CMSE 381: HW9

1 (30 pts) Exercise 7.9.1

2 (10 pts) Exercise 7.9.2

3 (10 pts) Exercise 7.9.3

4 (10 pts) Exercise 7.9.4

5 (20 pts) Exercise 7.9.9

6 (20 pts) Exercise 7.9.11

7 (Challenging problem) *Derivation of smoothing splines* Suppose that $N \geq 2$, and that g is the natural cubic spline interpolant to the pairs $\{x_i, z_i\}_{i=1}^N$, with $a < x_1 < \cdot < x_N < b$. This is a natural spline with a knot at every x_i; being an $N-$dimensional space of functions, we can determine the coefficients such that it interpolates the sequence z_i exactly. Let $\tilde{g}$ be any other differentiable function on $[a,b]$ that interpolates the N pairs.

(a). Let $h(x) = \tilde{g}(x) - g(x)$. Use integration by parts and the fact that g is a natural cubic spline to show that

$$\int_a^b g''(x)h''(x)dx = -\sum_{j=1}^{N-1} g'''(x_j^+)\{h(x_{j+1} - h(x_j)\} = 0$$

(b). Hence show that

$$\int_a^b \tilde{g}''(t)^2 dt \geq \int_a^b g''(t)^2 dt,$$

and that equality can only hold if h is identically zero in $[a,b]$.

(c). Consider the penalized least squares problem

$$\min_f \left[\sum_{i=1}^N (y_i - f(x_i))^2 + \lambda \int_a^b f''(t)^2 dt \right].$$

Use (b) to argue that the minimizer must be a cubic spline with knots at each of the x_i.