventory movement description which contains "Usage", "Purchase and Usage" and "Return". These are inventory movements which are important for us to calculate the monthly demand of each FFF-class/product.
5. To calculate the monthly demand for a FFF-class/masked part number, we took the summation of the "Quantity" column for each month.

Take note: Quantity with inventory movement description "Usage" and "Purchase and Usage" are positive. Quantity with inventory movement description "Return" gives a negative number. Assuming that "Return" inventory movement are all due to failed quality checks and that the deficit demand will be satisfied in the future months, by taking the summation of quantity of these 3 inventory movements will give us the monthly demand for each FFF-class/ masked part number.

6. After computing, we realized that there is a significant number of FFF-class/masked part numbers with few data points (<=5) given to us (monthly demand), which may not be statistically favourable. Hence, we further filter our data to keep only those that contains all 24 months of data.
7. Lastly, we realised that some of the FFF-Class/Masked Part Number could contain different levels of important levels and transactions for different months due to varying external reasons such as the urgency of customers in needing that part etc. Hence, for simplicity, we assumed that if there is an entry that contains "High" importance, we will classify this FFF-class/part number as "High". This same assumption applies to "Medium" and "Low".



2.2. Pre-Analysis

Before conducting our analysis, knowing that COVID-19 has took a big hit on the aviation industry which may have resulted in an enormous change in the inventory movement and demand for various aircraft parts, consumables, and tools, our group have decided to first perform data visualization on the data spanning across both years to get a sensing of how different (if any) the dataset in 2020 (greatly impacted by COVID) is from the dataset in 2019 (before COVID).

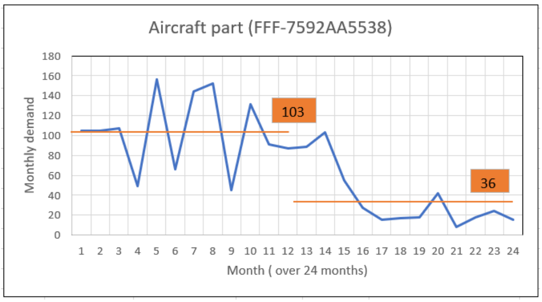
- We first compare the annual demand between 2019 and 2020 for each FFF-Class/masked part number for their respective item category.
- Then we plotted time-series graphs to see how the monthly demand have changed across the 2 years (before vs after COVID pandemic) for different items in each item category.

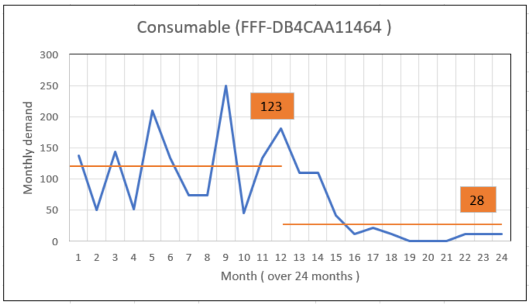
Masked FFF Class	Item Category	totaldemand2019	totaldemand2020
FFF-7592AA5538	Aircraft	1238	431
FFF-MS35AA15274	Aircraft	1237	134
FFF-BACCAA9865	Aircraft	1211	333
FFF-BACRAA10257	Aircraft	1201	76
FFF-NAS6AA43180	Aircraft	1199	73

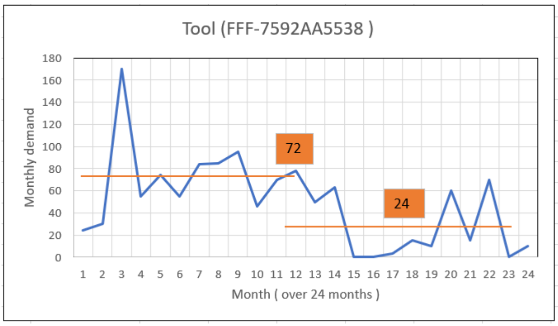
Masked FFF Class	Item Category	totaldemand2019	totaldemand2020
FFF-DB4CAA11464	Consumables	1485.0	343
FFF-TSDSAA18126	Consumables	1186.0	277
FFF-TSDSAA18070	Consumables	1016.0	502
FFF-TSDSAA18094	Consumables	906.0	355
FFF-2E98AA2311	Consumables	850.0	254

Masked FFF Class	Item Category	totaldemand2019	totaldemand2020
FFF-4801AA3847	Tool	22790	4189
FFF-6515AA4669	Tool	9600	3007
FFF-TS27AA26151	Tool	4596	16
FFF-TS27AA26152	Tool	2044	21
FFF-M819AA14345	Tool	866	253

The orange lines represent the annual average demands for the 2 years (2019,2020) and their corresponding value.







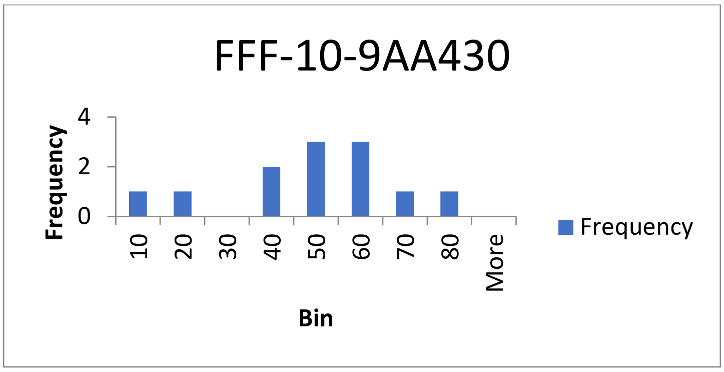
From this, we found that generally, the average demand in 2020 were drastically lower than the average demand in 2019, as shown in the figures above. As a result, we feel that by combining both sets of data in our analysis may be inaccurate, especially when COVID is settling down and the aviation industry is slowly recovering and will be operating like how they used to. Hence, we have decided to only use the 2019 data (before COVID) in our analysis and calculations. As we only have 1-year worth of data (few data points for each product), we further filter out those FFF-class/parts number that have all 12 points of data for each month (those with known demand for each month in 2019). Hence, we will only be performing our analysis on the remaining 828 products. Our general method to finding optimal Q and R will be illustrated below. We will be using 2 FFF-classes (**FFF-10-9AA430 and FFF-0212AA112**) to illustrate our general approach. Both FFF-classes are of "high" importance, which means we must ensure that the primary item of the 2 classes should never run out.

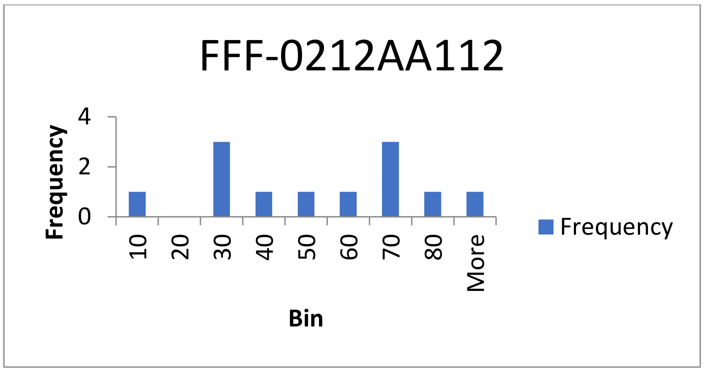
2.3. DEMAND MODEL

The mean and standard deviation for each class is computed as follows:

FFF-class	Mean	Stdev	No. months
FFF-10-9AA430	45.083	18.754	12
FFF-0212AA112	45.833	24.679	12

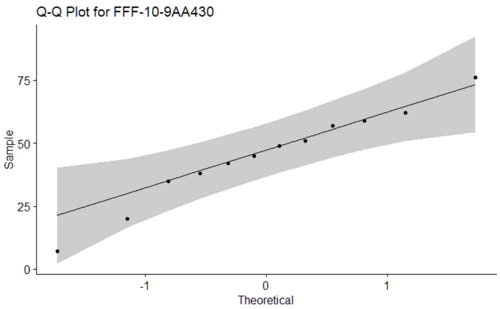
First, we want to determine the distribution of our demand so that we can make an estimate on our calculations. We first do so by plotting histograms with the demand data that we have to get a sensing of the demand distribution.

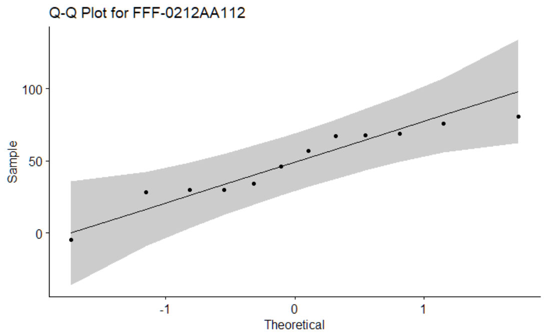




From the histogram, while the demand of FFF-10-9AA430 seems to appear to fit a normal distribution, the demand of FFF-0212AA112 does not seem to fit normal distribution. This could be due to the lack of data points that we have.

Hence, to ascertain that the demand for each of the 2 classes is normal, we used Q-Q plots. As seen from figure below, Q-Q plot may also not give us definite answer as to whether demand data is normally distributed. This can be seen from data points that do not lie exactly on the straight line.





Though we can make an estimate about the normality of our distribution based on these plots, we want to have a definite answer to our question. Our demand data is monthly data and hence there are only 12 data points for each inventory item. This is too small for Central Limit Theorem to be applied and therefore "Chi-squared test" is not suitable. "Wilk Shapiro test[1]" is what we use for small data points just like ours. First, we set up a hypothesis testing with alpha value of 5%:

H_0: Demand is normally distributed **H_1: Demand is not normally distributed**

We used R-programming to determine p-value for Wilk Shapiro test. For "FFF-10-9AA430" and "FFF-0212AA112", we get p-value = 0.07308 and 0.31750 respectively. Since p-value is higher than alpha value of 5% (0.0500), we do not reject our null hypothesis H_0 and hence we can conclude with 95% confidence that the demand data for "FFF-10-9AA430" and "FFF-0212AA112" are normally distributed).

2.4. Inventory Model

Based on the main 4 models learnt during the course, we consider all of them and (Q, r) model is the most suitable to the dataset. We did not consider EOQ and Base Stock models because they are part of (Q, r) model. Likewise, Newsvendor model assumes zero supply lead time and hence does not fit to our dataset as there is a supply lead time for our inventory items. Our decision is illustrated in the table shown below.

Parameters	(Q, r) model	Our data
Demand	Random	Random
Lead time	*Fixed & Known*	*Not Fixed & Known*
Holding cost	Known/Estimated	Estimated
Stockout cost	Known/Estimated	Estimated
Fixed setup cost	*Known*	*Unknown*

[1] "Normality Test in R." *STHDA*, Apr. 2021, www.sthda.com/english/wiki/normality-test-in-r.

Independent products	Yes	Yes
Batch ordering	Yes	Yes

As illustrated in the table above, our data fits (Q, r) model very closely except for lead time and fixed setup cost. Thus, we will assume our lead time to be 30 days and our fixed setup cost to be $500.

2.5. Q* and r*

To formulate (Q, r) model, we seek values of Q* and r* to solve for:

Minimize {fixed setup cost + stockout cost + holding cost}

The corresponding optimal solutions are: $Q^* = \sqrt{\frac{2AD}{h}}$ *and* $r^* = \theta + z\sigma$ (Derivations in appendix)

Parameters needed in determining Q* and r*:

- Demand (D): the mean of demand per month
- Holding cost (h): assumed to be $\frac{20\%}{12}$ per month of the unit cost of inventory item
- Fixed cost (A): assumed to be fixed at $500
- Expected demand during supply lead time (θ): mean demand monthly as supply lead time is assumed to be 1 month
- Standard deviation of demand (σ)
- Stockout cost (k): unit stock out cost = max. unit cost – min. unit cost (because SIAEC will have to pay a premium price if they run out of stock and they need that item urgently)
- $z = \frac{kD}{hQ+kD}$

Decision Variables	FFF-10-9AA430	FFF-0212AA112
Q^*	243	175
r^*	78	64

Table 1: Optimal Reorder Quantities and Reorder points, rounded up

3. Performance Measures Analysis

Performance	FFF-10-9AA430	FFF-0212AA112
Expected Stockout	0.05785	1.14
Expected leftover Inventory	154.43	106.61
Fill Rate	0.998	0.9765
Total Expected Cost	216.20	327.30

Table 2: Performance measure of optimal Q and r

Both of the 2 classes are of "high" importance level, hence, to ensure that the primary product of the 2 classes never run out, we recalculated the new reorder point, r_{new}, to ensure that the probability of no stockout (fill rate) is as close as possible to 1 (i.e., expected stock out levels to be as close to 0 as possible). For calculation purposes, we equate fill rate to be 0.9999 (99.99%).

By using fill rate (exact) formula, we equate S(Q, r) = 0.9999 and keeping Q^* unchanged and we want to find a reorder point, r_{new}, that will give us a fill rate of 99.99% :

$$0.9999 = 1 - \frac{1}{Q^*}\left[B(r_{new}) - B(r_{new} + Q^*)\right]$$

Where: $$B(r_{new}) = (\theta - r_{new})\Phi\left[\frac{r_{new}-\theta}{\sigma}\right] + \sigma\varphi\left[\frac{r_{new}-\theta}{\sigma}\right]$$

$$B(r_{new} + Q^*) = (\theta - r_{new} - Q^*)\Phi\left[\frac{r_{new} + Q^* - \theta}{\sigma}\right] + \sigma\varphi\left[\frac{r_{new} + Q^* - \theta}{\sigma}\right]$$

For FFF-0212AA112: $$0.9999 = 1 - \frac{1}{175}\left[B(r_{new}) - B(r_{new} + 175)\right]$$

For FFF-10-9AA430: $$0.9999 = 1 - \frac{1}{243}\left[B(r_{new}) - B(r_{new} + 243)\right]$$

By using excel solver:

FFF-class	FFF-10-9AA430	FFF-0212AA112
r_{new}	95	118

Table 3: New reorder points, rounded up

Performance	FFF-10-9AA430	FFF-0212AA112
Expected Stockout	0.004508	0.004883
Expected leftover inventory	171.01	157.11
Fill Rate	0.9999	0.9999
Total Expected Cost	223.73	390.37

Table 4: Performance measure of optimal Q and r_{new}

After equating the fill rate to 99.99%, the expected stockout level has become very close to 0 as shown in table 4. However, as expected, to increase the fill rate and to reduce expected stockout levels will come at the expense of having more excess inventory leftover and an increased in total expected cost.

For FFF-0212AA112:

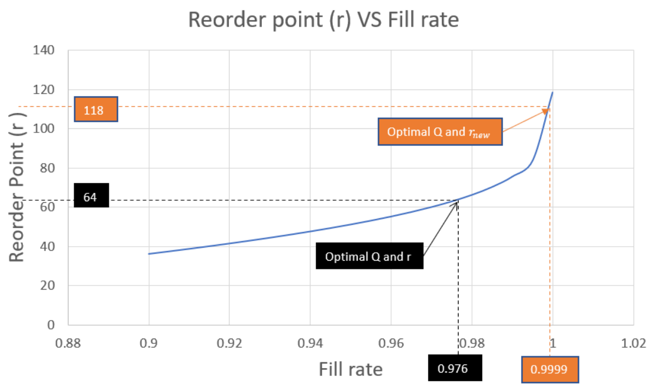


Figure 1: Plot of Reorder point (r) against fill rate for FFF-0212AA112

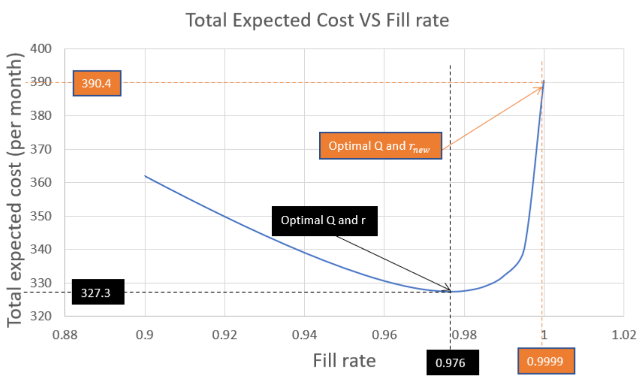


Figure 2: Plot of Total Expected Cost (per month) against fill rate for FFF-0212AA112

For FFF-10-9AA430:

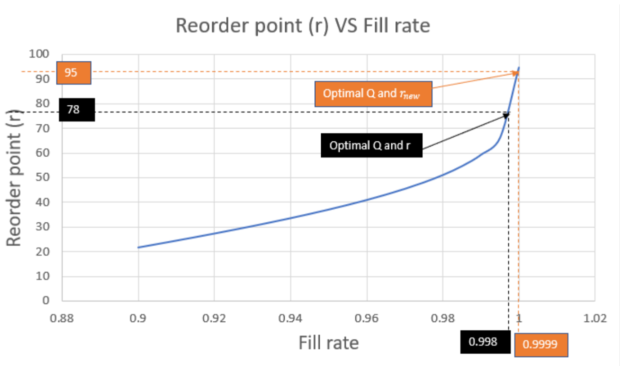


Figure 3: Plot of Reorder point (r) against fill rate for FFF-10-9AA430

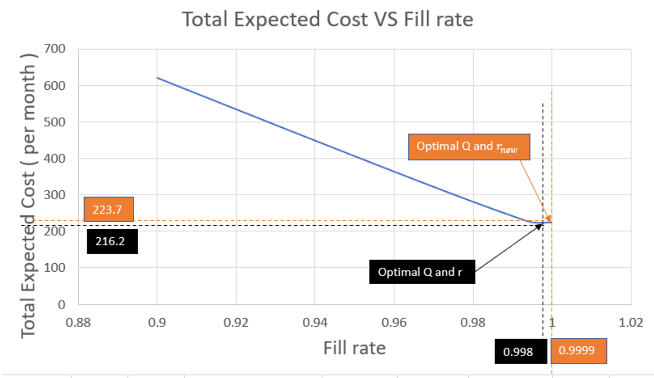


Figure 4: Plot of Total Expected Cost (per month) against fill rate for FFF-10-9AA430

As shown in Fig 2, our optimal Q and r gives the lowest total expected cost of \$327.30 per month for class FFF-0212AA112, but only has a fill rate of 97.6%.

To ensure fill rate of 99.99% for class FFF-0212AA112, SIAEC should pay an estimated additional amount of \$(390.4-327.3) = \$ 63 per month.

As shown in Fig 4, our optimal Q and r gives the lowest total expected cost of \$216.20 per month for FFF-10-9AA430 class and has a very high fill rate of 99.8%.

To ensure fill rate of 99.99% for FFF-10-9AA430, SIAEC should pay an estimated additional amount of \$(223.7-216.2) = \$ 7.52 per month.

In conclusion, to ensure that both primary products of the 2 FFF-Classes will not run out of stock (99.99% fill rate) while minimising unnecessary costs, the final optimal reorder quantity and reorder point for both classes are shown below.

FFF-class	Q^*	r^*
FFF-10-9AA430	243	95
FFF-0212AA112	175	118

3.1. Sanity check for our new reorder point:

	FFF-10-9AA430	FFF-0212AA112
Sample size, n	12	12
Sample mean, $\bar{x}$	45.083	45.833
Sample standard deviation, s	18.754	24.679

Since the sample size (n) is small and population variance (σ^2) is unknown, but the demand distribution is normal, we can use the t-distribution to find the Confidence Interval of the demand.

$$\bar{X} - t_{n-1,\alpha/2}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1,\alpha/2}\frac{S}{\sqrt{n}} \qquad \text{Eqn 1}$$

At 95% 2-sided Confidence level, substituting:

- $\alpha = 0.05$, Degree of freedom of (12-1) = 11 and the respective $\bar{x}$ and s values into Eqn 1,

For FFF-10-9AA430:

FFF-10-9AA430			
Confidence Interval			
Population variance is unknown, but demand is normally distributed.			
Hence, we use t-distribution to calculate the confidence interval.			
At 95% confidence interval :			
degree of freedom (n-1):	11	n:	12
alpha:	0.05		
t_11_0.025:	-2.20098516		
Upper bound:	56.99917809		
Lower bound:	33.16748858		

The CI that we obtained for the demand of FFF-10-9AA430 is [33.167, 56.999].

FFF-0212AA112:

Confidence Interval			
Population variance is unknown, but demand is normally distributed.			
Hence, we use t-distribution to calculate the confidence interval.			
At 95% confidence interval :			
degree of freedom (n-1):	11	n:	12
alpha:	0.05		
t_11_0.025:	-2.20098516		
Upper bound:	64.51371923		
Lower bound:	33.15294744		

The CI that we obtained for the demand of FFF-0212AA112 is [33.153, 64.154].

Conclusion:

With 95% confidence, the true monthly demand lies within the CI calculated. Hence, since our reorder points r for both FFF-classes are higher than their respective upper bounds of the 95% CI, we can be confident that on average we are able to meet demand during the 1 month of supply lead time.

4. Data/Logistics Challenge

Not all inventory items have a supply lead time of 30 days as what we assumed in our analysis. For lower supply lead time, the optimal reorder points calculated will be overestimated while higher supply lead time will be underestimated.

Moreover, we do not have the actual data on the exact holding cost and hence this will affect our actual optimal order quantity and optimal reorder point.

Finally, there is no exact data on fixed cost (A), and the stockout costs. This will be another factor that will affect Q and r values calculated.

5. Conclusion

All in all, we had based our monthly demand across the respective part numbers to be of Normal Distribution. Using the (Q,r) model that we have set up, we are able to target our fill rate to be of 99.8% and 97.6% respectively for the 2 part numbers we have drawn up previously. However, more information is required to better model Singapore Airlines Engineering Company's situation, including having more concrete holding costs, stockout costs and targeted monthly average demands. These data is essential and critical for us to produce a more reliable and useful optimal policy.