CMSE 381 MIDTERM

Feb 19, 2020 Time allowed: 70 minutes

Name: _____

Pledge: *I have neither given nor received any unauthorized aid during this exam.*

Signature _____

Instructions:

For all the questions, show all your work in the space provided. You will NOT receive credit if you do not justify your answers.

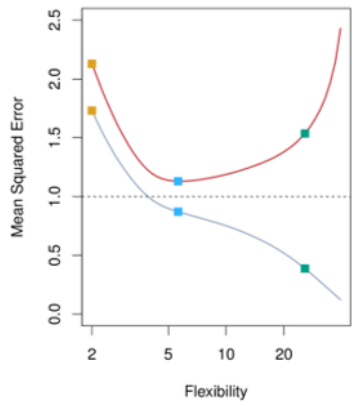
This is a closed book and closed notes examination.

Please budget you time so that you have sufficient time to do all the questions.

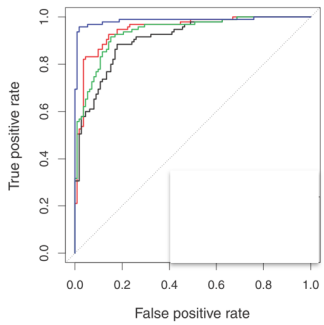
Good luck!

1. (3 pts) In a marketing setting, we have demographic information for a number of potential customers. We may wish to understand which types of customers are similar to each other by grouping individuals according to their observed characteristics. Is this a supervised or unsupervised learning? Explain your choice.

2. (5 pts) The following figure displays the average training and testing MSEs as a function of model flexibility. Explain the meaning of the three curves and justify your answer.



3. (5 pts) We are trying to predict whether a patient will have a heart attach within one year. We have tried four different methods on a training dataset and have the following ROC curves



Which curve has the best performance on this training set? Justify your answer.

4. (7 pts) Assume the true model is

$$Y = f(X) + \epsilon.$$

We have a set of training data Tr, which is used to fit a model $\hat{f}(x)$, and a new testing data (x_0, y_0). Here, we assume x_0 are fixed. Derive

$$E\left[\left(y_0 - \hat{f}(x_0)\right)^2\right] = Var(\hat{f}(x_0)) + [Bias(\hat{f}(x_0))]^2 + Var(\epsilon).$$

5. We assume $Y = \beta_0 + \beta_1 X + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$. We collect a set of i.i.d. training data $\{(x_1, y_1), \ldots, (x_n, y_n)\}$ and want to fit a linear regression.

   a. (5 pts) Derive the $\hat{\beta}_0$ and $\hat{\beta}_1$ which minimize the RSS.

   b. (5 pts) Let $\epsilon_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$. Prove that $\sum_{i=1}^{n} \epsilon_i x_i = 0$.

   c. (5 pts) If X is the ethnicity variable including three levels {Asian, African American, Caucasian }, how can we include it into the linear model?

6. We know to build a classification model with $Y \in \{0, 1\}$ and $X \in \mathbb{R}^p$. We collect a set of i.i.d. training data $\{(x_1, y_1), \ldots, (x_n, y_n)\}$, where $x_i \in \mathbb{R}^p$.

   a. (3 pts)First, we want to fit a logistic regression model. Write down the logistic regression model.

   b. (7 pts) As discussed in the class, we will use maximum likelihood framework to estimate the parameters in the logistic regress model. Write out the log likelihood of the training data.

   c. (5 pts) Using logistic regression, we normally will classify a data as follows

$$Y = \begin{cases} 1 & \text{if } \Pr(Y = 1|X = x) \geq 0.5 \\ 0 & \text{otherwise.} \end{cases}$$

   Assuming we want to classify a patient into either HIV positive ($Y = 1$) or HIV negative ($Y = 0$). In this case, we have worse consequence to have a Type II error than Type I error. To address this, how will you modify the model above to predict Y to reduce the type II error? Justify your answer.

   d. (Extra 5 pts) When we use LDA to perform the classification, the classifier will be

$$Y = \begin{cases} 1 & \text{if } \delta_1(x) \geq \delta_0(x) \\ 0 & \text{otherwise.} \end{cases}$$

   Prove that this is equivalent to the classifier in (c). Namely, when $\delta_1(x) \geq \delta_0(x)$, $\Pr(Y = 1|X = x) \geq 0.5$. Here,

$$\delta_k(x) = x^T \Sigma^{-1} \mu_k - \frac{1}{2} \mu_k^T \Sigma^{-1} \mu_k + \log \pi_k.$$

   And if $X \sim N(\mu, \Sigma)$, then its probability density function for $X = x$ is

$$f(x) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)}$$