

Examples of Discrete and Continuous Probabilities

Calculations by Using R

Calculating the expected value of a discrete RV X with the corresponding probabilities p:

`crossprod(x,p)`

Calculating the standard deviation of a discrete RV X in R:

`sqrt(crossprod(x^2,p)-(crossprod(x,p))^2)`

For both discrete and continuous distributions, the following four functions are available:

1. **d...()**: Calculates the probability mass function (discrete) or the probability density function (continuous). **Example: dbinom()**
2. **p...()**: Calculates the cumulative probabilities for both discrete and continuous distributions. **Example: ppois()**
3. **q...()**: Given a cumulative probability, calculates the value of the random variable (for both discrete and continuous distributions). **Example: qnorm()**
4. **r...()**: Generates a specified number of the random variable according to the specified distribution (for both discrete and continuous distributions). **Example: rbeta()**

Example: The popularity rate of the president is 42%. A random sample of 5 citizens is selected.

(a) What is the probability that no one in this sample is in favor of the president?

Success: the citizen is in favor

$P = 0.42$

$n=5$

$P(X=0)?$

```
> dbinom(0,5,0.42)
[1] 0.0656356768
```

Interpretation: In 6.6% of all possible samples of 5 citizens, there will be no one in the sample in favor of the president.

(b) What is the probability that at most 3 in the sample are in favor?

$P(X \leq 3)$

```
> pbinom(3, 5, 0.42)
[1] 0.8966916928
```

Example: On the average, Stefan Curry makes 7 three-pointers per game that he plays.

(a) What is the probability that in the game tonight he will succeed in 8 of his three-pointer attempts?

Poisson

Mean $\mu = 7$

$P(X=8)=?$

```
> dpois(8, 7)
[1] 0.1303774322
```

In about 13% of his games, he succeeds in exactly 8 three-pointer attempts.

(b) What is the probability that he succeeds in more than 5 attempts?

$P(X > 5) = ?$

$= 1 - P(X \leq 5)$

```
> 1 - ppois(5, 7)
[1] 0.6992917238
```

Interpretation: In almost 70% of the games, he succeeds in his 3-pointer attempts more than 5 times.

Example: What is the probability that after having 9 boys, a couple will have a girl on the 10th trial?

Geometric

$p=0.5$

$x=10$

```
> dgeom(10, 0.5)
[1] 0.00048828125
```

This happens in almost 5 in 10000 cases.

Example: Hypergeometric Distribution:

- (i) The population is of a finite size N .
 - (ii) There are K successes in the population (e.g., they carry a common genetic trait, or they may be infected with a certain virus).
 - (iii) Consequently, there are $N-K$ Failures in the population.
 - (iv) A random sample of size n is selected from the population.
 - (v) The Hypergeometric RV X is the number of successes in the sample.
- `dhyper(x,K,N-K,n)`**
(in Excel: `=hypgeom.dist(x, n, K, N,0)`)

Example: A jar contains 50 marbles of which 20 are blue. 10 marbles are randomly removed from the jar. What is the probability that there are 3 blue marbles in this sample?

```
N=50
K=20
N-k = 50 -20 = 30
n=10
P(X=3)=?
> dhyper(3,20,30,10)
[1] 0.2259296294
```

Example: The Continuous Uniform Distribution between two values a and b:

$P(X < x)$ = $P(X \leq x)$ = **`punif(x,a,b)`**

Mean = $(a+b)/2$

If a and b are not indicated as inputs, then the standard uniform formulas are obtained ($a=0$ and $b=1$)

Example: The Daily revenue at a convenient store is modeled according to the uniform distribution between \$1500 and \$2500.

- (a) What is the probability that the revenue will be less than \$2000 tomorrow?
 $P(X < 2000) =$
> **`punif(2000,1500,2500)`**
[1] 0.5
- (b) What is the probability that the revenue will be more than \$1800 tomorrow?
 $P(X > 1800) =$
> **`1-punif(1800,1500,2500)`**
[1] 0.7

- (c) What is the probability that the revenue will be between \$1700 and \$2200 tomorrow?

```
P(1700<X<2200)=  
> punif(2200,1500,2500) - punif(1700,1500,2500)  
[1] 0.5
```

Example: The Normal Distribution

Probability density function: **dnorm(x,μ, σ)**

Probability cumulative function $P(X \leq x) =$ **pnorm(x,μ, σ)**

Inverse Problems: $x =$ **qnorm(prob,μ,σ)**

Generates N random numbers: **rnorm(N,μ, σ)**

Note: If μ and σ are not indicated as inputs, then the standard normal formulas are obtained ($\mu = 0$ and $\sigma = 1$)

Example: The distribution of the weights of newborn is normal with a mean of 6.2 lbs and a standard deviation of 1.2 lbs.

- (a) What percentage of the newborns are less than 6 lbs?

```
P(X<6) =  
> pnorm(6,6.2,1.2)  
[1] 0.4338161674  
=43%
```

- (b) What percentage are heavier than 10 lbs?

```
P(X>10) =  
> 1-pnorm(10,6.2,1.2)  
[1] 0.0007709847845  
  
= 0.08%
```

- (c) What percentage are between 7 lbs and 9 lbs?

```
P(7<X<9) =  
> pnorm(9,6.2,1.2)-pnorm(7,6.2,1.2)  
[1] 0.2426772089  
  
= 24%
```

(d) Find the 90th percentile of the weights distribution.

```
> qnorm(0.9,6.2,1.2)
```

```
[1] 7.737861879
```

= 7.7 lbs